Fracture Toughness of Ceramic Composite Containing Damage Eutectics and Transformation Particles

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Abstract. The toughness of eutectic ceramic composites are obtained by multiple toughening mechanisms involving crack-bridging and pull-out of rod-shaped eutectics, as well as stress-induced transformation toughening. In the loading procedure, damage will emerge in the rod-shaped eutectic.The transformation toughening of eutectic ceramic composites are obtained by stress-induced transformation mechanisms. Firstly, defining a parabola transformation yield function, the transformation plastic strain increment is gotten by transformation plastic potential function. Secondly, the transformation region is determined by the stress field associated with a crack involves I-II combination. Thirdly, on base of weighted function method, the screening impact of transformation particles for mixed-mode I-II crack is gained. The result shows that the transformation toughening is associated with the fraction of the transformation particles, the elastic modulus of composite ceramic and the half width of transformation region.

1. Introduction

In various industrial fields, there is a great need for materials having high strength combined with high toughness. Directionally solidified eutectics contain a large amount of clean interfaces between two strongly-bonded phases with typical inter-phase spacing in the micron range, and these characteristics result in an improvement of some material properties. For instance, rods of oxide/oxide eutectics present a smooth surface and exhibit high strength and toughness, chemical stability in oxidizing environments as well as excellent thermal shock resistance [1]. The increase of the hardness or strength of the ceramics could be attributed to nano-submicron interphase spacing and the refinement of the eutectic grains, whereas high-energy, high-angle boundaries between rod-shaped grains could also introduce strong toughening mechanisms involving crack-bridging and pull-out of rod-shaped grains [2]. Experiments showed that there were two kinds of fracture models – fracture in the rod-shaped eutectics and fracture in inter-eutectics regions. Because of the presence of nano-submicrometer t-ZrO2 fibers and inter-phase spacing in the colony as well as micrometer t-ZrO_2 spherical grains in the inter-colony region, intensive coupled toughening of residual stress toughening, transformation toughening and transformation- induced microcrack toughening mechanisms was bound to occur [3]. The toughing mechanism of this composite material is not clear. To address these issues, we develop the transformation toughing model of the eutectic composite ceramic.

2. The breaking stress of the damage eutectic

Composites mainly composed of randomly-oriented rod-shaped eutectic grains; within the rod-shaped grains, aligned nano-micron fibers are embedded. Overlooking the resistance of crystal lattice against dislocation motion, the micro strength formation of rod-shaped eutectics is computed by the dislocation pileup theory [4]. The damage variables are defined by the microstructure of rod-shaped eutectic with parallel nano/micro-fibers. The maximum strain criterion is used for determining the loading function. According to the attenuation characteristic of eutectic rigidity, the critical fracture stress of the damage rod-shaped eutectic is obtained by damage variable maximizing.

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$$\sigma_{du} = \frac{2E'_{22}}{n(AE'_{22} + E_b\upsilon_{21})\tanh(nl/d)}\left(1 - \sqrt{4f_d/\pi}\right)^2 \sqrt{\frac{3E_0\gamma_0}{2d}} \tag{1}$$

The breaking stress of the damage eutectic depends on the fraction, shape and diameter of fiber, and on the elastic modulus and free surface energy of matrix.

3. Damage eutectic toughening

At the crack surface, the bridging stress is [5]:

$$\sigma_f(L_s) = \frac{\sigma_f}{2}\left(\sqrt{1 + \frac{16E'_{22}\tau u}{R\sigma_f^2 \sin\alpha}} + 1\right) \tag{2}$$

where, σ_f is the load undertaken by damage rod-shaped eutectic at far distance from crack, R is the radius of the damage rod-shaped eutectic, τ is the shear stress on the sliding part of the damage rod-shaped eutectic, u is the crack opening displacement. α is the angle between the bridging damage eutectic and crack. While the crack opening is limited by the damage rod-shaped eutectic, some debonding of the damage rod-shaped eutectic takes place. The toughening can be discussed in two ways. In one, the load carried by the rod-shaped eutectic in the crack wake to produce a crack-closing force. This force reduces the stress intensity in front of the crack. Further, because of residual thermal stress, a frictional stress exists across some rod-shaped eutectic. The frictional work of damage eutectic pull-out adds to the material toughness. Consider the orientation and fraction of the damage rod-shaped eutectic. Suppose that the direction of the damage rod-shaped eutectic is three-dimensional position of completely random distribution. Based on the energy dissipation, taking aspect ratio $\lambda = L/(2R)$, the bridging toughening value of the damage rod-shaped eutectic can be calculated

$$\Delta K_{C1} = \sqrt{\frac{\chi f_f L\sigma_{du}^3}{2\lambda\tau}} \tag{3}$$

The constant χ is determined by E'_{22} and E [5].

