EXPLORING DIFFERENT TYPES OF KNOWLEDGE REQUIRED FOR MATHEMATICS TEACHING IN SELECTED SCHOOLS IN MTHATHA.

by

GODSWAY KOFI SENOO

(214571806)

A research thesis submitted in fulfilment of the requirement for the degree of

MASTER OF EDUCATION (M.Ed)

(Mathematics Education)

at

UNIVERSITY OF KWAZULU- NATAL
(EDGEWOOD)

SUPERVISOR: Professor Vimolan Mudaly

DECEMBER 2018

ABSTRACT

The study sought to explore different types of knowledge for teaching mathematics in selected schools in Mthatha in South Africa. A survey design which used both quantitative and qualitative aspects of research was used in the study. Task sheets, observational schedules and interview schedules were used to collect data. Participants were made up of 6 Grade 9 mathematics teachers from 6 schools out of 25 schools in circuit 3 in the Mthatha District of Education. Descriptive statistics and content analysis were used to analyse the data. Frequency tables, pie charts and histogram graphs were used to present quantitative data based on observation while verbal quotes were presented to support themes that emerged from qualitative data gleaned from task sheets and interview schedules. These were analysed by means of content analysis. The findings of the study revealed that mathematics knowledge (conceptual understanding, procedural fluency, strategic competence, adoptive reasoning and productive disposition), knowledge of instructional practices (curriculum, tasks and tools for teaching) were limited with regard to most of the teachers in the Mthatha District. These related to many factors such as unqualified mathematics teachers, lack of in-service training, inadequate teaching and learning material, teachers' attitudes towards mathematics etc. It was recommended that the Department of Education should organize regular in-service training for mathematics teachers in order to improve the quality of mathematics teaching and also keep mathematics teachers updated. There is also a need for department to supply adequate teaching and learning resources to schools in order to improve teachers' knowledge of instructional practices. Furthermore, teachers need to update themselves in order to acquire sound pedagogical content knowledge for effective teaching. The Department of Education should try to motivate mathematics teachers by financing their efforts to upgrade themselves. Moreover there is a need for the department to strengthen their supervision team in order to monitor mathematics teachers in schools. This will help teachers to prepare better for mathematics lessons.

DECLARATION

I, Godsway Kofi Senoo, sincerely and solemnly declare that this is my own work and that all other sources that I have used or quoted have been indicated and acknowledged by means of complete references. I also indicate that I have not submitted this study previously for a degree at any University.

CANDIDATE'S NAME : Godsway Kofi Senoo

CANDIDATE'S NUMBER : 214571806

CANDIDATE'S SIGNATURE : **……………………………………**

DATE : **…………………………………….**

SUPERVISOR'S NAME : Prof. Vimolan Mudaly

SUPERVISOR'S SIGNATURE : **…………………………………**

DATE : **…………………………………..**

ACKNOWLEDGEMENT

I would first of all like to express my gratitude to Almighty God for his guidance throughout this study.

I also wish to render a special thanks to my supervisor, counsellor and mentor Professor Vimolan Mudaly for his support, constructive criticism, patience and ongoing advice towards the success of this study.

My profound appreciation and sincere thanks go to all those who gave their time and assistance towards the completion of this study.

Special thanks go to the Eastern Cape Department of Education, Mthatha District of Education, principals and teachers of various schools for granting me permission to conduct this study.

I would like to express my appreciation to my lovely wife Mrs Patience Senoo and Rev. Charles Koramoah Appiah for their corrections.

Lastly, I would like to express my sincere appreciation to my editor Jill D'Eramo for her wonderful work.

DEDICATION

This study is dedicated to my dear wife, Patience Seyram Senoo, my children, Joshua Ken and Juanita Karen for their moral support and assistance throughout this study.

TABLE OF CONTENTS

LIST OF APPENDICES

LIST OF TABLE

LIST OF FIGURES

ACRONYMS AND ABBREVIATIONS

ANA	Annual National Assessment
CDE	Centre for Development and Enterprise
DoBE	Department of Basic Education
DoE	Department of Education
E	Evident
FET	Further Education and Training
GET	General Education and Training
NE	Not Evident
PCK	Pedagogical Content Knowledge
SE	Slightly Evident
VE	Very Evident

CHAPTER 1

INTRODUCTION

1.1 THE PROBLEM AND CONTEXT

The process of learning mathematics in school is enormously complex. Children need to learn and understand Mathematics with the sense that we live in a Mathematical world and everyone needs Mathematics at all times and anywhere. Thinking and solving problems mathematically in work place is in high demand. In view of this mathematics knowledge needs to improve in order to reduce learners' poor performance. In light with this, the study sought to explore different types of knowledge teachers demonstrate in class during teaching and learning mathematics. This chapter presents the problem and its context. It covers the background to the study, statement of the problem, sub-research questions, objectives, rationale for the study, theoretical framework and the significance of the study.

1.2 BACKGROUND TO STUDY

Learners are performing poorly in the National Senior Certificate Examination (CDE) as far as mathematics is concerned and this poses problems for educators, universities and employers (CDE, 2011). According to the (CDE, 2011), records have shown the mathematics learners who passed with 40% and above from 2008 to 2011 are shown on the table 1.2 below:

Table 1.2: Candidates' performance in mathematics, 2008-2011 at 40% and above level.

Year	Total number of candidates registered in mathematics	Percentage of candidates who got passes above 40% pass level
2008	300008	29.9
2009	290407	29.4
2010	263034	30.9
2011	224635	30.1

(Reports on National Senior Certificate Examination, 2011).

According to Shepherd (2012), in terms of mathematics and science knowledge, South Africa was placed 137th out of 139 countries by world economic forum. Considering the fact that the South African government spends more on education than any other country in Africa, South African learners still perform poorly. The World Economic Forum (WEF) Global Information Technology Report 2013 ranks South Africa's Mathematics and Science learners second last in the world (Shepherd, 2012).

This poses many questions for the stakeholders, one being "What is going on in the classroom? What are the key challenges facing education and training and how can they be addressed? What content and pedagogical content knowledge do teachers possess in order to be classified as effective?

According to (Shepherd, 2012), in education, stakeholders normally use the quality of the teacher to determine the performance of the learner. Shepherd (2012) highlights the point that South African parents in 2009 who preferred not to let their wards to attend the nearest available education institution were about 13%; these households cited "poor quality of teaching as the reason for doing so" (Shepherd, 2012, p. 2).

Shepherd (2012) alluded to the fact that the quantity and quality of South Africa school teachers were ranked low in term of learner performance in mathematics and science compared to other developing countries. Many mathematics and science teachers cannot manage the subjects properly and poor teaching by teachers in schools also contributes to the poor performance in mathematics and the sciences (Shepherd, 2012).

According to Rigelman (2007), mathematics representation and explanations learners receive from the teachers in the classroom are characterised by the teachers' conceptual understanding or knowledge of the subject. Flexible and fluent thinkers frequently solve mathematical problems effectively. They are optimistic in the use of their abilities, knowledge, methods and techniques to achieve result (Rigelman, 2007). This has been confirmed by Mishra & Koehler (2006) in that teaching is a complex exercise that demands an interweaving of diverse aspects of specialised knowledge. It is helpful for teachers to consider that problem solving in mathematics requires deep mathematical thinking (Stacey, 1989).

According to Etkina (2010), a teacher should be able to acquire exceptional knowledge and abilities that integrate content of the learning area that they are teaching (Etkina, 2010). This is the reason why Kilpatrick, Swafford & Findell (2001) say that knowledge of the mathematical concepts, facts and ideas to be taught are able to underpin mathematical proficiency.

Hill, Rowan & Ball (2005) conducted research to find out whether teachers' mathematical knowledge required for teaching contributed to what learners gained in mathematics. It was found out that teachers' mathematical knowledge was significant for their learners' achievement. The study shows the importance of teachers' knowledge which is needed to be explored further. Shepherd (2012) also investigated "the impact of teacher subject knowledge on learner performance in South Africa." This study revealed that deep knowledge and understanding of subject matter taught were important, but of greater importance was the ability of the teacher to transfer or inculcate that information in a meaningful way to learners. This study, however, created room for investigation to find out what types of knowledge mathematics teachers possessed? (Shepherd, 2012).

Sibuyi (2012) in another study investigated the type of pedagogical knowledge which effective teachers displayed with regard to teaching quadratic functions in Grade 11 using two teachers. It was found that the two teachers displayed adequate knowledge of the subject's content knowledge on quadratic equation but the teachers' knowledge of instructional strategies were limited. Shuhua (2000) views that "if you want to give the students one cup of water, you should have one bucket of water of your own" (p. 12). This means more knowledge and skills are needed as a mathematics teacher before standing in front of learners, especially in South Africa at present.

According to Kilpatrick et al (2001), effective, well prepared and hard working teachers tend to understand clearly what to do in the classroom; they have positive mindset towards their learners and manage them appropriately, they accept responsibilities when it comes to challenges from their learners and effectively work towards progress, growth and development. Studies have revealed that it is very important to train and equip teachers since they assist them to possess sufficient knowledge to teach with confidence and effectiveness. It is necessary for teachers to understand mathematics they teach by taking into consideration their learners' current mathematics thinking in order to adopt strategies to address them.

1.3 STATEMENT OF THE PROBLEM

It has been highlighted in the background to the study that this trend in poor performance in mathematics across the nation is becoming alarming. This poses many problems for stakeholders such as educators, universities, employers etc., despite the fact that mathematics as a subject is the foundation of the country's economy. Thus, development of the country depends on mathematics as far as technology is concerned. In this regard, there were many unanswered questions. Though many researches have delved into the problem in order to find solutions, it is necessary to continue delving until a solution is found. The present study therefore attempted to explore different types of mathematics knowledge teachers demonstrate in class with special reference to the Mthatha District.

1.4 OBJECTIVES OF THE STUDY

The study was designed:

1. To qualitatively explore mathematics teachers' mathematical knowledge using mathematics topics such as measurement, algebraic expressions, algebraic equations, and so on, in Grade 9.
2. To provide descriptive ways of assisting the existing mathematics teachers to improve their teaching knowledge for effective teaching.

1.5 RESEARCH SUB-QUESTIONS

The current study sought to answer the following sub-research questions

1. What types of knowledge are necessary for teaching Mathematics?
2. What kinds of knowledge do the selected teachers demonstrate in their teaching of mathematics?
3. How do they use this knowledge in their teaching?

1.6 THE THEORETICAL FRAMEWORK

Research requires a theoretical base which can be used to collect and analyse data. Theory enables coherence and systematic research work (Orodho, 2004). This study is based on

Kilpatrick's et al (2001) model of teaching for mathematical proficiency (Kilpatrick et al 2001). The theory suggests that "Mathematical Proficiency provides a better way to think about Mathematics learning than narrower views that leave out key features of what it means to know and be able to do Mathematics". It continues to explain that a child begins to learn Mathematics before he/she starts schooling, this tells us the kind of knowledge he acquires to support his/her learning process in school. It also implies that the child's social setting affects his/her learning process as far as Mathematics is concerned. In other words, social settings create basic knowledge for one another. Young children have the ability to acquire fundamental competencies in mathematics and these early basic foundations enable them to understand more complex mathematics in future (Kilpatrick et al 2001; Oppermann, Anders, & Hachfeld, 2016). Studies on teachers' knowledge have revealed that effective and skilful teachers can change mathematical ideas by ways that are influential in pedagogy and adaptive to learners' abilities and backgrounds (Miheso-O'Connor Khakasa & Berger, 2015). This indicates that teachers with good PCK can find means of building upon mathematical knowledge children bring to school and this study suggests that there is a need to explore different knowledge that teachers have to transform in learners. Teachers' professional knowledge is taken as the basic requirement for laying quality foundation as far as early childhood education is concerned (Miheso-O'Connor Khakasa & Berger, 2015). It is very important for teachers to have more opportunities to learn about learners' mathematics thinking and to critically find ways of assisting its progress and growth.

The theory reflects "three kinds of knowledge that are evident for teaching school mathematics." These are "knowledge of mathematics, knowledge of students and knowledge of instructional practices" (p. 370). It explains that this knowledge is necessary since they are the cornerstone for teaching mathematics for proficiency (p. 373). According to Lee et al. (2018), knowledge of the teacher is the basis of instructional practices in their learning environment. Also, teachers with high subject matter knowledge can maintain and develop good mathematics performance of their learners than those who lack or have limited subject matter knowledge (Lee et al., 2018). In other words, teachers who are lacking mathematics teacher knowledge can have negative impact on the learners' mathematics skills and performance (Jacobson & Kilpatrick, 2015).

The theory also explains five components necessary for anyone to be successful in learning mathematics. These components according to Kilpatrick et al (2001, p. 16) are Conceptual understanding, Procedural fluency, Strategic competency, Adaptive reasoning and Productive disposition.

According to the theory, the “five strands are interwoven and interdependent in the development of proficiency in mathematics” (p. 116). It argues that helping learners to acquire mathematical proficiency as a teacher, it is very important to call for instructional programs that talk to all the five strands (p. 116).

The model further explains that conceptual understanding, as the first component of mathematical proficiency, also enables learners to remember what they learnt. This is owing to the fact that; ideas, facts and procedures that are acquired or mastered with assimilation and critical thinking are linked; they can easily be recollected and used, and they can be reconstructed and restructured when failed to recall. The most appropriate way of using connections is when they join related concepts and methods correctly. This has been supported by the social constructivist approach by saying that a constructivist teacher makes a learning environment suitable for the learners to become involved in exciting activities that boosts, motivates and facilitates learning (Vygotsky, 1978).

Kilpatrick et al (2001) emphasize that teaching and learning for mathematical proficiency offers the chance for the teachers and learners to “add significantly to the knowledge base on teaching and learning mathematics” (pp. 358). Thus, when the five strands are used interchangeably by teachers and learners wisely, they help to add to the existing knowledge as far as mathematics is concerned. It also states that learners become serious and learn effectively when they are confronted with high-order questions that focus on critical thinking and problem- solving as well as skill development. This is proved and confirmed by many researches.

This theory applies to the study since it relates to different knowledge that should be used by teachers and learners in order to attain mathematical proficiency. Learners can only do well and take an interest in mathematics if attaining mathematical proficiency becomes the target. Mathematical proficiency however, cannot be attained in isolation if the knowledge used in class is not addressed. The types of knowledge that are used in class therefore need to be monitored for effective teaching and learning.

1.7 THE SIGNIFICANCE OF THE STUDY

This study is expected to benefit learners, teachers, curriculum designers, researchers, parents, the Department of Education (DoE) and the government of South Africa. The following paragraphs carry the details.

The study is expected to identify different types of mathematical knowledge that are demonstrated by mathematics teachers during teaching so as to compare them with the required knowledge that is supposed to be used. This helps to eliminate certain flaws in teaching mathematics in Grade nine in the Mthatha District. Addressing this knowledge may result in effective teaching and learning of mathematics that may help eradicate poor performance.

The study will assist teachers to identify certain flaws in teaching their learners. Some of these flaws may have contributed to poor performance in mathematics in the past. It may therefore help with appropriate policies, strategies, and teaching methods in order to improve effective teaching of the subject for better result. Using appropriate mathematics knowledge in teaching may also change learners' attitude positively towards mathematics which will help improve performance. It may further encourage teachers to take in-service training for mathematics teaching in the district seriously.

This study is also expected to help curriculum designers to plan appropriately towards teaching of mathematics. It is necessary that curriculum designers take many factors into consideration before implementation, especially when it comes to curriculum knowledge, as explained by Kilpatrick et al (2001). Teachers are required to be supported with tools, ideas etc as far as instructional practices are concerned. Such ideas should be found in approved textbooks and workbooks which are accompanied by teaching and learning support material. When these aspects are well planned, it helps teachers to implement the expected knowledge in teaching the subject.

