

The effect of non-linear inertia on dynamic response of asymmetric multi-story buildings

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Received 28 March 2018; accepted 4 April 2018

DOI <https://doi.org/10.21595/vp.2018.19856>



Abstract. The goal of this paper is to inspect the influence of non-linear inertia on dynamic response of multi-story asymmetric buildings. In this study unlike conventional linear approach, the non-linear inertial terms are considered in the equations of motion. For considering non-linear inertia, the motion equations are derived in local rotary coordinates system. Also, the stiffness and damping parameters are defined in the rotary coordinates system. In the novel approach which is proposed in this paper, the motion equations are different with conventional linear approach. Numerical examples are presented to compare the response of the proposed model and conventional linear model. The modeled building response is derived under harmonic excitation. The results show that if non-linear inertia is considered, the dynamic response of asymmetric multi-story buildings may become different with conventional linear approach.

Keywords: non-linear inertia, asymmetric buildings, harmonic excitation.

1. Introduction

Torsional motions in asymmetric buildings and can cause severe damages [1]. So, it is important to carefully investigate the behavior of asymmetric buildings under dynamic loads. Many researchers have studied the effects of non-linear inertia on behavior of mechanical systems. Mayet and Ulbrich studied the non-linear detuning of centrifugal pendulum vibration absorber [2]. Mamandi et al. studied the non-linear behavior of an inclined beam subjected to a moving load [3]. In the field of structural engineering little attention has been paid to non-linear inertia and a few number of researchers have studied the effect of non-linear inertia on dynamic response of structural systems. Amin Afshar and Aghaei Pour studied the inertia non-linearity in irregular-plan isolated structures under seismic excitations [4]. Amini and Amin Afshar studied the effect of non-linear inertia on dynamic response of single story asymmetric building [5].

The effect of non-linear inertia in dynamic response of multi-story asymmetric buildings has not been studied enough. In this study non-linear inertia is considered in equations of motion and for considering non-linear inertia the equations of motion in each story is defined in local rotary coordinates system.

2. Non-linear differential equations of motion

As seen in Fig. 1(a), a single story building is subjected to ground excitation in excitation in X and Y directions. The floor mass center is denoted by C.M. and C.R. represents the center of stiffness of the story. Based on Amini and Amin Afshar approach, the non-linear equations of motion in local rotary xyz coordinates system can be expressed as (see Fig. 1(b)) [5]:

$$m\ddot{u}_x + C_x\dot{u}_x + K_x u_x = -m(-2\dot{u}_y\dot{\theta} - u_y\ddot{\theta} - u_x\dot{\theta}^2 + \ddot{u}_{gX}\cos\theta + \ddot{u}_{gY}\sin\theta), \quad (1)$$

$$m\ddot{u}_y + C_y\dot{u}_y + K_y u_y + e_x K_y \theta = -m(2\dot{u}_x\dot{\theta} + u_x\ddot{\theta} - u_y\dot{\theta}^2 - \ddot{u}_{gX}\sin\theta + \ddot{u}_{gY}\cos\theta), \quad (2)$$

$$mr^2\ddot{\theta} + C_\theta\dot{\theta} + (K_{\theta R} + K_y e_x^2)\theta + e_x K_y u_y = 0. \quad (3)$$

In Eqs. (1)-(3), e_x is the distance between C.M. and C.R. It is assumed that C.R. is located on X axis. The value of e_x can be calculated by:

$$e_x = \frac{\sum_{j=1}^N x_j k_{yj}}{\sum_{j=1}^N k_{yj}}, \quad (4)$$

where k_{yj} represents the stiffness of the j th element resisting in Y direction. In Eqs. (1)-(3), m denotes the total mass of the floor and r is the floor radius of gyration about the mass center, \ddot{u}_{gX} and \ddot{u}_{gY} are the ground accelerations in X and Y directions, u_x and u_y are displacements of the floor mass center in the x and y directions and θ is the rotation of the floor about the z axis. Also, K_x and K_y are total stiffness of the story in the x and y directions. The parameter $K_{\theta R}$ is the torsional stiffness of the story about C.R. and can be calculated by:

$$K_{\theta R} = \sum_{j=1}^M K_{xj} y_j^2 + \sum_{j=1}^N K_{yj} (x_j - e_x)^2, \quad (5)$$

where K_{xj} represents the stiffness of the j th resisting element in X direction. Moreover C_x , C_y and C_θ are damping coefficients [5].

