

# Nonlinear dynamics and control of electrodynamic tether for deorbiting space debris

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**Abstract.** The ever increasing population of space debris poses a great threat to the sustainable development of space industry. Electrodynamic tether has been recognized as a promising technology for the active removal of space debris from overpopulated orbital regions. A typical electrodynamic tether system consists of two end-bodies connected by a conductive tether in space. The electric current flowing in the tether will interact with the magnetic field of the Earth to generate the Lorentz force, by which the system can be deorbited almost without expending propellant. The dynamics and control of any electrodynamic tether system is highly nonlinear by nature and have two critical aspects for practical application: the deployment of electrodynamic tether and the attitude stability during the deorbiting process. This paper summarizes some recent efforts made to address these two issues by the authors' research team in Nanjing University of Aeronautics and Astronautics. Moreover, some open problems deserving future investigation are discussed.

**Keywords:** nonlinear dynamics, control, electrodynamic tether, deorbit, space debris.

## 1. Introduction

Electrodynamic Tether (ET) refers to a long and conductive cable connecting satellites or other space objects to each other [1]. The charge exchange between an ET system and the ionosphere can be attained by means of electric contact devices, such as electron gun or hollow electrode, etc. For instance, in the case of an insulated ET with its two ends exposed for charge exchange, electrons can be collected by a contact device at one end and then expelled back into the ionosphere by using an electron gun at the other end. Consequently, a continuous current flow can be established in the ET provided that a compatible electric field is applied along the tether. The magnetic field of the Earth will interact with the electric current flowing in the ET to generate the Lorentz force, by which the system orbit can be changed almost without expending any propellant. In the absence of external power supply, the electric current in the ET flows in the same direction as the motional electric field such that the ET system operates in a 'drag mode' of converting its kinetic energy to electrical energy, thereby deorbiting itself. On the contrary, in a 'thruster mode', the direction of electric current can be reversed by applying an external power supply so as to convert the electrical energy to kinetic energy, thereby raising the orbit of the ET system. Over the last three decades, there have been many proof-of-concept missions flown in orbit to demonstrate some key aspects of the ET technology [1-3].

Among its diverse potential applications, ET has been especially recognized as a promising technology for the active removal of space debris from overpopulated orbital regions. The ever increasing population of space debris poses a great threat to the sustainable development of space industry [4, 5]. As compared to the conventional means relying on chemical propellants, the technology of ET gains special advantage in deorbiting large-size space debris, e.g. defunct satellite or used rocket up-stage, due to its compact size, low power consumption, and technical simplicity. Although technically appealing for deorbiting application, the development of ET technology poses many challenging and interesting problems at the same time. One point of great concern is that the librations of the tether can grow dramatically in the deployment stage. Besides, special attention should be paid to the inherent instability during deorbiting process since the

Lorentz force acting on the tether may lead to significant energy injection into attitude motions. Hence, active control actions usually have to be taken for such a nonlinear and under-actuated system to prevent it from becoming unstable. This paper summarizes some recent research efforts made on the nonlinear dynamics and control of ET systems by the authors' team in Nanjing University of Aeronautics and Astronautics. In the following, Section 1 reviews some recently developed control laws, which explicitly include the tension constraint, for deploying the tether in a stable and fast manner; Section 2 focuses on the output feedback control problem of the electric current for deorbiting ET systems, along with a discussion on the nonlinear state estimation of attitude motions. Finally, some open problems deserving future investigation are discussed.

## 2. Tether deployment

As a prerequisite of performing deorbiting task, the tether in the ET system has to be deployed to a commanded length. The deployment process under the gravity gradient remains stable provided that the speed of tether deployment is kept at sufficient slow levels all the time. However, it is not easy to achieve a stable deployment in a simultaneously fast manner since the rapid change of tether length may induce significant tether librations and vibrations due to the effects of Coriolis acceleration. Although the control action can be applied by installing and using thrusters at the end-bodies, a simpler but still effective way, especially tailored for deployment control, is to regulate the tension in the tether. Besides, extra control actuation can be achieved by regulating the electric current in the ET, thereby changing the Lorentz force.