The study also aimed to contribute to the development of the education system, especially mathematics education and various aspects regarding the introduction of the new curriculum by the South African Department of Basic Education. It will enable the department of education to monitor and organise in-service training and workshop for the teachers on a regular basis in the district and in the country as a whole.

The study will serve as a baseline for future researchers in the area of mathematics teaching, knowledge and performance. It will also serve as a basis for further research in the various departments in schools, colleges and universities in the country. Furthermore, this study will give the parents some understanding concerning the knowledge of mathematics teachers so that they can find a way of assisting their children in their mathematics school activities at home. Parents should assist them to be abreast of the system so that they know what is necessary to facilitate effective mathematics teaching and learning in schools.

Finally, findings that are discussed in this study may be used by the government in the implementation of its strategies in minimizing the appalling failure rate in mathematics in the country.

1.8 DELIMITATION OF THE STUDY

The study was conducted in circuit 3 in the Mthatha Education District in the Eastern Cape Province. Circuit 3 comprises 25 schools. It focused on six Grade 9 teachers from different schools in that Circuit. The selected teachers were chosen from two high-achieving schools, two average-performing schools and two low-performing schools in the Education District. These schools were identified by using the Annual National Assessment (ANA) results in the circuit through the mathematics subject advisor. These participant teachers were used to establish different types of knowledge teachers demonstrate when following the mainstream educational programmes in the district.

1.9 LIMITATIONS OF THE STUDY

The study was conducted in one district (Mthatha) only; the findings cannot be generalised and it can only be used within the context of the Mthatha District which falls under Department of Education. The findings of the study can however, serve as a starting point for further research projects in mathematics education in schools in South Africa.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Chapter Two looked at the related literature reviewed on the existing knowledge based on the different types of knowledge offered by the teachers in teaching of mathematics in schools. Chapter further highlights on critical aspects of different types of knowledge and their relevance to the teaching and learning of mathematics. This chapter also focuses on the importance of mathematics teachers' understanding of mathematics subject matter and having a sense of efficacy in the teaching and learning of mathematics, various teaching methods that can be used in class by mathematics teachers, an overview of the quality of mathematics teachers' content knowledge in the subject and a conclusion.

2.2 A CRITICAL LOOK AT DIFFERENT TYPES OF KNOWLEDGE AND ITS RELEVANCE TO TEACHING AND LEARNING OF MATHEMATICS

Shulman's (1986) introduction of Pedagogical Content Knowledge (PCK) really highlighted on different views of researches on how teachers are knowledgeable in their subject matter and the significance of using this knowledge for teaching mathematics effectively. Teachers need to understand and be conversant with two types of knowledge: "(a) content, also known as "deep" knowledge of the subject itself, and (b) knowledge of curricular development" (Shulman, 1986, p. 4). Content knowledge encompasses the "structure of knowledge" including "the theories, principles and concepts of a particular discipline. Content knowledge also deals with the teaching process, including the most useful forms of representing and communicating content and how students' best learn the specific concepts and topics of a subject." According to Shulman (1986), "if beginning teachers are to be successful, they must wrestle simultaneously with issues of pedagogical content (or knowledge) as well as general pedagogy (or generic teaching principles)". The model of Pedagogical Reasoning which was developed by Shulman includes "a cycle of several activities that a teacher should complete for good teaching: This includes: comprehension, transformation, instruction, evaluation, reflection, and new comprehension" (Shulman, 1986, p. 4).

Solis (2009) concurred with Shulman's (1986) idea of pedagogical content knowledge refers to teachers' interpretation and transformation of subject-matter knowledge that is appropriate for promoting learner learning. He (Solis, 2009) further went to say that the key elements of pedagogical content knowledge of teachers in supporting learners' learning revolve around the following:

"(i) knowledge of representations of subject matter (content knowledge); (ii) understanding of students' conceptions of the subject and the learning and teaching implications that were associated with the specific subject matter;(iii) general pedagogical knowledge (or teaching strategies); (iv) curriculum knowledge; (v) knowledge of educational contexts; and (vi) knowledge of the purposes of education" (Solis, 2009, p.2).

Solis's (2009) study directs teachers to incorporate in their teaching, the idea of pedagogical content knowledge that should embrace theory and practice gained from ongoing teaching activities.

Furthermore, he indicated that teachers' progress in pedagogical content knowledge in most cases influenced by some factors such as the background of the teacher as well as the environment in which they work. However, whatever the case may be, teachers' method of teaching should change the lives of learners for the better. Teaching of content and pedagogy should be taught by taking into consideration their purpose in the learning process; thus, learners have to put into practice what they were taught. Teachers' actions of presenting a lesson will be determined by the level of their pedagogical content knowledge, establishing the fact that this is very important component of their learning process.

A study conducted by Sibuyi (2012) on the type of Pedagogical knowledge which effective teachers display with regard to teaching quadratic functions in Grade 11 using two teachers found out that the two teachers displayed adequate knowledge of the subject's content knowledge on quadratic equation but their (teachers') knowledge of instructional strategies were limited during presentation of their lessons.

According to Matthew (2011), Shulman's (1986) introduction of PCK points out the importance of teachers' subject knowledge and pedagogy which were carefully delved into by many researchers. The practical result of such researches was establishment of teacher education and training programs which were influenced by either subject matter or pedagogy. In order to help solve this problem, a proposal by Matthew (2011) suggests that one should take into account the necessary relationship between the two concepts when it comes to introducing the idea of Pedagogical Content Knowledge (Matthew, 2011).

This knowledge comprises of understanding what teaching techniques and methods suit the content, and likewise, assimilating how elements of the content can be organized for effective teaching. PCK is characterized by the presentation and creation of concepts or ideas, learning strategies, knowledge of interpreting concepts and knowledge of learners' previous knowledge. It also includes knowledge of teaching strategies that combine appropriate conceptual representations to deal with learner challenges and misconceptions and promote effective learning in class (Matthew, 2011).

Kilpatrick et al (2001) also introduced three kinds of knowledge for teaching school mathematics. These are "knowledge of mathematics, knowledge of learners and knowledge of instructional practices" (p. 370).

2.2.1. Knowledge of mathematics

Mathematical knowledge includes knowledge of mathematical facts, concepts, procedures and their relationship. It also includes knowledge of the ways that mathematical ideas can be shown and displayed, and knowledge of mathematics as a discipline, and in particular, how mathematical knowledge is produced or discovered (Kilpatrick et al 2001, p. 370). Acquiring knowledge for teaching also involves more than having knowledge of mathematics for oneself. It is necessary for a teacher to be conversant with concepts constructively and follow the procedures without fault, but teachers also need to investigate the conceptual foundations of that knowledge. They point out that mathematical content must be understood by mathematics teachers in context to enable them to clarify and unpack ideas in a particular manner (Kilpatrick et al, 2001). Unless teachers are ready to maintain that there is professional knowledge that is necessary for effective teaching and which can be supported with evidence, teachers will carry on with common voice among many competing to maintain what they are capable of knowing or understanding (Ball, Hill and Bass, 2005).

2.2.2. Knowledge of learners

The way learners understand mathematics includes general knowledge of how different kinds of mathematical concepts or ideas are developed in learners over a period of time as well as specific knowledge of how to determine where, in a developmental trajectory, a learner might be (Kilpatrick et al 2001, p. 371). Example; guiding learners to undergo mathematics projects, investigations, group discussions, playing mathematics games and so on. Ball, Hill and Bass (2005) pointed out that "although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement," however, "the actual nature and extent of that knowledge--whether it is simple skills at the grade they teach, or complex and professionally specific mathematical knowledge--is largely unknown"(P. 17).

2.2.3. Knowledge of instructional practices

"Knowledge of instructional practice includes knowledge of curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms that support the development of mathematical proficiency" (Kilpatrick et al 2001, p. 372). This implies that knowledge of instructional practice begins when information gotten by the teacher is used to inculcate new knowledge into the learners. This is done by taking into consideration the level of the learners and the content knowledge of the teacher. Effective teaching and learning revolves around knowledge of instructional practices.

2.3 KILPATRICK'S MODEL OF TEACHING AND SOCIAL CONSTRUCTIVISM

According to Jawarsky (1994), Ernest (1991) came up with two key features as an account of social constructivism which are described firstly as active constructivism of knowledge, typically concepts and hypotheses, on the basis of experiences and previous knowledge. The features provide the basis for understanding and serve the purpose of guiding future action. This is similar to the idea of Kilpatrick et al (2001) which says that knowledge that has been gained with assimilation supplies the foundation for promoting new knowledge and for solving new, complex and unfamiliar problems. This implies that new knowledge that has been generated is grounded and fully depended on one's previous knowledge.

Secondly, social constructivists point out that experience and interaction play an essential role in both physical and social worlds. This is confirmed by Kilpatrick et al (2001) in the definition of adaptive reasoning as "the capacity to reason logically about the relationships among concepts and situations" (p.129). It is what social constructivists refer to as interaction with the physical and social world. Thus, new knowledge acquired in mathematics is based on previous knowledge experiences in mathematics according to the social constructivists' perspective which is confirmed by Kilpatrick et al's (2001) model of mathematical proficiency. This is similar to the views of Kim (2001, p. 2) in that social constructivism throws more light on the significance of culture and context in having an idea about what is happening in society and generating knowledge based on this idea. This shows that teaching and learning mathematics in a classroom is related to real life situations as far as society is concerned. Kilpatrick et al (2001, p. 157) further explains that children start to assimilate mathematical knowledge well before they step into elementary school. Children usually develop basic skills, concepts and misconceptions right from the beginning of their initial schooling within their society, where they develop basic prior knowledge as indicated by constructivist ideology (Kilpatrick et al, 2001).

Kilpatrick et al (2001, p. 5) also contend that "at all ages, students encounter quantitative situations outside of school from which they learn a variety of things about number. Their experiences include, for example, noticing that a sister received more candies, counting the stairs between the first and second floors of an apartment, dividing a cake so every-one gets the same amount and figuring out how far it is to the bus stop". That is why social constructivists say that there is an active constructivism of knowledge, typically concepts and hypotheses, on the basis of experience and previous knowledge which provide understanding in future. This was earlier corroborated by Vygotsky (1978) who maintained that cognitive functions begin within, and must therefore be clarified as a product of social interaction. He also assumed that learning was not solely the assimilation and acquisition of new knowledge by learners; instead, it was the situation by which learners were racially mixed into knowledge of the community. Kilpatrick et al (2001) therefore explain conceptual understanding as "an integrated and functional grasp of mathematical ideas" (p. 118) and they (Kilpatrick et al) view concepts and ideas from a social perspective; that is, that learners base their understanding on the concepts and ideas acquired from the society which stimulate their “appetite” for learning mathematics effectively in the classroom. This helps learners to relate to real life situations in the environment. Knowledge is therefore actively constructed by learners in response to interaction with environmental stimuli. Vygotsky (1978) emphasized the duty of language and culture in cognitive development, when saying language and culture play very important role in the development of human thinking as well as how they (humans) view the world. This has been supported by Jaworsk (1994) who emphasises the necessity of merits and demerits of shaping as far as language is concerned since this is the area human culture takes place and the situation where rules govern language are built by an individual with the vigorous functional results surfaced around us in human society. In addition, It Jaworsk (1994) explains that socio cultural and socio emotional contexts of learning highlight the control role of language in learning and points out the learner as an interactive co-constructor of knowledge. This is confirmed by Kilpatrick et al (2001, p. 343) when stating that “in the mathematics class the teacher naturally interacts differently with different learners. Sometimes, however, differential interactions are associated not with differences in mathematical ability or accomplishment but with differences in learners’ social class, ethnicity, language, or gender”. They (Kilpatrick et al, 2001) continue to elaborate by stating that effective teachers demonstrate an understanding of their learners’ backgrounds and experiences, link classroom content to those experiences and use familiar cultural patterns absorbed by them (children). This supports Vygotsty’s (1978) views which states that all cognitive functions start within the body, and later expanded as products of, social interactions. He contended that learning is not just an assimilation and accommodation of new knowledge by learners but is the procedure by which learners are racially mixed into the community of knowledge. He (Vygotsky) further explained that the ability of every child to develop culturally happens in two levels, thus, first, on social level and later on the individual level. This means that individuals are developed culturally by starting to learn from the society through their own internal drive. In other words, one can only learn with enthusiasm if he/she is interested. It is believed that these levels are usually applied equally to individual observation, to logical reasoning, and to the development of concepts and ideas (Vygotsky, 1978). This also implies that knowledge is not merely built in the life of a learner but it is built with understanding which mean that there should always be pre-knowledge which will bring out new knowledge. In other words, there must be an existence of old knowledge from which new knowledge will be "extracted" and it is done through interaction with the environment.

One of the assumptions of social constructivism is learning. According to Kim (2001) social constructivists believe learning as a social procedure. It does not happen only within an individual, nor is it a passive growth of behaviours that modelled by outside forces. This was supported by Vygotsky (1978) in that knowledge development is a social phenomenon; learners are sometimes motivated by reward given by the knowledge community which includes knowledge of the teacher. In most cases, knowledge is actively constructed by the learner, however learning also depends to a significant extent on the learner's internal drive to understand and promote the learning process. Kilpatrick et al (2001, p. 123) agree with this by saying that understanding enables "learning skills easier, less susceptible to common errors, and less prone to forgetting." In other words, a certain level of skill is demanded to acquire many mathematical concepts with understanding, and using procedures can help build up and develop that understanding. They (Kilpatrick et al, 2001) continue by saying that children do not easily forget what they previously learnt when they are taught the new ones. Instead they consider using either the old concept and procedure or the new one to find solution to a problem based on the challenges they might be facing. They (Kilpatrick et al, 2001) state that if a person starts learning with concentration and understanding this can help him/her to learn more efficiently (Kilpatrick et al, 2001, p.123). This aligned with social constructivism in that learning depends, to a significant extent, on the learner's internal drive to understand and promote the learning process.

According to social constructivists, perceiving learning as an individual and social process enables teachers to understand their learners. This is because; the construction of an individual and the society in the learning process gives teachers a conceptual framework for effective teaching and learning. It is believed that a constructivist teacher needs to plan ahead in order to provide reliable teaching and learning materials that supports learners learning by engaging and directing them in an interesting activities or problem-solving in order to arouse their learning interest at all times. In other word, it is the duty and responsibility of a constructivist teacher to make sure boredom is avoided during teaching and learning process. The teacher does not just stand aloof and observe children explore and discover; instead, the teacher may often guide learners as they approach problems. Kilpatrick et al (2001, p. 32) confirm this by stating that “the teacher of mathematics plays a critical role in encouraging learners to maintain positive attitudes towards mathematics. How a teacher views mathematics and its acquisition affects that teaching practice, which ultimately affects not only what the learners think but how they view themselves as mathematics learners.” Moreover they say that teachers and learners agree among themselves about the norms of conduct in the learning environment, and when norms allow learners to enjoy doing mathematics irrespective of the challenges and sharing their ideas with others, they realise themselves of having the ability to understand. This shows that meaningful learning happens if there is mutual collaboration between learners and the teacher. This is supported by Prawat (1992) and Brown et al (1989 b) who “view the teacher as a kind of expert guide who helps learners as novices traverse new cognitive territory while enculturating them into a particular disciplinary community”. That is why Kilpatrick et al (2001) say that one’s teaching depends on what is learned. Selecting the content, taking decisions on the means of introducing it and finding out how much time to apportion to it are means in which learning is influenced by how the teacher understands and interacts with the content. In other words, learning according to social constructivists’ perspective is corroborated by Kilpatrick et al (2001): it can be done effectively when responsible learners are guided by a well- organized teacher in order to come up with social concepts and ideas.

