Helicopter landing gear vibration analysis using Lagrange method

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Abstract. In this paper, landing dynamics for a helicopter fitted with an oleo pneumatic landing gear in the front and tail gears has been analysed with a seven degree of freedom mathematical model. Dynamic equations have been developed using Lagrange principle to investigate the effects of vibrations on the structure during landing phase of the helicopter. The bounce, roll, pitch, yaw acceleration response of helicopter landing obtained by numerical simulation in Matlab/Simulink. The developed model by Lagrange method will be useful to measure the vibration levels on landing and taxing of helicopter with uncertainties. This approach is also helpful to optimize the stiffness and damping properties of landing gear.

Keywords: helicopter landing gear, vibration, Lagrange principle, simulation, landing gear properties.

1. Introduction

Landing is the most critical operational phase of a helicopter. During landing of helicopter on the uneven surface, a large amplitude of vibrations transmitted to the helicopter fuselage structure causing safety and comfort problems to crew and passengers. Moreover, the helicopter also involves complex dynamics of main rotor, tail rotor, engine transmission system and flying dynamics. Landing gear plays an important role in absorbing the shocks and vibrations due to touch down impact and rolling on the deck or un even surface. So, the performance efficiency of the shock absorber is very much essential to meet the variable damping requirements to cope with varying operating conditions. Peter J. F. O. Reilly [1] presented an analytical model for dynamic analysis of landing and taking off of the aircraft from a moving deck. Fu Shang et al. [2] derived the SH-2Fship board dynamic model to prescribe the safe environment for taking off and landing of a helicopter from the flight deck. In his study, energy method used for deriving equations of motion for ship board dynamic motion and assumptions to develop a dynamic model of helicopter. Bernard [3] studied analytic approach of helicopter ship testing simulation for an EH101 helicopter and CPF ship model. Black well and Feik [4] have described a mathematical model of a helicopter landing on an arbitrarily moving deck. The mathematical model of two main oleo and single tail oleo developed and analyzed for oleo load and oleo deflection for deck landing. Kim and Tilbury [5] presented a six degree of freedom mathematical model for a helicopter to investigate the combined action between fly bar and the main rotor blades make it easier for a pilot to fly. Li pan and Chen Renliang [6] presented a helicopter comprehensive analysis by developing a mathematical model with the feature of flexibility and mathematical simplicity. Tulio Salazar [7] developed a mathematical model for a helicopter with a main rotor hub and tail rotor configuration to simulate flying behavior of the helicopter. An active landing gear behavior was demonstrated by Sivakumar and Haran [8] by developing six degree of freedom mathematical model of full aircraft with tricycle landing gear configuration. The author has investigated non-linear analysis of oleo pneumatic landing gear in [8]. Most of the papers have shown the two degree of freedom mathematical model of single landing gear. This model does not include pitch, roll and yaw degrees-of-freedom. The effects of longitudinal and lateral interconnections cannot be studied in the two degrees-of-freedom system model. In this study, the seven degree of freedom

math's model of helicopter with landing gear has been developed using Lagrange method for landing dynamics of helicopter.

2. Mathematical model formulation

The full helicopter dynamic model built by considering typical US-Navy sea hawk helicopter version. The helicopter fuselage body or concentrated mass M is free to bounce, roll, pitch and yaw. The fuselage mass M is connected to the three lumped masses m_t, m_l and m_r which are tail, front left and front right landing gears piston and tire viewed from front. The front landing gears are called left and right main landing gears and the rear one is called tail landing gear. They are free to bounce vertically with respect to the fuselage mass. The full helicopter model contains four degrees-of-freedom for the helicopter fuselage mass (bounce, roll, pitch, yaw), and three degrees-of freedom for the vertical motions of the tail landing gear tire mass and for the front main landing gears tire masses. In this work, the seven degrees-of-freedom vibration model of the full helicopter as indicated in figure1.This model represents bounce, pitch, roll and yaw motion of the helicopter as x, α, γ, φ about the axes respectively. The displacements and ground excitations of the left, right and tail main landing gears as x_1, x_2, x_3 and y_1, y_2, y_3.