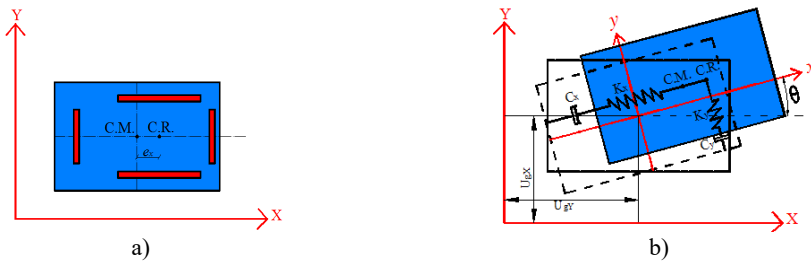


Fig. 1. a) The plan of asymmetric building, b) global (XYZ) and local (xyz) coordinates systems;
— – structure elements [4]

The motion equations of the multi-story building will be derived according to Eqs. (1)-(3). The equations of motion of the i th story will be derived in the local rotary $x_i y_i z_i$ system of coordinates. As seen in Fig. 2 the $x_i y_i z_i$ coordinates system is located on the base of the building and rotates by an angle θ_i . It should be noted that in the conventional linear approach the motion equations are derived in fixed XYZ system of coordinates. The motion equations of the i th story can be expressed as [6]:

$$M_i a_i^i + C_i^i \dot{U}_i^i + K_i^i \bar{U}_i^i - C_{i+1}^i \dot{U}_{i+1}^i - K_{i+1}^i \bar{U}_{i+1}^i = 0, \quad (6)$$

where $M_i a_i^i$ denotes the inertial forces, which is exerted to the i th floor. Also the term $K_i^i \bar{U}_i^i$ is the force of resisting elements in $i - 1$ th story which is applied to the i th floor and the term $K_{i+1}^i \bar{U}_{i+1}^i$ is the force of resisting elements in $i + 1$ th story which is exerted to the i th floor. Also, the terms $C_i^i \dot{U}_i^i$ and $C_{i+1}^i \dot{U}_{i+1}^i$ denote the damping forces. In Eq. (6), M_i denotes the mass matrix of the i th floor. The matrix M_i can be expressed as:

$$M_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i r_i^2 & 0 \\ 0 & 0 & m_i \end{bmatrix}, \quad (7)$$

where m_i is the mass of the i th floor and r_i is the i th floor radius of gyration about the mass center. In Eq. (6), a_i^i represents the total acceleration of the i th floor center of mass in the local $x_i y_i z_i$ coordinates system. The vector a_i^i can be calculated by [6]:

$$a_i^i = \begin{Bmatrix} a_{xi}^i \\ a_{\theta i} \\ a_{yi}^i \end{Bmatrix} = \begin{Bmatrix} \ddot{u}_{xi}^i \\ \ddot{\theta}_i \\ \ddot{u}_{yi}^i \end{Bmatrix} + \begin{Bmatrix} -2\dot{u}_{yi}^i \dot{\theta}_i - u_{yi}^i \ddot{\theta}_i - u_{xi}^i \dot{\theta}_i^2 \\ 0 \\ 2\dot{u}_{xi}^i \dot{\theta}_i + u_{xi}^i \ddot{\theta}_i - u_{yi}^i \dot{\theta}_i^2 \end{Bmatrix} + \begin{Bmatrix} \ddot{U}_{Xg} \cos \theta_i + \ddot{U}_{Yg} \sin \theta_i \\ 0 \\ -\ddot{U}_{Xg} \sin \theta_i + \ddot{U}_{Yg} \cos \theta_i \end{Bmatrix}, \quad (8)$$

