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The Quantitative Diagnosis Method of Rubbing Rotor System

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Abstract. The dynamics of the rubbing rotor system is analyzed by applying harmonic balance method. The relationship between harmonic components in the response of the rubbing rotor system and the dynamic stiffness matrix of the fault free rotor system is revealed, based on which a new model based method for rubbing identification is presented. By applying this method, the fault location and rubbing forces of the single rubbing rotor system can be identified by using vibration data of only two nodes, the rubbing locations and rubbing forces of the double rubbing rotor system can be identified by using vibration data of three nodes. The numerical simulations and experiments on the rotor test-rig are carried out to verify the efficiency of the present method.

1. Introduction

Rotor-to-stator rub is a common fault in rotor system, which can cause catastrophic failure if undetected. The early detection of the existing of rubbing, and then obtaining the exact information about the fault location and the fault extent can significantly improve the safety issue in the operation of the rotating machinery.

Extensive researches have been done on the forecast and detection of the rotor-to-stator rub in the past few decades. In general, there are mainly two types of diagnosis methods for rotor-to-stator rub: one is the signal-based method, and the other is the model-based method. The signal-based method uses vibration response signals and signal processing techniques such as FFT, STFT, CWT for the detection of rubs. A lot of researches have been conducted in this area, such as references [1-3]. Enough signal-based diagnosis tools have been provided to judge the existence of the rubbing fault, but the existing signal-based identification methods are difficult to detect the rubbing location and the fault extent.

Model-based identification methods, on the contrary, can provide quantitative information about the fault, which includes the rubbing location and the fault extent. Excellent examples can be seen in references [4-7]. In this literatures, only the fundamental harmonic components are used in the model based diagnosis process. The sub and super harmonic components commonly exist in the response of the rubbing rotor systems, but are rarely utilized.

So in this paper the relationship between the sub or super harmonic components in the response of the rubbing rotor system and the dynamic stiffness matrix of the fault free rotor system is revealed, based on which a new model based method for rubbing identification is presented. By applying this method, the location and rubbing forces of the single rubbing rotor system can be identified by using sub or super harmonic components of only two node's responses, the rubbing locations and the rubbing force of the double rubbing rotor system can be identified by using sub or super harmonic responses of three nodes. To validate the efficiency of the present method, numerical studies and experiments on rotor test-rig are carried out.

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2. The mathematical model of the locally rubbing rotor system

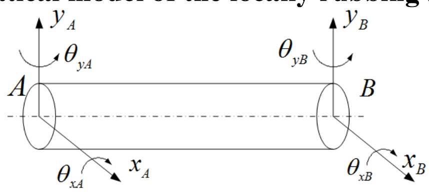

Figure 1. The finite element model of the rotor segment.

The rotor is made up of elastic segments and each segment has two translational and two rotational degrees of freedom for bending mode at each node. The generalized displacement of each element is:

$$\mathbf{y}_r = \begin{bmatrix} x_A - jy_A & \theta_{yA} - j\theta_{xA} & x_B - jy_B & \theta_{yB} - j\theta_{xB} \end{bmatrix}^{\mathrm{T}} \tag{1}$$

The translating mass matrix, rotating mass matrix, stiffness matrix and gyroscopic matrix are shown in reference [8]. The finite element equation is

$$\mathbf{M}\ddot{\mathbf{R}}_r + \mathbf{C}\dot{\mathbf{R}}_r + \mathbf{K}\mathbf{R}_r = \mathbf{U} \tag{2}$$

where, $\mathbf{M}$ is the mass matrix, $\mathbf{C} = \mathbf{D} + \omega\mathbf{G}$ is the damping and gyroscopic matrix, ω is the rotating speed, $\mathbf{K}$ is the stiffness matrix, $\mathbf{R}_r$ is the response vector, $\mathbf{U}$ is the exciting force vector. The dimensions of the matrixes are $2J\times 2J$ and the dimensions of the vectors are $2J\times 1$.