The stiffness and damping constants of left shock strut, right and tail gear shock struts indicated as k_1, k_2, k_3 and G_1, G_2, G_3. The stiffness and damping constants of left gear tire, right and tail gear tires is denoted as c_1, c_2, c_3. and G_x, G_y, G_z.The tail landing gear is located at a distance of d from centre of gravity (C.G) position. The main landing gears positioned at e-distance from C.G. b-distance from C.G to left main landing gear. a-distance from C.G to right main landing gear. The dynamic equations have been derived by considering x, φ, α, γ for bounce, yaw motion along vertical axis, pitch about lateral axis and roll about longitudinal axis. In this study, helicopter aerodynamic forces and rotor thrust are not taken in to account. In this work, the helicopter landing dynamics equations are written by using Lagrange principle from the model as in Eqs. (1)-(15).
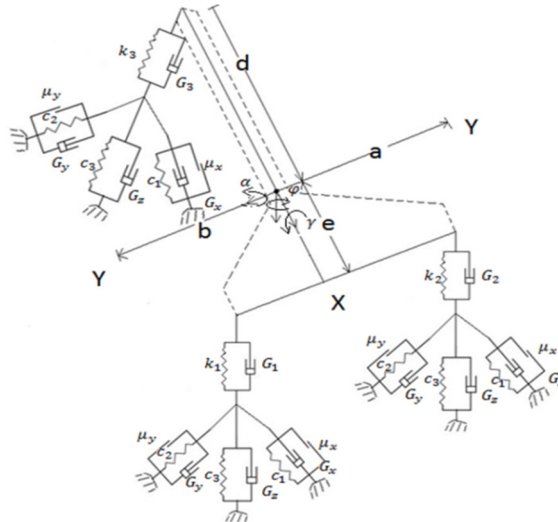

Fig. 1. Mathematical model of helicopter with landing gears

Total kinetic energy (K) is calculated as:

$$K = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I_{xx}\dot{\gamma}^2 + \frac{1}{2}I_{yy}\dot{\alpha}^2 + \frac{1}{2}I_{zz}\dot{\varphi}^2 + \frac{1}{2}m_l\dot{x_1}^2 + \frac{1}{2}m_r\dot{x_2}^2 + \frac{1}{2}m_t\dot{x_3}^2. \tag{1}$$

Total potential energy (U) is calculated as:

$$U = \frac{1}{2}k_1(x - x_1 - b\gamma - e\alpha + h\varphi)^2 + \frac{1}{2}k_2(x - x_2 - a\gamma - e\alpha + h\varphi)^2$$
$$+ \frac{1}{2}k_3(x - x_3 + d\alpha + h\gamma + j\varphi)^2 + \frac{1}{2}c_3(x_1 - y_1) + \frac{1}{2}c_2(x_1\cos\theta - y_1\cos\theta)^2$$
$$+ \frac{1}{2}c_1(x_1\cos\theta - y_1\cos\theta)^2 + \frac{1}{2}c_3(x_2 - y_2)^2 + \frac{1}{2}c_2(x_2\cos\theta - y_2\cos\theta)^2$$
$$+ \frac{1}{2}c_1(x_2\cos\theta - y_2\cos\theta)^2 + \frac{1}{2}c_3(x_3 - y_3)^2 + \frac{1}{2}c_2(x_3\cos\theta - y_3\cos\theta)^2$$
$$+ \frac{1}{2}c_1(x_3\cos\theta - y_3\cos\theta)^2 + \mu_x R_N(x_1\cos\theta - y_1\cos\theta) + \mu_y R_N(x_1\cos\theta - y_1\cos\theta)$$
$$+ \mu_x R_N(x_2\cos\theta - y_2\cos\theta) + \mu_y R_N(x_2\cos\theta - y_2\cos\theta) + \mu_x R_N(x_3\cos\theta - y_3\cos\theta)$$
$$+ \mu_y R_N(x_3\cos\theta - y_3\cos\theta). \tag{2}$$

