Scientia Magna
Vol. 5 (2009), No. 1, 29-36

On Smarandache least common multiple ratio

S.M. Khairnar†, Anant W. Vyawahare‡ and J.N.Salunke♯

† Department of Mathematics, Maharashtra Academy of Engg., Alandi, Pune, India.
‡ 49, Gajanan Nagar, Wardha Road, Nagpur-440015, India.
♯ Department of Mathematics, North Maharashtra University, Jalgoan, India.
E-mail: smkhairnar2007@gmail.com vvishwesh@dataone.in.

Abstract Smarandache LCM function and LCM ratio are already defined in [1]. This paper gives some additional properties and obtains interesting results regarding the figurate numbers. In addition, the various sequaences thus obtained are also discussed with graphs and their interpretations.

Keywords Smarandache LCM Function, Smarandache LCM ratio.

§1. Introduction

Definition 1.1. Smarandache LCM Function is defined as $SL(n) = k$, where $SL : N \longrightarrow N$
(1) n divides the least common multiple of $1, 2, 3, \cdots, k$,
(2) k is minimum.

Definition 1.2. The Least Common Multilpe of $1, 2, 3, \cdots, k$ is denoted by $[1, 2, 3, \cdots, k]$, for example $SL(1) = 1,\ SL(3) = 3,\ SL(6) = 3,\ SL(10) = 5,\ SL(12) = 4,\ SL(14) = 7,\ SL(15) = 5, \cdots$.

Definition 1.3. Smarandache LCM ratio is defined as

$$SL(n, r) = \frac{[n, n-1, n-2, \cdots, n-r+1]}{[1, 2, 3, \cdots, r]}.$$

Example.

$$SL(n, 1) = n,$$

$$SL(n, 2) = \frac{n.(n-1)}{2},\ n \geq 2,$$

$$SL(n,3) = \begin{cases} \dfrac{n(n-1)(n-2)}{6}, & \text{if n is odd, } n \geq 3 \\ \dfrac{n(n-1)(n-2)}{12}, & \text{if n is even, } n \geq 3 \end{cases}$$

Proof. Here we use two results:
1. Product of LCM and GCD of two numbers = Product of these two numbers,
2. $[1, 2, 3, \cdots, n] = [[1, 2, 3, \cdots, p], [p+1, p+2, p+3, \cdots, n]]$.

Now,

$$SL(n,3) = \frac{[n, n-1, n-2]}{[1,2,3]}, \tag{1}$$

Here, $[n, n-1, n-2] = \left[n,\ \frac{(n-1)(n-2)}{(n-1, n-2)}\right]$.
But $(n-1, n-2) =$ GCD of n-1 and n-2, which is always 1.
Hence, $[n, n-1, n-2] = [n, (n-1)(n-2)]$ and clearly $[1,2,3] = 6$.

At $n = 3 : (1) \Rightarrow SL(3,3) = \frac{[3,2,1]}{[1,2,3]} = \frac{3 \times 2 \times 1}{6} = \frac{6}{6} = 1.$

At $n = 6 : (1) \Rightarrow SL(6,3) = \frac{[6,5,4]}{[1,2,3,4]} = \frac{6 \times 5 \times 4}{12} = 10.$

Hence, $SL(n,3) = \begin{cases} \frac{n(n-1)(n-2)}{6}, & \text{if n is odd} \\ \frac{n(n-1)(n-2)}{12}, & \text{if n is even} \end{cases}$

is proved.

Similarly $SL(n,4) = \begin{cases} \frac{[n, n-1, n-2, n-3]}{[1,2,3,4]}, & \text{for } n \geq 4 \\ \frac{n.(n-1).(n-2).(n-3)}{24}, & \text{if 3 does not divides n} \\ \frac{n.(n-1).(n-2).(n-3)}{72}, & \text{if 3 divides n} \end{cases}$

Similarly, $SL(n,5) = \frac{n.(n-1).(n-2).(n-3).(n-4)}{360}$, with other conditions also.
Here, we have used only the general valuesof LCM ratios given in ([2] and [3]).
The other results can be obtained similarly.

