

The sensitivity analysis of the translation and the rotation angle of the first-order mode shape of the joints in frame structures

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Abstract. When damage occurs, there are changes in the structural stiffness. This causes changes in the vibrational information, such as translation and rotation angle of the joints in the structure. This paper presents a sensitivity study of translation and slope of the first-order mode shape of the joints in the frame structure, and which will contribute to structural damage identification further. Starting from the equation of natural vibration of frame structure, and according to the characteristics of the stiffness matrix, derive sensitivity coefficient expression of translation and slope of first-order mode shape of the joints in the structure, and obtain the first-order relative sensitivity coefficient of the first story is always smaller than zero when single column of the first story damage occurs. The numerical analysis of the four stories three spans frame shows consistent results with the formula derivation.

Keywords: sensitivity analysis, translation, rotation angle.

1. Introduction

The modal parameters of the structure (such as frequency, modal shape and modal damping) are the function of structural physical parameters (such as mass, stiffness and damping), so the change of structural physical parameters will lead to the changes of vibration modal parameters of structure. In order to correctly establish the relationship between the dynamic properties of structure and damage of structure, it is necessary to conduct the damage sensitivity analysis of the dynamic characteristics.

Zhao and DeWolf [1] derived sensitivity coefficients of natural frequency, vibration mode, and the modal flexibility, which was applied to the spring-mass model. It was concluded that the modal flexibility is more sensitive to the damage. Zhu et al. [2, 3] used optimization method to identify the damage of periodic structure based on sensitivity analysis of modal parameters of periodic structures. Li et al. [4] derived sensitivity coefficient expressions of structure natural frequency, mode shapes and mode shape slopes based on the characteristics of shear frame structural stiffness matrix. Yang [5] presented a mixed sensitivity method that combines eigenvalue and flexibility sensitivity to identify structural damage. Chen [6] analyzed the effectiveness of damage localization method based on the change in the value of the slope of the first mode. Yam et al. [7] studied the sensitivity of static and dynamic parameters to damage occurring in plate-like structures and proposed corresponding damage indices.

The first-order mode shape slope is a good index of damage detection for shear frames, which can be used to detect the damage of story stiffness [8]. As for the general frame, there is not only the translation but also rotation angle. Thus the damage of the beam and column will cause the changes of the slope and rotation angle.

In this paper sensitivity study of the translation and slope of the first-order mode shape of first story is conducted for further damage detection of general frame structures.

2. Sensitivity analyses

In the sensitivity analysis, the sensitivity coefficient of the structure modal parameter is divided

into absolute sensitivity coefficient and relative sensitivity coefficient. Absolute sensitivity coefficient is defined as the derivative of structure modal parameters with respect to structural physical parameters, and relative sensitivity coefficient is defined as the ratio of the rate of change of structure modal parameter to the rate of change of structural physical parameter.

2.1. Sensitivity analyses of translation of mode shapes

The equilibrium equation for undamped structural vibration is:

$$(K - \omega^2 M)\varphi_i = 0, \tag{1}$$

where K and M are the stiffness and mass matrices, respectively, ω_i is the i th circular frequency, and φ_i is the i th mode shape corresponding to ω_i , which is normalized to be the unit-mass mode shape, i.e.:

$$\varphi_i^T M \varphi_i = 1, \tag{2}$$

the derivative of Eq. (1) with respect to any beam or column stiffness, k_t , is:

$$\left(\frac{\partial K}{\partial k_t} - 2\omega_i \frac{\partial \omega_i}{\partial k_t} M\right)\varphi_i + (K - \omega_i^2 M)\frac{\partial \varphi_i}{\partial k_t} = 0. \tag{3}$$

And based on vibration theory, the mode shapes of a structure are independent of each other, and thus a complete set of mode shapes can be used to constitute the sensitivity coefficients of the i th mode shape. Therefore, the absolute sensitivity coefficients of the mode shapes can be defined as follows:

$$\frac{\partial \varphi_i}{\partial k_t} = \sum_{r=1}^N \beta_r \varphi_r, \tag{4}$$

both sides of Eq. (3) are premultiplied by the transposed vector of the r th mode shape, φ_r^T . Making use of the orthogonal properties of the mode shapes, Eq. (3) can be rewritten as:

$$\varphi_r^T \frac{\partial K}{\partial k_t} \varphi_i + \varphi_r^T (K - \omega_i^2 M) \frac{\partial \varphi_i}{\partial k_t} = 0, \quad r \neq i, \tag{5}$$

because $\varphi_r^T K = \varphi_r^T \omega_r^2 M$, Eq. (5) is simplified:

$$\varphi_r^T \frac{\partial K}{\partial k_t} \varphi_i + (\omega_r^2 - \omega_i^2) \varphi_r^T M \frac{\partial \varphi_i}{\partial k_t} = 0, \quad r \neq i. \tag{6}$$

