TWO FORMULAS FOR SMARANDACHE LCM RATIO SEQUENCES

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Abstract In this paper, a reduction formula for Smarandache LCM ratio sequences SLR(6)and SLR(7) are given.

Keywords: Smarandache LCM ratio sequences; Reduction formula.

§1. Introduction

Let $(x_1, x_2, ..., x_t)$ and $[x_1, x_2, ..., x_t]$ denote the greatest common divisor and the least common multiple of any positive integers $x_1, x_2, ..., x_t$ respectively. Let r be a positive integer with $r > 1$. For any positive integer n, let $T(r, n) = \frac{[n,n+1,...,n+r-1]}{[1,2,...r]}$, then the sequences $SLR(r) = \{T(r, n)\}_\infty$ is called Smarandache LCM ratio sequences of degree r. In reference $[1]$, Maohua Le studied the properties of $SLR(r)$, and gave two reduction formulas for SLR(3) and SLR(4). In this paper, we will study the calculating problem of $SLR(6)$ and $SLR(7)$, and prove the following:

Theorem 1. For any positive integer n, we have the following result:
If $n \equiv 0, 15 \bmod 20$, then

$$SLR(6) = \frac{1}{7200}n(n+1)(n+2)(n+3)(n+4)(n+5);$$

If $n \equiv 1, 2, 6, 9, 13, 14, 17, 18 \bmod 20$, then

$$SLR(6) = \frac{1}{720}n(n+1)(n+2)(n+3)(n+4)(n+5);$$

If $n \equiv 5, 10 \bmod 20$, then

$$SLR(6) = \frac{1}{3600}n(n+1)(n+2)(n+3)(n+4)(n+5);$$

If $n \equiv 3, 4, 7, 8, 11, 12, 16, 19 \bmod 20$, then

$$SLR(6) = \frac{1}{1440} n(n+1)(n+2)(n+3)(n+4)(n+5).$$

Theorem 2. For any positive integer n, we have the following

If $n \equiv 0, 24, 30, 54 \bmod 60$, then

$$SLR(7) = \frac{1}{302400} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 1, 13, 17, 37, 41, 53 \bmod 60$, then

$$SLR(7) = \frac{1}{5040} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 2, 8, 16, 22, 26, 28, 32, 38, 46, 52, 56, 58 \bmod 60$, then

$$SLR(7) = \frac{1}{20160} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 3, 27, 51 \bmod 60$, then

$$SLR(7) = \frac{1}{30240} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 4, 10, 14, 20, 34, 40, 44, 50 \bmod 60$, then

$$SLR(7) = \frac{1}{100800} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 5, 25, 29, 49 \bmod 60$, then

$$SLR(7) = \frac{1}{25200} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 6, 12, 18, 36, 42, 48 \bmod 60$, then

$$SLR(7) = \frac{1}{60480} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 7, 11, 23, 31, 43, 47 \bmod 60$, then

$$SLR(7) = \frac{1}{10080} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 9, 45 \bmod 60$, then

$$SLR(7) = \frac{1}{75600} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 15, 39 \bmod 60$, then

$$SLR(7) = \frac{1}{151200} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 19, 55, 59, 35 \bmod 60$, then

$$SLR(7) = \frac{1}{50400} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6);$$

If $n \equiv 21, 33, 57 \bmod 60$, then

$$SLR(7) = \frac{1}{15120} n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6).$$

§2. Proof of the theorem

To complete the proof of Theorem, we need following several simple Lemmas.

Lemma 1. For any positive integer a and b, we have $(a, b)[a, b] = ab$.

Lemma 2. For any positive integer s with $s < t$, we have

$$(x_1, x_2, ..., x_t) = ((x_1, ..., x_s), (x_{s+1}, ..., x_t))$$

and

$$[x_1, x_2, ..., x_t] = [[x_1, ..., x_s], [x_{s+1}, ..., x_t]].$$

Lemma 3. For any positive integer n, we have

$$T(4, n) = \begin{cases} \frac{1}{24}n(n+1)(n+2)(n+3), & \text{if } n \equiv 1, 2 \bmod 3; \\ \frac{1}{72}n(n+1)(n+2)(n+3), & \text{if } n \equiv 0 \bmod 3. \end{cases}$$