According to Prawat (1992), it is very necessary to consider classroom environment as a place where knowledge is acquired by individuals through different means such as exploration of ideas, asking questions, solving difficult or high order questions and making mistakes. Kilpatrick et al (2001) explain that “learning is not an all-or-none phenomenon; as it proceeds, each strand of mathematical proficiency should be developed in synchrony with the others.” (p.115).They believe progress in children is a gradual process; one of the challenges teachers go through in pre-Kindergarten to the 8^{th} grade is to see that learners are making progress along every strand and not just one or two. One of the beliefs of social constructivism is for the teacher to create a very conducive learning environment that paves way for learners’ engagement in an interesting activity to encourage self-learning among learners.

Learning mathematics according to the social constructivist perspective is interaction with others. There are many advantages that one can get from the implementation of discussion in the mathematics classroom. Participation in group discussions offers an opportunity to learners to be creative and transfer what they were thought or learnt in classroom and in so doing build a strong foundation for communicating ideas orally.

This is confirmed by Kilpatrick et al (2001) when they maintain that a classroom norm can be established in which learners are expected to prove and clarify ideas in order to clear their thoughts, sharpen their reasoning abilities and boost their conceptual understanding. Clearly, discussion in mathematics class increases learners' ability to test their ideas, synthesize the ideas of others, and build deeper understanding of what is being taught and learnt in class. Vygotsky (1978, p. 85) says that the level of potential development is the level of achievement that the learner is having the capacity of reaching under the direction of teachers with peers. The learner has the ability to think and solve problems with understanding at this level that they are actually not capable of solving or assimilating at their level of actual development. Interaction is needed during the learning process. It clears misconceptions and helps learners to be confident. One of the best ways to help learners to be positive minded towards learning process is to engage them in several discussions. In other words, learners become interested and enjoy learning mathematics when they interact with the teacher and peers on regular basis. According to Kilpatrick et al (2001), in mathematics classes the teachers naturally interact differently with different learners. Sometimes, however, the way and manner learners interact are associated with their social class, ethnicity, language or gender not with difference in mathematical ability or accomplishment.

It is very important for the learners to interact with knowledgeable people in the society to enable them understand their society. It is not possible to understand important symbol system in the society and master how to use them without interacting with more knowledgeable members in the society. Young children begin to think critically by interacting with experience people (Kim, 2001). It was justified by Kilpatrick et al (2001) concur when saying learners need to prove their points mathematically and also make others to understand. Mathematical ideas develop in a child over a period of time as well as particular knowledge of how to regulate where in a developmental trajectory a child might be. This includes familiarity with usual challenges that learners have with some mathematical concepts and procedures and it involves knowledge about learning to understand and about the kinds of experiences, designs and ways that arouse learners' critical thinking and learning of mathematics.

Teachers and learners are considered to be active in making sense in context which always enables them to understand each others' words and actions as they interact (Jawarsky, 1994). This is because the success of instruction depends critically on the quality of the interaction of teachers and learners around the content. Moreover the most successful and efficient teachers are not simply tactful of "the cultural diversity of their students but use that diversity to enrich the learning experiences they provide to the class as a whole" (Kilpatrick et al 2001, p. 344).

Another point worth mentioning is reality. One of social constructivism's assumptions is reality. Social constructivists assume that reality is developed through human activity. According to Jawarsky (1994), the best thing to do is to develop mathematical concepts or ideas in such a way that they match with what the world experiences on daily basis. Learners need to see reality of mathematics as they do mathematics. According to constructivists, reality can only be built and established through human activity. Reality cannot be innovated or invented; it does not exist prior to its social invention. This is because reality can only be perceived and believed in existence. A perception of a real object is one of the special features of human. Mathematics must be seen with sense and meaning which bring about its reality. Kilpatrick et al (2001) say learners who are capable of developing a productive disposition always have confident in themselves as far as knowledge and ability are concerned. They believe that with determination and conscious effort they can learn since mathematics is both reasonable and intelligible. Mathematics should make sense according to people who are proficient in mathematics that they can figure it out (reality), that they can come out with solutions of mathematical problems by putting more effort on them, and that becoming mathematically proficient is worth the effort. The reality of mathematics can be seen if teaching and learning of mathematics is incorporated with a real object in the environment, according to social constructivists' perspective. Mathematics facts or ideas may be objective when they are usually acknowledged and when they are not matter for an individual preference or opinion (Jawarsky, 1994).

2.4 THE IMPORTANCE OF MATHEMATICS TEACHERS' UNDERSTANDING OF MATHEMATICS SUBJECT MATTER AND HAVING A SENSE OF EFFICACY IN THE TEACHING AND LEARNING OF MATHEMATICS

To teach mathematics to all learners according to today's level of education, teachers have to deeply assimilate subject matter in order to enable them in assisting learners to develop useful cognitive maps, connect one concept to another, and to clear every misconception that are being conceived by the learners. Teachers need to envisage or perceive how mathematics concepts connect across fields and to everyday life. Teachers can only make ideas or concepts available to others if they have good pedagogical content Knowledge (Shulman, 1987).

According to Kilpatrick et al (2001, p. 338), teachers who are effective and highly productive in class appear more confident in the teaching and learning environment. They also appear to be more positive and less critical with their learners, they appear to be good instructors in class, to be more accepting and effective in solving problems in class, and they also appear to be more supportive in development and growth. Teachers' senses of efficacy emphasize the necessity of training teachers so that they can obtain adequate mathematical knowledge to do quality teaching of mathematics with confidence.

Shuhua (2000) as already stated also points out that "if you as a mathematics teacher want to give the learners one cup of water, you should have one bucket of water of your own" (p. 12); more knowledge and skills are needed as a mathematics teacher before standing in front of learners to teach.

Shepherd (2012) investigated the impact of teachers' subject knowledge on learner performance in South Africa. This study revealed that deep knowledge and understanding of subject matter taught were important, but of greater importance was the ability of the teacher to transfer that information in a meaningful way to learners. This study however created a room for investigation: to find out what types of knowledge mathematics teachers possess (Shepherd, 2012).

According to Etkina (2010), a teacher should be able to acquire special understandings and abilities so that he/she can integrate the content of the learning area that is being taught. This is the reason why Kilpatrick et al (2001) concur and say that deep understanding of the content to be taught is very important in promoting mathematical proficiency. Hill, Rowan and Ball (2005), prior to this, had conducted research to find out whether mathematics teachers' level of understanding mathematics for instruction contributed to what learners gained in mathematics. It was realised that mathematics teachers' knowledge was significant for learners' progress and achievement. Their study showed the importance of teachers' knowledge and this needs to be explored further.

According to Rigelman (2007), mathematics representation and teachers' explanations to learners during teaching and learning are characterised by the teachers' conceptual understanding or knowledge of the subject. Flexible and fluent thinkers are effective mathematical problem-solvers. They use their knowledge and processes with confident (Rigelman, 2007). This was confirmed by Mishra and Koehler (2006) in that teaching demands a combination of many categories of specialised and skilful knowledge since is a complicated practice. It is helpful for teachers to consider that problem-solving in mathematics requires deep mathematical thinking (Stacey, 1989).

2.5 VARIOUS TEACHING METHODS THAT CAN BE USED IN CLASS BY MATHEMATICS TEACHERS

Methodology refers to the means or a strategy adopted by the teacher in trying to impact on knowledge for the learner during teaching and learning. Asikhia (2010, p. 4) explains teaching method as the technique or preparation that enables teachers to decide the approach they can use to accomplish the expected objectives. This includes the way teachers plan to use various techniques and strategies related to the subject matter, teaching tools and teaching materials that enable the teacher to achieve objectives of the lesson. An evaluation may be carried out at the end of the lesson to assess learners' performance in terms of behaviour or instructional objectives. What is necessary for the teacher to do is to examine his/her teaching methods and techniques instead of looking at learners as the cause of low performance (Asikhia, 2010).

According to Asikhia (2010, p. 2), most unqualified and unprepared mathematics teachers blame learners rather than themselves when learners are not able to do well during evaluation at the end of the lesson or in examination. “Teachers’ planning should include: choice of appropriate teaching material, choice of appropriate teaching method, intensive research on the topic to be taught and determination of the objectives for the lesson” (Asikhia, 2010, p. 2).

Douglas (2002) states that teaching strategies comprise interventions used by a teacher to come out with student learning. The learning must have taken place as a function or a correlate of the approaches that the teacher used. Four components are involved in teaching: the learner, the teacher, the curriculum, and the learned repertoire. The foundation of learning involves at least these four components of instruction that integrate both learner and teacher interaction and it stimulates learners’ interest during teaching and learning.

Ozkan (2011, p. 1) states that “teaching methods can best be defined as the types of principles and methods used for instruction.” There are different kinds of teaching techniques such as participation, demonstration, recitation, rote and relational learning etc but teachers can only use them appropriately after assessing the information or skill they want to pass on to the learners. Learners progress, achievement and success may largely depend on the way teachers effectively handle these teaching methods and techniques.

According to Ozkan (2011), different approaches or ways of learning are known as learning styles. It is necessary for teachers to evaluate the learning styles of their learners and become used to strategies and techniques that are helpful and productive for each learners learning style. Ozkan claims that the most common learning styles are visual, auditory and tactile/kinesthetic. Visual learners have a potential to think in pictures using visual aids such as overhead slides, diagrams, videos, handouts, etc. Auditory learners are comfortable with learning through listening to tapes, discussions with peers, attending lectures regularly etc. Tactile/kinesthetic learners enjoy and become active in learning through moving, touching, conducting experiments, doing science project, involving actively in exploration of the world etc.

According to Gill (2015, p. 1), an effective teaching style allows learners in the learning process and enables them to develop ability to think critically. Traditional teaching styles have evolved with the advent of differentiated instruction, prompting mathematics teachers to adjust their styles toward learners' learning needs. Gill (2015, p. 1) lists the following as some of the teaching styles and strategies mathematics teachers use in the classroom:

2.5.1. Authority, or lecture style: According to Gill (2015), the authority model in the teaching and learning situation refers to teacher-centered or one-way presentations, which usually involves lengthy lecture sessions whereby the teacher dominates the teaching aspect. In this case, learners only listen to the teacher and take notes or record information during teaching process. This type of teaching style is also known as traditional method of teaching. This is only applicable for certain higher-education disciplines where the learning environment is arranged with large groups of learners. This type of teaching style is mostly suitable for subjects such as history and geography that usually calls for learners to learn some fundamental rules, dates and names by heart. While this teaching technique can produce result when combined with other more hands-on methods, it is often not effective when used alone as it does not give learners the opportunity to practice taught concepts. To use the lecture method effectively, the teacher has to pair it with another instructional method (Schreiner, 2015, p. 1). In mathematics classroom, this type of teaching style is not applicable in the sense that it does not allow learners to think critically. Teacher-learner interaction is more important in the mathematics classroom since it helps learners to develop conceptual understanding (Gill, 2015).

2.5.2. Demonstrator or coach style: This type of teaching strategy allows the teachers to exhibit their teaching skills by showing learners what they need to understand in the teaching and learning process. In most cases, this type of style helps teachers to get an opportunity to apply various strategies and techniques such as lectures, multimedia presentations and demonstrations. One of the disadvantages of using this style is that it cannot cater for learners' individual needs in larger class even if it is compatible for teaching mathematics and other subjects such as music, physical education, arts and crafts (Gill, 2015, p. 1).

2.5.3. Facilitator or activity style: In the normal classroom settings, teachers should not be seen to dominate teaching whereby learners are seen as passive learners. They (teachers) should serve as facilitators to learners and guide teaching and learning whereby learners become active in learning. Facilitators help to "promote self-learning and help students develop critical thinking skills and retain knowledge that leads to self-actualization." This helps in training of learners to ask questions and helps develop skills to find answers and solutions through exploration of problems. It further guides teachers to interact with learners and prompt them (learners) towards innovation and invention (Gill, 2015, p. 2).

2.5.4. Delegator or group style: This teaching style is suitable for the curriculum that demands lab activities, such as chemistry and biology, and/or subjects that warrant peer feedback, such as a debate and creative writing which support learners' conceptual understanding. This is a form of guided discovery and inquiry-based learning which place the teacher in an observer position to inspire learners by working together to find solution to a problem (Gill, 2015, p. 2).

2.5.5. Hybrid or blended style: Hybrid or blended style of teaching is an integrated approach that brings together the teachers' personality and interests with learners' needs that yields to the demands of the curriculum. This usually involves the inclusive approach of combining teaching style clusters that enables teachers to strategise their teaching methods which embrace learner needs and appropriate subject matter. Studies have recommended this type of teaching style in mathematics classroom since it runs the risk of engaging learners in too many activities at the same time (Gill, 2015).

The following statement was, however, indicated by Gill (2015, p. 6) to describe the nature of hybrid style of teaching:

"Parents take a decidedly proactive role in child-learning techniques. The traditional authoritative/expert, or lecture style, has come under attack by some parents — and contemporary educational leaders — who emphasize that a more diverse approach to teaching is necessary to engage students." "This is compounded by the rise of "tiger moms", a term made popular by parents devoted to improving the quality of education with laser-precision focus on A-list schools and a highly competitive job market."

One way of using different learning styles is to ensure that classrooms are interactive enough for the benefit of learners. **In interactive classrooms,** laptops and tablets, videoconferencing and podcasts play a vital role in teaching. This style assists teachers to evaluate their learners' knowledge or performance while they are learning with technology in mind. Sometimes, teachers wait for their learners' assessment results, only to come across knowledge gaps that should have been identified during the stage of active learning (Gill, 2015, p. 6).

Another way a teacher can use hybrid teaching style for effective teaching is through **knowledge and information.** Knowledge in this way means a complete understanding, or full comprehension, of a particular subject. A blend of teaching styles that incorporate facilitator, delegator, demonstrator, and lecturer techniques help many learners to acquire in-depth knowledge and to master a given subject. This is completely different from passive learning, which typically entails memorizing facts, or "information, with the short-term objective of scoring well on tests" (Gill, 2015, p. 6). Information here means the correct ideas that the teacher conveys to his/her learners during teaching and learning to eliminate misconceptions. This may include correct word problem activities that help to develop learners understanding of concepts.

2.5.6. Differentiated instruction style: This style refers to the situation where by a teacher keeps all learners in mind when preparing his/her lesson plans, workbook exercises, lectures and interactive learning. This style helps to embrace diverse classrooms for learners at all learning levels and from various backgrounds which further encourages individual differences during teaching and learning (Gill, 2015).

According to Schreiner (2015, p. 1), the following teaching strategies and style are also used by mathematics teachers when teaching mathematics:

2.5.7. Cooperative Learning: In this learning environment, learners are encouraged and also given an opportunity to learn from one another through collaboration. Through cooperative learning, teachers give learners the opportunity to discuss in groups in order to acquire new knowledge from their peers. Most of the teachers usually incorporate cooperative learning into their teaching and learning process by letting the learners to work in groups through completing assignments in pairs or small groups. In this situation, teachers are advised to carefully select groups by pairing stronger learners with weaker rather than allowing learners to select their own pairings (Schreiner, 2015).

2.5.8. Model-Lead-Test Instruction: The Model-Lead-Test approach to mathematics teaching is a highly-structured model which involves three-step processes of teaching mathematics; thus, a mathematics strategy must be first taught by the teacher to learner. They (teachers) then lead by drilling the strategy with the learners through discussion, questioning and answering with the entire class or in small groups. The teacher then concludes this teaching method by assessing learners' understanding of the particular concept under discussion. This approach of teaching is considered to be successful if learners have mastered the strategy taught by the teacher and use it to solve problems without any difficulty (Schreiner, 2015).

2.5.9. Constructivist Techniques: This technique makes teachers to involve learners in meaningful learning by inculcating their (teachers) knowledge into the learners. Constructivist teaching enables teachers to engage learners in activities such as investigations and projects in order for them (learners) to use the results to reach their own conclusions. The learner can relate the method to the knowledge previously acquired and then apply it in real life situations. This technique can be effective in teaching some mathematics concepts, but teachers can face many challenges in using this technique when teaching basic mathematics skills, such as addition or subtraction, as these skills are so basic that learners cannot easily develop how they came about but rather been told about them and also use them accordingly (Schreiner, 2015).