§2. Sets of $SL(n, r)$ [2]

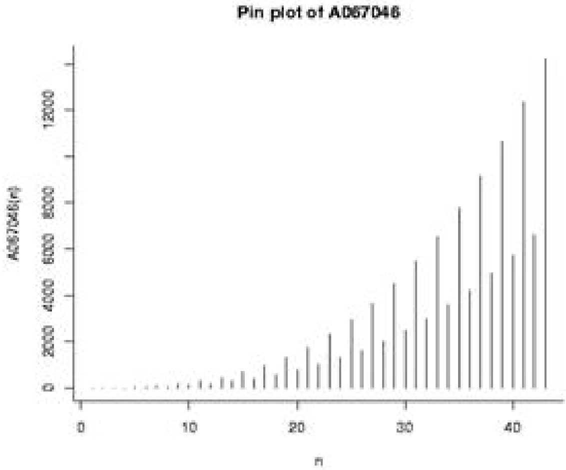
(1) $SL(n,1) = \{1,2,3,4,5,6,\cdots,n,\cdots\}$ It is a set of natural numbers.

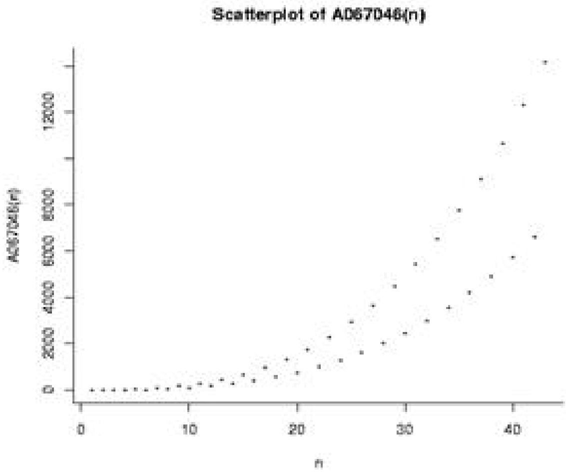
(2) $SL(n,2) = \{1,3,6,10,\cdots,\frac{n(n-1)}{2},\cdots\}$ It is a set of triangular numbers.

(3) $SL(n,3) = \{1,2,10,10,35,28,84,\cdots,\frac{n(n-1)(n-2)}{12},\cdots\}$.

This set, with more elements, is $\{1,2,10,10,35,28,84,60,165,110,286,182,455,$
$280,680,408,969,570,1330,770,1771,1012,2300,1300,2925,1638,3654,2030,4495,2480,$
$5456,2992,6545,3570,7770,4218,9139,4940,10660,5740,12341,6622,14190,\cdots\}$.

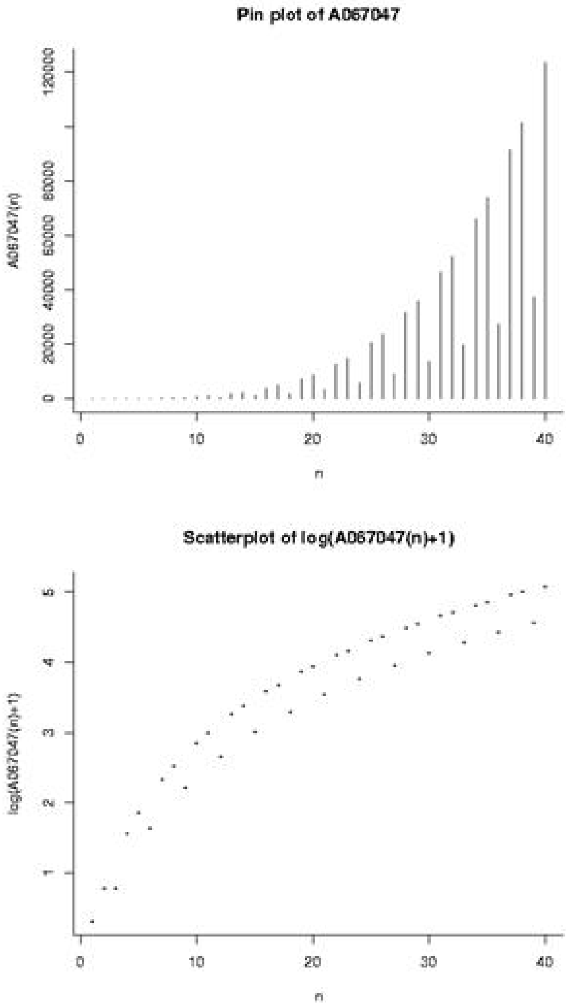
Its generating function is $\frac{x^4+2x^3+6x^2+2x+1}{(1-x^2)^4}$.





