Simplified modal analysis of multi-storey RC buildings for application in seismic retrofitting

M. Hajjar[1,*], B.A. Izzuddin[1], L. Macorini[1] and M. Mabsout[2]

[1] Computational Structural Mechanics Group, Imperial College London, London, UK.
m.hajjar17@imperial.ac.uk,
b.izzuddin@imperial.ac.uk, l.macorini@imperial.ac.uk
[2] Department of Civil Engineering, American University of Beirut, Beirut, Lebanon.
mounir@aub.edu.lb

Abstract

This paper presents a novel simplified approach for determining the fundamental vibration characteristics of reinforced concrete (RC) buildings for application in the seismic retrofitting of substandard multi-storey RC building structures. Even when retrofitted, such structures would likely respond inelastically under a major earthquake, where the seismic response is most realistically represented by computationally expensive nonlinear dynamic analysis. Towards reducing such computational demands, the capacity spectrum method (CSM) offers a practical design and assessment approach which utilises nonlinear static pushover analysis in conjunction with the dynamic characteristics of a structure to establish whether the seismic capacity of the structure meets the demands of seismic ground motion. In this respect, the dynamic characteristics of a structure can be ordinarily obtained by performing conventional direct modal analysis which requires the assembly of mass and stiffness matrices to solve the eigenvalue problem. This paper presents the development of a simplified numerical approach that determines the translational modes of vibration and the associated periods for a retrofitted building structure in both horizontal directions in a simplistic manner without assembling the mass and stiffness matrices. The accuracy of the proposed simplified approach is verified for realistic structures via comparisons against direct eigenvalue analysis. This, along with planned research on simplified pushover analysis, paves the way for the practical seismic retrofitting of substandard reinforced concrete buildings within the CSM framework.

Key words: *(RC) buildings; seismic retrofitting; capacity spectrum method; modal analysis*

1. Introduction

The seismic retrofitting of existing vulnerable buildings is currently a process that relies heavily on engineering judgment and layout constraints, which can be an onerous task. Even when retrofitted, most structures including RC buildings would likely respond inelastically under a major earthquake [1], where the seismic response is most realistically represented by computationally expensive nonlinear dynamic analysis. Towards reducing such computational demands, the capacity spectrum method (CSM) is an already established nonlinear pushover analysis which utilises the dynamic characteristics of a structure to establish a graphical plot of the seismic capacity of the structure against the demands of seismic ground motion [2, 3]. The CSM is based on the simplified assumption that the maximum lateral storey drifts are governed by deformations of the fundamental mode of the originally elastic system. Therefore, the procedure requires finding the fundamental vibration mode, which is ordinarily obtained using conventional direct modal analysis that relies on the assembly of mass and stiffness matrices to solve the eigenvalue problem [4]. This typically imposes a significant computational bottleneck for practical application in the optimal design of retrofitting, where numerous retrofitting strategies would typically need to be considered. In that regard, this paper presents the development of a simplified numerical approach that determines the translational modes of vibration and associated periods for a retrofitted multi-storey RC building structure in both horizontal directions. The proposed approach avoids the need for assembling the mass and stiffness matrices of the building structure, offering a practical tool for the design of seismic retrofitting. The developed approach sets the ground for further planned research on simplified pushover analysis, which paves the way for the practical seismic retrofitting of substandard reinforced concrete buildings within the CSM framework.

2. Problem definition

The vibration characteristics of a structure are mainly obtained by solving the main eigenvalue problem ($K\phi = \lambda m\phi$) which characterises the free vibration of an undamped system [4], noting that the eigenvalue λ is equal to ω^2, with ω being the natural circular frequency of vibration. The eigenvalue problem can be reduced to the form ($[K - \lambda m]\,\phi = 0$). Apart from the trivial solution ($\phi = 0$), solving the problem for multi-storey RC building is too onerous due to the massive computational requirements besides the challenge imposed when assembling the stiffness (K) an mass (m) matrices.

Of the typical approaches to solve eigenvalue problems such as the above, the inverse vector iteration method [4] is an iterative procedure which aims at finding an eigenpair (eigenvalue and eigenvector) that satisfy the main equation ($K\phi = \lambda m\phi$). To illustrate the method, multiplying both sides of the equation ($K\phi = \lambda m\phi$) by ϕ^T converts the matrix products into scalars, and the factor λ in the following equation:

$$\lambda = \frac{\phi^T K \phi}{\phi^T m \phi} \tag{1}$$

is called Rayleigh's quotient [4] which has the inherent property that when ϕ is an eigenvector of the equation, λ becomes the corresponding eigenvalue.

