THERE ARE INFINITELY MANY SMARANDACHE DERIVATIONS, INTEGRATIONS AND LUCKY NUMBERS

Pantelimon Stănică and Gabriela Stănică

March 1, 2002

Abstract

A number is said to be a *Smarandache Lucky Number* (see [3, 1, 2]) if an incorrect calculation leads to a correct result. In general, a *Smarandache Lucky Method or Algorithm* is said to be any incorrect method or algorithm, which leads to a correct result. In this note we find an infinite sequence of *distinct* lucky fractions. We also define a lucky product differentiation and a lucky product integration. For a given function f, we find *all* other functions g, which renders the product lucky for differentiation/integration.

Keywords. Smarandache Lucky Numbers, Fractions, Lucky Derivatives, Lucky Integrals

1 Introduction

A number is said to be a *Smarandache Lucky Number* (see [2]) if an incorrect calculation leads to a correct result. For example, in the fraction $64/16$ if the 6's are incorrectly cancelled the result $4/1 = 4$ is correct. (We exclude trivial examples of the form $400/200$ where non-aligned zeros are cancelled.)

In general: *The Smarandache Lucky Method/Algorithm/Operation/etc.* is said to be any incorrect method or algorithm or operation, which leads to a correct result. The wrong calculation is funny, and somehow similarly to the students' common mistakes, or to produce confusions or paradoxes. In [1] (see also [2], [3]), the authors ask the questions: *Is the set of all fractions, where an incorrect calculation leads to a correct result, finite or infinite? Can someone give an example of a Smarandache Lucky Derivation, or Integration, or Solution to a Differential Equation?*

In this note we give an infinite class of examples of each type. In fact, given a real-valued function f, we find all examples for which an incorect differentiation/integration, in a product with f, leads to a correct answer.

2 Main Results

Let f, g be real-valued functions. Define the incorect differentiation as follows:
$\frac{d_O\,(f(x)\cdot g(x))}{dx} = \frac{df(x)}{dx}\cdot\frac{dg(x)}{dx}$.
We prove

Theorem 1. *Let $f : \mathbf{R} \to \mathbf{R}$. The functions $g : \mathbf{R} \to \mathbf{R}$, satisfying $\frac{d_O(f(x)\cdot g(x))}{dx} = \frac{df(x)\cdot g(x)}{dx}$ are given by*

$$g(x) = c\cdot e^{\int \frac{f'(x)}{f'(x)-f(x)}dx},$$

where c is a real constant.

Proof. Since $\frac{d_O(f(x)\cdot g(x))}{dx} = f'(x)g'(x)$, we need to find all functions such that

$$f'(x)g'(x) = f'(x)g(x) + f(x)g'(x),$$

by the product rule for differentiation. Thus, we need

$$g'(x)(f'(x)-f(x)) = g(x)f'(x) \iff \frac{g'(x)}{g(x)} = \frac{f'(x)}{f'(x)-f(x)},$$

from which we derive

$$g(x) = c\cdot e^{\int \frac{f'(x)}{f'(x)-f(x)}dx}.$$

$\square$

Examples.

1. Take $f(x) = x$, then $g(x) = c \cdot e^{\int \frac{1}{1-x} dx} = c \cdot e^{\ln(1/(1-x))} = c \cdot \frac{1}{1-x}$.
2. Take $f(x) = x^2$, then

$$g(x) = c \cdot e^{\int \frac{2x}{2x - x^2} dx} = c \cdot e^{\ln \frac{x}{x-2} + \ln \frac{1}{x^2 - 2x}} = \frac{c}{(x-2)^2}.$$

From the previous theorem we derive an equivalent result on lucky integration. The incorect integration is defined by: *the integral of a product is the product of integrals.*

Theorem 2. *Given a real-valued function f, the functions g such that the integral of the product of f and g is the product of the integral of f and integral of g are given by*

$$g(x) = \frac{c f(x)}{f(x) - \int f(x) dx} \cdot e^{\int \frac{f(x)}{f(x) - \int f(x) dx} dx}.$$

Proof. Similar to the proof of Theorem 1. $\square$

Obviously, the previous theorem is an example of a lucky differential equation, as well.