Graph of SL(n,3)

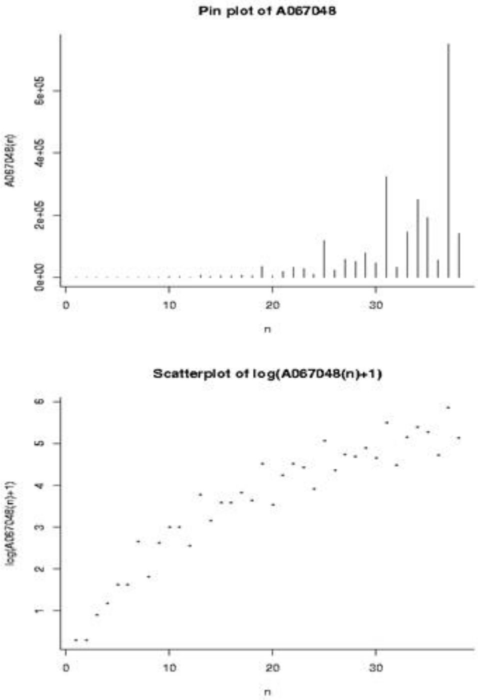
Physical Interpretation of Graph of SL(n,3): This graph, given on the next page, represents the V-I characteristic of two diodes in forward bias mode. It is represented by the equation:
$I = I_0\{\exp(\frac{eV}{kBT}) - 1\}$, a rectifier equation, where,
I_0 = total saturation current,
e = charge on electron,
V = applied voltage,
kB = Boltzman's constant, and
T = temperature.
Here V is positive. X-axis represents voltage V and Y-axis is current in mA.
Also, this graph represents harmonic oscillator: Kinetic energy along Y-axis and velocity along X-axis.



Graph of SL(n,4)

(4) $SL(n,4) = \{1, 5, 5, 35, 70, 42, \cdots, \dfrac{n(n-1)(n-2)(n-3)}{72}, \ldots\}$,
This set, to certain terms is $\{1, 5, 5, 35, 70, 42, 210, 330, 165, 715, 1001, 455,$
$1820, 2380, 1020, 3876, 4845, 1995, 7315, 8855, 3542, 12650, 14950, 5850, 20475, 23751, 9135,$
$31465, 35960, 13640, 46376, 52360, 19635, 66045, 73815, 27417, 91390, 101270, 37310, 123410, \cdots\}$.

Physical Interpretation of Graph of SL(n,4): This graph, the image of graph about a line of symmetry $y = x$, is a temperature-resistance characteristic of a thermister.
Its equation is $R = R_0. \exp[\beta(\frac{1}{T} - \frac{1}{T_0})]$, where
R_0 = resistance of room temperature,
R = resistance at different temperature,
β = constant,
T_0 = room temperature.



Graph of SL(n,5)

Temperature T, in Kelvin units, along X-axis and resistance R, in ohms, along Y-axis, β value lies between 3000 and 4000.

The above equation can be put as $R = C.e^{\frac{\beta}{T}}$.

Its another representation is potential energy (in ergs, along Y-axis) of system of spring against extension (in cetimeters along X-axis) for different weights.

The second graph below is characteristic curve of V_{CE} against I_{CE} at constant base current I_B.

(5) $SL(n,5) = \{1,1,1,7,14,42,42,\cdots,\frac{n(n-1)(n-2)(n-3)(n-4)}{360},\cdots\}$ This set, to certain terms, is $\{1,1,7,14,42,42,462,66,429,1001,1001,364,6188,1428\cdots\}$

Physical Interpretation of Graph of SL(n,5): The second graph of $\{SL(n,5)\}$, given above, represents the V-I characteristic of two diodes in reverse bias mode. It is represented by the same equation mentioned in graph of $SL(n,3)$ with a change that V is negative.

Hence, $-V \geq \frac{4kBT}{e}$, and that $\exp(\frac{-eV}{kBT}) \leq 1$, so that $I = I_0$.

This shows that the current is in reverse bias and remains constant at I_0, the saturation current, until the junction breaks down. Axes parameters are as above.

Similarly for the other sequences.