Total Dissipative energy (D) is calculated as:

$$D = \frac{1}{2}G_1(\dot{x} - \dot{x}_1 - b\dot{\gamma} - e\dot{\alpha} + h\dot{\varphi})^2 + \frac{1}{2}G_2(\dot{x} - \dot{x}_2 - a\dot{\gamma} - e\dot{\alpha} + h\dot{\varphi})^2$$
$$+ \frac{1}{2}G_3(\dot{x} - \dot{x}_3 + d\dot{\alpha} + h\dot{\gamma} + j\dot{\varphi})^2 + \frac{1}{2}G_z(\dot{x}_1 - \dot{y}_1)^2 + \frac{1}{2}G_y(\dot{x}_1\cos\theta - \dot{y}_1\cos\theta)^2$$
$$+ \frac{1}{2}G_x(\dot{x}_1\cos\theta - \dot{y}_1\cos\theta)^2 + \frac{1}{2}G_z(\dot{x}_2 - \dot{y}_2)^2 + \frac{1}{2}G_y(\dot{x}_2\cos\theta - \dot{y}_2\cos\theta)^2$$
$$+ \frac{1}{2}G_x(\dot{x}_2\cos\theta - \dot{y}_2\cos\theta)^2 + \frac{1}{2}G_z(\dot{x}_3 - \dot{y}_3)^2 + \frac{1}{2}G_y(\dot{x}_3\cos\theta - \dot{y}_3\cos\theta)^2$$
$$+ \frac{1}{2}G_x(\dot{x}_3\cos\theta - \dot{y}_3\cos\theta)^2. \tag{3}$$

By applying Lagrange method:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_k}\right) - \frac{\partial K}{\partial q_k} + \frac{\partial D}{\partial \dot{q}_k} + \frac{\partial U}{\partial q_k} = Q_k, \tag{4}$$
$$q_k = f(x, x_1, x_2, x_3, \gamma, \alpha, \varphi, y_1, y_2, y_3),$$
$$\dot{q}_k = f(\dot{x}, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{\gamma}, \dot{\alpha}, \dot{\varphi}, \dot{y}_1, \dot{y}_2, \dot{y}_3),$$
$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_k}\right) = m\ddot{x} + I_{xx}\ddot{\gamma} + I_{yy}\ddot{\alpha} + I_{zz}\ddot{\varphi} + m_l\ddot{x}_1 + m_r\ddot{x}_2 + m_t\ddot{x}_3, \tag{5}$$
$$\frac{\partial D}{\partial q_k} = -G_1 b\dot{x} + G_1 b\dot{x}_1 + G_1 b^2\dot{\gamma} + G_1 be\dot{\alpha} - G_1 bh\dot{\varphi} - G_1\dot{x}e + G_1\dot{x}_1 e + G_1 be\dot{\gamma}$$
$$-G_1 he\dot{\varphi} + G_1 e^2\dot{\alpha} + G_1\,\dot{x}h - G_1\dot{x}_1 h - G_1 b\dot{\gamma}h - G_1 e\dot{\alpha}h + G_1 h^2\dot{\varphi} + G_2 a\dot{x} - G_2 a\dot{x}_2$$
$$+G_2 a^2\dot{\gamma} - G_2 ae\dot{\alpha} + G_2 ah\dot{\varphi} - G_2\dot{x}e + G_2\dot{x}_2 e - G_2 ae\dot{\gamma} - G_2 he\dot{\varphi} + G_2 e^2\dot{\alpha} + G_2\,\dot{x}h \qquad (6)$$
$$-G_2\dot{x}_2 h + G_2 a\dot{\gamma}h - G_2 e\dot{\alpha}h + G_2 h^2\dot{\varphi} - G_3 d\dot{x} + G_3 d\dot{x}_3 - G_3 dh\dot{\alpha} - G_3 dh\dot{\gamma} + G_3 dj\dot{\varphi}$$
$$+G_3 d^2\dot{\alpha} + G_3\,\dot{x}r - G_3\dot{x}_3 r + G_3 r\dot{\gamma}h + G_3 d\dot{\alpha}r + G_3 r^2\dot{\varphi},$$
$$\frac{\partial U}{\partial q_k} = -k_1 bx + k_1 bx_1 + k_1 b^2\gamma + k_1 be\alpha - k_1 bh\varphi - k_1 xe + k_1 x_1 e + k_1 be\gamma - k_1 he\varphi$$
$$+k_1 e^2\alpha k_1 xh - k_1 x_1 h - k_1 b\gamma h - k_1 e\alpha h + k_1 h^2\varphi + k_2 ax - k_2 ax_2 + k_2 a^2\gamma - k_2 ae\alpha$$
$$+k_2 ah\varphi - k_2 xe + k_2 x_2 e - k_2 ae\gamma - k_2 he\varphi + k_2 e^2\alpha + k_2 xh - k_2 x_2 h + k_2 a\gamma h \qquad (7)$$
$$-k_2 e\alpha h + k_2 h^2\varphi + k_3 hx - k_3 hx_3 + k_3 h^2\gamma + k_3 dh\alpha + k_3 rh\varphi + k_3 xd - k_3 x_3 d$$
$$+k_3 dh\gamma + k_3 dr\varphi + k_3 d^2\alpha + k_3 xr - k_3 x_3 r + k_3 r\gamma h + k_3 d\alpha r + k_3 r^2\varphi.$$