When thermal residual stresses are present, this eutectic pull-out work can be appreciable. The residual stresses are manifested by a clamping force between eutectic and particles that overcome in accomplishing their final separation. The work done in this separation represents additional fracture work. The additional fracture work can be calculated on the basis of it arising from a frictional clamping force. That work Dividing by $dS = \pi R^2/4$ (the acrossal area of the pulled-out eutectic) yields the fracture work per unit area. This can be equated to ΔJ. Only a fraction f_f of eutectics is subject to crack bridging. The pull-out work per unit area is transformed as

$$\Delta J_2 = \frac{5f_f\mu\sigma_n L^2}{64R} \tag{4}$$

Analysis of the added toughness proceeds along lines used in analysing bridging toughening. We use $\Delta K_C = (E\Delta J)^{1/2}$ and $\lambda = L/(2R)$. The pull-out toughening value of the damage rod-shaped eutectic can be expressed as

$$\Delta K_{C2} = \sqrt{\frac{5f_f\mu\sigma_n E'_{22}\lambda L}{32}} \tag{5}$$

Note that there is an implicit damage effect in equation (5). The elastic modulus E'_{22} varies inversely with damage variable. Thus, provided we are in the damage variable domain where pull-out toughening exists, the toughening is smaller for composite ceramic with the damage rod-shaped eutectic.

4. Transformation toughening and fracture toughness of ceramic composite

In the ceramic composite containing damage eutectics and transformation particles, the transformation may be triggered by the stress field associated with a crack. As the crack advances, tetragonal particles transform to the monoclinic form in a zone above and below the fracture plane. The work expended in effecting this transformation adds to the material toughness.

For the transformation toughening, the comprehensive transformation criterion [6] was used to describe the plastic behavior of partially stabilized zirconias. The transformation yield condition was defined by the macro equivalent stress and average stress. For the tectic ceramic composite, there are rod-shaped eutectics around transformation particles, so the transformation yield condition is not only related to the macro equivalent and average stresses, but also the difference between the maximum tensile stress and compressive stress. Based on the experiment, the parabola transformation yield function is defined as follow:

$$F = a\sigma_0 + J_2 - k^2 = 0 \tag{6}$$

where, $a = (\sigma_1 - \sigma_3)/3$, σ_1 is the maximum tensile stress and σ_3 the maximum compressive stress determined by the stress field associated with a crack. σ_0 is the bulk stress, i.e. $\sigma_0 = \sigma_1 + \sigma_2 + \sigma_3$. k is material constant. J_2 is the second stress invariant, i.e. $J_2 = {S_{ij}S_{ij}}/{2}$, and $S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_0\,\delta_{ij}$, the stress deviator tensor.

According to potential function method, the gradient of plastic strain tensor are gotten from Eq. (6):

$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial F}{\partial \sigma_{ij}} = d\lambda\left(a\delta_{ij} + S_{ij}\right) \tag{7}$$

In the general case of material fracture, the stress field associated with a crack involves I-II combination.

$$\begin{cases} \sigma_x = \dfrac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left[1-\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right] - \dfrac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\sin\dfrac{\theta}{2}\left[2+\cos\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}\right] \\ \sigma_y = \dfrac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left[1+\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right] + \dfrac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} \\ \tau_{xy} = \dfrac{K_{\mathrm{I}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} + \dfrac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left[1-\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right] \\ \sigma_z = \upsilon\left(\sigma_x + \sigma_y\right) \end{cases} \tag{8}$$

Substituting equation (8) into equation (6), we can obtain the radius of transformation region of stationary crack

$$r = \left(\frac{K_{\mathrm{I}}}{a}\right)^2 \frac{e^2}{2\pi\left[2(1+\upsilon)^2 g^2 + b^2 e - 2(1+\upsilon)g\sqrt{(1+\upsilon)^2 g^2 + b^2 e}\right]} \tag{9}$$

where, $b=\dfrac{k}{a}$, $e=e_1+\alpha e_2+\alpha^2 e_3$, $\alpha=\dfrac{K_{II}}{K_I}$, $g=\cos\dfrac{\theta}{2}-\alpha\sin\dfrac{\theta}{2}$,
$\mathrm{e}_1=\dfrac{1}{3}\left(4\upsilon^2-4\upsilon+1\right)\cos^2\dfrac{\theta}{2}+\dfrac{1}{4}\sin^2\theta$, $\mathrm{e}_2=\dfrac{1}{3}\left(-4\upsilon^2+4\upsilon-1\right)\sin\theta+\dfrac{1}{2}\sin 2\theta$,
$\mathrm{e}_3=\dfrac{1}{3}\left(4\upsilon^2-4\upsilon+1\right)\sin^2\dfrac{\theta}{2}-\dfrac{3}{4}\sin^2\theta+1$.