§3. Properties [3]

Murthy [1] formed an interserting triangle of the above sequences by writing them vertically, as follows:

1										
1	1									
1	2	1								
1	3	3	1							
1	4	6	2	1						
1	5	10	10	5	1					
1	6	15	10	5	1	1				
1	7	21	35	35	7	7	1			
1	8	28	28	70	14	14	2	1		
1	9	36	84	42	42	42	6	3	1	
1	10	45	60	210	42	42	6	3	1	1

1. Here, the first column and the leading diogonal contains all unity.
The second column contains the elements of sequence $SL(n, 1)$.
The third column contains the elements of sequence $SL(n, 2)$.
The fourth column contains the elements of sequence $SL(n, 3)$.
and similarly for other columns.
2. Consider that row which contains the elements 1 1 only as first row.
If p is prime, the sum of all elements of p^{th} row $\equiv 2(mod\ p)$.
If p is not prime, the sum of all elements of 4^{th} row $\equiv 2(mod\ 4)$.
The sum of all elements of 6^{th} row $\equiv 3(mod\ 6)$.
The sum of all elements of 8^{th} row $\equiv 6(mod\ 8)$.
The sum of all elements of 9^{th} row $\equiv 5(mod\ 5)$.
The sum of all elements of 10^{th} row $\equiv 1(mod\ 10)$.

§4. Difference

We have,
$SL(n, 2) - SL(n - 1, 2) = SL(n - 1, 1)$.
This needs no verification.

Also, $$SL(n, 3) - SL(n - 1, 3) = \frac{n(n-1)(n-2)}{6} - \frac{(n-1)(n-2)(n-3)}{6} = \frac{(n-1)(n-2)}{2} = SL(n - 1, 2).$$

Similarly,

$$SL(n,4) - SL(n-1,3) = SL(n-1,3)$$

$$SL(n,5) - SL(n-1,5) = SL(n-1,4).$$

Hence, in general,

$$SL(n,r) - SL(n-1,r) = SL(n-1,r-1),\ r < n.$$

§5. Summation

Adding the above results, we get,

$$\sum_{r=2}^{\infty} SL(n,r) = n-1,\ n > 1.$$

§6. Ratio

We have,

$$\frac{SL(n,3)}{SL(n,2)} = \frac{n-2}{3},$$

$$\frac{SL(n,4)}{SL(n,3)} = \frac{n-3}{4}, \frac{SL(n,5)}{SL(n,4)} = \frac{n-4}{5}.$$

In general,

$$\frac{SL(n,r+1)}{SL(n,r)} = \frac{n-r}{r+1}.$$

§7. Sum of reciprocals of two cosecutive LCM ratios

We have,

$$\frac{1}{SL(n,2)} + \frac{1}{SL(n,3)} = \frac{n+1}{3 \cdot SL(n,3)},$$

$$\frac{1}{SL(n,3)} + \frac{1}{SL(n,4)} = \frac{n+1}{4 \cdot SL(n,4)},\ \frac{1}{SL(n,4)} + \frac{1}{SL(n,5)} = \frac{n+1}{5 \cdot SL(n,5)}.$$

In general,

$$\frac{1}{SL(n,r)} + \frac{1}{SL(n,r+1)} = \frac{n+1}{(r+1) \cdot SL(n,r+1)}.$$

§8. Product of two cosecutive LCM ratios

1. $SL(n,1) \cdot SL(n,2) = \dfrac{n^2(n-1)}{2!}$

2. $SL(n,2) \cdot SL(n,3) = \dfrac{n^2(n-1)^2(n-2)}{2! \cdot 3!}$

3. $SL(n,3) \cdot SL(n,4) = \dfrac{n^2(n-1)^2(n-2)^2(n-3)}{3! \cdot 4!}$

4. $SL(n,4) \cdot SL(n,5) = \dfrac{n^2(n-1)^2(n-2)^2(n-3)^2(n-4)}{4! \cdot 5!}$

In general,

$$SL(n,r) \cdot SL(n,r+1) = \frac{n^2 \cdot (n-1)^2 \cdot (n-2)^2 \cdot (n-3)^2 \cdot \cdots \cdot (n-r+1)^2 \cdot (n-r)}{r! \cdot (r+1)!}.$$

References

[1] Amarnath Murthy, Some notions on Least common multiples, Smarandache Notions Journal, **12**(2001), 307-308.

[2] Maohua Le, Two Formulas of Smarandache LCM Ratio Sequences, Smarandache Notions Journal, **14**(2004), 183-185.

[3] Wang Ting, Two Formulas of Smarandache LCM Ratio Sequences, Scientia Magna, **1**(2005), No.1, 109-113.