The matrix formulation is written as (8):

$$M\ddot{X} + C\dot{X} + KX = 0, \tag{8}$$

$$[M] = \begin{bmatrix} M & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{xx} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_l & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_t \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\gamma} \\ \ddot{\alpha} \\ \ddot{\varphi} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + [C]$$

$$= \begin{bmatrix} S_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_7 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\gamma} \\ \dot{\alpha} \\ \dot{\varphi} \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + [K]$$

$$= \begin{bmatrix} T_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & T_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & T_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & T_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & T_7 \end{bmatrix} \begin{Bmatrix} x \\ \gamma \\ \alpha \\ \varphi \\ x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \{0\},$$

where:

$S_1 = \dot{x}(-G_1 b - G_1 e + G_1 h + G_2 a - G_2 e + G_2 h - G_3 d + G_3 h + G_3 r)$,
$S_2 = \dot{\gamma}(G_1 b^2 + G_1 be - G_1 bh + G_2 a^2 - G_2 ae + G_2 ah - G_3 dh + G_3 h^2 + G_3 rh)$,
$S_3 = \dot{\alpha}(G_1 be + G_1 e^2 - G_1 eh - G_2 ae + G_2 e^2 - G_2 eh - G_3 d + G_3 d^2 + G_3 dh)$,
$S_4 = \dot{\varphi}(-G_1 bh - G_1 eh + G_1 h^2 + G_2 ah - G_2 eh + G_2 h^2 + G_3 dr + G_3 rh + G_3 r^2)$,
$S_5 = \dot{x}_1(G_1 b + G_1 e - G_1 h)$, $\quad S_6 = \dot{x}_2(-G_2 a + G_2 e - G_2 h)$, $\quad S_7 = \dot{x}_3(G_3 d - G_3 h - G_3 r)$,
$T_1 = x(-k_1 b - k_1 e + k_1 h + k_2 a - k_2 e + k_2 h - k_3 d + k_3 h + k_3 r)$,
$T_2 = \gamma(k_1 b^2 + k_1 be - k_1 bh - k_2 ae + k_2 ah + k_2 a^2 + k_3 h^2 + k_3 rh + k_3 dh)$,
$T_3 = \alpha(+k_1 be + k_1 e^2 - k_1 eh - k_2 ae + k_2 e^2 - k_2 eh + k_3 d^2 + k_3 dh + k_3 dr)$,
$T_4 = \varphi(-k_1 bh - k_1 he + k_1 h^2 + k_2 ah - k_2 eh + k_2 h^2 + k_3 dr + k_3 rh + k_3 r^2)$,
$T_5 = x_1(k_1 e - k_1 h + k_1 b)$, $\quad T_6 = x_2(-k_2 a + k_2 e - k_2 h)$, $\quad T_7 = x_3(-k_3 h - k_3 r - k_3 d)$.