where u_{xi}^i and u_{yi}^i are displacements of the i th floor mass center in x_i and y_i directions (see Fig. 2), \ddot{U}_{Xg} and \ddot{U}_{Yg} are the ground translational accelerations. Moreover in Eq. (6), K_i^i is the stiffness matrix of the i th story. The matrix K_i^i is defined in the directions of $x_i y_i z_i$ coordinates system. The matrix K_i^i can be expressed as:

$$K_i^i = \begin{bmatrix} K_{xi}^i & 0 & 0 \\ 0 & K_{\theta i} & K_{yi}^i e_{xi} \\ 0 & K_{yi}^i e_{xi} & K_{yi}^i \end{bmatrix}, \quad (9)$$

where K_{xi}^i and K_{yi}^i are the i th story stiffness in the x_i and y_i directions. The parameters K_{xi}^i and K_{yi}^i are defined in the rotary $x_i y_i z_i$ coordinates system. The parameter $K_{\theta i}^i$ is the torsional stiffness of the i th story about the i th story center of stiffness. Also, e_{xi} is the distance between the mass center and the center of the stiffness of the i th story. In Eq. (6), C_i^i is the damping matrix of the i th story. The matrix C_i^i can be expressed as:

$$C_i^i = \begin{bmatrix} C_{xi}^i & 0 & 0 \\ 0 & C_{\theta i} & C_{yi}^i e_{xi} \\ 0 & C_{yi}^i e_{xi} & C_{yi}^i \end{bmatrix}, \quad (10)$$

where C_{xi}^i , C_{yi}^i and $C_{\theta i}$ are the i th story damping coefficients. These coefficients are defined in $x_i y_i z_i$ coordinates system. If U_{Xi} and U_{Yi} represent the displacements of the i th floor center of mass in the X and Y direction (see Fig. 2), the relationship between $[U_{Xi}, \theta_i, U_{Yi}]^T$ and $[u_{xi}, \theta_i, u_{yi}]^T$ vectors can be expressed as:

$$U_i^i = \begin{Bmatrix} u_{xi}^i \\ \theta_i \\ u_{yi}^i \end{Bmatrix} = \begin{bmatrix} \cos \theta_i & 0 & \sin \theta_i \\ 0 & 1 & 0 \\ -\sin \theta_i & 0 & \cos \theta_i \end{bmatrix} \begin{Bmatrix} U_{Xi} \\ \theta_i \\ U_{Yi} \end{Bmatrix} = Q^i U^i, \quad (11)$$

where Q^i is the rotation matrix about the z_i axis by an angle θ_i . Also, the vector U_{i+1}^{i+1} is defined by:

$$U_{i+1}^{i+1} = [u_{x_{i+1}}^{i+1}, \theta_{i+1}, u_{y_{i+1}}^{i+1}]^T. \quad (12)$$

Moreover in Eq. (6), \bar{U}_i^i and \bar{U}_{i+1}^i are defined by:

$$\bar{U}_i^i = U_i^i - U_{i-1}^i, \quad (13)$$

$$\bar{U}_{i+1}^i = U_{i+1}^i - U_i^i, \quad (14)$$

where the vector U_{i-1}^i denotes the $i - 1$ th floor displacement. Also U_{i+1}^i is the $i + 1$ th floor displacement vector. The vectors U_{i-1}^i and U_{i+1}^i , are defined in the rotary $x_i y_i z_i$ coordinates system. The relationship between U_{i+1}^i and U_{i+1}^{i+1} can be expressed as:

$$U_{i+1}^i = \begin{Bmatrix} u_{x_{i+1}}^i \\ \theta_{i+1}^i \\ u_{y_{i+1}}^i \end{Bmatrix} = \begin{bmatrix} \cos \bar{\theta}_{i+1} & 0 & -\sin \bar{\theta}_{i+1} \\ 0 & 1 & 0 \\ \sin \bar{\theta}_{i+1} & 0 & \cos \bar{\theta}_{i+1} \end{bmatrix} \begin{Bmatrix} u_{x_{i+1}}^{i+1} \\ \theta_{i+1}^{i+1} \\ u_{y_{i+1}}^{i+1} \end{Bmatrix} = (\bar{Q}_i^{i+1})^T u_{i+1}^{i+1}, \quad (15)$$

where $\bar{\theta}_{i+1}$ is defined by $\bar{\theta}_{i+1} = \theta_{i+1} - \theta_i$.

Now in Eq. (6), the matrix K_{i+1}^i will be derived. The vector F_{i+1}^{i+1} is defined by:

$$F_{i+1}^{i+1} = K_{i+1}^{i+1} \bar{U}_{i+1}^i. \quad (16)$$

As it was mentioned before, the vector F_{i+1}^{i+1} denotes the force of the resisting elements in $i + 1$ th story which is applied to the $i + 1$ th floor. Vector F_{i+1}^{i+1} is defined in $x_{i+1}y_{i+1}z_{i+1}$ coordinates system. It is obvious that the reaction of the vector F_{i+1}^{i+1} is applied to the i th floor. To define the vector F_{i+1}^{i+1} in $x_iy_iz_i$ system of coordinates, it can be rotated about z_i axis by an angle $-\bar{\theta}_{i+1}$. So, following equation can be written:

$$F_{i+1}^i = -(\bar{Q}_i^{i+1})^T F_{i+1}^{i+1}, \quad (17)$$

where vector F_{i+1}^i is the $i + 1$ th story resisting elements force which is applied to the i th floor. Vector F_{i+1}^i is defined in $x_iy_iz_i$ coordinates system. As it was mentioned earlier, in Eq. (6) vector F_{i+1}^i is defined by:

$$F_{i+1}^i = -K_{i+1}^i \bar{U}_{i+1}^i. \quad (18)$$

According to Eqs. (17)-(19) the term $-K_{i+1}^i \bar{U}_{i+1}^i$ can be expressed as:

$$\begin{aligned} F_{i+1}^i &= -(\bar{Q}_i^{i+1})^T F_{i+1}^{i+1} = -(\bar{Q}_i^{i+1})^T K_{i+1}^{i+1} (\bar{U}_{i+1}^i) = -(\bar{Q}_i^{i+1})^T K_{i+1}^{i+1} (\bar{Q}_i^{i+1} (\bar{U}_{i+1}^i)) \\ &= -\left((\bar{Q}_i^{i+1})^T K_{i+1}^{i+1} (\bar{Q}_i^{i+1}) \right) \bar{U}_{i+1}^i, \end{aligned} \quad (19)$$

and so K_{i+1}^i can be expressed as:

$$K_{i+1}^i = (\bar{Q}_i^{i+1})^T K_{i+1}^{i+1} (\bar{Q}_i^{i+1}). \quad (20)$$

Also, similar method can be used to derive C_{i+1}^i and it can be expressed as:

$$C_{i+1}^i = (\bar{Q}_i^{i+1})^T C_{i+1}^{i+1} (\bar{Q}_i^{i+1}), \quad (21)$$

where C_{i+1}^{i+1} is the $i + 1$ th floor damping matrix which is defined in $x_{i+1}y_{i+1}z_{i+1}$ coordinates system.