The inverse vector iteration method starts by assuming a unit value of λ and selecting an arbitrary normalised eigenvector ϕ_j, then calculates the displacement vector $\overline{\phi}_{j+1}$ that balances the applied inertia force $m\phi_j$ ($K\,\overline{\phi}_{j+1} = \lambda m\phi_j = m\phi_j$), which permits to estimate the eigenvalue by evaluating Rayleigh quotient (λ_{j+1}) as:

$$\lambda_{j+1} = \frac{\overline{\phi}_{j+1}^{\,T}\, K\, \overline{\phi}_{j+1}}{\overline{\phi}_{j+1}^{\,T}\, m\, \overline{\phi}_{j+1}} = \frac{\overline{\phi}_{j+1}^{\,T}\, m\, \phi_j}{\overline{\phi}_{j+1}^{\,T}\, m\, \overline{\phi}_{j+1}} \tag{2}$$

The displacement vector obtained is then normalised, and the whole process is repeated until convergence of the eigenvector and eigenvalue.

The inverse vector iteration method is fundamental to the developed approach, though it is employed within a simplified framework that avoids assembling the stiffness matrix. In this respect, the iterative approach starts with an arbitrary mode shape, and the output is a displacement vector that balances the applied forces associated with the input mode vector.

3. Proposed approach and verification

A simplified numerical approach for establishing the fundamental mode of vibration has been developed. The approach relies on the inherent concept of the inverse iteration method, commencing with an arbitrary mode shape consisting of two translational and one planar rotational acceleration components at a predefined point across all the floors of the building (Figure 1).

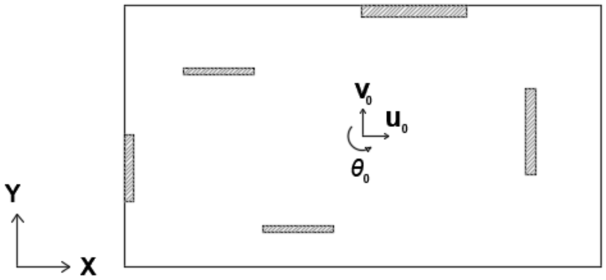

Figure 1: Typical floor plan – components of mode shape applied at floor level

Assuming that the floor slab acts as a rigid diaphragm, the associated inertia forces are determined accordingly. Equilibrium is satisfied as the storey forces are distributed in proportion to the respective stiffness of the walls, noting that the stiffness is approximated assuming the rotation at the top of wall to be restrained and neglecting the torsional stiffness of the walls. Next, compatibility of wall displacements has to be enforced to ensure that the walls are deforming in compliance with the top and bottom rigid floor diaphragms. Therefore, the force distribution at each level is re-established, and corrections are applied in order to restore compatibility conditions. In the process, correcting the forces at the current level alters the distribution of forces in the lower levels, as the moment produced at the base of the floor carries over to lower levels and as such it alters the distribution of forces at these levels (Figure 2). Considering this effect is crucial for maintaining the rotational equilibrium at the interface between the floors, and the procedure may not converge at all if this effect is overlooked. Furthermore, as the forces are corrected at the current floor level, the effect of having a rotational flexibility at the base of floor is taken into account as the latter increases the translational and rotational flexibilities at the top of the floor.

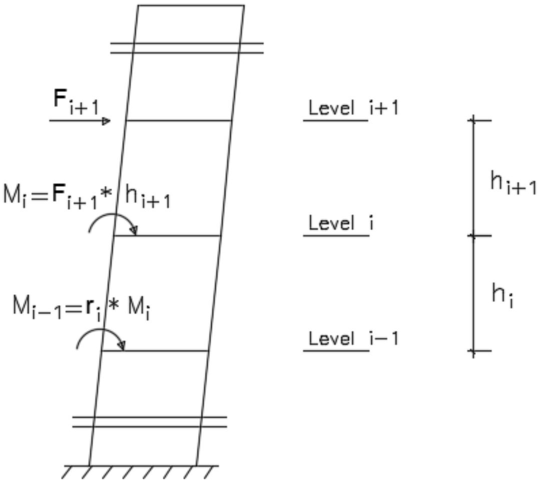

Figure 2: Typical wall elevation – effect of force correction on lower floors

After successfully satisfying equilibrium and compatibility requirements, the floor resultant displacements are calculated, and an estimate of the eigenvalue is obtained by evaluating Rayleigh's quotient as outlined before.

