Vibration fatigue reliability analysis of aircraft landing gear based on fuzzy theory under random vibration

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Abstract. The failure of aircraft landing gear (ALG) is major caused by vibration fatigue. And its main failure mode is fatigue fracture. Currently the reliability of ALG is usually calculated by the stress strength interference (SSI) model, which is based on the binary state assumption. While in reality, the strength is degraded with time and the boundary of the failure and success is blur, so the binary state assumption is deviated from the fact. To overcome this problem, this paper uses the membership function (MF) to represent fuzzy safe state which caused by the strength degradation under the failure mode of vibration fatigue. Moreover, a fuzzy reliability model (FRM) of ALG is proposed based on fuzzy failure domain (FFD). Finally, the feasibility of method is tested through a simulation example. By comparing the simulation results (SRs) of the FRM with SRs of the static SSI model and the dynamic SSI model, the rationality of the method is verified. The FRM can calculate the reliability without the gradual degradation processes, thus it is used more widely.

Keywords: vibration fatigue, fuzzy reliability analysis, SSI model, membership function.

1. Introduction

With the high requirement for safe operation of ALG, the issue of structure reliability under vibration fatigue becomes more and more prominent. ALG is one of the key components of aircraft. Due to the complexity of landing gear structure and nonlinear factors, it is difficult to analysis the reliability of ALG directly from the dynamics. Usually the structure reliability of ALG is calculated by the static SSI model which is widely used in the calculation of structural reliability [1-4]. In this model a certain distribution is used to represent the stress and strength. When the stress suffered by products is greater than the strength, mechanical structure are likely to fail. As a matter of fact, this model is based on the binary state assumption which is deviated from the reality under the vibration fatigue. Because the strength is degraded with time under the vibration fatigue failure mode and the boundary of the failure and success is blur. So the binary state assumption is deviated from the fact. As a result, the reliability estimated by the static SSI model may deviate from the practical situation [4-6].

The failure mode of ALG is fatigue failure which is a depletion process, so the boundaries of safety and failure become blurred. In this case, it is difficult to determine the safety or failure. Therefore, it's necessary to do the research of the reliability model based on fuzzy reliability under the vibration fatigue. Until now some scholars have done considerable in this field.

For depletion failure, the strength is usually degenerated with stress [7-10]. Gao proposed the dynamic reliability analysis method of degraded mechanical components based on process probability density function of stress [7]. Zhang established reliability calculation model of gears considering strength degradation [10].

Degenerated SSI model solves the problem of strength degradation with stress, but ignores the ambiguity of security status. Hence, it is necessary to combine fuzzy mathematics with reliability during studying structural reliability under the failure mode of vibration fatigue.

In recent years, some efforts have been made to analyze fuzzy reliability of various engineering systems by employing the fuzzy set theory introduced by Zadeh in 1965 [11]. Ma et al. proposed a method to analyze the dynamic response and reliability of fuzzy-random truss systems under the stationary random load [12]. Roberts presented an approach to analyzing fuzzy system reliability

by modeling a single fuzzy probability distribution for a component from some possible distributions [13]. Zhou put forward a fuzzy reliability model of pressure piping system considering both the randomness of assessment parameters and the fuzziness of failure areas [14]. Li and Kapur made a lot in the continuous-state reliability measures based on fuzzy sets [15]. Kumar researched the fuzzy system reliability evaluation using time-dependent intuitionistic fuzzy set [16]. Huang proposed a method of reliability assessment for fuzzy multi-state systems [17], and did the reliability analysis of structure for fuzzy safety state [18]. Gao established the fuzzy dynamic reliability model of dependent series mechanical systems [19]. Lv studied the numerical method and fuzzy model for the fuzzy reliability [20, 22].

However, these models mainly focused on static fuzzy reliability analysis of systems. Fuzzy dynamic reliability models of mechanical systems under the vibration fatigue are seldom reported. Besides, when considering the depletion failure of ALG, the existing static fuzzy models cannot be directly extended to fuzzy dynamic reliability models of mechanical systems.

This paper takes the fuzziness of depletion failure under vibration fatigue into account and provides a mathematical tool to deal with the uncertainty involved in reliability models. Due to the limitations of technology and cost, it is difficult or impossible to acquire accurate distribution of stress at each time instant with statistical tools. We transfer the random vibration into a generalized stress. Then use the MF to represent the fuzziness of security status [21]. The MF defines how each element in the fuzzy set is mapped to the degree of membership ranging from 0 to 1, where 1 represents that the corresponding elements completely belong to the fuzzy set, 0 means that the corresponding elements are completely not in the fuzzy set, and values between 1 and 0 represent that the corresponding elements partially belong to the fuzzy set. In this paper, FRM of ALG is established through the FFD. Finally by comparing SRs of the FRM with SRs of the static and dynamic SSI model, the rationality of the method is verified.