The Eqs. (9)-(15) can be solved separately in Matlab/Simulink as:

$$m\ddot{x} + S_1 \dot{x} + T_1 x = 0, \tag{9}$$
$$I_{xx}\ddot{\gamma} + S_2 \dot{\gamma} + T_2 \gamma = 0, \tag{10}$$
$$I_{yy}\ddot{\alpha} + S_3 \dot{\alpha} + T_3 \alpha = 0, \tag{11}$$
$$I_{zz}\ddot{\varphi} + S_4 \dot{\varphi} + T_4 \varphi = 0, \tag{12}$$
$$m_l \ddot{x}_1 + S_5 \dot{x}_1 + T_5 x_1 = 0, \tag{13}$$
$$m_r \ddot{x}_2 + S_6 \dot{x}_2 + T_6 x_2 = 0, \tag{14}$$
$$m_t \ddot{x}_3 + S_7 \dot{x}_3 + T_7 x_3 = 0. \tag{15}$$

3. Landing simulation

The Simulink model of typical helicopter with landing gear system has been developed from the dynamic equations written in Section 2 as shown in Fig. 2. Several trials of numerical simulations can be done by using this Simulink model set up. The helicopter of fuselage mass 9000 kg is considered for numerical simulation. The helicopter and landing gear parameters used

for numerical simulation has taken from Blackwell and Feik [4] tabulated in Table1 and Table 2.

As per FAR 23.725, recommended free drop height range for both MLG and NLG is minimum 9.2 in and maximum 18.7 in. In this simulation, the higher value of 0.4 m is taken for landing. simulation in Matlab/Simulink environment and landing gear touches the runway at 2.5 s on even ground. During landing simulation both the main landing gears touch first and the tail landing gear later. The vertical acceleration, pitch, roll, yaw acceleration responses due to touch down impact of the main landing gear in time domain is presented in the Fig. 3-6.
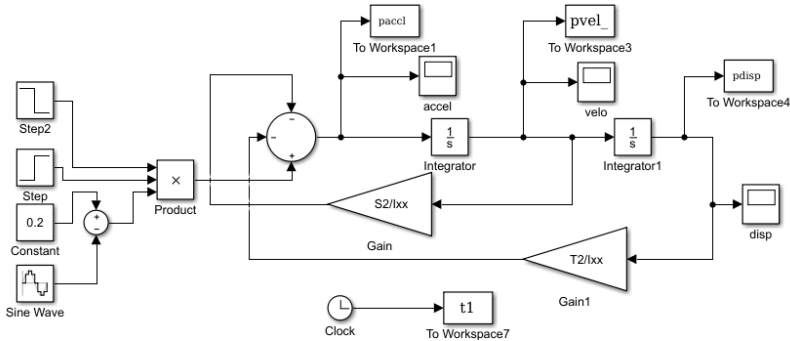

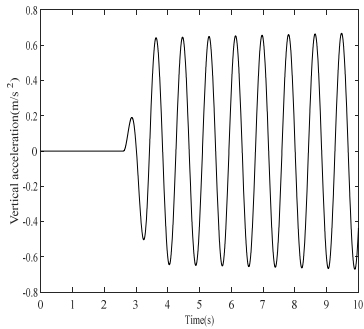**Fig. 2.** Simulation set up in Matlab/Simulink environment

Fig. 3. Bounce acceleration response
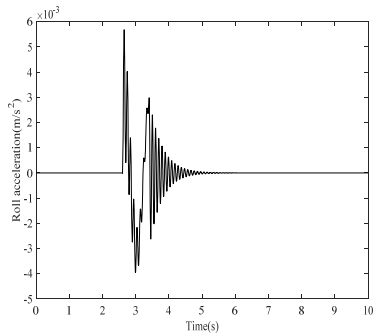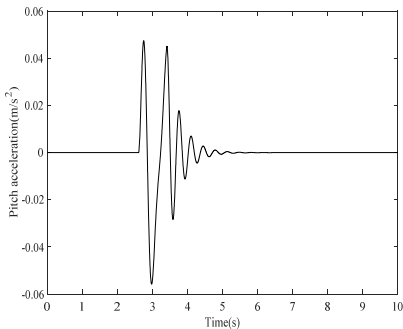

Fig. 4. Roll acceleration response

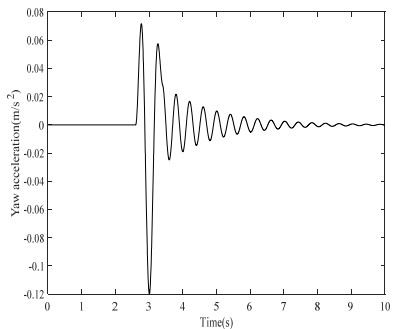**Fig. 5.** Pitch acceleration response

Fig. 6. Yaw acceleration response

Results shows that vertical acceleration is not settled within a short time. The bounce, roll, pitch and yaw acceleration levels are damped within 2 to 4 secs. The settling time is very less for the helicopter to become stable. The trend of the results for a typical simulation is qualitatively matching with the reference given in [4]. It is very difficult to do the real time experimental validation with the typical helicopter. In future study, the typical helicopter has been developed in other Multi body dynamics software's like ADAMS or LMS to do landing dynamics.