The developed procedure has been implemented using the symbolic computational tool Maple [5] and tested for a 10-storey building (Figure 3), where the results have been shown to perfectly correlate with conventional eigenvalue analysis performed using ETABS (Figure 4).

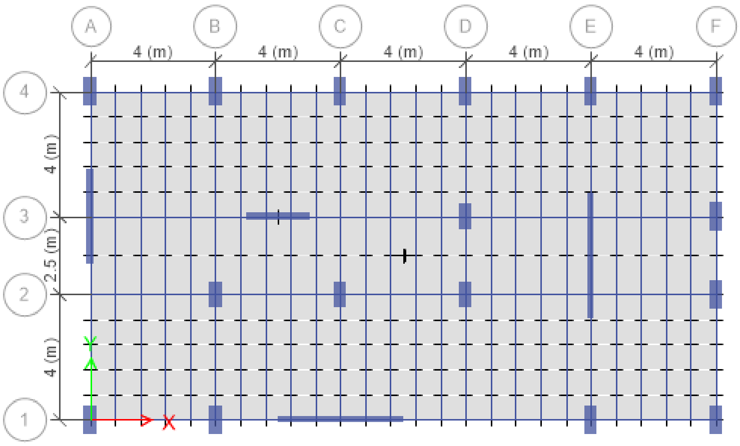

Figure 3: Test building – typical floor plan

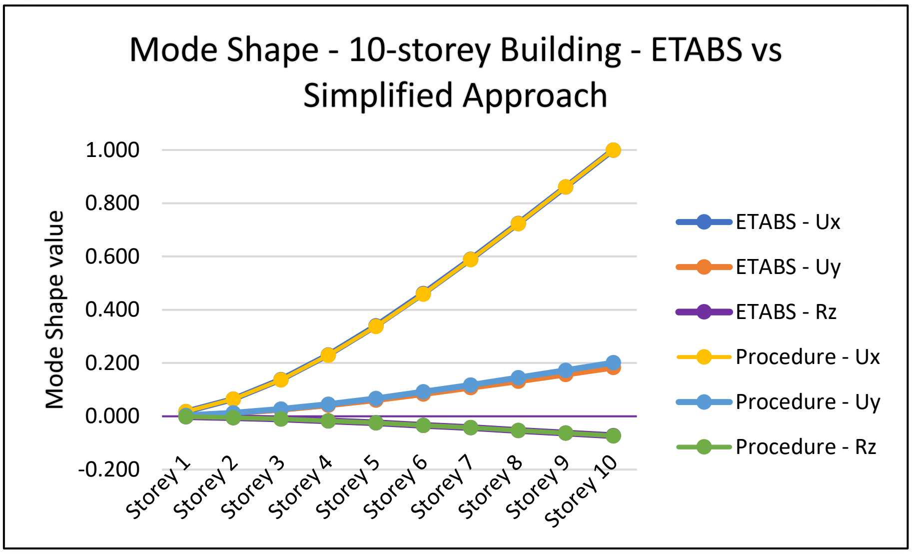

Figure 4: Mode shape results – ETABS vs. simplified approach

4. Conclusions

A new approach has been developed for simplified evaluation of the fundamental vibration mode of multi-storey building structures in which the lateral resistance system consists mainly of shear walls. The developed approach offers a practical and reliable alternative to conventional eigenvalue analysis, avoiding the assembly of the stiffness and mass matrices and direct solution at the overall system level. Furthermore and most importantly, the powerful advantage of the approach lies in the fact that the iterations required to solve the static problem are coupled with the iterations required to find the actual mode shape. After satisfying equilibrium of applied loads with the internal forces in the walls at each floor, the developed procedure proceeds on a floor by floor basis, starting from the base and moving up the levels, in order to enforce compatibility taking into consideration all the correction effects described earlier. Hence, after every iteration for correcting the forces to satisfy compatibility, the floor displacements are normalised and used as the new input mode shape vector for the next iteration until convergence. This substantially reduces the computational requirements, with the approach requiring only a few iterations to converge to the fundamental mode shape and the corresponding fundamental frequency. The developed approach will be complemented with planned research on simplified pushover analysis, paving the way for the practical seismic retrofitting of substandard RC buildings within the CSM framework.

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