As the crack advance, tetragonal particles transform to the monoclinic form in a zone lying $\pm H$ above and below the fracture plane. The H can be determined for equation (4). Let $d(r\sin\theta)/d\theta=0$, we get $\theta=\theta_m$. Then $H=(r\sin\theta)_{\theta=\theta_m}$. The radius of transformation region of steady-state growing crack is given:

$$r=\begin{cases}\left(\dfrac{K_{\mathrm{I}}}{a}\right)^2\dfrac{e^2}{2\pi\left[2g^2+b^2e-2g\sqrt{g^2+b^2e}\right]} & 0\le\theta\le\theta_m\\ H/\sin\theta & \theta_m\le\theta\le\pi\end{cases}\tag{10}$$

On base of weighted function, the transformation fracture enhancements of mode I and mode II are calculated as follow [7, 8]

$$dK_{\mathrm{I}}=\int_s\vec{h}_{\mathrm{I}}(s,c)\vec{T}(s)ds\tag{11}$$

$$dK_{\mathrm{II}}=\int_s\vec{h}_{\mathrm{II}}(s,c)\vec{T}(s)ds\tag{12}$$

Here s is the boundary of transformation region. c is the length of crack. $\vec{h}_I(s,c)$ and $\vec{h}_{II}(s,c)$ are the weight functions of mode I and II respectively. The components of surface force $\vec{T}(s)$ on the boundary are expressible in the form

$$T_i=\frac{E}{3(1-2\upsilon)}\varepsilon_{kk}^p\delta_{ij}n_j+\frac{E}{1+\upsilon}e_{ij}^p n_j\tag{13}$$

where n_j is the direction consine. The plastic strain tensor are gotten.

$$\begin{cases}\varepsilon_{kk}^p=f_t\varepsilon_{kk}^T\\ e_{ij}^p=f_t\varepsilon_{kk}^T S_{ij}/(3a)\end{cases}\tag{14}$$

In equation ε_{kk}^T is the corresponding martensite transformation strain, f_t is the fraction of the triclinic phase in the distance $\pm H$ above and below the fracture plane that transforms martensitically.
We can acquire the weight function of mode I [7]

$$\vec{h}_{\mathrm{I}}=\frac{1}{2(1-\upsilon)\sqrt{2\pi r}}\begin{Bmatrix}\cos\dfrac{\theta}{2}\left(2\upsilon-1+\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)\\ \sin\dfrac{\theta}{2}\left(2-2\upsilon-\cos\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}\right)\end{Bmatrix}\tag{15}$$

The weight function of mode II is [8]

$$\vec{h}_{\mathrm{II}} = \frac{1}{2(1-\upsilon)\sqrt{2\pi r}} \begin{Bmatrix} \sin\frac{\theta}{2}\left(2-2\upsilon+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \\ \cos\frac{\theta}{2}\left(1-2\upsilon+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \end{Bmatrix} \tag{16}$$

Substituting equations (13) and (15) into equation (11), we have

$$\Delta K_I = \frac{f_t E \varepsilon_{kk}^T}{6(1-\upsilon)\sqrt{2\pi}} \iint_A \left\{ r^{-\frac{3}{2}} \cos\frac{3\theta}{2} + \left(\frac{K_{II}}{a}\right) \frac{r^{-2}}{3(1+\upsilon)\sqrt{2\pi}} (e_{11}+e_{12}) \right\} dA \tag{17}$$

here, $e_{11} = -\frac{9}{4}\cos\theta + \left(8\upsilon^2 - 3\upsilon - 2\right)\cos^2\theta + \left(2\upsilon^2 - \frac{3}{2}\upsilon + \frac{13}{4}\right)\cos\theta + \frac{3}{2}v + 1$,

$e_{12} = \frac{K_{\mathrm{II}}}{K_{\mathrm{I}}}\left[\frac{27}{4}\cos^3\theta - \left(8\upsilon^2 - 3\upsilon - 2\right)\cos^2\theta + \left(2\upsilon^2 - \frac{5}{2}\upsilon - \frac{15}{4}\right)\cos\theta + 4\upsilon^2 - \frac{3}{2}\upsilon - 1\right]$

where, A is the area of transformation region.