Modern styles of teaching are focused on group discussions, projects, experiments and investigations. Teaching styles such as modelling, coaching, and test preparation through rubrics scaffolding are alternative teaching styles that are adapted into constructivist methods of teaching in order to enhance effective teaching and teaching. They are also incorporated into the teaching and learning to promote student maximum participation and increase the use of hybrid approach to teaching. One of the concerns raised about constructivist method of teaching is that it tends to benefit extrovert and group-oriented learners more than introverts. However, it is considered that introverts do not learn by observing (Gill, 2015).

2.6 AN OVERVIEW OF THE QUALITY OF MATHEMATICS TEACHERS' CONTENT KNOWLEDGE IN THE SUBJECT

According to Shepherd (2012), stakeholders in education always assume that the performance of the learner completely depends on the quality of the teacher. Shepherd (2012) highlights the fact that South African households in 2009 that did not allow their wards to attend nearby available education institution comprised approximately 13%; these households cited "poor quality of teaching as the reason for doing so" (Shepherd, 2012, p. 2). Shepherd (2012) alluded to the fact that the quantity and quality of South Africa school teachers ranked low in terms of student performance in mathematics and science compared to other developing countries. Many mathematics and science teachers cannot manage the subjects properly and also poor teaching by teachers in school contributes to the poor performance in mathematics and the sciences (Shepherd, 2012).

As already touched on, according to Shepherd (2012), South Africa was placed 137th out of 139 countries in terms of mathematics and science knowledge by the world economic forum. Considering the fact that the South African government spends more on education than any other country in Africa, South African learners still perform poorly. It may also interest one to know that The World Economic Forum (WEF) Global Information Technology Report 2013 ranks South Africa's Mathematics and Science learners second last in the world (Shepherd, 2012).

As stated earlier, learners are performing poorly in the National Senior Certificate Examination (CDE) as far as mathematics is concerned and this poses problems for educators, universities and employers. Shay (2019) confirms this by stating that matric mathematics pass rate is creating a lot of problems for universities with the sense that mathematics is always at lowest performing levels. Thus, the percentage pass rate has always been between 30 and 35%. According to Shay (2019), these challenges need solution as soon as possible since mathematics helps one to think critically and systematically to solve problem. It is also a subject that leads to many professions such as engineering, commerce and health science (Shay, 2019).

Asikhia (2010) comments that no profession has suffered a greater reversal of fortune than teaching; this has affected the commitment expected of mathematics teachers. It means that the quality of service given by a lackadaisical mathematics teacher could have negative effects on the learners as far as academic progress and achievement are concerned. How does one clarify a situation whereby high school learners receive an average of 125 hours and 150 hours of teaching as against 250 hours and 300 hours, respectively, per term?

2.7 CONCLUSION

The discussion on the literature was based on the existing knowledge about the different types of knowledge for mathematics teaching in schools. The literature review also included each of the following in the discussions: a critical look at various types of mathematics knowledge and its relevance to the teaching and learning of mathematics, the Importance of Mathematics teachers' understanding of mathematics Subject matter and having a sense of efficacy in the Teaching and learning of mathematics, various teaching methods that can be used in class by mathematics teachers and an overview of the quality of mathematics teachers' content knowledge in the subject.

CHATER 3

THE RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

Chapter three highlight on the following issues: the research design, research methodology and the paradigm of the study.

3.2 RESEARCH PARADIGMS

According to LeCompte and Schensul (1999, p. 41), a research paradigm constitutes the way the world is perceived; interpreting what is seen; and decoding which of the things seen by the researcher are real, valid and important to document. What participants knew about the different types of knowledge for mathematics teaching in schools, their beliefs and assumptions were imperative in this study. Participants' assumptions were considered by the researcher in the selection of the methodology for this study and how the research questions were selected.

This study was underpinned by a qualitative investigation framed within an interpretive paradigm.

3.2.1 Interpretive paradigm

According to Cohen and Manion (1994, p. 36), the main endeavour within the context of the interpretive paradigm is "to understand the subjective world of human experience." In order to maintain the standard of the phenomenon under investigation, effort must be made to "get inside" the research subject in order to "understand from within".

The interpretive paradigm is identified by individual interest; in this situation the teacher who should try to understand the learner from within in order to help him/her for better understanding. How a teacher views mathematics and the different types of knowledge for mathematics teaching in the classroom ultimately affects not only what the learners know but how they view and assess themselves as learners of mathematics (Brown and Atkins, 1988).

According to Krauss (2005), the interpretivist paradigm is an approach to research where apart from our perceptions interpretivists/qualitative researchers do not believe that there is a single unitary reality. Instead, they highlight a realistic ontology that posits that there is no objective reality; they support several realities which are socially designed by individuals from within their own contextual interpretations.

3.3 THE RESEARCH METHODOLGY

Research methodology refers to the range of approaches used in educational and other research to collect data which are to be made available as a basis for inference and interpretation. The aim of research methodology is therefore to enable us assimilate, in a wide range as possible, not the products of scientific inquiry but the process itself (Louse, Lawrence and Keith, 2009, p. 47).

The research methodology used in this study was qualitative research. This method was used to help understand participants' views and beliefs with regards to the different types of mathematical knowledge that are used for teaching in schools. According to Damon (2001, p. 1), "qualitative research emphasizes linguistic data, as opposed to numerical data. Research of this type tends to be less objective than numerical data, yet it has the ability to describe phenomena in real-world language. Qualitative research methods have the additional advantage of gathering subjective data that can come directly from the source being investigated".

According to White (2005, p. 82), in qualitative research studies, there is greater flexibility in both the methods and the research process. Typically, a qualitative researcher uses a laid-down-procedure that is not established prior to the commencement of the research. He/she takes decisions concerning the data collection strategies or techniques in the process of the study.

Qualitative research is a type of scientific research which consists of an investigation that seeks answers to research questions in a systematic way, systematically uses a pre-defined set of procedures to answer the questions, produces results that were not found in advance, endeavours to envisage a given research problem or topic from the perspectives of the local population it is involved in and it is especially effective in obtaining culturally-specific information about the values, opinions, behaviours and social context of a particular population (Mack, Cynthia, Kathleen, Greg, and Emily, 2011, p.1).

3.4 THE RESEARCH DESIGN

This research study was a case study which aligns with an interpretive orientation. According to Cohen, Manion and Morrison (2000, p. 180), a case study is “a careful study of some social units that attempts to determine what factors led to its success or failure”. It allows for one aspect of a case to be investigated in some depth within the limited time of the study.

According to Judith (2004, p. 12), the case study approach is suitable for individual researchers because it gives an opportunity for one aspect of a problem to be studied in some in-depth within a limited time scale. The case study can be described as an umbrella term for a family of research methods having in common the decision to focus on an inquiry around an issue. In a case study, evidence is collected systematically, the relationship between variables is studied and the study is methodologically planned which help the researcher to dig deeper into the phenomenon under study. Normally, many researchers use observation and interview in case study, but it must be noted that no method is ruled out. Methods of collecting information are carefully chosen if they are suitable for the task.

3.5 PARTICIPANTS (SAMPLING)

The purposive sampling technique was used to select six Grade 9 mathematics teachers each from the following three categories: two (2) mathematics teachers from two high-achieving schools, two (2) mathematics teachers from two average-performing schools and two (2) mathematics teachers from two low-performing schools in the Mthatha Education District. These schools were identified by relying on the Annual National Assessment (ANA) results in the circuit.

According to White (2005, p. 120), purposive sampling completely depends on the decision of the researcher in that a sample is designed of components that comprise the most representative characteristics of the population or the most definite attributes of the population. Considering the researcher’s information of the population, a decision was made about which subjects should be chosen to give the most appropriate information to deal with the aims and objectives of the case under investigation.

Purposive sampling sizes are normally discovered "on the basis of theoretical saturation" (the situation of collecting data when another data does not add meaning to the research questions). Purposive sampling is therefore become clearer when a data assess and the analysis is performed simultaneously with data collection (Mack et. al. 2011, p. 5).

Considering the fact that data collection procedure demands teachers to endeavour to express their own pedagogical content knowledge in mathematics, Grade 9 mathematics teachers are suited to this methodology. The 9th grade is the last stage of the General Education and Training (GET) band. Teachers teaching in these grades are expected to lay a good foundation which will prepare learners for Grade 10 which is the start of the FET band.

3.6 DATA GENERATION

The researcher motivated the interviewees by explaining to them that the information that would be gathered would be valuable to school principals, the Department of Education, mathematics teachers, subject advisors and mathematics learners and other professionals in the education system. Each data collection method or tool (task sheet, observation of participants in class or interview) started with a contextualizing statement such as: "There is much discussion in education about the different types of knowledge for mathematics teaching in schools. I am conducting a study to find out about the different types of knowledge for mathematics teaching in schools in the Mthatha Education District. I am interested in exploring your ideas and experiences on such phenomenon; therefore, I would like you to tell me about your experiences with regard to the different types of knowledge necessary for mathematics teaching in schools in the district, and why this is so.

The following three different types of data-collection tools were used in this study:

3.6.1 Task Sheet

Task questions were drawn from Grade 9 mathematics text books and syllabus. The task followed the format of both multiple choice and open-ended questions. The multiple choice questions tested the teachers' basic understanding of mathematics concepts that they needed to use to build a mathematics foundation for their learners. Another test was the subjective test designed to unfold Kilpatrick et al's (2001) strands of mathematical proficiency, which are "conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition." This exercise was done to explore Pedagogical Content Knowledge and understanding of teachers in mathematics.

3.6.2 Lesson Observation

One lesson was observed taught by each of the participant teachers during the research study and an observation schedule was used in the process of teaching and learning. The schedule was planned according to the five strands of teaching mathematics for proficiency. This helped to analyse teachers' mathematical knowledge critically. It also helped to focus on teachers' mathematical knowledge during the lesson observation.

The lessons were also video-taped for the purpose of reflection; in this case, participant teachers and the researcher replayed the tapes of individual participants teaching for the purpose of reflection and discussion. An opportunity was given to each participant teacher to clarify some points about his or her lesson. Each participant was requested to make use of a personal journal as a reflective practitioner. This meant, participants had to record their own experiences and were afforded a voice concerning their experiences.

According to Louse, et al. (2009, p. 47) one of the peculiar features of observation as a research approach is that it gives an investigator "the opportunity to gather live data from naturally occurring social situations. In this way the researcher can look directly at what is taking place in situ rather than relying on second-hand accounts."

Robson (2002, p. 310) maintains that what teachers do may "differ from what they say they do, and observation provides a reality check; observation also enables a researcher to look afresh at everyday behaviour that otherwise might be taken for granted, expected or go unnoticed." This means that observation approach helps to gather some helpful or useful information which may enrich the research process. In other words, observation enables the researcher to gather some necessary data which might be ignored by the researcher.

3.6.3 One-on-one interviews

The face-to-face, in-depth individual interviews were conducted in a semi-structured manner after observation which allowed for more flexibility between the researcher (the interviewer) and the participants (the interviewees). This semi-structured interview provided an opportunity for the participants to express themselves openly, thus leading to “rich” information that was collected (McMillan and Schumacher, 2006, p. 207).

According to Maree (2007), an interview is a situation where a two-way conversation happens between the researcher (interviewer) and research participant (interviewee) in order to gather information for the study. It also enables the researcher to acquire more knowledge such as ideas beliefs, news, suggestions and behaviours about the research participants. Participants were informally interviewed when the written expression of their mathematical knowledge was either unclear or requested classification by verbal explanation. The informal interview, also known as the unstructured interview, was an unrestricted strategy to interviewing in which questions are asked from the background of the current study (Cohen and Manion, 1994).

The main reason for the interview was to give research participants the chance to further explain in writing their pedagogical knowledge in mathematics regarding questions on the topic.

Since the sole purpose of these interviews was to seek further clarification of the mathematical knowledge under discussion both in the task sheet and the lesson observation, more discussion strategies were adopted. Also, interviews were confined to those specific situations where participants’ written or explanatory mathematical knowledge was either confusing or needed clarification by verbal explanation.

Participants who demonstrated interesting mathematical knowledge in the task sheet, when teaching and during in-depth interviews were audio-taped. Thereafter tapes were transcribed. An analysis of transcriptions, in terms of answering questions and teaching using appropriate mathematical knowledge added significantly to the richness of the research data.

The audiotape recorder was switched on immediately the interview began so as to capture the direct words of the respondents. The advantage of this form of data collection was that the researcher could concentrate on the responses of the respondents, rather than trying to write down what they said. Data were collected by means of semi-structured interviews which allowed the generation of first-hand, in-depth, rich, unexpected and relevant information from the interviewees. The length of each interview was about 30 to 45 minutes and these were conducted after school hours to avoid the disruption of lessons.

3.7 DATA ANALYSIS

3.7.1 Mathematical response task sheet

Based on the five interwoven propose teaching framework by Kilpatrick et al (2001), for mathematical proficiency was used to analyse each response by the participants. That is, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition were used for each analysis (Kilpatrick et al, 2001). The creation of additional categories became necessary for solution strategies which have not been mentioned by Kilpatrick et al (2001) but considered to be important during analysis of this study. For mathematics proficiency, descriptive statistics were used to ascertain mathematical proficiency and isolate areas of strengths and weaknesses. Descriptors and solution strategies were used together to generate a rich profile for each research participant as well as for each individual response task.

3.7.2 Lesson observation

Each participant teacher was critically observed according to the types of knowledge being demonstrated in class during teaching. In this case, each participant's observational schedule after the lesson was described according to the different types of knowledge such as content knowledge, pedagogical knowledge, knowledge of curriculum, and so on that the teacher displayed. This was done in conjunction with Kilpatrick et al's (2001) five strands for mathematical proficiency since they are interrelated with the type of knowledge portrayed. Each participant teacher was identified on the observational schedule by letter of the alphabet and numbers; For example, school 1 participant A, school 2 participant B and so on were used on the observational schedule for description.

3.7.3 One-on-one interviews

The researcher sat face to face with the research participants during the interview. Every interview was started by an introduction and small talks in order to create conducive atmosphere.

Thereafter, permission was sought to start recording while the interview questions were ready to engage the participant. The questions were selected according to the answers each participant gave. In some instances, more questions which were not part of the questionnaire were asked for the purpose of clarification.

The researcher, after gathering the data, analyzed it by reading through the data, and becoming familiar with it and then identifying the main themes. Through the process of coding, the researcher placed the raw data that were transcribed into logical, meaningful categories and examined them in a holistic fashion. The next stage was to re-examine the themes, to categorise and then interpret and synthesize the organised data into general conclusions or for better understanding.

According to White (2005, p. 22) qualitative data analyses involve becoming use to the data in depth to give detailed descriptions of the situation, participants and activities categorizing and coding pieces of data and physically grouping the above into themes and then interpreting and synthesizing the organized data into understanding.

3.8 LIMITATIONS

Since the study was conducted in one education district (Mthatha) only, the findings cannot be generalised and were only used within the context of the Mthatha District. However, the findings of the study could serve as a starting point for further research projects in mathematics education in schools in South Africa.

3.9 VALIDITY AND RELIABILITY

McMillan and Schumacher (2006, p. 10) state that, in a qualitative study the technical features of instruments such as validity and reliability are not used. Nevertheless, the more general ideas of appropriateness of the inferences (validity) and error in collecting information (reliability) are still important.

The task sheet, observational schedule and interview schedules were piloted first before they were used in order to ensure the validity and reliability of the questions. According to Edwin and Vanora (2001), “the term pilot study is used in two different ways in social science research. It can refer to so-called feasibility studies which are small scale version[s], or trial run[s], done in preparation for the major study"(p. 1). However, a pilot study can also be the "pre-testing or 'trying out' of a particular research instrument."