2. The fuzzy failure domain's mathematical description of aircraft landing gear

FFD's mathematical description of ALG is the model which considers the gradual degradation law of strength under vibration fatigue to the reliability. We assume the generalized strength of ALG is R and the generalized stress is S which are two independent data.

The probability density function of generalized strength R is $f_R(R)$.

While the probability density function of Stress S is $f_S(S)$. In addition, the safe state is defined as $R - S$, and its MF is $\mu_{\hat{m}}(m)$. Then the mathematical model of FFD M can be represented by Eq. (1):

$$M = \iint f_R(R) f_S(s) \mu_{\hat{m}}(m) dRdS. \tag{1}$$

3. Model the fuzzy reliability of aircraft landing gear

The distribution function of generalized strength R and generalized stress S are $f_R(R)$, $f_S(S)$. R and S are independent.

Fuzzy security state is $m = R - s$, its MF is $\mu_{\hat{m}}(m)$. Then the fuzzy failure rate and fuzzy reliability could be formulated as Eq. (2) and Eq. (3) based on the probability of fuzzy events:

$$P_f = \frac{1}{M} \iint_{R<S} f_R(R) f_S(S) \mu_{\hat{m}}(m) dRdS, \tag{2}$$

$$P_R = \frac{1}{M} \iint_{R>S} f_R(R) f_S(S) \mu_{\hat{m}}(m) dRdS. \tag{3}$$

4. Model the fuzzy reliability of aircraft landing gear

Reliability of ALG is calculated by determining the distribution and the parameters in distribution, such as the MF of fuzzy safe state.

4.1. Generalized stress

Due to the limitations of technology and cost, it is difficult or impossible to acquire accurate distribution of stress at each time instant with statistical tools. An alternative method is equivalent the random vibration stress to a generalized stress which is shown in Fig. 1. The above curve in picture a) is one-tenth of the below curve which is the random vibration stress. Fatigue failure is the structure damage in alternating stress which is difficult to direct analysis. Therefore, we equivalent the random vibration to a generalized stress through data processing and analysis and obtain the probability density function which is shown in picture b).
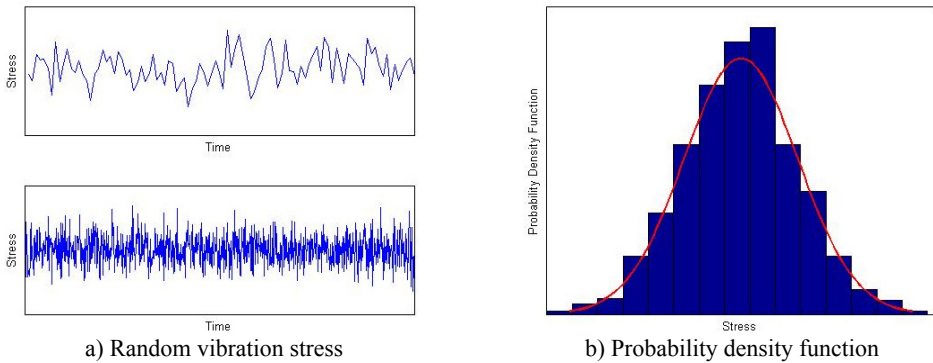

a) Random vibration stress   b) Probability density function

Fig. 1. Equivalent diagram of generalized stress

4.2. Selecting of membership function

The failure mode of ALG is fatigue failure which is depletion-type failure. In its interference area, the stress suffered by products is greater than the strength and mechanical structure are likely to fail. What's more, the failure rate increases with the increase of the difference between generalized strength and generalized stress which are shown in Fig. 2. b) is the picture which enlarge the box in a).
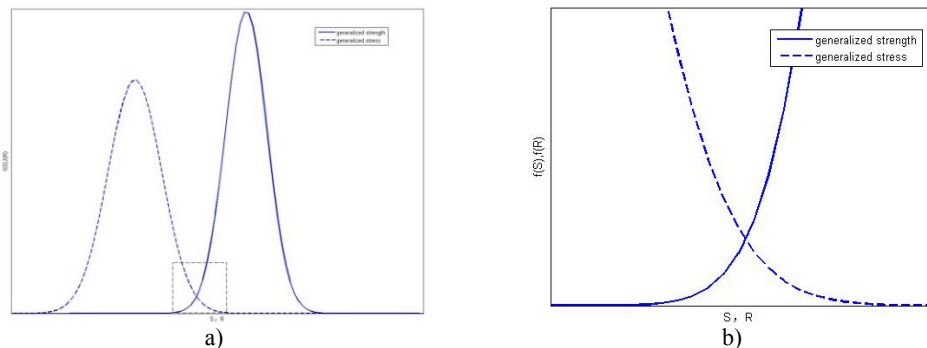

a)   b)

Fig. 2. The interferogram of generalized strength and generalized stress

Accordingly, in this paper, the partial large normal distribution membership function is adopted to describe the fuzzy characteristics of safe state in the fuzzy reliability calculation.