Considering the stimulation caused by unbalance, the expression of $\mathbf{U}$ is:

$$\mathbf{U} = \begin{bmatrix} A_1 e^{j\omega t + \varphi_1} & 0 & \ldots & A_{2J-1} e^{j\omega t + \varphi_{2J-1}} & 0 \end{bmatrix}^{\mathrm{T}} \tag{3}$$

where, $A_i \left(i = 1, \ldots, J\right)$ is the eccentricity concentrated on node i, and $\varphi_i \left(i = 1, \ldots, J\right)$ is the phase of the unbalance force.

Assuming the rub happens on node L_1 of the rotor system, the dynamic model of the fault rotor system is [9]:

$$\mathbf{M}\ddot{\mathbf{R}}_r + \mathbf{C}\dot{\mathbf{R}}_r + \mathbf{K}\mathbf{R}_r = \mathbf{U} - \mathbf{T}\left(\mathbf{P}_{x1} + j\mathbf{P}_{y1}\right)e^{j(\omega t - \xi)} \tag{4}$$

where, $\mathbf{T} = \begin{bmatrix} \overbrace{0\ldots 0}^{2L_1-2} & 1 & 0\ldots 0 \end{bmatrix}^{\mathrm{T}}$ and P_{x1}, P_{y1} are force components in x and y direction of the rubbing force at node L_1.

(4) - (2) obtains:

$$\mathbf{M}\Delta\ddot{\mathbf{R}}_r + \mathbf{C}\Delta\dot{\mathbf{R}}_r + \mathbf{K}\Delta\mathbf{R}_r = -\mathbf{T}\left(\mathbf{P}_{x1} + j\mathbf{P}_{y1}\right)e^{j(\omega t - \xi)} \tag{5}$$

3. The characteristics of the harmonic components of the equivalent system

Consider the steady state situation of the rubbing rotor system, the responses of the equivalent system are in steady state, and the responses can be decomposed in to harmonic components

$$\Delta\mathbf{R} = \Delta\mathbf{R}_0 + \Delta\mathbf{R}_1 e^{j\omega t} + \Delta\mathbf{R}_2 e^{2j\omega t} + \Delta\mathbf{R}_3 e^{3j\omega t} + \ldots \tag{6}$$

The equivalent forces, which are all in the right side of equation (5), can be expanded to

$$-\mathbf{T}\left(\mathbf{P}_{x1} + j\mathbf{P}_{y1}\right)e^{j(\omega t - \xi)} = \mathbf{T}\left(A_0 + A_1 e^{j\omega t} + A_2 e^{2j\omega t} + A_3 e^{3j\omega t} + \ldots\right) \tag{7}$$

By applying the harmonic balance criteria from equation (5), for each harmonic component, one gets the following equation

$$\begin{aligned}\Delta\mathbf{R}_0 &= \mathbf{K}^{-1}\mathbf{T}A = \mathbf{E}(0)\mathbf{T}A_0 = \mathbf{E}_{2L_1-1}(0)A_0\\ \Delta\mathbf{R}_1 &= \left[-\mathbf{M}\omega^2 + j\omega\mathbf{C} + \mathbf{K}\right]^{-1}\mathbf{T}A_0 = \mathbf{E}(j\omega)\mathbf{T}A_0 = \mathbf{E}_{2L_1-1}(j\omega)A_0\\ \Delta\mathbf{R}_l &= \left[-\mathbf{M}l^2\omega^2 + j\omega l\mathbf{C} + \mathbf{K}\right]^{-1}\mathbf{T}A_l = \mathbf{E}(jl\omega)\mathbf{T}A_0 = \mathbf{E}_{2L_1-1}(jl\omega)A_0\end{aligned} \tag{8}$$

where, $\mathbf{E}(jl\omega) = \left[\mathbf{K} + j\omega l\mathbf{C} - \omega^2 l^2\mathbf{M}\right]^{-1}$ and $E_{2L_1-1}(jl\omega)$ is the $2L_1-1^{\text{th}}$ column of $\mathbf{E}(jl\omega)$.

