THE SQUARES IN THE SMARANDACHE FACTORIAL PRODUCT SEQUENCE OF THE SECOND KIND

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Abstract . In this paper we prove that the Smarandache factorial product sequence contains only one square 1.

Key words . Smarandache product sequence, factorial, square.

For any positive integer n, let

$$F(n)=\prod_{k=1}^{n} k!-1. \tag{1}$$

Then the sequence $F=\{F(n)\}_{n=1}^{\infty}$ is called the Smarandache factorial product sequence of the second kind (see [2]). In this paper we completely determine squares in F. We prove the following result.

Theorem . The Smarandache factorial product sequence of the second kind contains only one square $F(2)=1$.

Proof. Since $F(1)=0$ by (1), we may assume that $n>1$. If $F(n)$ is a square, then from (1) we get

$$a^2=\prod_{k=1}^{n} k!, \tag{2}$$

where a is a positive integer. By [1,Theorem 82], if p is a prime divisor of a^2+1, then either $p=2$ or $p\equiv 1 \pmod 4$. Therefore, we see from (2) that $n<3$. Since $F(2)=1$ is a square, the theorem is proved.

References

[1] G.H Hardy and E.M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, Oxford, 1937.

[2] F. Russo, Some results about four Smarandache U-product sequence, Smarandache Notions J. 11(2000)42-49.

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