MF of the fuzzy safe state is:

$$\mu_{\hat{m}}(m) = \begin{cases} 0, & m \le \mu, \\ 1 - \exp\left[-\left(\dfrac{m-\mu}{\sigma}\right)^2\right], & m > \mu, \end{cases} \tag{4}$$
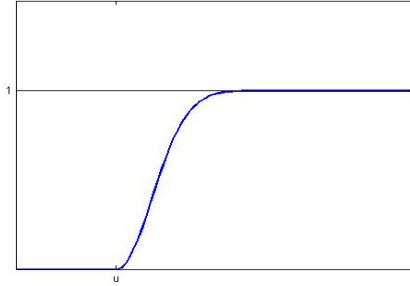
where $m = R - s$, $\mu = \mu_R - 3\sigma_R$.

Fig. 3. Membership function of partial large Normal distribution

4.3. Calculation of the fuzzy reliability model

Based on the selecting of membership function and the parameters, the failure rate P_f of ALG can be calculated by Eq. (5) from the Eq. (2), to (4):

$$P_f = \frac{1}{M} \int_0^\infty f_R(R) \int_R^\infty f_s(s)\,\mu_{\hat{m}}(m)\,dSdR. \tag{5}$$

Let be:

$$G(R) = \int_R^\infty f_S(s)\mu_{\hat{F}}(m)\,ds = \int_R^\infty \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left\{-\frac{(s-\mu_s)^2}{2\sigma_s^2}\right\} \cdot \left(1 - \exp\left[-\left(\frac{m-\mu}{\sigma}\right)^2\right]\right) ds. \tag{6}$$

Then:

$$P_f = \frac{1}{M} \int_0^\infty f_R(R)\,G(R)\,dR = \frac{1}{M} \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left\{-\frac{(R-\mu_R)^2}{2\sigma_R^2}\right\} G(R)\,dR. \tag{7}$$

The reliability P_R is:

$$P_R = 1 - P_f. \tag{8}$$

The calculation of FFD M is familiar to the failure rate. The reliability also can be calculated directly from the Eq. (3) to (5).

5. Numerical example

The strength of ALG obeys the normal distribution which average value and variance are 630 MPa and 44.1. The average value and variance of the external stress are 750 MPa and 37.5. The generalized stress and generalized strength interference cure is shown in Fig. 5.
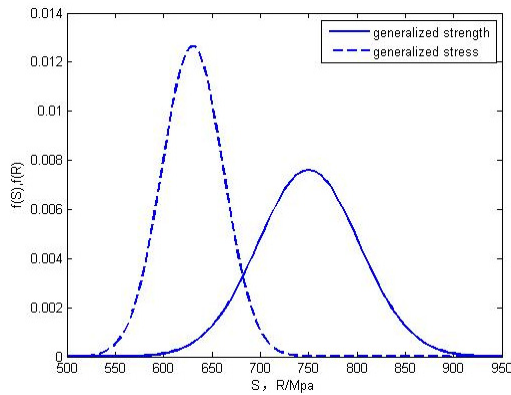
SRs shown in Table 1.

Fig. 5. SSI Graph of ALG

Table 1. Result of fuzzy reliability model

Reliability index	Fuzzy failure domain M	Failure rate	Reliability
Simulation result	0.9342	0.0393	0.9607

6. Conclusions

By transferring the random vibration stress to a normal distribution and taking the fuzziness of safe state into consideration, this paper established the FRM of ALG based on the static SSI model. Finally two conclusions are drawn at the end of the dissertation:

1) In this paper, MF in fuzzy mathematics illustrates the fuzzy failure state of ALG during the calculation. Furthermore, fuzzy reliability model is established by vibration fatigue failure mode without knowing the law of strength degradation. Accordingly, when strength degradation law does not known, this model has stronger applicability. So it's a method that can be widely used.

2) This fuzzy reliability model can be extended to other situation, for example, the distribution of strength and stress that obeys the exponential distribution, Weibull distribution or Log-normal distribution. Furthermore, for the situation of random stress – fuzzy strength and fuzzy stress – random strength, the fuzzy model also works.

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