3 There Are an Infinity Number of ... Lucky Numbers

To avoid triviality, we exclude among the lucky numbers, those which are constructed by padding at the end the same number of zeros in the denominator and numerator of a fixed fraction (e.g., $\frac{3000}{11000}$). We also exclude 1's, that is $\frac{ab \cdots x}{ab \cdots x}$.

The fact that there are an infinity of lucky fractions is not a difficult question (even if they are not constructed by padding zeros or they come from 1). Our next result proves that

Theorem 3. *Let the fraction* $\frac{99\cdots 96}{24\cdots 99}$ *(same number of digits). By cancelling as many 9's as we wish (and from any place, for that matter), we still get* 4.

Proof. Let $n+1$ be the number of digits in the numerator (or denominator) of the given fraction. We write it as

$$\begin{aligned}&\frac{9\cdot 10^n + 9\cdot 10^{n-1}+\cdots+9\cdot 10+6}{24\cdot 10^{n-1}+9\cdot 10^{n-2}+\cdots+9}\\ &=\frac{3\cdot\frac{10^n-4}{9}}{8\cdot 10^{n-2}+3\cdot\frac{10^{n-2}-1}{10-1}}=\frac{10^n-4}{24\cdot 10^{n-2}+10^{n-2}-1}\\ &=\frac{10^n-4}{\frac{1}{4}10^n-1}=4.\end{aligned}$$

We see that by cancelling any number of digits of 9, we get a fraction of the same form. □

In the same manner we can show (we omit the proof)

Theorem 4. *Define the fractions* $\frac{33\cdots 32}{8\cdots 33}$ *(the numerator has one digit more that the denominator), respectively,* $\frac{6\cdots 64}{16\cdots 6}$ *(same number of digits),* $\frac{9\cdots 95}{19\cdots 9}$ *(same number of digits),* $\frac{6\cdots 65}{26\cdots 6}$ *(same number of digits),* $\frac{9\cdots 98}{49\cdots 9}$ *(same number of digits),* $\frac{77\cdots 75}{217\cdots 7}$, $\frac{13\cdots 34}{3\cdots 34}$ *(same number of* 3*'s). By cancelling as many* $3's$, *respectively,* $6's, 9's, 6's, 9's, 7's, 3's$, *as we wish, we get the same number, namely* 4, *respectively,* $5, \frac{5}{2}, 2, \frac{25}{7}, 4$.

Other examples of lucky numbers are given by taking the above fractions and inserting zeros appropriately. We give

Theorem 5. *The following fractions are also lucky numbers*

$$\frac{b0\cdots 0xy}{a0\cdots 0wz}$$

(same number of zeros), where $1\le a,b,w,x,y,z\le 9$ *are integers,* $\frac{xy}{wz}$ *are the fractions from the previous theorem equal to* $\{2, 5/2, 4, 5\}$ *and* $\frac{b}{a}$ *is equal to that same reduced fraction. When* $\frac{xy}{wz}=\frac{25}{7}$, *then* a,b *are not digits, rather they are integers such that* $\frac{b}{a}=\frac{25}{7}$.

You might think that these are the only lucky numbers. That is not so. Our last theorem will present an infinite number of distinct *lucky* numbers.

Theorem 6. *Take any reduced fraction* $\frac{b}{a}$. *Then, the following sequence of fractions is a sequence of lucky numbers* $\frac{b0\cdots 0b}{a0\cdots 0a}$. *Assuming the denominator (numerator) has k more digits than the numerator (denominator), then the numerator (denominator) has k more zeros in it. Since* $\frac{b}{a}$ *was arbitrary, we have an infinite number of lucky fractions.*

Example. Let $\frac{b}{a} = \frac{11}{7}$. Then we build the following sequence of lucky numbers

$$\frac{11}{7}, \frac{11011}{7007}, \frac{110011}{70007}, etc.$$

References

[1] C. Ashbacher, "Smarandache Lucky Math", *Smarandache Notions Journal*, Vol. 9, p. 155, Summer 1998.

[2] E. Burton & M. Perez, Some Notions and Questions in Number Theory, vol. III, 2000.

[3] F. Smarandache "Collected Papers" (Vol. II), University of Kishinev, p. 200, 1997.

Address:
Auburn University Montgomery
Department of Mathematics
Montgomery, AL 36124-4023, USA
e-mail: *stanica@strudel.aum.edu*