The design or instructions' questions that were altered during the course of the investigation meant a further pilot study was conducted before the tasks were presented to the research participants. In all cases, teachers who were used for the pilot studies did not form part of the main investigation. Copies of transcripts were given to the participants for the interpretive validity of this research (Maxwell, 1992). All the participants were treated with sensibility and integrity and a letter was sent to DoE and principals requesting permission to conduct research in their respective schools.

Validity is the measure of ability of an instrument to measure what is it expected to measure. In quantitative research validity might be modified through cautious sampling, appropriate instrumentation and appropriate statistical treatments of the data (Louis, Lawrence and Keith, 2009, p. 105).

The researcher was able to minimize the threat to validity and reliability by selecting a suitable time scale, by making sure that there were enough resources for the required research that was undertaken, by choosing suitable methods and techniques to answer the research questions, by selecting an appropriate instrument for gathering the type of data needed and by using an appropriate sample.

To ensure validity and reliability, the researcher followed proper procedures when collecting the information from the participants throughout the interview. The researcher also considered the issue of bias throughout the data-collection stage. Furthermore, to ensure validity and reliability, participants were given sufficient time during the test, or task, during the observation period and during the interview session to answer the questions.

The researcher was able to ensure reliability through the elimination of causal errors that were influenced by the results. To ensure validity, the researcher formulated techniques that checked the credibility of data in order to eliminate or control the distorting effect of personal bias upon the logic of the evidence. According to White (2005, p.193), by reliability it is understood that the researcher's conclusion - true or correct - responds to the actual state in reality. The division of reliability includes internal reliability and external reliability.

According to Andy (2006, p.1) "reliability is generally understood to represent the probability that repeating the research procedure under the same conditions would produce similar results. While it often refers to the consistency of the measuring instrument, it may also relate to the people conducting the research".

3.10 ETHICS

Ethical decisions are the values that are considered by the researchers and their communities or research sites and inform the negotiation which takes place between the researchers, sponsors, research participants and those who control the access to information which the researchers seek. The amount of control the researchers can exercise over the research process also influences the exercise of ethical decisions themselves (Mack, Cynthia, Kathleen, Greg, and Emily, 2011, p. 8).

The issue of Ethics is considered to play a vital role in the social sciences as far as any research investigation is concerned (Cohen and Manion, 1994). Maree (2010) highlights the view that the most importance aspects of ethical issues include the issue of confidentiality of the results, the findings of the study and the protection of the participants' identities. This also includes obtaining letters of consent, obtaining permission to be interviewed and undertaking to destroy audiotapes.

According to Babbie (2008, p. 67), informed consent is the criterion emphasizing the importance of both accurately informing the subjects or respondents as to the essence of the research and obtaining their oral or written permission to partake in the research process. Those in authority too need to be presented with informed consent form to enable them to grant the researcher permission to conduct the research.

Permission to conduct the study in the secondary schools was sought and obtained from the Department of Education, principals and the mathematics teachers. Participants were assured that no names of individual participants would be reflected in the results of the study and participants were also assured of the strictest confidentiality and anonymity in the study. The participants were permitted to withdraw from the study at any time without penalty or victimization and they were also protected from any forms of abuse.

Tool that is used to ensure that people understand the meaning of taking part in a particular research study so they can take decision on their own accord to either participate or not is called informed consent. Informed consent is one of the essential tools that are used to establish the fact that respect is given to people in the process of the research. Informing people about the research in the way they can understand is the first task of achieving informed consent; thus research participants should be told what they need to do throughout the research process including time required, risk involved, and advantages. Participants should be aware of the fact that taking part is voluntary and that one can decide to withdraw anytime he or she wishes with no harmful consequence and how privacy will be protected. It also includes obtaining letters of consent, seeking permission to be interviewed, undertaking to destroy audiotapes, and so on (Mack, Cynthia, Kathleen, Greg, and Emily, 2011, p. 8).

This requests a commitment to ensure that the identities of the research participants are protected by telling them not to write names on the questionnaires or mention their names during interviews. The status of every research participant in this case was respected. Adherence to this concept established the fact that participants were not used simply as a means to accomplish research aims and objectives (Mack, Cynthia, Kathleen, Greg, and Emily, 2011, p. 9).

The researcher made sure that a given participant's response having been identified was not made public. The researcher tried to make sure that no information about participants that will tarnish their image or jeopardize their relationships and jobs would be known to the public (Maree, 2010, p. 41). Identified research records have to be kept confidential whether or not an explicit pledge was given. According to Maree (2010, p. 41) essential ethical aspects include "the issue of confidentiality of the results and findings of the study and the protection of the participants' identities." This also includes obtaining letters of consent, seeking permission to be interviewed, undertaking to destroy audiotapes, and so on.

Participants could make up their minds not to partake in the study or choose to drop out during the study. The researcher did not put pressure on any identified participant to take part in the study. According to Orb, Eisenhauer and Wynaden (2001, p. 5), "respect for people is the recognition of participants' rights, including the right to be informed about the study, the right to freely decide whether to participate in a study, and the right to withdraw at any time without penalty."

CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

The main reason for this study was to explore the types of mathematical knowledge that a few chosen teachers in the Mthatha District demonstrated in some aspects of mathematics content. The data discussed in this chapter were gathered through task sheets, observations and interviews using teachers from six selected schools. The first section deals with the task sheet responses followed by observations and, finally, the interviews. The discoveries from this investigation are presented within the context of the research questions posed in Chapter one that guided this study. The research questions focused on the types of knowledge necessary for teaching mathematics, the kinds of knowledge that is used and the way the knowledge is being used by the selected teachers.

4.2 TASK SHEET ANALYSIS

The questions on the task sheets provide different ways in which teachers and content interact to produce different opportunities for effective teaching. Each answer reveals the kinds of mathematical knowledge teachers demonstrated in the process of teaching and learning of mathematics in content and method. What happens in class to promote the development of mathematical proficiency may thus be understood through examining how teachers and learners (during teacher observation) as well as content interact in different contexts to produce meaningful learning.

The identified themes based on knowledge of mathematics (content) and knowledge of instructional practices (method) follow Kilpatrick et al’s (2001) five strands of mathematics proficiency and these are: conceptual understanding, procedural fluency strategic competence, adaptive reasoning and productive disposition.

4.2.1 Question 1.1

If the answer to a sum is $\frac{3}{4}$, what will the numbers be? Explain your answers.

Responses:

Teacher A: $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

Teacher B: $\frac{1}{2}+\frac{1}{4}$ $\rightarrow$ *half of an orange and a quarter of an orange put together gives three quarters.*

Teacher C: $\frac{1}{4}$ *and* $\frac{2}{4}$ *or any fractions equivalent to the stated ones. Add numerators when denominators are the same.*

Teacher D: $\frac{1}{2}+\frac{1}{4}$ *or* $\frac{2}{4}+\frac{1}{4}$ *as the answer is* $\frac{3}{4}$*. It means fractional values are added, sum is the result of addition.*

Teacher E: The numbers could be $\frac{1}{4}+\frac{2}{4}$ *this is because addition of these fractions will result in the answer* $\frac{3}{4}$*.*

Teacher F: $\frac{1}{2}+\frac{1}{4}$ *because* $\frac{3}{4}$ *is* $\frac{1}{4}$ *three times i.e.* $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ *and two quarters is* $\frac{1}{2}$*.*

In attempting to answer this question, some teachers provided single solutions without any explanation for their choice of answer. Teacher A, for instance, simply wrote $\frac{1}{4}+\frac{2}{4}$. Teacher A provided no explanation but for a graduate teacher this might have been too simple and hence an explanation may have been seen to be unnecessary and not worth his/her time. What should be questioned though is the fact that all these teachers provided single solutions. In establishing that the sum of numbers is $\frac{3}{4}$, the teachers could have provided the answer $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, as used by Teacher F as an explanation to justify his solution of $\frac{1}{2}+\frac{1}{4}$. It was evident that all teachers saw the sum as that of adding only two numbers. Perhaps one could have also considered $-\frac{1}{4}+1$ as a solution. The answers could have constituted various combinations such as $\frac{1}{8}+\frac{3}{8}+\frac{1}{4}$, $\frac{1}{12}+\frac{1}{3}+\frac{1}{3}$, $\frac{1}{16}+\frac{3}{16}+\frac{2}{4}$ and so on. It was noted that none of the teachers thought of using diagrams to clarify answers. Perhaps this was due to lack of good pedagogical content knowledge.

This question was attempted by all the six teachers but the concept was not properly explained, revealing that teachers A, B, C, D, E and F did not demonstrate deep understanding of the subject matter. The procedural fluency that was supposed to be demonstrated in order to establish understanding was simply not evident. Some drawings, such as number lines and Cuisenaire rods, could have been used to clear every misconception that might have been developed in the early grades in order to solve the problem systematically to arrive at the answer.

4.2.2 Questions 1.3

What is the difference between $x^2 \div x^3$ and $x^3 \div x^2$?

Responses:

Teacher A: $x^2 \div x^3 = x^{2-3} = x^{-1} = \frac{1}{x}$ *and* $x^3 \div x^2 = x^{3-2} = x^1$

The difference is that the first will give you a negative exponent of -1 while the second will give you a positive exponent of 1.

Teacher B: $x^2 \div x^3$ *gives the reciprocal of* x *and* $x^3 \div x^2$ *gives* x.

Teacher C: $x^2 \div x^3 = x^{-1} = \frac{1}{x}$ *it gives an answer with a negative exponent that is a fraction; when written with a positive exponent* $x^3 \div x^2 = x$ *it gives a positive exponent.*

Teacher D: $\frac{x^2}{x^3} = \frac{1}{x} = x^{-1}$ *and* $\frac{x^3}{x^2} = x^1$*. In the first expression we have a remainder in the denominator, while in the second one we have an extra* x *in the numerator.*

Teacher E: The result of $x^3 \div x^2 = x^1$ *whereas the result of* $x^2 \div x^3 = x^{-1}$.

Teacher F: $x^2 \div x^3 = x^{2-3} = x^{-1}$, $x^3 \div x^2 = x^{3-2} = x^1$, $x^2 \div x^3 = \frac{1}{x}$ *and* $x^3 \div x^2 = x$

In attempting to provide an answer to this question, some of the teachers provided correct solutions to the question in different ways.

The concept of exponent in this question was not properly explained; teachers B, C, D, E and F got it right but without proper explanation, thus, the concept of positive and negative exponents was not addressed; That is, $\frac{1}{x}$ and x /denominator and numerator/ $x\, exponent - 1\, and\, x\, exponent\, 1$. Insufficient mathematical knowledge was reflected. It could be that the question was too simple for them or may also mean that they lacked good pedagogical content. One could even have considered solving the question in a different way to arrive at another solution such that $\frac{X^2}{X^3} - \frac{X^3}{X^2} = \frac{1}{x} - \frac{x}{1} = \frac{1-x^2}{x} = \frac{(1-x)(1+x)}{x}$.

4.2.3 Question 1.4

Which is bigger x^2 or x^3?

Responses:

Teacher A: x^3 i.e. x cubed is bigger than x squared because x^3 is raised to exponent 3 whilst x^2 is x to the power 2.

Teacher B: x^3, the bigger the exponent the bigger the number or letter.

Teacher C: $x^3$is bigger because you multiply x by itself twice where as: x^2 you multiply x by itself. You can relate this with numbers.

Teacher D: $x^3 >: x^2$, the exponent determines the number of times the base has : x^2 multiplied itself, so $x^3 >: x^2$.

Teacher E: $x^3$is bigger than: x^2. If and only if x is a positive number.

Teacher F: x^3 is bigger because x^3= x three times i.e. $x \times x \times x$ while: $x^2 = x \times x$

In attempting to answer this question, some teachers provided straight forward answers without exploring the answers in diverse ways; for instance, teachers A, C and E could have used several examples that could confirm their decisions before drawing conclusions. Perhaps they thought that the question was too straight forward and easy enough to handle. Teachers B, D and F also provided answers by assumption without exploring or proving them. Perhaps they might also have thought of a simple solution to the question or lacked of good PCK. Teacher E could have given some examples to support his answer. Graduate teachers should have given in-depth answers that some teachers lack good PCK.

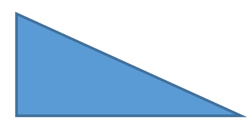
The concept of positive, negative and fraction base that determine the result, irrespective of number of exponent, was not considered by the five participant teachers. Participant teachers provided the result by looking at the exponents. Also, Procedural fluency in terms of giving an example that could help the participants in order to establish a concrete answer was not shown either. In fact, this question, which seems very simple, was poorly answered by the participant teachers. Content/indepth knowledge is needed to come up with a solution, thus, if x $is\ negative, x^2\ will\ be\ bigger\ than\ x^3$. Example if $x = -2, (-2)^2 = 4\ while\ (-2)^3 = -8,$ but if $x\ is\ positive, x^3$ is bigger than x^2. Example: $2^3 = 8$ while 2^2 =4 In terms of fraction or negative exponent, x^2 will always be bigger than x^3. Example: if $x = \frac{1}{2}$ then $x^2 = \frac{1}{4}$ and $x^3 = \frac{1}{8}$. This could reflect lack of mathematical knowledge (concept, procedure, strategy and reasoning).

4.2.4 Question 1.5

If one angle of a triangle equals 45°, is it possible to find other angles? If so, give an example.

Responses:

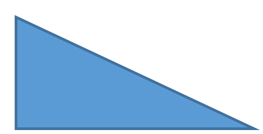
Teacher A: Yes, it is possible; for example a triangle has a sum total of 180° interior angle.



Teacher B: Yes, if the right-angled triangle is also an isosceles triangle.

Teacher C: It is possible when you are dealing with the isosceles triangle.

Teacher D: No, at least to calculate one triangle we need to have two angles known as well as the type of triangle you are working with.



Teacher E: Yes, because the sum of angles of any triangle is 180°.

Teacher F: Yes, one of the angles is 90° for example

In trying to solve this problem, the answers provided showed that teachers did not explore the problem in order to come up with a concrete solution; For instance, teacher A simply wrote one property of a triangle to prove the answer. He could have explored further in order to provide a concrete solution to the problem. The teacher however, might have thought that the question was very easy to solve. He could have thought of other types of triangle in order to come up with concrete answer. Teacher C and F were also supposed to explore further since they understood the concept and could have given examples of isosceles triangle and right angled triangle, respectively. Teacher D and E simply provided the answer without much investigation. This might be owing to a lack of good mathematical knowledge.

The question was not answered properly by all the participants. This question calls for conceptual understanding, procedural fluency and reasoning, thus, the concept of different types of triangles and their properties is needed. Procedures in calculating interior angles of every triangle using adoptive reasoning are supposed to be used. None of the participant teachers was able to answer this question fully though some tried to provide the solution. Right angled triangle and isosceles triangles are possible while scalene and equilateral triangles are impossible, therefore, the answer cannot be yes/no because it depends on the type of triangle you are dealing with.

It is possible to find other angles in some triangles such as a right angled triangle and an isosceles triangle. In a right angled triangle, $45 + 90 + x = 180, therefore\ x = 45$; also, in an isosceles triangle, two angles are equal, thus, $45 + 2x = 180,\ and\ x = 67{,}5$.

4.2.5 Question 1.6

Assuming a learner in your class answers $\sqrt{49}$ as -7, is the learner right or wrong?

Responses:

Teacher A: $(-7)^2 = 49$*, the answer may depend on the square root sign e.g.* $\pm\sqrt{49}$ *means* ± 7*, i.e. +*7 *or -*7 *will be correct for the answer so -*7 *can be accepted since* -7×-7 *will give you* 49 *and again square root is a perfect square of two similar numbers.*

Teacher B: The learner is correct because $7 \times 7 = 49$ *and* $-7 \times -7 = 49$ *also.*

Teacher's approach to this question was not convincing. Thus, mathematical knowledge was limited.