Assuming the l^{th} harmonic component response of node i is $\Delta\mathbf{r}_i e^{jl\omega t}$, and that of node k is $\Delta\mathbf{r}_k e^{jl\omega t}$. It can be deduced from equation (8) that:

$$\frac{\Delta\mathbf{r}_k}{\Delta\mathbf{r}_i} = \frac{E_{k,2L_1-1}(jl\omega)}{E_{i,2L_1-1}(jl\omega)} \tag{9}$$

This means that the ratio of the harmonic components responses of any two nodes equals to the ratio of the corresponding elements in the $2L_1-1^{\text{th}}$ column of the dynamics stiffness matrix of the pre-fault rotor system.

Equation (10) can be obtained from equation (9):

$$\delta_n = \frac{\Delta\mathbf{r}_k}{\Delta\mathbf{r}_i} = \frac{E_{2n-1,i}(jl\omega)}{E_{2n-1,\mathrm{k}}(jl\omega)}, \quad n = 1, 2, ..., J \tag{10}$$

$\mathrm{abs}(\delta_L)$ will be the minimum of all the $\mathrm{abs}(\delta_n)$. So the rubbing location can be found when the minimum of $\mathrm{abs}(\delta_n)$ be found.

After the location L_1 has been identified, the l^{th} harmonic component response of every node of the system can be obtained as:

$$\Delta\mathbf{R}(jl\omega) = \mathbf{E}_{2L_1-1}(jl\omega)\frac{\Delta\mathbf{r}_i(jl\omega)}{E_{i,2L_1-1}(jl\omega)} \tag{11}$$

Assuming there are N^{th} order harmonic components that are evident in the response, then the rubbing force can be deduced as:

$$\Delta\mathbf{F} = \sum_{l=0}^{N}\mathbf{H}_{2L_1-1}(jl\omega)\Delta\mathbf{R}(jl\omega) \tag{12}$$

where $\mathbf{H}_{2L_1-1}(jl\omega)$ is the $2L_1-1^{\text{th}}$ row of $\mathbf{H}(jl\omega) = \left[-\mathbf{M}l^2\omega^2 + j\omega l\mathbf{C} + \mathbf{K}\right]$.

Sometimes there are two rubbing locations exist in the system, which may happen on node L_1 and L_2. In this case, the following equation can be obtained from equation (8):

$$\frac{\Delta\mathbf{r}_k}{\Delta\mathbf{r}_i} = \frac{\alpha E_{k,2L_1-1}(jl\omega) + \beta E_{k,2L_2-1}(jl\omega)}{\alpha E_{i,2L_1-1}(jl\omega) + \beta E_{i,2L_2-1}(jl\omega)} \tag{13}$$

α and β can be obtained by:

$$\begin{Bmatrix}\alpha\\ \beta\end{Bmatrix} = \begin{bmatrix}E_{k,2L_1-1}(jl\omega) & E_{k,2L_1-1}(jl\omega)\\ E_{i,2L_1-1}(jl\omega) & E_{i,2L_1-1}(jl\omega)\end{bmatrix}\begin{Bmatrix}\Delta\mathbf{r}_k\\ \Delta\mathbf{r}_i\end{Bmatrix} \tag{14}$$

So in this case, harmonic component $\Delta\mathbf{r}_h$ of node h is needed for the rubbing location identification:

$$\delta_{n_1 n_2} = \frac{\Delta\mathbf{r}_h}{\Delta\mathbf{r}_i} - \frac{\alpha E_{h,2L_1-1}(jl\omega) + \beta E_{h,2L_2-1}(jl\omega)}{\alpha E_{k,2L_1-1}(jl\omega) + \beta E_{k,2L_2-1}(jl\omega)}, \quad n_1 = 1, 2, ..., J, \; n_2 = 1, 2, ..., J \tag{15}$$

4. The identification method based on super-harmonic components

A simple rubbing position and forces identification method is presented based on the property of the rubbing rotor system. The procedures are as follow:

(1) Obtain the vibration response at any two nodes of the rubbing rotor system, for example, the nodes at the bearings position. Obtain one sub or super harmonic components of the two signals, for example l^{th} harmonic components, mark it as $\Delta\mathbf{r}_i = y_{ix}(l\omega) - jy_{iy}(l\omega)$ and $\Delta\mathbf{r}_k = y_{kx}(l\omega) - jy_{ky}(l\omega)$.