Table 1. Helicopter parameters

Description	Symbol	Value	Units
Helicopter fuselage mass	M	9000	kg
Mass moment of inertia about XX axis	I_{xx}	9.0e3	kg.m²
Mass moment of inertia about YY axis	I_{yy}	6.3e4	kg.m²
Mass moment of inertia about ZZ axis	I_{zz}	6.0e4	kg.m²
Distance from C.G to tail gear	d	3.3	m
Distance from C.G to horizontal axis of main gear	e	1.5	m
Distance from C.G to left main landing gear	a	1.4	m
Distance from C.G to right main landing gear	b	1.4	m

Table 2. Helicopter landing gear parameters

Parameter	Main gear	Tail gear	Units	Parameter	Main gear	Tail gear	Units
k_1	3.6e5	–	N/m	c_2	4.5e5	6.0e5	N/m
k_2	5.0e6	–	N/m	c_3	1.2e6	1.0e6	N/m
k_3	–	2.0e6	N/m	G_x	2.0e4	2.0e4	N.s/m
G_1	2.2e4	–	N.s/m	G_y	2.0e4	2.0e4	N.s/m
G_2	4.0e4	–	N.s/m	G_z	1.0e4	1.0e4	N.s/m
G_3	–	3.0e4	N.s/m	μ_x	0.7	0.7	–
c_1	1.4e6	8.0e5	N/m	μ_y	0.7	0.7	–

4. Conclusions

In this research work, helicopter with landing gear seven degree of freedom mathematical model using Lagrange method has been developed to investigate dynamic response. The response of bounce, pitch, roll, yaw accelerations are obtained to verify the developed model. The responses are stable. This numerical simulation procedure is helpful to check the vibration levels of aircraft during landing and taxing on the various ground uncertainties. By using this model, the stiffness and damping coefficient of landing gear component obtained experimentally during drop test can be evaluated theoretically. This procedure enables the designer to optimize the design parameters of landing gear in the initial design stage itself for achieving better performance of landing gears.

References

[1] **O'Reilly P. J.** Aircraft /deck interface dynamics for destroyers. Marine Technology, Vol. 24, Issue 1, 1987, p. 15-25.

[2] **Me Fu-Shang, Baitis Erich, Myers Milliam** Analytical Modeling of SH-2F Helicopter Ship Board Operation. AGARD, AD-A2448690, 1992.

[3] **Ferrier Bernard, Polvi Henry, Thibodeau Francois A.** Helicopter/Ship Analytic Dynamic Interface. AD-A244869, 1992.

[4] **Black Well J., Feik B. A.** A Mathematical Model of the on Deck Helicopter/Ship Dynamic Interface. Aerodynamics Technical Memorandum, AR-005-549, 1988.

[5] **Kim S. K., Tilbury D. M.** Mathematical modeling and experimental identification of a model helicopter. AIAA Modeling and Simulation Technologies Conference and Exhibit, Guidance, Navigation, and Control and Co-located Conferences, 1998.

[6] **Pan Li, Chen Renliang** A mathematical model for helicopter comprehensive analysis. Chinese Journal of Aeronautics, Vol. 23, 2010, p. 320-326.

[7] **Salazar Tulio** Mathematical Model and Simulation for a Helicopter with Tail Rotor, Advances in Computational Intelligence, Man-Machine Systems and Cybernetics. Beijing, China, 2010.

[8] **Sivakumar S., Haran A. P.** Mathematical model and vibration analysis of aircraft with active landing gears. Journal of Vibration and Control, Vol. 21, Issue 2, 2015, p. 229-245.

[9] **Sivakumar S., Syed Haleem M.** Non-linear vibration analysis of oleo pneumatic landing gear at touchdown impact. Mathematical Models in Engineering, Vol. 4, Issue 2, 2018, p. 89-97.