Teacher C: Correct because $\sqrt{49}$ *is the number that you multiply by itself to give* 49, *so* -7×-7 = 49.

Teacher D: The learner is correct because $(-7)^2$ *will give the same result as* 49. *The* $\sqrt{49}$ = ±7.

Teacher E: Wrong, because square root of a positive number gives a positive answer which is 7.

Teacher F: Yes, the learner is correct. The square root of a number is either positive or negative.

In attempting to provide answers to this question, most of the teachers could not explain, and therefore prove, that they understood the concept; for instance, teacher B, C, D, and F could not explain their answers is that 7×7= 49 and -7×-7 = 49 so these are not convincing answers. This could reflect lack of concept and adaptive reasoning. They might also have thought the question was too simple. Teacher A managed to explain the answer to his best ability but Teacher E could not explain his answer though he has some ideas. Teacher could have justified the answer by giving an example. It was therefore evident that these teachers saw the question as square root and perfect square. Perhaps one could have thought $x^2 = 49$ gives $x = \pm \sqrt{49}$ = -7 or +7.

This question was answered by all the participants but none got it correct. Clearly, mathematical knowledge concerning this question was not demonstrated by the participant teachers. There should be a concept of $\sqrt{x}$ and $x^2 = 49$. The learners should be taught that $\sqrt{49}$ gives positive 7 while an equation such as $x^2 = 49$ gives $x = \pm \sqrt{49}$ = -7 or +7. The answer to the question is therefore positive 7.

4.2.6 Question 1.7

The perimeter of a rectangular plot of land is $29.5m$. If the length is increased by $2m$ and the breadth is reduced by $1m$, the area of the plot remains unchanged. Demonstrate this.

Responses:

Teacher A: $p = 2(L+B) \rightarrow 29.5m = 2l + 2b.\ l = 29.5 \div 2 = 14.75;$ $b = 29.5m \div 2 = 14.75m\ \therefore\ l = 14.75m + 2m = 16.75m;\ l = 14.75m - 1 = 13.75m\ \therefore\ A = 16.75m \times 13.75 = 230.3m^2$

Teacher B: $(l+2)+(b-1) =$

Teacher C: No answer

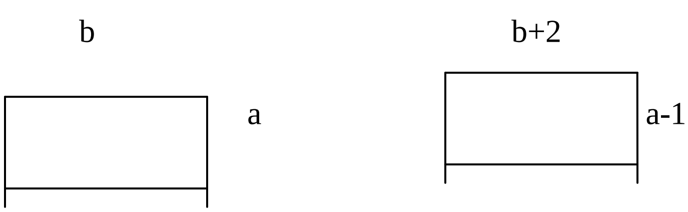
Teacher D: $P = 2(l+b) \rightarrow 29.5m = 2(l+b)\ ;\ l+b = 29.5 = 14.75;\ \ l + 2m + b - 1m = 14.75;\ l + b + b - 1m = 14.75;\ 2(l + 2m^2 + b - 1m) = 29.5m\ ;\ 2(l + b + 1m) = 29.5$

Teacher E: No answer

Teacher F: $A = l + w;\ A = [(14.75+2) - l](14.75 - 1 - w) = (16.75 - 1)(13.75 - w)$

In attempting to solve this question, none provided answers to the question. Perhaps this was due to a lack of good pedagogical content knowledge; for instance, teacher A tried but deviated along the way. Teachers B, D and F could not handle it. Teachers C and E did not attempt the question. This could, however have been solved by at least two of the graduate teachers.

Every participant teacher who attempted the question was wrong. This question needs conceptual understanding, procedural fluency, strategic competence and adaptive reasoning. Teachers who tried to provide a solution to the question deviated. The question could have been solved by formulating two equations (reflecting concept and fluency) that could have been solved simultaneously (adoptive reasoning/ content knowledge).



a= breadth and b= length (concept/ content knowledge)

This means 2a+2b= 29,5m since perimeter (P) = 2(a+b) (Reflecting procedural fluency)

a+b = 14,75 equation (1)

Also, A = ab and (b+2) (a-1) since Area = LxB

Therefore, ab = (b+2)(a-1)

Therefore, 2a-b = 2 equation (2) solving equation 1 and 2 simultaneously to get a and b (strategic competence and adaptive reasoning).

4.2.7 Question 1.11

Find the remainder when $x^2 - x + 1$ is divided by $x + 1$. How do you explain this?

Responses:

Teacher A: Using long division for the explanation, the remainder is $+3$ *i.e.* $x^2 - x + 1 \div x + 1 = x - 2$, *remainder 3, the child must know the divisor, quotient and the remainder by using the factor theorem.*

Teacher B: $x^2 - x = x.x \times (x + 1) = -x^2 +. x^2 - x - (x^2 + x) = -2x - 1. -2x \div x - 2. -2 \times (x + 1) = -2x - 2. -2x - 1 - (-2x - 2) = 1.$ *Remainder*$= 1.$

Teacher C:

$$\begin{array}{r} x - 2 \\ x + 1\sqrt{x^2 - x + 1} \\ \underline{x^2 + x} \\ \underline{-2x + 1} \\ 2x - 2 \\ +3 \end{array}$$

Use long division to divide, then divide the dividend by the divisor following the method of division.

Teacher D:

$$\begin{array}{r} x - 2 \\ x + 1\sqrt{x^2 - x + 1} \\ x^2 + x \\ \pm 2x + 1 \end{array}$$

$$2\,x - 2$$

$$3$$

Teacher E: The remainder is 3. This is done by using the long division method or factor theorem.

Teacher F: Use normal division to explain like $9 \div 2$ *using long division.*

In attempting to solve this question, teachers' approaches differed from one another; for instance, teacher A and E provided the answer without showing any work but were able to tell how to approach the question. Possibly they thought showing work was unnecessary and not worth their time. It could also be that they were not sure of the procedures needed to arrive at the answer; they wrote the remainder as 3 and added the method to be used. Teachers B and F could not solve the question let alone provide an answer to the question. Perhaps they had forgotten the concept, were confused or they lacked good pedagogical content knowledge. This is unusual for graduate teachers. Teacher C and D were able to provide the solution to the question using similar methods and procedures. It was evident that these two teachers understood the concept. Perhaps they could have considered using the synthetic division method as one of the methods to solve the problem; that is, $x^2 - x + 1 : x + 1$ can be solved using the synthetic division method as follow;

$$\begin{array}{r|rrr} -1 & 1 & -1 & 1 \\ & & -1 & 2 \\ \hline & 1 & -2 & 3 \end{array}$$

The remainder is 3 as indicated in red.

This question needs conceptual understanding, procedural fluency and adaptive reasoning. Example: Using long division as one of the popular methods:

$$x - 2$$

$$x + 1\sqrt{\,x^2 - x + 1}$$

$$x^2 + x$$

$$-2\,x+1$$
$$-2\,x-2$$
$$3$$

The remainder is therefore +3. In this case, the learner should understand the divisor, quotient and remainder, using the factor theorem.

4.2.8 Question 1.12

Is $12x^3y^2z^5$ a factor of $24x^4y^5z^6$? How do you know?

<u>Responses:</u>

Teacher A: By using indices to check as they would not be remainder going into it exactly. $24 \div 12 \times x^4 \div x^3 \times y^5 \div y^2 \times z^6 \div z^5 = 2x^{4-3}y^{5-2}z^{6-5} = 2x^1y^3z^1 = 2xy^3z$

This teacher demonstrated clear mathematical knowledge for this question.

Teacher B: Yes it is. Multiplication of two factors = the multiple.

Teacher C: Yes because $12x^3y^2z^5$ *is a factor that when multiplied by* $2xy^3z$ *gives* $24x^4y^5z^6$.

Teacher D: $\frac{24x^4y^5z^6}{12x^3y^2z^5} = 2xy^3z$. *If you multiply* $2xy^3z$ *by* $12x^3y^2z^5$, *it gives* $24x^4y^5z^6$ *when proves that* $12xy^3z$ *is the factor.*

Teacher E: Yes, because it is divisible.

Teacher F: Yes, because 12 *is a factor of* 24 *and* xyz *are indices.*

In trying to provide a solution to this question, some teachers provided the solution without showing their work; for instance, teachers B and E just wrote: ‘yes, multiplication of two factors equal multiplication and, yes, because it is divisible.’ Perhaps they thought the question was too easy to show work. Teacher F, on the other hand, provided an explanation to the answer without solving the problem. This was not enough; he was supposed to come up with a clear answer by first solving the problem. May be he thought the question was too easy. Teachers A, C and D were able to come up with the solution by using the correct procedure but different methods; for instance, teacher C was able to use the answer to prove the question. This shows clear understanding of the concept as well as procedures.

4.2.9 Question 1.13

Is $3p^2$ a factor of $6p^4$? Explain your answer.

<u>Responses:</u>

Teacher A: By using the method of exponent or powers to divide exactly without any remainder: $6 \div 3 \times p^4 \div p^2 = 2p^{4-2} = 2p^2$ *laws of indices when dividing we subtract once the bases are the same.*

Teacher B: Yes, each factor of $3p^2$ *can divide* $6p^4$.

Teacher C: Yes, because $3p^2 \times 2p^2$ *gives*$6p^4$.

This teacher solved the question with limited mathematical knowledge, thus, concept and reasoning had been shown but procedure was not shown.

Teacher D: $\frac{6p^4}{3p^2} = 2p^2$ *as long we don't have a remainder.*

Teacher E: Yes, because $3p^2$ *is divisible by* $6p^4$.

Teacher F: Yes, because 3 *is a factor of* 6 *and* p^4 *is an index that has a factor* p^2.

This question is similar to question 1.12 and the answers participant teachers provided followed the same trend. Surprisingly, the knowledge demonstrated in question 1.12 was the same in this question, thus, in trying to provide a solution to this question; some teachers provided the solution without showing the work; for instance, teacher B and E just wrote 'yes, multiplication of two factors equal multiplication and yes, because it is divisible.' Perhaps, they thought that the question was so that they did not need to show work. Teacher F also provided an explanation to the answer without solving the problem. This was not enough; he was supposed to come up with clear answer by solving the problem. There again, he might have thought the question was too easy. Teachers A, C and D were able to come up with the solution by using correct procedures but different methods; for instance, teacher C was able to use the answer to prove the question.

This shows clear understanding of the concept as well as procedures. The approaches to this question can be compared to question 1.12 which confirmed that teachers A, C and E lacked good pedagogical content knowledge.

Concept of common factors and common multiples were needed for this question. Procedural fluency was also essential as far as solving this problem was concerned. All the participant teachers got the answer correct with some explanations appropriate to their knowledge.

4.2.10 Question 1.14

If the x-intercept is 4, what will the y-intercept be? Explain.

Responses:

Teacher A: When the x-intercept is 4 the y-intercept can either take a negative or positive value.

Teacher B: y-intercept$= 0$. On the x-intercept, $y = 0$.

Teacher C: The y-intercept could be any other integer.

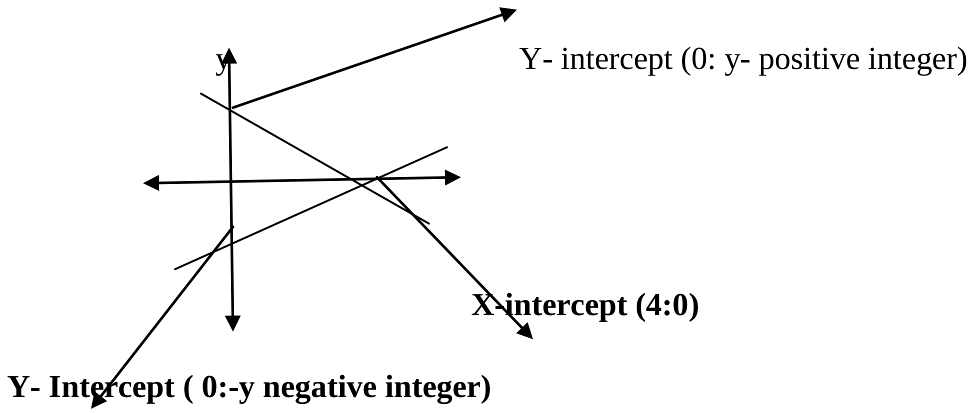
Teacher D: No answer

Teacher E: No answer

Teacher F: Depends on the equation of the line or function.

In attempting to answer this question, most of the teachers seemed to be confused, for instance, teacher B and F provided answers which were not correct. This could be that they were confused about the question or they did not have the required concept/ PCK to approach the question. Teachers D and E could not provide answers to the question either. Perhaps they did not understand the question or their pedagogical content knowledge needed to provide a solution to the question was inadequate. Teachers A and C provided solutions to the question without much explanation. Possibly they felt the question was very simple and hence an explanation was unnecessary. They could have considered using a diagram to provide a solution to the question coupled with a clear explanation; the diagram could have been a guide for them to provide a solution with confidence.

When this question is critically analyzed, it is clear that its concept and procedures cannot be ignored as far as functions and relationships are concerned. Unfortunately, only two teachers were able to understand this concept. Let us see how this question can be approached:



Here, y-intercept could therefore be any positive or negative integer on the y-axis.

4.2.11 Question 1.15

The diameter of the object is $7cm$. The height of the object is $5.5cm$. Identify the geometric object with an explanation.

Responses:

Teacher A: No answer

Teacher B: Cylinder; it has two circles and a height.

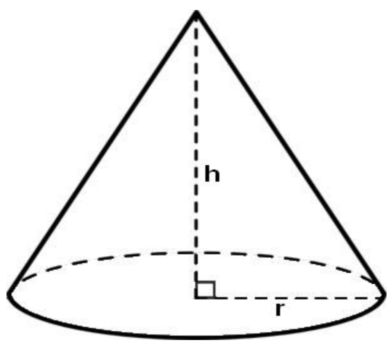
Teacher C: Cylinder. The diameter is found in a circle so therefore the base is the circle and there's a height that shows the figure is the 3D shape.

Teacher D: This is a cylinder; only in circles we have a diameter; the base of an object is a circle.

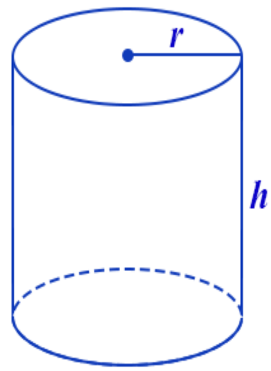
Teacher E: Cylinder, which has a circle, a base and perpendicular height.

Teacher F: The object is a cylinder because it has a diameter and a height.

Five participants (B, C, D, E and F) were able to partially answer this question with an explanation but without proper analysis. This could be that participant teachers saw the question as very simple and hence did not need further exploration. It could also mean that teachers were not used to a cone as being one of the geometric figures. This question needs a concept of all geometric figures and their properties before a conclusion can be established. Five teachers mentioned the cylinder without considering the cone as having a diameter and height. Teacher A did not attempt the question. This could be owing to a lack of conceptual understanding and adaptive reasoning. It could, however, have been an oversight. Overall, this question was poorly answered, taking into consideration that it should not have flawed a graduate teachers. It was actually an easy question but answered poorly by graduate teachers.



Cone



Cylinder

Radius = $\frac{d}{2}$ and Diameter = 2r, therefore, the cone and the cylinder can have the same height as well as the same diameter. A good conceptual understanding helps to develop the ability to solve problems.

4.2.16 Question 1.16

Use the inter-quartiles to determine if there are any outliers in the following data series. 5; 20; 6; 5; 7; 8; 15.