Notably, the design of tension control law is subject to the physical constraint of positive tension since the tether cannot sustain any compression. Recently, numerical optimization techniques, e.g. Nonlinear Model Predictive Control (NMPC), have gained a great attention for dealing with the tension and other constraints arising from the control problem of tether deployment [6, 7]. Such optimization-based strategies provide a generally applicable framework for not only optimizing the control performance measured by a cost function, but also accommodating the nonlinear dynamics and complex constraints of the ET system. However, the conventional formulations of NMPC require to solve a complete nonlinear programming problem within a single sampling interval. This requirement usually makes the NMPC schemes too computationally demanding to be implemented by using the limited computational power available from an on-board hardware platform.

To deal with the computational challenge of conventional NMPC strategies, an algorithm of Online Quasi-Linearization Iteration (OQLI) has been proposed to online solve the NMPC laws for both the deployment and retrieval processes of a space tether system [8]. The basic idea of the OQLI algorithm is to convert the original nonlinear optimal control problem into a series of linear optimal control problems scheduled to be solved one by one at consecutive instants. In other words, the OQLI algorithm requires to solve only a constrained Quadratic Programming (QP) problem instead of a full nonlinear programming problem at each time step, thereby achieving a significant reduction in computational cost with respect to the conventional NMPC schemes. The computational performance of the OQLI algorithm was demonstrated for a case study of tether deployment control, where the simulation results were obtained with 20 discretization nodes and the Intel i7-4600U processor. It was found from the numerical results that the time cost for each online solution based on the OQLI algorithm was no more than 13 milliseconds and had an averaging value below 10 milliseconds. Moreover, the significant advantage in computational cost of the OQLI algorithm has also been demonstrated in a comparison with a conventional quasi-linearization iteration scheme, which involved a series of quasi-linearization iterations in each sampling interval so as to meet a convergence criterion.

For some small-size and low-cost ET systems, the available computational power or other hardware resources may become so limited that only feedback control law in a simple analytical form can be implemented. However, it is not easy to synthesize an analytical feedback control law for such an under-actuated and nonlinear system with tension constraint. Over the last decades,

numerous attempts have been made to devise analytical control laws for stabilizing the deployment process of space tether system [1, 9]. Most of the investigations treated the deployment control as an unconstrained problem without taking the constraint of positive tension into explicit consideration. Consequently, the design parameters of the unconstrained tension controllers have to be carefully selected so as to avoid the violation of tension constraint. It is obvious that such a parameter selection is valid case by case and sensitive to the change of the initial conditions and physical properties of the ET system. To circumvent this issue, a tension control law in an analytical feedback form, which explicitly included the constraint of positive tension by a strictly increasing saturation function, has been proposed for deploying the tether in a stable and fast manner [10]. The control law relied only on the feedback information of tether length and its rate, and had no requirement of measuring the tether libration states. It was demonstrated numerically that the constrained control law took less time to accomplish the deployment task in a comparison with a recently developed fractional-order control strategy [11]. Moreover, a great deal of simulations gave a statistical view that the constrained controller had a quite loose restriction on the initial conditions of the deployment process for guaranteeing the convergence of the system trajectory.

The analytical feedback control law with the tension constraint was further extended for stabilizing a space-tug system, in which a ‘tug’ spacecraft was towed to a space debris through an inert tether and actuated by thrusting force to deorbit the orbit of the combined system [12]. By the exploitation of a dynamic extension technique, the tension controller for the space-tug system was reformulated in a velocity-free form which had no requirement of measuring the length rate. The tension control law can also be applied to stabilize the general three-dimensional deployment motions of the ET system. However, in the presence of out-of-plane libration, it is hard for the pure tension control law to completely diminish the three-dimensional motions in a short amount of time [13]. To address this problem, a feedback control law has been proposed to gain a better control performance by augmenting the tension control with the action of Lorentz force [14]. The combined control law was in an analytical form which accounted for not only the tension constraint but also the bounds on the strength of electric current. Simulation results showed the advantage of the combined strategy in decay rate of libration angles over the pure tension control scheme.