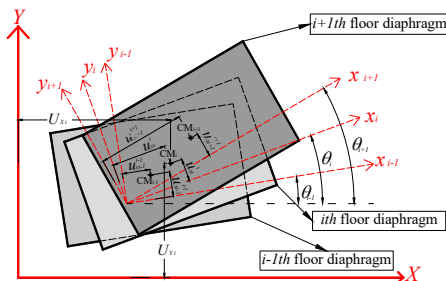


Fig. 2. Local rotary $x_iy_iz_i$ coordinates system; CM_i denotes the i th floor center of mass

3. Numerical studies

Here, for comparing the response of proposed model and conventional linear model a five story building model is considered. The modeled building properties are listed in Table 1. It should be noted that the modeled structures is a scaled buildings. The damping ratio of the modeled building is selected to be 0.5 %. Moreover, the natural frequencies of the first three modes are listed in Table 2. The ground harmonic excitation in X and Y directions is selected to be:

$$\ddot{U}_{gX} = A \sin(\omega t) \cos(\beta), \quad \ddot{U}_{gY} = A \sin(\omega t) \sin(\beta), \quad (22)$$

where the parameter A denoted the amplitude of the excitation and β is the excitation arrival angle with respect to X direction.

Table 1. Modeled building properties

Story No.	Mass (N/(cm/s ²))	K_x and K_y (N/cm)	$K_{\theta R}$ (N.cm)	e_x (cm)	R (cm)
1	0.1	142.1	32095	9.67	20
2	0.1	142.1	32095	9.67	20
3	0.1	142.1	32095	9.67	20
4	0.1	142.1	32095	9.67	20
5	0.1	142.1	32095	9.67	20

Table 2. Modeled building natural frequencies in first three modes

ω_1 (rad/s)	ω_2 (rad/s)	ω_3 (rad/s)
5.39	10.73	12.29

The response of top floor mass center is shown in Fig. 3. It is seen that the response of linear and non-linear models is quite different. The response of top floor mass center in linear model and X direction is zero. But in non-linear model the top story mass center oscillates in X direction. In non-linear model, after initial steps of excitation the amplitude of the response in X direction increases. Moreover, in non-linear model the amplitude of the response in Y direction reaches a constant value after initial oscillations. It is seen that when the amplitude of excitation is increased, the difference between linear and non-linear model responses become more.

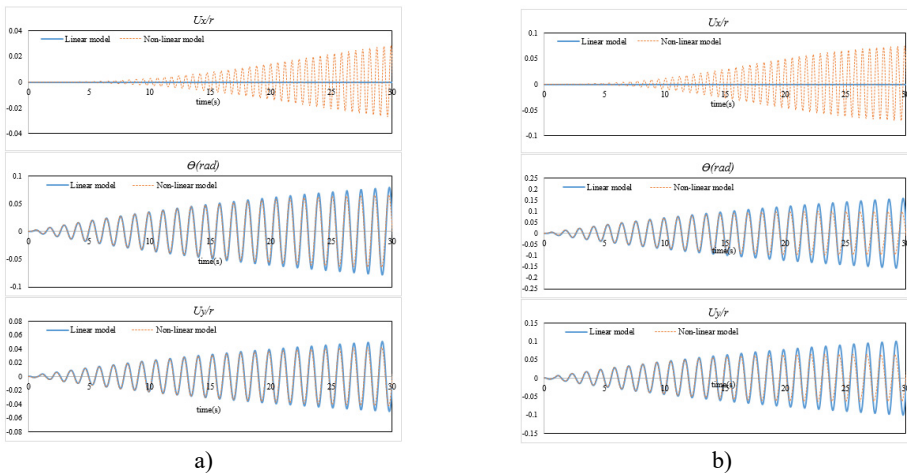


Fig. 3. Top story response of the modeled building: a) $A = 0.005$, $\beta = 90^\circ$ and $\omega = \omega_1$,
 b) $A = 0.01$, $\beta = 90^\circ$ and $\omega = \omega_1$

In Fig. 4 the response of the top floor mass center is shown. In this figure the parameter β is selected to be near to zero. In Fig. 4(a), the response of the both linear and non-linear models are

identical in X direction and the difference between these two models can be ignored. In the response of Y direction and θ , linear and non-linear models have the same response in initial steps of excitation. But after initial steps of oscillation the response of Y direction and θ increases.