(2) Obtain the dynamic stiffness matrix $\mathbf{E}(\mathrm{j}l\omega)$ of the former linear system. It can be calculated from structure parameters or measure from model testing.

(3) Build the following equation:

$$\delta_n = \frac{\Delta\mathbf{r}_k}{\Delta\mathbf{r}_i} - \frac{E_{2n-1,i}(jl\omega)}{E_{2n-1,\mathrm{k}}(jl\omega)}, \qquad (n = 1, 2, ..., J) \tag{16}$$

and calculate δ_n when $n = 1,\ 2,\ \ldots, J$.

(4) Obtain the minimum δ_n, then n is the position where rotor-to-stator rub occurs.

(5) Obtain the rubbing forces using equation (12).

The procedures for the identification of two rubbing are almost the same, except that vibration responses of three nodes are needed, and equation (15) should be used instead of equation (16).

5. The numerical example of the identification method

The rotor system used to test the identification method is shown as follows:

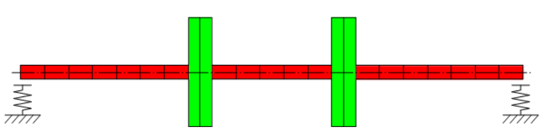

Figure 2. Sketch and finite element model of the rotor system.

The diameter of the shaft is 10mm and the length is 420mm. The diameters of the discs are 80mm with the thickness of 20mm respectively. The rotor system is divided into 23 segments with 24 nodes. Assuming the elastic modulus of the steel material is 2.1×10^{11}Pa, the stiffness of each supporting is 7×10^{6}N/m, the eccentricity of the rotor system is at node 9, which is $m_r = 5\times10^{-6}$N/m in magnitude.

Assuming the rubbing occurs at node 12. The contact stiffness is 2×10^{5}N/m and the clearance is $e = 2\times10^{4}$N/m. Using the present identification method, when the rotating speed is ω=300rad/s, the change of $\mathrm{abs}(\delta_n)$ is shown in Fig. 3(a). It can be seen from Fig. 3(a) that the minimum $\mathrm{abs}(\delta_n)$ is at node 12, which means that rub occurs at node 12. The identified rubbing forces are compared with the actual rubbing forces in Fig. 3(b) and (c), in which the actual forces are drawn in real lines and the identified results are drawn in dashed lines. It can be seen that the identified results confirm the actual forces very well. This proved the validity of the present method.

The method is still effective when two rubs happen at the same time. Assuming the rubs occur at node 12 and node 18, the rubbing type is partial rub and the rubbing angle are $0° \le \alpha_1 \le 45°$, $0° \le \alpha_2 \le 45°$ respectively. Fig. 4(a) is the contour plot of the $\mathrm{abs}(\delta_{n_1 n_2})$ when ω=600rad/s, and

Fig. 4(b) and (c) are the identified rubbing forces at node 12. It can be seen that the identified results confirm the actual results very well, which proved the validity of the present method again.

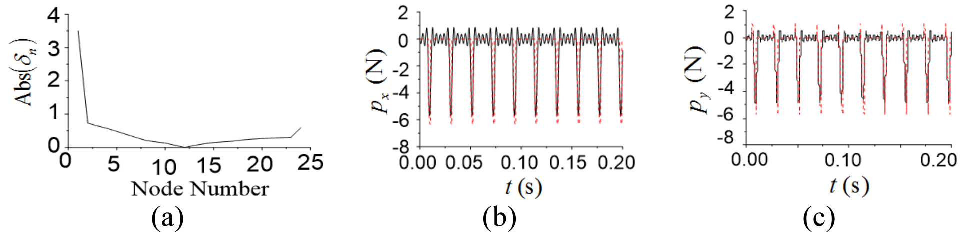

Figure 3. The identification results when ω=300rad/s.