### 3. ET-based deorbit

Recent investigations of ET systems have heavily focused on its potential application for deorbiting large space debris, with focus on the dynamics and control concerning the attitude motions. It has been widely recognized that attitude instability may be caused by the continuous energy injection into the attitude motions from the Lorentz force [15]. A great deal of control strategies have been proposed to achieve the deorbit of ET system in a stable and efficient manner, by actively regulating the electric current, e.g. based on a simple logic of on-off switch [16] or a more complex control law in a continuous form [3]. It is worthy to note that most of these control schemes relied on the real-time measurement or estimation of the libration angles and even their change rates. Hence, one point of practical importance for a safe and successful deorbit is how to measure or estimate the libration angles and their rates by using the limited resources available from an on-board hardware system.

A few of strategies based on the theory of Kalman filter have been proposed for the state estimation of space tether systems, where the available measurements were supposed to be the relative positions of two end-bodies obtained from the Global Positioning System (GPS) [17] or the three-axis measurement of tether tension [18], etc. One problem arising from the potential application of such filters is that the assumption of Gaussian noise made by these designs can hardly be met in practice. Besides, the convergence characteristics of the estimated states have not been clarified in depth for the Kalman filter designs of the ET systems. As an alternative to the Kalman estimators, a deterministic observer has recently been proposed for estimating the angular

velocities of an ET system in an inclined and elliptical orbit [19]. It was supposed in the work that the estimations of the attitude angles had been obtained by using the GPS measurements of the end-body positions. By applying the technique of Immersion and Invariance (I&I), the exponential convergence of the velocity estimation errors can be theoretically guaranteed by the nonlinear observer irrespective of the control input. The rapid and smooth convergence of the velocity estimator has also been demonstrated via numerical case studies subject to measurement noise.

A research effort has also been made to remove the requirement of measuring full states by exploiting the technique of Moving Horizon Estimation (MHE), by which both the tether angles and their rates can be estimated from the measurements of the relative position between the two end-bodies [20]. The MHE estimators were combined with a full-state feedback MPC controller so as to develop the control law of the electric current in an output feedback form. The MHE-MPC scheme is versatile for not only dealing with the nonlinearity and constraints but also optimizing the specified performance index of the deorbiting task. Both the MPC and MHE problems were formulated on moving horizons as continuous optimization problems, which were discretized first on a basis of a multiple-interval pseudospectral algorithm and then solved by an off-the-shelf nonlinear programming solver. For the sake of safety, conventional MPC schemes usually require to impose some kinds of hard state constraints on the libration angles and/or their rates for the deorbiting control. However, the imposition of hard state constraints may lead to not only the infeasibility of the optimization problem but also an increase of the involved computational loads. To address this issue, the hard state constraints were removed from the MPC formulation through an online gain adaptation of the MPC cost function. It was demonstrated by numerical simulations that the tether libration can be stabilized during the deorbiting process by the MHE-MPC strategy even with only output feedback.

#### 4. Conclusions

The technology of ET promises a vast of potential applications, e.g. the propellant-free deorbit of large space debris. Although technically appealing, the dynamics and control of any ET system are highly nonlinear and under-actuated by nature, and especially complex in the deployment and deorbiting stages. Besides, it is even more demanding to account for the requirements of enforcing the tension constraint and estimating the unmeasurable states. The challenging and interesting topics have attracted considerable attention from the academia and industry, and gained significant progresses in different aspects over the last decades. Among them, a few of recent research efforts were summarized in this paper, with focus on the deployment and deorbiting control problems. However, there are still many open problems deserving future investigation. For example, there exist high levels of uncertainties and disturbances in the deploying mechanism, sensors and current regulating system, etc. It brings great challenges to not only the dynamic modeling but also the robust and adaptive control of ET system. Moreover, substantial extensions are also required to handle the flexible tether and space environment in a more realistic manner.

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