DEPASCALISATION OF SMARANDACHE PASCAL DERIVED SEQUENCES AND BACKWARD EXTENDED FIBONACCI SEQUENCE

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Given a sequence S_b (called the base sequence).

$b_1, b_2, b_3, b_4, \ldots$

Then the Smarandache Pascal derived Sequence S_d

$d_1, d_2, d_3, d_4, \ldots$ is defined as follows: **Ref [1]**

$d_1 = b_1$

$d_2 = b_1 + b_2$

$d_3 = b_1 + 2b_2 + b_3$

$d_4 = b_1 + 3b_2 + 3b_3 + b_4$

...

$$d_{n+1} = \sum_{k=0}^{n} {}^nC_k . b_{k+1}$$

Now Given S_d the task ahead is to find out the base sequence S_b . We call the process of extracting the base sequence from the Pascal derived sequence as **Depascalisation.** The interesting observation is that this again involves the Pascal's triangle though with a difference.

We see that

$b_1 = d_1$

$b_2 = -d_1 + d_2$

$b_3 = d_1 - 2d_2 + d_3$

$b_4 = -d_1 + 3d_2 - 3d_3 + d_4$

...

which suggests the possibility of

$$b_{n+1} = \sum_{k=0}^{n} (-1)^{n+k} \cdot {}^{n}C_k \cdot d_{k+1}$$

This can be established by induction.

We shall see that the depascalised sequences also exhibit interesting patterns.

To begin with we define The **Backward Extended Fibonacci Sequence (BEFS)** as Follows:

The Fibonacci sequence is

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, . . .

In which $T_1 = 1$, $T_2 = 1$, and $T_{n-2} = T_n - T_{n-1}$, $n > 2$ (A)

Now If we allow n to take values 0 , -1 , -2 , . . . also , we get

$T_0 = T_2 - T_1 = 0$, $T_{-1} = T_1 - T_0 = 1$, $T_{-2} = T_0 - T_{-1} = -1$, etc. and we get the Fibonacci sequence extended backwards as follows { T_r is the r^{th} term }

. . . T_{-6} T_{-5}, T_{-4}, T_{-3}, T_{-2}, T_{-1}, $\underline{T_0}$, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, . . .

. . . -8, 5, -3, 2 , -1, 1 <u>0</u>, 1, 1, 2 , 3, 5 8, 13, 21, 34, . . .

1. **Depascalisation of the Fibonacci sequence:**

The Fibonacci sequence is

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, . . .

The corresponding depascalised sequence $S_{d(-1)}$ comes out to be

$S_{d(-1)} \longrightarrow$ 1, 0, 1, -1, 2, -3, 5, -8, . . .

It can be noticed that , The resulting sequence is nothing but the **BEFS rotated by 180^0 about T_1** . and then the terms to the left of T_1 omitted. { This has been generalised in the Proposition 2 below.}

It is not over here. If we further depascalise the above sequence we get the following sequence $S_{d(-2)}$ as

1 , -1, 2, -5, 13, -34 , 89 , -233

This can be obtained alternately from the Fibonacci Sequence by:

(a) Removing even numbered terms.

(b) Multiplying alternate terms with (-1) in the thus obtained sequence.

Propositions:

Following two propositions are conjectured on Pascalisation and Depascalisation of Fibonacci Sequence.

(1) If the first r terms of the Fibonacci Sequence are removed and the remaining sequence is Pascalised, the resulting Derived Sequence is $\mathbf{F_{2r+2}}, \mathbf{F_{2r+4}}, \mathbf{F_{2r+6}}, \mathbf{F_{2r+8}}, \ldots$ where F_r is the r^{th} term of the Fibonacci Sequence.

(2) In the **FEBS** If we take T_r as the first term and Depascalise the Right side of it then we get the resulting sequence as the left side of it (looking rightwards) with T_r as the first term.

As an example let $r = 7$, $T_7 = 13$

$\ldots T_{-6}\ T_{-5},\ T_{-4},\ T_{-3},\ T_{-2},\ T_{-1},\ T_0,\ T_1,\ T_2,\ T_3,\ T_4,\ T_5,\ T_6,\ \underline{T_7},\ T_8,\ T_9, \ldots$

$\ldots -8,\ 5,\ -3,\ 2,\ -1,\ 1\ \ 0,\ 1,\ 1,\ 2,\ 3,\ 5\ \ 8,\ \ \underline{13},\ 21,\ 34,\ 55,\ 89, \ldots$

→→→→→→→→→→→

depascalisation

The Depascalised sequence is

13, 8, 5, 3, 2, 1, 1, 0, 1, -1, 2, -3, 5, -8 . . .

which is obtained by rotating the FEBS around 13 (T_7) by 180^0 and then removing the terms on the left side of 13.

One can explore for more fascinating results.

References:

[1] "Amarnath Murthy", 'Smarandache Pascal derived Sequences', SNJ, 2000.