Lemma 4. For any positive integer n, we have

$$T(5, n) = \begin{cases} \frac{1}{1440}n(n+1)(n+2)(n+3)(n+4), & \text{if } n \equiv 0, 8 \bmod 12; \\ \frac{1}{120}n(n+1)(n+2)(n+3)(n+4), & \text{if } n \equiv 1, 7 \bmod 12; \\ \frac{1}{720}n(n+1)(n+2)(n+3)(n+4), & \text{if } n \equiv 2, 6 \bmod 12; \\ \frac{1}{360}n(n+1)(n+2)(n+3)(n+4), & \text{if } n \equiv 3, 5, 9, 11 \bmod 12; \\ \frac{1}{480}n(n+1)(n+2)(n+3)(n+4), & \text{if } n \equiv 4 \bmod 12; \\ \frac{1}{240}n(n+1)(n+2)(n+3)(n+4), & \text{if } n \equiv 10 \bmod 12. \end{cases}$$

The proof of Lemma 1 and Lemma 2 can be found in [3], Lemma 3 was proved in [1]. Lemma 4 was proved in [4].

In the following, we shall use these Lemmas to complete the proof of Theorem 1. In fact, from the properties of the least common multiple of any positive integers, we know that

$$\begin{aligned} &[n, n+1, n+2, n+3, n+4, n+5] = [[n, n+1, n+2, n+3, n+4], n+5] \\ &= \frac{[n, n+1, n+2, n+3, n+4](n+5)}{([n, n+1, n+2, n+3, n+4], n+5)}. \end{aligned} \tag{1}$$

Note that $[1, 2, 3, 4, 5, 6] = 60$, $[1, 2, 3, 4, 5] = 60$ and

$$([n, n+1, n+2, n+3, n+4], n+5)$$

$$= \begin{cases} 5, & \text{if } n \equiv 0, 20, 30, 50 \bmod 60; \\ 6, & \text{if } n \equiv 1, 13, 31, 49 \bmod 60; \\ 1, & \text{if } n \equiv 2, 6, 8, 12, 14, 18, 24, 26, 32, 38, 42, 44, 48, 54, 56 \bmod 60; \\ 4, & \text{if } n \equiv 3, 11, 23, 27, 39, 47, 51, 59 \bmod 60; \\ 3, & \text{if } n \equiv 4, 16, 22, 28, 34, 46, 52, 58 \bmod 60; \\ 10, & \text{if } n \equiv 5, 45 \bmod 60; \\ 2, & \text{if } n \equiv 9, 17, 21, 29, 33, 41, 53, 57 \bmod 60; \\ 15, & \text{if } n \equiv 10, 40 \bmod 60; \\ 20, & \text{if } n \equiv 15, 35 \bmod 60; \\ 12, & \text{if } n \equiv 7, 19, 31, 43 \bmod 60; \\ 30, & \text{if } n \equiv 25 \bmod 60; \\ 60, & \text{if } n \equiv 55 \bmod 60. \end{cases} \quad (2)$$

Now Theorem 1 follows from Lemma 3.

The proof of Theorem 2 is similar to the proof of Theorem 1. From the properties of the least common multiple of any positive integers, we know that

$$[n, n+1, n+2, n+3, n+4, n+5, n+6]$$

$$= [[n, n+1, n+2, n+3, n+4, n+5], n+6]$$

$$= \frac{[n, n+1, n+2, n+3, n+4, n+5](n+6)}{([n, n+1, n+2, n+3, n+4, n+5], n+6)}. \quad (3)$$

Note that $[1, 2, 3, 4, 5, 6, 7] = 420$, and

$$([n, n+1, n+2, n+3, n+4, n+5], n+6)$$

$$= \begin{cases} 6, & \text{if } n \equiv 0, 12, 36, 48 \bmod 60; \\ 1, & \text{if } n \equiv 1, 5, 7, 11, 13, 17, 23, 25, 31, 35, 37, 41, 43, 47, 53, 55 \bmod 60; \\ 4, & \text{if } n \equiv 2, 10, 22, 26, 38, 46, 50, 58 \bmod 60; \\ 3, & \text{if } n \equiv 3, 15, 21, 27, 33, 45, 51, 57 \bmod 60; \\ 10, & \text{if } n \equiv 4, 44 \bmod 60; \\ 12, & \text{if } n \equiv 6, 18, 30, 42 \bmod 60; \\ 2, & \text{if } n \equiv 8, 16, 20, 28, 32, 40, 52, 56 \bmod 60; \\ 15, & \text{if } n \equiv 9, 39 \bmod 60; \\ 20, & \text{if } n \equiv 14, 34 \bmod 60; \\ 5, & \text{if } n \equiv 19, 29, 49, 59 \bmod 60; \\ 30, & \text{if } n \equiv 24 \bmod 60; \\ 60, & \text{if } n \equiv 54 \bmod 60. \end{cases} \quad (4)$$

Now Theorem 2 follows from Theorem 1.

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