Responses:

Teacher A: $5; 5; 6; \mathbf{7}; 8; 15; 20$ *Median*$= 7$ $Q_1 = 5$ $Q_3 = 15$ $IQR = Q_{3-}Q_1$ *therefore interquartile range = upper quartile* Q_3 – *lower interquartile* Q_1 $IQR = Q_{3-}Q_1 = 15 - 5 = 10$. *There are outliers since the spread for the series are wide.*

Teacher B: 15 *and* 20 *are outliers.*

Teacher C: $5; 5; 6; 7; 8; 15$. *The outliers are between* 8 *and*15.

Teacher D: Range$= 20 - 5 = 15$, *median*$= 7$

Teacher E: $Q_1 = 5$

$Q_3 = 15$

$\Rightarrow 1.5 \times 10 = 15$

Hence there are no outliers

Teacher F: $5; 5; 6; 7; 8; 15; 20$

Median $= 7$

Lower interquartile$= 5$

Upper quartile$= 15$

15 *and* 20 *are outliers*

In attempting to find a solution to this question, it was obvious that most teachers did not understand the question; for instance, teachers B and C were not able to provide a solution to the problem or they thought the question was straight forward, hence did not need to be analyzed before solving. It could also mean that they lacked concept, procedures and adaptive reasoning. Teacher D and E could not understand the question properly. Possibly they had not encountered such a question before. It could also mean that they lacked concept as far as the topic is concerned. Teacher E thought the question was easy but could not provide the expected solution compared to what teacher A did though there was evidence of E's conceptual understanding. Teacher A was able to solve the question with understanding; that is, pedagogical content knowledge was clearly demonstrated by teacher A.

This question needed conceptual understanding, procedural fluency, strategic competence and adaptive reasoning. The approach to this question by some teachers was very disappointing. Note the following:

Arrange the data in ascending order;

5 ; **5** ; 6 ; **7** ; 8 ; **15** ; 20

Lower quartile (Q_1) median (Q_2) upper quartile (Q_3)

Therefore $Q_1 = 5$, $Q_2 = 7$ and $Q_3 = 15$

IQR = $Q_3 - Q_1 \approx$ 15-5 = 10, and there are outliers between 8 and 15, 15 and 20.

Looking at the participants' answers it revealed that most teachers displayed limited mathematical knowledge about the question. Teachers need to have curriculum knowledge to meet the requirements. This knowledge is essential to keep learners informed.

4.2.12 Question 1.19

Given $a = 22 + b$, determine whether:

A. a is greater than b

B. b is greater than a

C. b = 18

D. Cannot tell which is greater

<u>Responses:</u>

Teacher A: No answer

Teacher B: A

Teacher C: A

Teacher D: A

Teacher E: No answer

Teacher F: A

In attempting this question, teachers B, C, D and F were able to get the answer correct without proving their stands. There should have been proof to show the conceptual understanding. Possibly they saw the question as being simple. It could also mean that they were not used of proving multiple choice answers due to the nature of the question. They could have considered explaining the answer as; if $a = 22 + b$, this means that a will always be more than b with 22 or if $b = a - 22$, which means b will always be less 22 than a, thus, knowledge of representations of subject matter is required to solve the question. Teacher A and E did not attempt the question. This could have been an oversight or lack of good pedagogical content knowledge.

4.2.13 Question 1.20

If p and q are positive integers ($p \neq 0; q \neq 0$), how many pairs ($p; q$) satisfy $4\,p + 5q = 150$?

Responses:

Teacher A: Two pairs$4(15) + 15(18) = 60 + 90 = 150$*; $p = 15$ and $q = 18$, coordinates numbers p and q positive numbers.*

Teacher B: 1

Teacher C: No answer

Teacher D: $(0{:}\,30); (5{:}\,26); (10{:}\,22); (15; 18); (20{:}\,14); (25{:}\,10); (30{:}\,6)$

Teacher E: $(0{:}\,30)$ *(37.5; 0)*

Teacher F: Two under changeable pairs

In attempting to provide a solution to this question, it was observed that teachers had different views about the question; for instance, teacher A provided a single solution without trying to explore further for more solutions. Maybe the question was too simple and hence the teacher did not see the need to explore further. It could also mean that he did not understand the question before he started to solve it. It might also have been due to lack of good pedagogical content knowledge. Teacher D gave $(0:30);(5:26);(10:22);(15;18);(20:14);(25:10);(30:6)$ as answers. The answers provided by teacher D were correct but not complete. The teacher could have explored further in order to complete the task. This might have been an oversight. Teachers B, C, E and F were not able to understand the question let alone provide the answers. This could have been lack of good PCK. Thus, teachers lacked concept, procedure fluency and adaptive reasoning. This is because the question required conceptual understanding and procedural fluency. This question was poorly answered by the participant teachers. None of them was able to get the answer correct since they lacked PCK and procedural fluency. Teachers B, E and F proved that many mathematics teachers did not do not have sufficient mathematics knowledge. This question could easily have been solved using substitution. Consider the equation: $4p + 5q = 150$. The following pair can be used; (35;2), (30;6), (25;10), (20;14), (15;18), (5;26), (10;22) and (30;6) ; thus, 8 pairs satisfy the equation. The standard of this question was too easy for a graduate teacher to get it wrong.

METHODS IN TEACHING

4.2.14 Question 2.2

Explain how you would help your Grade 9 children to understand the names of the side of the right-angled triangle relative to the angle under consideration.

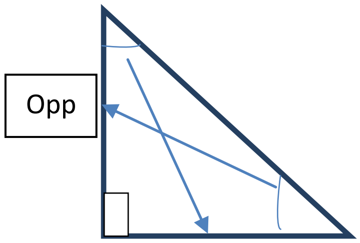
<u>Responses:</u>

Teacher A: After cutting out the shape from the paper, they will see a longer side and you tell them that the side is called hypotenuse, the side directly across the hypotenuse you let them know is the opposite and the down one is called adjacent.

Teacher B: The side that is opposite to the angle under consideration is called the opposite side.

Teacher C: Two perpendicular sides that make an angle of 90°. The longest side that is opposite to 90° is referred as the hypotenuse.

Teacher D: Identify the longest side of the right-angled triangle; ask the learners where to find the longest side of a right-angled triangle.

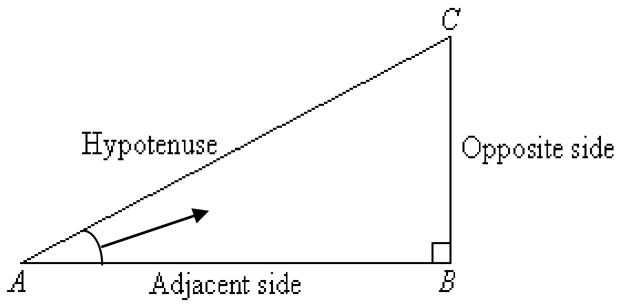


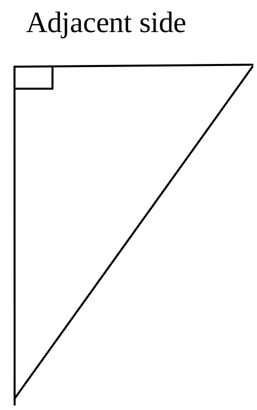
An angle will be placed inside the triangle as shown beside to demonstrate opposite and adjacent and the side opposite the 90° as hypotenuse. Hypotenuse does change.

Teacher F: No answer.

Teacher E was able to help the learners with the aid of a diagram. Thus, teacher E displayed good pedagogical content knowledge and instructional practice concerning the topic but did not demonstrate enough; thus, in establishing good and concrete understanding, he could perhaps have used many diagrams of showing right angled triangles to demonstrate the topic. It could have been an oversight or possibly, they did not perceive the use of diagrams. Teachers A, B, C, and D could not handle the question as expected, though they tried. This could have been a lack of good PCK and instructional practice. Teacher F was not able to touch the question. The question was probably too difficult for him. Perhaps he had limited instructional practice regarding the topic.

Learners may not have been able to grasp the concept and flow of the question since the introduction was not clear. This question needs many diagrams depicting right-angled triangles with different shapes. Also, right angled should have been explained before dealing with the sides. Conceptual understanding, procedural fluency, strategic competence and adaptive reasoning (mathematical knowledge) are needed for this topic to be introduced. The diagrams show a right-angled triangle in different shapes and positions. The learners should understand that a right-angled triangle is the triangle which has one angle of 90^o. The side which is opposite the right angle is the longest side and it is called the hypotenuse. The opposite side is the one across from a given angle, and an adjacent side is next to a given angle (conceptual understanding/PCK, procedure and reasoning).





4.2.15 Question 2.3

The volume V of a cylindrical bin (open at one end) of a radius r and height is V=$\pi r^2 h$. Describe how you would lead Grade 9 learners to discover for themselves that the volume of a right cone having a radius and height as the cylinder is $\frac{1}{3}\ \pi r^2 h$.

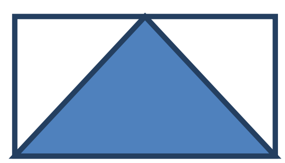
<u>**Responses:**</u>

Teacher A: No answer.

Teacher B: V = Area × Height

$= \pi r^2 h.$

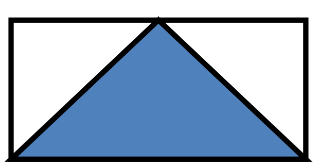
Cylinder is opened to form a rectangle and divided into 3 triangles with the shaded triangle forming the cone 1 out of 1 is $\frac{1}{3}\ \pi r^2 h.$



Teacher C: No answer.

Teacher D: No answer.

Teacher E

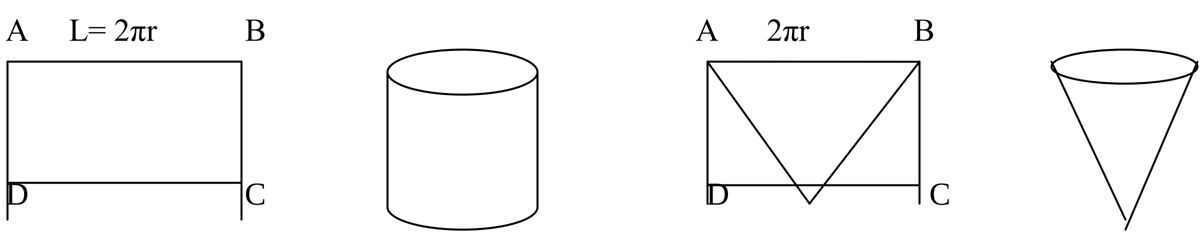


The cone will be placed in a cylindrical bin and water will be poured till the bin gets filled up. When measured, the learners are going to discover that the cone covers $\frac{1}{3}$ of the space of the cylinder. Hence $\frac{1}{3}\ \pi r^2 h$ is the volume.

Teacher F: Not clear.

In trying to provide an answer to this question, teacher B and E managed to answer the question though not accurately. They demonstrated mathematical knowledge and instructional practices. Teachers A, C, D and F could not answer this question. This may be due to a lack of pedagogical content knowledge and knowledge of instructional practices. This may also mean that they were not conversant with the question. Some may even have been confused with the question. It was evident that many learners would take a long time to understand the concept or would simply never understand it. In establishing the answer to this task, one could have considered diagrams and practical demonstrations when introducing this concept, thus, real/concrete and semi-concrete teaching materials are needed before introduction of this topic. Example: the volume of a cone can be developed by comparing the cone with a cylinder of the same height and same base. With firm cardboard, make a cylinder and cone of the same height and same base and leave one end of each solid open.

Rectangular shape Cylinder Triangle Cone



Fill the open cone with sand and empty the contents into the open cylinder with the same radius and height. Repeat this till the cylinder is full. It will be noticed that you need three times the content of the cone to fill the corresponding cylinder (concept/PCK, procedural, strategic competence and adaptive reasoning).

Therefore, 3 times the volume of the cone is equal to the Volume of the cylinder.

Therefore, Volume of the cone = $\frac{1}{3}$ of the volume of the cylinder.

But Volume of the cone = $\pi r^2 h$. Therefore, Volume of the cone = $\frac{1}{3}\pi r^2 h$ (adaptive reasoning).

4.3 OBSERVATIONAL SCHEDULE

VE: Very Evident E: Evident SE: Slightly Evident NE: No Evident

4.3.1 SECTION A: Teaching for Thinking

4.3.2 Part 1 : **Questioning**

	VE in %	E in %	SE in %	NE in %
The teacher's questioning style encourages confidence in the learners	**33**	**50**	**17**	**0**
The teacher's questioning style encourages learners to participate in the learning	**0**	**83**	**17**	**0**
The teacher's questions are clear	**17**	**66**	**17**	**0**
The teacher's questions are relevant	**17**	**50**	**33**	**0**
The teacher's questioning shows continuity	**0**	**50**	**50**	**0**
The questioning style encourages learners' understanding	**0**	**67**	**33**	**0**
The questioning style encourages learners to transfer.	**0**	**50**	**50**	**0**

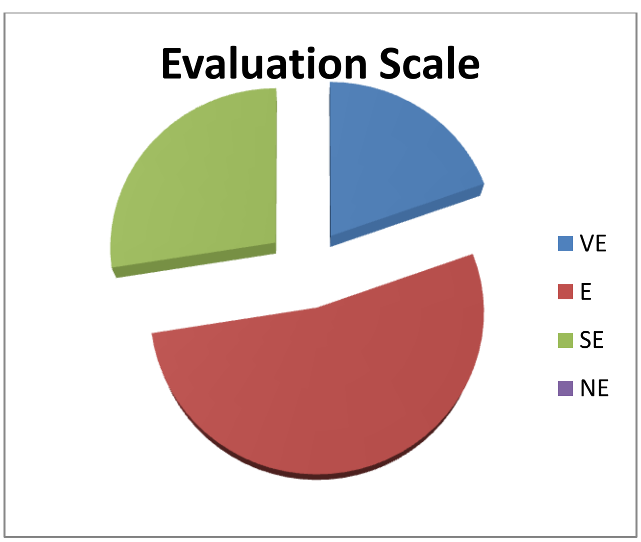


Figure 4.3.1: Average of teachers' questioning style in percentage

Figure 4.3.1 shows that most of the participant teachers asked questions (52.7%) during teaching but only a few questions (19.5%) were clear and relevant to the topic under discussion during observation; for instance, most of the questions asked by some teachers were general questions designed to create a cordial relationship between teacher and learners such as "What level do you expect in mathematics by the end of the year?" "What do you want to be in future?" "How many of you have finished my assignment?" and so on. It could be that some of the teachers did not prepare well before the lesson. It could also mean that some teachers lacked good instructional practices. Some may have also lacked good pedagogical content knowledge. Content knowledge deals with the instruction procedure, involving the best way of representing and communicating content and how learners' best understand the specific concepts and topics of a subject (Shulman, 1986, p. 4). This indicates that learners may become confused along the way when teaching and learning mathematics is in progress especially if the teacher does not prepare well before the class. It is very necessary for a teacher to interact with learners in a context. The most important aspect of what goes on in a class is to make learners understand the topic under discussion. Confusion in one concept may impact other subsequent concepts. Also, progression may be affected. Many may not be able to understand the next topic if there was confusion in the previous one. A learner should be able to strategize and progress in solving a mathematical problem if the previous topics are well understood.