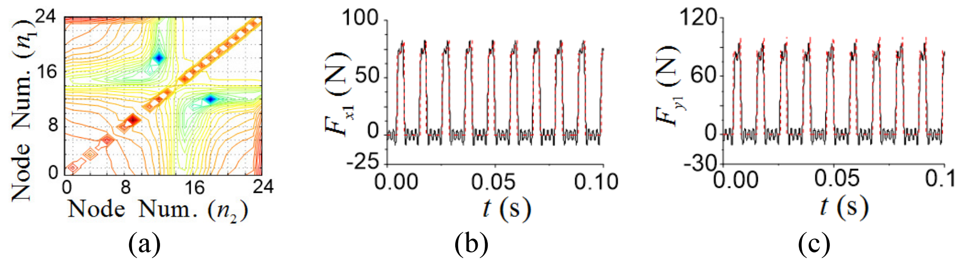

Figure 4. The identification results when ω=600rad/s.

6. Experiment results

The experimental work are carried out on the Bently rotor test-rig. The general view of the rotor rig is shown in Fig. 5. The displacement sensors are installed on nodes 4 and 18 and are used to detect displacements, and the collector is B&K3560B. Rotor-to-stator rub will happen when the rubbing rod and the shaft contacts. So the rubbing is of partial rub type.

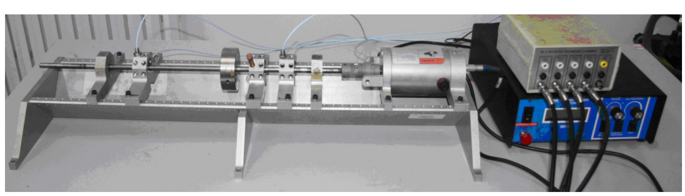

Figure 5. General view of the experimental rotor rig.

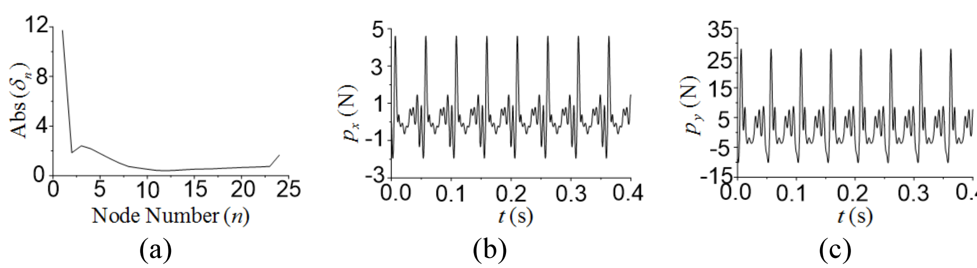

Figure 6. The identification results when ω=20Hz.

Firstly the vibration signals of nodes 4 and 18 are measured when the rotating speed is ω_1=20Hz. By adopting the identification method presented in this paper and the 3X super-harmonic component as a reference, the change of $\text{abs}(\delta_n)$ is shown in Fig. 6(a). It can be seen from Fig. 6(a) that the $\text{abs}(\delta_{12})$ is the minimum, which means that the rubbing occurs at node 12. Fig. 6(b) and (c) are the identified results of the rubbing forces. It's difficult to measure the actual rubbing forces, so the identified results cannot be compared with the actual forces. But the identified results have the characteristic of rubbing forces such as periodicity, and the dwell period can be seen clearly on the figures.

7. Discussion

The efficiency of the present method has been verified by numerical studies and experiments. But the present method is only fit for rotor systems with steady state responses. This is due to that the sub or super harmonic components can only be obtained from steady state responses. For the various motion types such as quasi-periodic and chaotic motion, the method is invalid.

The advantage of the method is that only two sensors are enough to detect the location of rubbing fault. For the large-scale rotating machinery, the sensors can only be installed near bearings or on very few other places. The present method is very convenience in these cases.

8. Conclusions

In this paper, a novel approach has been proposed for rubbing location detection in rotor systems. This new approach was developed based on the properties of the high-order harmonic components of the system response. By applying this method, the single rubbing location and rubbing force can be identified by using vibration data of only two nodes, the double rubbing locations and the rubbing forces on each location can be identified by vibration data of three nodes. So the proposed method is much simpler than the existing model-based identification method. Both simulation studies and experimental work based on a rubbing rotor system have verified the effectiveness of the new approach.

The present approach could also be applied to detect the rub or impact locations of other practical MDOF structures such as beams, truss and so on. Only one time sinusoidal excitation and two places of vibration measurements are required.

Acknowledgements

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