For the first story column, substituting Eq. (4) into Eq. (6) and making use of the orthogonal properties of the mode shapes leads to:

$$\beta_r = \frac{2E(6\varphi_{i,1}\varphi_{r,1} + 3\varphi_{i,1}\varphi_{r,n+s}l_1 + 3\varphi_{i,1}\varphi_{1,n+s}l_1 + 2\varphi_{i,n+s}\varphi_{r,n+s}l_1^2)}{l_1^3(-\omega_r^2 + \omega_i^2)}, \quad r \neq i, \tag{7}$$

where n and m are the number of stories and the number of spans respectively, s is the s th column in the first story, which is no more than $(m + 1)$. E and l_1 are respectively the elastic modulus and the length of the column. And when $r = i$:

$$\beta_r = 0. \tag{8}$$

Thus, the first-order absolute sensitivity coefficient of the mode shapes can be summarized as follows:

$$S_{it}^M = \frac{\partial \varphi_i}{\partial k_t} = \sum_{r=1}^N \beta_r \varphi_r, \tag{9}$$

where $\beta_r = \begin{cases} \frac{2E(6\varphi_{i,1}\varphi_{r,1}+3\varphi_{i,1}\varphi_{r,n+s}l_1+3\varphi_{i,1}\varphi_{1,n+s}l_1+2\varphi_{i,n+s}\varphi_{r,n+s}l_1^2)}{l_1^3(-\omega_r^2+\omega_i^2)}, & (r \neq i), \\ 0, & r = i, \end{cases}$ and the first-order

relative sensitivity coefficient of the mode shapes can be summarized as follows:

$$\bar{S}_{it}^M = \lim_{\Delta k_t \rightarrow 0} \frac{\frac{\Delta \varphi_i}{\varphi_i}}{\frac{\Delta k_t}{k_t}} \approx \frac{\partial \varphi_i}{\partial k_t} \cdot \frac{k_t}{\varphi_i}. \tag{10}$$

2.2. Sensitivity analyses of mode shape slopes

For general frames, the mode shape slope of the j th story in the i th mode is defined as:

$$\varphi'_{ij} = \frac{\varphi_{ij} - \varphi_{i(j-1)}}{h_j}, \tag{11}$$

where h_j is the height of the j th story. Taking the first derivatives of Eq. (11) with respect to k_t leads to:

$$S_{ij,t}^S = \frac{\partial \varphi'_{ij}}{\partial k_t} = \frac{1}{h_j} \left(\frac{\partial \varphi_{ij}}{\partial k_t} - \frac{\partial \varphi_{i(j-1)}}{\partial k_t} \right), \tag{12}$$

where $S_{ij,t}^S$ is the first-order absolute sensitivity coefficient of the mode shape slope of the j th story in the i th mode with respect to the t th column or beam stiffness. And the first-order relative sensitivity coefficient:

$$\bar{S}_{ij,t}^S = \lim_{\Delta k_t \rightarrow 0} \frac{\frac{\Delta \varphi'_{ij}}{\varphi'_{ij}}}{\frac{\Delta k_t}{k_t}} \approx \frac{\partial \varphi'_{ij}}{\partial k_t} \cdot \frac{k_t}{\varphi'_{ij}} = \frac{k_t}{\partial \varphi_{ij} - \partial \varphi_{i(j-1)}} \left(\frac{\partial \varphi_{ij}}{\partial k_t} - \frac{\partial \varphi_{i(j-1)}}{\partial k_t} \right), \tag{13}$$

when $j = 1$:

$$\bar{S}_{i1,t}^S \approx \frac{\partial \varphi_{i1}}{\partial k_t} \cdot \frac{k_t}{\varphi_{i1}}, \tag{14}$$

when $i = 1$:

$$\frac{\partial \varphi_{11}}{\partial k_t} \cdot \frac{k_t}{\varphi_{11}} = \frac{k_t}{\varphi_{11}} \cdot \sum_{r=2}^N \beta_r \varphi_{r1} = \frac{k_t}{\varphi_{11}} \cdot (\beta_2 \varphi_{21} + \beta_3 \varphi_{31} + \beta_4 \varphi_{41} + \dots). \tag{15}$$

According to the analysis of the examples, we can get nearly accurate answer of $\frac{\partial \varphi_{11}}{\partial k_t}$ when $r = n$. As for $\beta_r \varphi_{r1}$, suppose:

$$\beta_r \varphi_{r1} < 0, \tag{16}$$

$$\beta_r \varphi_{r1} = \frac{2E(6\varphi_{1,1}\varphi_{r,1} + 3\varphi_{1,1}\varphi_{r,n+s}l_1 + 3\varphi_{r,1}\varphi_{1,n+s}l_1 + 2\varphi_{1,n+s}\varphi_{r,n+s}l_1^2)}{l_1^3(-\omega_r^2 + \omega_1^2)} \cdot \varphi_{r,1} < 0. \tag{17}$$