Substituting equations (13) and (16) into equation (12), we obtain

$$\Delta K_{II} = \frac{f_t E \varepsilon_{kk}^T}{6(1-\upsilon)\sqrt{2\pi}} \iint_A \left\{ r^{-\frac{3}{2}} \sin\frac{3\theta}{2} + \left(\frac{K_{II}}{a}\right) \frac{r^{-2}}{3(1+\upsilon)\sqrt{2\pi}} (e_{21}+e_{22}) \right\} dA \tag{18}$$

where, $e_{21} = \left[\frac{9}{4}\cos^2\theta - \left(8\upsilon^2 - 3\upsilon - 2\right)\cos\theta - 2\upsilon^2 + 2\upsilon - \frac{5}{4}\right]\sin\theta$,

$e_{22} = \frac{\bar{K}_{\mathrm{II}}}{\bar{K}_{\mathrm{I}}}\left[\frac{27}{4}\cos^3\theta - \left(8\upsilon^2 - 3\upsilon - 2\right)\cos^2\theta + \left(2\upsilon^2 - \frac{5}{2}\upsilon - \frac{15}{4}\right)\cos\theta + 4\upsilon^2 - \frac{3}{2}\upsilon - 1\right]$

For the giving $K_{\mathrm{II}}/K_{\mathrm{I}}$ and k/a, equation (17) is integrated in the transformation region of steady-state growing crack given by equation (10). We gain to the toughening effects of mode I crack.

$$\Delta K_I = \Delta_1 f_t E \varepsilon_{kk}^T \sqrt{H} \tag{19}$$

On integrating equation (18) in the transformation region of steady-state growing crack given by equation (10) the transformation region, we get the toughening effects of mode II crack

$$\Delta K_{II} = \Delta_2 f_t E \varepsilon_{kk}^T \sqrt{H} \tag{20}$$

Here Δ_1 and Δ_2 are constants relating to υ, $K_{\mathrm{II}}/K_{\mathrm{I}}$ and k/a. For mixed-mode I-II crack, using the strain energy release rate criterion, we obtain the fracture enhancement of transformation.

$$\Delta K = \frac{\Delta K_{\mathrm{I}} + \left(K_{\mathrm{II}}/K_{\mathrm{I}}\right)\Delta K_{\mathrm{II}}}{\sqrt{1+\left(K_{\mathrm{II}}/K_{\mathrm{I}}\right)^2}} \tag{21}$$

Substituting equations (19) and (20) into equation (21), the expression for the transformation-effected toughness can written as

$$\Delta K_{C3} = \Delta f_t E \varepsilon_{kk}^T \sqrt{H} \tag{22}$$

In equation (22), Δ_1 is a constant relating to υ, $K_{\mathrm{II}}/K_{\mathrm{I}}$ and k/a. For Al_2O_3-ZrO_2 ceramic composite containing damage Eutectics and transformation Particles, $\upsilon = 0.31$, $K_{\mathrm{II}}/K_{\mathrm{I}} = 0.5$, $k/a = 2.5$. We can determine Δ=0.0732, i.e.

$$\Delta K_{C3} = 0.0732 f_t E \varepsilon_{kk}^T \sqrt{H} \tag{23}$$

The transformation toughening is associated with the fraction of the transformation particles, the elastic modulus of composite ceramic and the half width of transformation region.

Experiments showed that there were two kinds of fracture models – fracture in the rod-shaped eutectics and fracture in inter-eutectics regions. Because of presence of nano-submicrometer fibers and inter-phase spacing in the eutectic as well as micrometer transformation particles in the inter-eutectic region, intensive coupled toughening of damage eutectic-induced crack bridging toughening, eutectic pull-out toughening and transformation toughening. For the coupled toughening mechanisms discussed above, the added toughness scales with the inherent matrix toughness (i.e. K_m). Thus, the fracture toughness K_C of ceramic composite is

$$K_C = K_m + \Delta K_{C1} + \Delta K_{C2} + \Delta K_{C3} \tag{24}$$

5. Conclusions

(1) According to the attenuation characteristic of eutectic rigidity, the critical fracture stress of the damage rod-shaped eutectic is obtained by damage variable maximizing. Bridging toughening mechanism and pull-out toughening mechanism of damage rod-shaped eutectics are constructed.

(2) Defining a parabola transformation yield function, the transformation plastic strain increment is gotten by transformation plastic potential function. Based on the strain energy release rate criterion, the method of weight function is used to predicate the fracture enhancement of mixed-mode I-II crack in eutectic ceramic composites.

(3) Based on the crack-bridging and pull-out toughening mechanisms of damage rod-shaped eutectics, as well as stress-induced transformation toughening mechanism, the added toughness scale with the inherent matrix toughness, the theoretical formula of fracture toughness of the eutectic ceramics composite is determined.

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