As seen in Fig. 4 when the excitation amplitude is increased, the difference between linear and non-linear models becomes more. In Fig. 4(b) when the amplitude of the response in X direction reaches a certain value; the amplitude suddenly decreases in X direction and again increases. Moreover, after initial steps of excitation the response of Y direction θ increases. When the energy amount in the dominant mode of X direction reaches a certain value, this mode is saturated, and the energy is transferred to other modes.

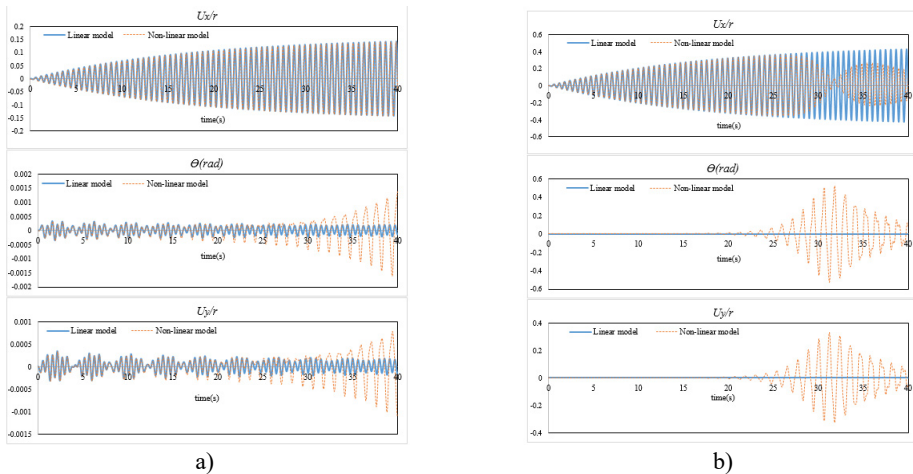


Fig. 4. Top story response of the modeled building: a) $A = 0.01$, $\beta = 0.05$ rad and $\omega = \omega_2$,
 b) $A = 0.03$, $\beta = 0.05$ rad and $\omega = \omega_2$

4. Conclusions

In this study unlike conventional linear approach, the non-linear inertia was considered in equations of motion of multi-story asymmetric building. For considering non-linear inertia, the equations of motion were derived in local rotary coordinates system. It was seen that the conventional linear approach is weak to model the behavior of asymmetric buildings and considering non-linear inertia leads to different response. In the modeled structure if the amplitude of the excitation is increased the difference between linear and proposed non-linear models become more. Also, in some cases energy transfer between the modes is observed.

References

- [1] **Amin Afshar M., Amini F.** Non-linear dynamics of asymmetric structures under 2:2:1 resonance. International Journal of Non-Linear Mechanics, Vol. 47, 2012, p. 823-835.
- [2] **Mayet J., Ulbrich H.** First-order optimal linear and nonlinear detuning of centrifugal pendulum vibration absorbers. Journal of Sound and Vibration, Vol. 335, 2015, p. 34-54.
- [3] **Mamandi A., Kargarnovin M. H., Younesian D.** Nonlinear dynamics of an inclined beam subjected to a moving load. Nonlinear Dynamics, Vol. 60, 2010, p. 277-293.
- [4] **Amin Afshar M., Aghaei Pour S.** On inertia nonlinearity in irregular-plan isolated structures under seismic excitations. Journal of Sound and Vibration, Vol. 363, 2016, p. 495-516.
- [5] **Amini F., Amin Afshar M.** Saturation in asymmetric structures under internal resonance. Acta Mechanica, Vol. 221, 2011, p. 353-368.
- [6] **Amini F., Rezazadeh H., Amin Afshar M.** Adaptive control of rotationally non-linear asymmetric structures under seismic loads. Structural Engineering and Mechanics, Vol. 65, Issue 6, 2018, p. 721-730.