Therefore:

$$\frac{(3\varphi_{1,1}\varphi_{r,n+s}l_1 + 2\varphi_{1,n+s}\varphi_{r,n+s}l_1^2)\varphi_{r,1}}{6\varphi_{1,1}\varphi_{r,1}^2 + 3\varphi_{r,1}^2\varphi_{1,n+s}l_1} > -1, \tag{18}$$

when $\varphi_{r,n+t}$, $\varphi_{r,1}$ symbol are the same, inequality (18) is correct. And when $\varphi_{r,n+t}$, $\varphi_{r,1}$ have different symbol:

$$\frac{(3\varphi_{1,1}l_1 + 2\varphi_{1,n+s}l_1^2)}{6\varphi_{1,1} + 3\varphi_{1,n+s}l_1} < -\frac{\varphi_{r,1}}{\varphi_{r,n+s}}. \tag{19}$$

According to the analysis of the examples, $|\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1}|$ is the minimum in all of $|\frac{\varphi_{i,1}}{\varphi_{i,n+s}l_1}|$ ($i = 1, 2, 3, \dots$). And making $-\frac{\varphi_{r,1}}{\varphi_{r,n+s}l_1}$ equals to $\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1}$, inequality (19) yields:

$$\frac{(3\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1} + 2)}{6\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1} + 3} < \frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1}, \tag{20}$$

$$\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1} > \frac{\sqrt{3}}{3} \approx 0.577. \tag{21}$$

Thus, inequality (16) is necessarily true when $\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1} > \frac{\sqrt{3}}{3}$. And according to the analysis of cantilever column, taking concrete frame columns minimum section 300 mm×300 mm, and the corresponding $\frac{\varphi_{1,1}}{\varphi_{1,n+s}l_1}$ is 0.669, which is more than $\frac{\sqrt{3}}{3}$. So inequality (16) is true, that is the first-order relative sensitivity coefficient of the first story is always smaller than zero when the first story column is damaged.

3. Numerical example

The proposed method is demonstrated using numerical example of three spans four stories frame, as shown in Fig. 1, which is referred to as model 1 hereinafter. All of the column and beam length is 3 m, and the cross section of the column is 500 mm×500 mm and cross section of the beam is 300 mm×500 mm. Other parameter information is in the Table 1.

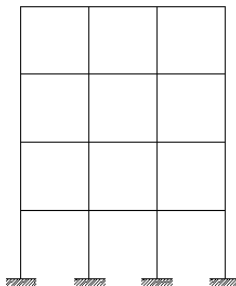


Fig. 1. Three spans four stories frame

Table 1. Parameter information of the frame

Elastic Modulus	Poisson's ratio	Density
2.486E+10	0.2	2400 kg/m ³

When the frame is damaged, stiffness of the column or beam will reduce. In the study, the reducing of the stiffness is simulated by the reducing of moment of inertia of column or beam. The following discussing the sensitivity of the first-order mode shape slope of first story when the first story column is damaged.

Assumes that the damage degree is 20 %, and the translation of the first order mode shape before and after the damage is shown in Table 2, and the slope of the first order mode shape before and after the damage is shown in Table 3.

Table 2. Translation of the first order mode shape before and after the damage

Story	The translation before damage	The translation after damage
1	0.001817	0.001849
2	0.004335	0.004349
3	0.006258	0.006252
4	0.007249	0.007231

Table 3. Slope of the first order mode shape before and after the damage

Story	The slope before damage	The slope after damage
1	0.000606	0.000616
2	0.000839	0.000833
3	0.000641	0.000634
4	0.000330	0.000326

The Table 3 shows that when the first story column is damaged the slope of the first order mode shape of first story will increase, which also means that the first-order relative sensitivity coefficient of mode shape slope of first story is smaller than zero.

From the above numerical example we can obtain that the first-order relative sensitivity coefficient of the first story is smaller than zero when the first story column is damaged, and this conclusion is the same with the results of the formula derivation.

4. Conclusions

Starting from the equation of natural vibration of frame structure, and according to the characteristics of the stiffness matrix, the paper has derived sensitivity coefficient expression of translation and slope of first-order mode shape of the joints in the structure. And some important observations can be pointed out:

1. When the single column of the first story damage occurs, the first-order relative sensitivity coefficient of the first story is always smaller than zero.
2. The damage of column of the first story can be detected by observing the changes of first-order mode shape slope of the first story.

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