SMARANDACHE FRIENDLY CUBE NUMBERS

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Abstract The main purpose of this paper is to introduce new concepts of Smarandache numbers, namely Smarandache Friendly Cube Numbers, and give definitions, curious note, theorem, conjectures, proposed future studies, and ask open problems.

Keywords: Smarandache Friendly Triple Cube Numbers; Smarandache Friendly Pairs Cube Numbers.

1.1 Definition. The positive integers of ordered triple (m, n, k) are called Smarandache Friendly Triple Cube Numbers, denoted by $SFTCN_{(m,n,k)}$, if the following conditions satisfy:

1) The sum of its digits (i.e. $m, n,$ and k) is cube.

2) The second integer (n) is formed by summing the digits of the first integer (m) after cubing it, and the integer (n) must be the reverse order of the first number (m).

3) The third integer (k) is obtained by cubing the second integer (n) and summing its digits, and (k) must equal the sum of its digits after cubing (k).

1.2 Example: $(53, 35, 26)$ is $SFTCN_{(53,35,26)}$, note the following conditions:

1) The sum of the digits of (53) is $5 + 3 = 8$(cube).

2) $(53)^3 = 148877$, then $1 + 4 + 8 + 8 + 7 + 7 = 35$, the digits sum is also cube $(3 + 5 = 8)$, and 35 is formed form the backorder of 53.

3) $(35)^3 = 42875$, then $4 + 2 + 8 + 7 + 5 = 26$, the sum digits of 26 are $2 + 6 = 8$, which is cube, and $26^3 = 17576$, $1 + 7 + 5 + 7 + 6 = 26$.

The proposed sequence of the

$$SFTCN_{(m,n,k)} := \{(10, 1, 1), (53, 35, 26), (62, 26, 26), (80, 8, 8), \cdots .$$

1.3 Conjectures:

1) The $SFTCN_{(m,n,k)}$ contains infinitely many triples.

2) The $SFTCN_{(m,n,k)}$ contains infinitely many triples that satisfy Transitive property, but there are exceptions such as the triple $(53, 35, 26)$.

1.4 Open problems:

1) What is the general formula of $SFTCN_{(m,n,k)}$?
2) What is the procedure that can verify $SFTCN_{(m,n,k)}$ (by using computer programming such as Maple, or Mathematica)?
3) How many triples prime are there in $SFTCN_{(m,n,k)}$?
4) What is the density of $SFTCN_{(m,n,k)}$?
5) Is there a relationship between $SFTCN_{(m,n,k)}$, and other Smarandache sequence (such as Smarandache cube-digital sequence [2])?
6) Are there such integers that satisfy $SFTCN_{(m,n,17)}$,and $SFTCN_{(m,n,71)}$?

2.1 Definition: Any two positive integers satisfy the following two conditions (are called Smarandache Friendly Pairs Cube Numbers, denoted by $SFPCN_{(m,n)}$):
1) The sum of its digits is cube.
2) The sum of its digits after cubing, equal itself.

2.2 Theorem. $SFPCN_{(m,n)}$ satisfies the Reflexive property.

Proof. suppose $m =_{ci} 10^i +_{ci-1} 10^{i-1} + \cdots +_{c1} 10 +_{c0}$, i.e. the decimal form of m.

Also, $n =_j 10^j +_{cj-1} 10^{j-1} + \cdots +_{c1} 10 +_{c0}$, i.e. the decimal form of n. Now by definition (condition 2) we must have

$$({}_{ci}10^i +_{ci-1} 10^{i-1} + \cdots +_{c1} 10 +_{c0})^3 = ({}_j10^j +_{cj-1} 10^{j-1} + \cdots +_{c1} 10 +_{c0})^3,$$

for all i and j. Hence, $m^3 = n^3$, i.e. $SFPCN_{(m,n)} = SFPCN_{(n,m)}$, which is the Reflexive property.

2.3 Example: Consider $SFPC_{(8,8)}$, then we have
1) 8 cube.
2) $8^3 = 512$, the sum of digits $5+1+2 = 8$, hence $(8, 8)$ is $SFPCN_{(8,8)}$.
Thus, the proposed sequence of the

$$SFPCN_{(m,n)} := \{(1, 1), (8, 8), (17, 17), (26, 26), \cdots\},$$

noting that 71, and 62 are not $SFPCN$.

2.4 Curious note: the number 27 is not $SFPCN_{(27,27)}$, but $27^3 = 19683$, i.e. $1 + 9 + 6 + 8 + 3 = 27$, in addition, the number 18 has this property; can you find another ones?

2.5 Conjecture: $SFPCN_{(m,n)}$ is a special case from $SFTCN_{(m,n,k)}$.

2.6 Open problems:
1) Is the sequence of the proposed $SFPCN_{(m,n)}$, finite or infinite?
2) What is the general formula of $SFPCN_{(m,n)}$?
3) What is the formula that connects $SFPCN_{(m,n)}$ and $SFTCN_{(m,n,k)}$?
4) Is there a relationship between $SFPCN_{(m,n)}$, $SFTCN_{(m,n,k)}$, and Smarandache Sequence of Happy Cube Numbers [1]?

3.1 Definition: The ordered pair of integers (m, c) is called Fixed Smarandache Friendly Pairs Cube Numbers (FSFPCN) if the following conditions satisfy:
1) The sum of its digits $(m,$ and $c)$ is cube.
2) c is a constant formed from the digits of m after cubing .
3.2 Examples: $(35, 26)$, $(26, 26)$, $(44, 26)$, $(62, 26)$, $(71, 26)$, and so on.

3.3 Open problems:
1) Are there other examples for different constants that satisfy $(FSFPCN)$?
2) Is there a relationship between

$$SFPCN_{(m,n)}, SFTCN_{(m,n,k)} \quad \text{and} \quad FSFPCN_{(m,c)}?$$

4.1 Proposed future studies: The author invites the researcher for more studies about the following new concepts:

4.1.1 Smarandache Friendly Pairs of 4-th powers Numbers (such as, $7^4 = 2401$, the sum of its digits = 7, also $22^4 = 234256$, so $2+3+4+2+5+6 = 22$, also $25^4 = 390625$, then $3 + 9 + 0 + 6 + 2 + 5 = 25$, $36^4 = 1679616$, hence $1 + 6 + 7 + 9 + 6 + 1 + 6 = 36$, and so on).

4.1.2 Smarandache Friendly Pairs of 5-th powers Numbers (such as, $28^5 = 17210368$, so $1 + 7 + 2 + 1 + 0 + 3 + 6 + 8 = 28$, i.e. the sum of its digits $= 28$, also $35^5 = 52521875$, hence $5 + 2 + 5 + 2 + 1 + 8 + 7 + 5 = 35$, and so on).

4.1.3 Smarandache Friendly Pairs of n-th powers Numbers.

References

[1] Jebreel Muneer, Smarandache Sequence of Happy Cube Numbers, Smarandache Notions Journal **14** (2004), 139-143.
[2] http://www.gallup.unm.edu/ smarandache.