4.3.3 Part 2: Teacher feedback

	VE in %	**E in %**	**SE in %**	**NE in %**
The teacher displays good listening skills	**0**	**83**	**17**	**0**
The teacher uses feedback to focus the learners' attention on the topic (concept)	**0**	**50**	**50**	**0**
Feedback from the teacher is used to build the learner's self esteem.	**0**	**50**	**50**	**0**
The teacher uses the learners' answers to elaborate on lesson issues.	**0**	**83**	**17**	**0**

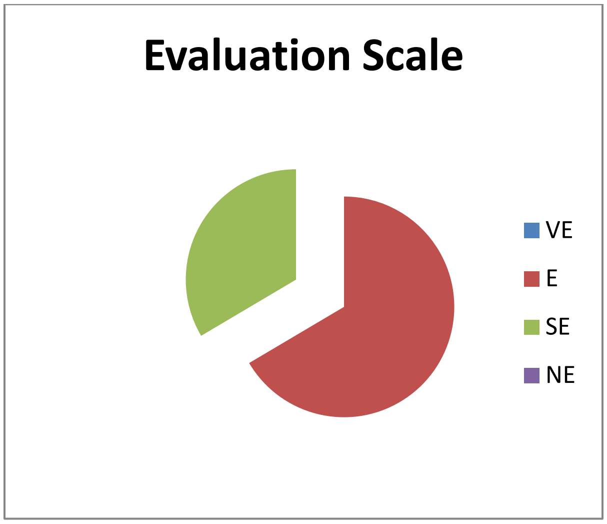


Figure 4.3.2: Average of how teachers give feedback in percentage

The graph indicates that participant teachers (66.5%) do give feedback to learners but much of the feedback does not focus the learners' attention on the concept under discussion since most of the questions given did not focus on the standard of the concept under discussion. During observation, it was noticed that most of the teachers could not finish marking exercises given to learners. This may have been due to the time factor, the sizes of the classes and may imply that none of the participant teachers was able to focus on the mathematics concept/knowledge strictly as far as giving feedback was concerned. This could be owing to a lack of good instructional practices. It could also mean that some teachers lacked good content knowledge. Possibly most teachers merely do routine work to meet the requirement of the Department of Basic Education without taking into consideration the needs of the learners. Exercises are supposed to be given according to the standard of the topic under discussion so that the feedback informs one about the learners' performance accordingly.

4.3.4 Part 3: Cooperative Learning

	VE in %	E in %	SE in %	NE in %
Effective use is made of pairs	**0**	**33**	**67**	**0**
They organised the seating of learners appropriately	**0**	**33**	**67**	**0**
The instructions are clear	**0**	**83**	**17**	**0**
Learners are given the opportunity during cooperative learning to use their ideas	**0**	**33**	**67**	**0**
The teacher plays a supportive role when the learners are working in groups	**33**	**0**	**67**	**0**
The teacher mediates feedback from the group effectively	**0**	**17**	**83**	**0**
The teacher makes sure that the learners have a record of work they have done together as a group or in pair.	**0**	**17**	**67**	**17**

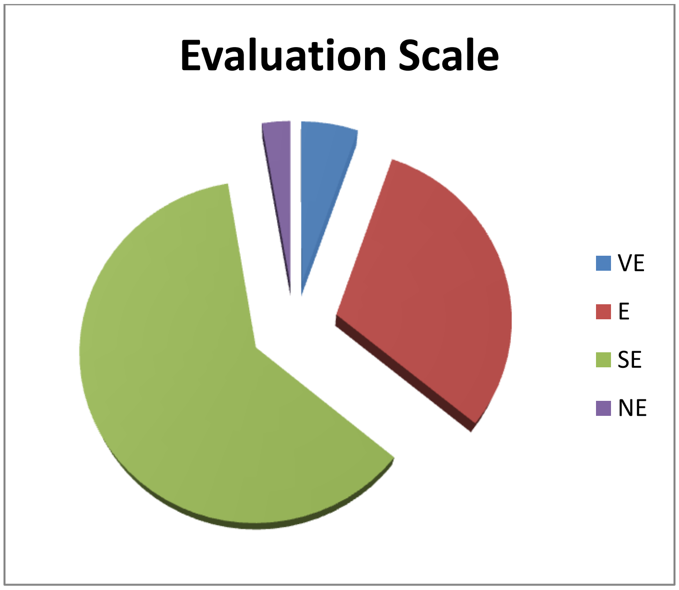


Figure 4.3.3: Average of how teachers engage in cooperative learning in percentage

Figure 4.3.3 shows that most of the participant teachers (64%) did not engage their learners in cooperative learning. This may be because of a lack of instructional practice, lack of space in class as well as ignorance and also lack of proper planning; for instance, during observation, it was realised that some teachers lacked organization skills which should have been part of instructional practices. This deprived learners of sharing ideas. Sometimes learners may better understand a mathematical concept, acquire deep knowledge and progress regarding the topic under discussion in groups rather than working or learning alone. Less than 50% of participant teachers gave learners an opportunity to learn in groups, according to the graph.

4.3.5 Part 4: The role of language in learning

	VE in %	**E in %**	**SE in %**	**NE in %**
The teacher creates time for the learners to talk about the topic	**0**	**67**	**33**	**0**
Learners are encouraged to talk in group	**0**	**33**	**67**	**0**
Learners are encouraged to use their own words (Mathematical terms)	**0**	**83**	**17**	**0**
There are opportunities for the learners to explain and elaborate on their own answers	**0**	**50**	**50**	**0**
There are opportunities for the learners to interpret the answers of others	**0**	**50**	**50**	**0**

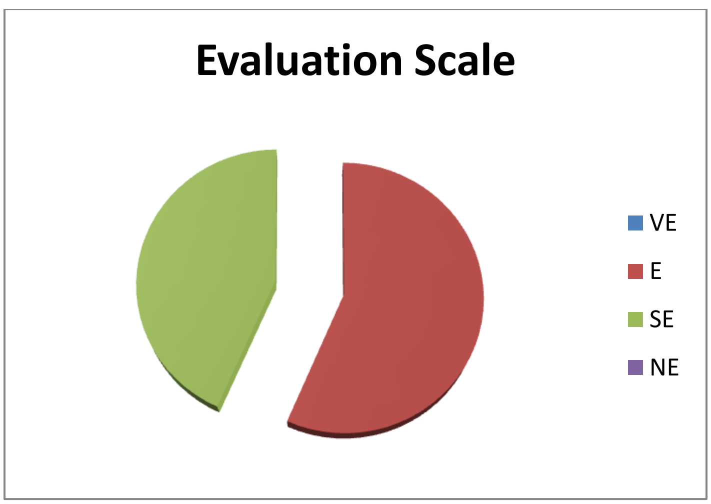


Figure 4.3.4: Average of how teachers use mathematical language in percentage

The figure indicates that mathematical language was used by participants but not sufficiently; 43.4% of the participants did not help their learners to understand mathematical terms, and neither did they (teachers) use them. This could be owing to a lack of preparation. It could also mean that some teachers lacked sound pedagogical content; for instance, teacher A could not explain “symmetry” to his learners when he was teaching transformation during the observation. Perhaps this teacher did not prepare his lesson before the class. It could also mean that he lacked the mathematical terms which form part of the required concept/pedagogical content knowledge. In another instance, teacher B could not explain “mean” to the learners though he was able to teach the calculation of “mean” correctly when teaching data handling. This could have been because of a lack of sound pedagogical content knowledge.

4.3.6 Part 5: The use of learning AIDS

	VE in %	**E in %**	**SE in %**	**NE in %**
The teacher use concrete learning AIDS	**17**	**33**	**50**	**0**
The teacher makes use of textbook	**33**	**50**	**17**	
The concrete AIDS helps the learners to understand the concepts	**0**	**50**	**50**	**0**
The textbook helps the learners to understand the concepts	**17**	**67**	**17**	**0**
The teacher takes the learners through concrete experiences to symbolic and then abstract experiences	**0**	**33**	**67**	**0**

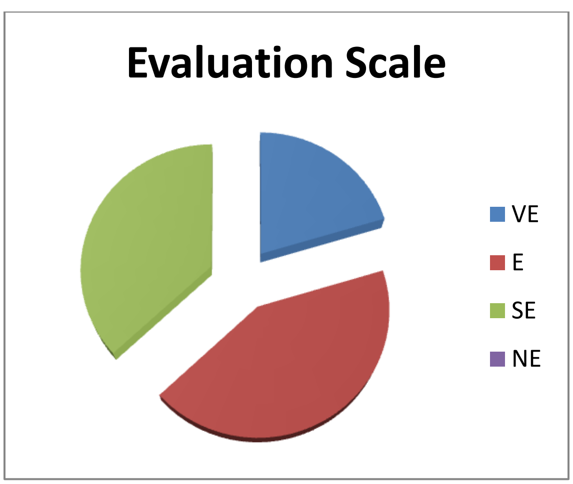


Figure 4.3.5: Average of the use of teaching and learning AIDS by teachers in percentage

Teaching and learning AIDS assist learners to retain the concept under discussion. Figure 4.3.5 reveals that about 37% of the participant teachers denied learners an understanding of mathematical concepts through teaching and learning aids. This may have been due to inadequate teaching and learning material in some schools. It could also mean that some teachers failed to prepare before going to class. Teachers may also have lacked good content knowledge; for instance, teacher E was teaching about measurement (surface area and volume of 3D shapes) so this concept could have been practically demonstrated using improvised teaching AIDS such as cans, boxes etc. This teacher used only textbooks and the chalkboard to teach the concept. When the teacher understands the concept, preparation should then be a priority.

4.3.7 SECTION B: Learner Behaviour

	VE in %	**E in %**	**SE in %**	**NE in %**
The learners are confident about answering questions	**0**	**67**	**33**	**0**
The learners participate actively in the learning process	**17**	**66**	**17**	**0**
The learners are willing to answer questions	**0**	**50**	**50**	**0**
There is evidence that the learners can work effectively in pairs/groups.	**0**	**17**	**83**	**0**

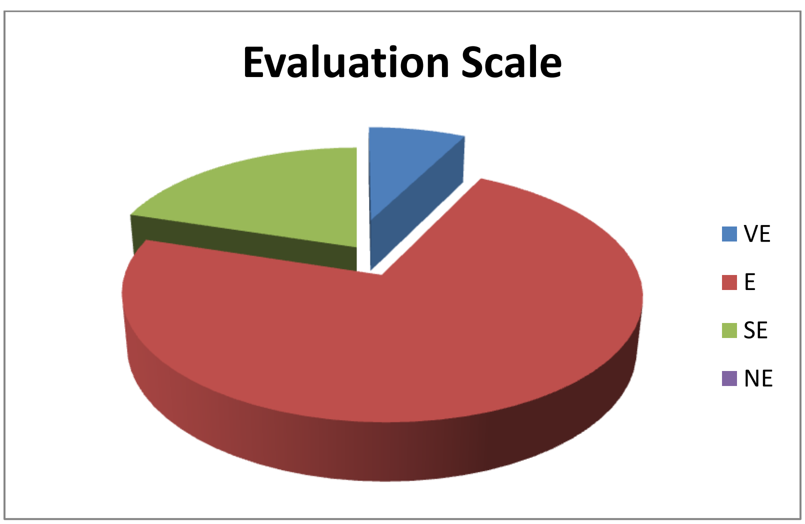


Figure 4.3.6: Average of how learners participate in lessons during teaching and learning

Figure 4.3.6 clearly shows 72.3% of the learners in participants' schools were willing and interested in acquiring mathematics knowledge but that knowledge was not available as far as mathematics was concerned, thus, teachers' limited knowledge of mathematics meant a limited impact on learners. It could be that some of these teachers' lack of mathematical knowledge was because they did not specialise in mathematics but were compelled to teach mathematics for numerous reasons. This could also mean that some mathematics teachers did not want to upgrade themselves in order to adapt to the new system.

4.3.8 SECTION C: Teacher/Learner Relationship

	VE in %	**E in %**	**SE in %**	**NE in %**
The teacher establishes a warm atmosphere	**33**	**50**	**17**	**0**
The teacher is open and flexible	**50**	**33**	**17**	0
Praise and encouragement is used appropriately	**0**	**50**	**50**	**0**

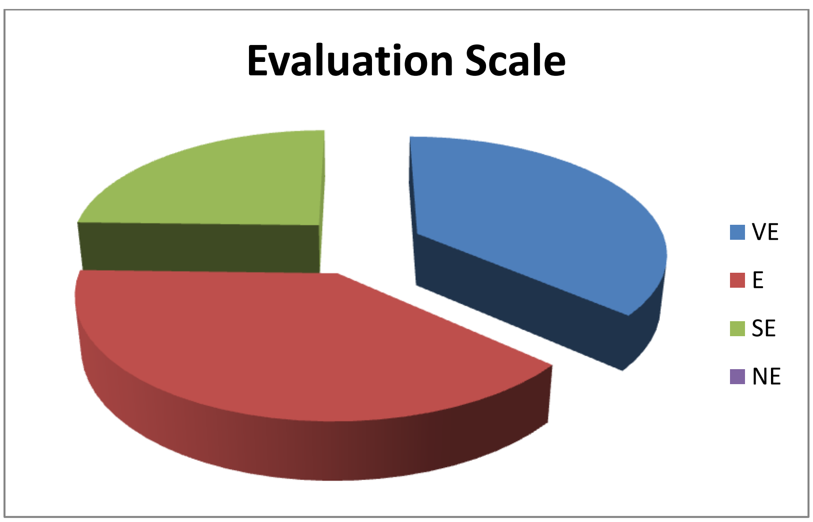


Figure 4.3.7: Average of teacher/learner relationship in percentage

The graph shows that there were cordial relationships between participant teachers and their learners; 75.5% of the teachers were open and flexible with learners which in essence should promote good teaching and learning of mathematics. Perhaps, teachers simply lacked good pedagogical content knowledge despite cordial relationships.

AVERAGE PERFORMANCE

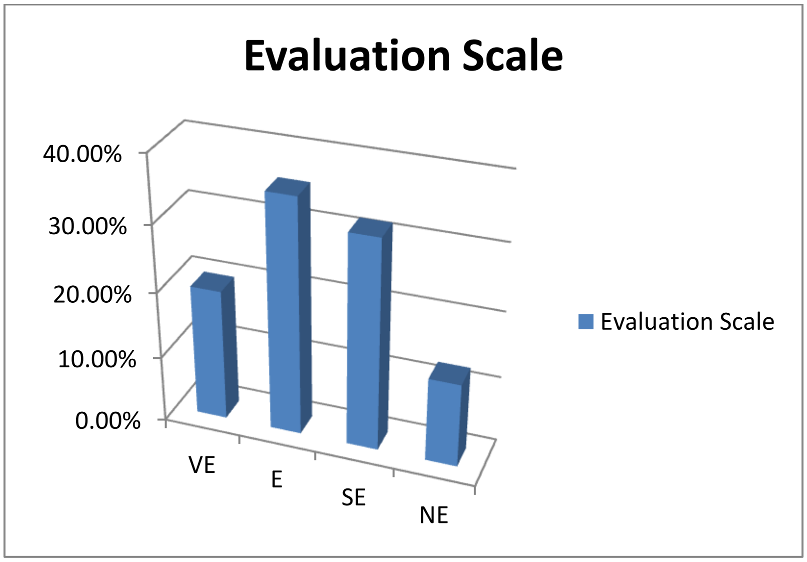


Figure 4.3.8: Overall average performance of teachers in percentage

Figures 4.3.8 shows the summary of the evaluation scale which reveals how various participant teachers demonstrated their mathematical knowledge in class. According to the graph, it is clear that "very evident" (VE) records 20% which is not a good sign. This means that teachers' mathematics knowledge was not enough to produce good performance from learners that would help improve general achievement. It could also mean that the pace at which learners were acquiring mathematics knowledge was limited and slow. Another answer that is necessary to consider is "not evident" which records 12.7% and worsens the case since, "slightly evident" (SE) totalled 31.8% neither is good for learners' mathematics skills. This revealed that learners' ability to acquire mathematical knowledge would take time.

4.4 ONE- ON-ONE- INTERVIEW

4.4.1 What is your highest qualification?

Teacher A: B.Ed. Honours in Mathematics and Science

Teacher B: Advance Certificate in Education (ACE)

Teacher C: Not Interviewed

Teacher D: B.Ed. Honours in Mathematics

Teacher E: B.Ed.

Teacher F: Not Interviewed

The answers to this question reveal that some of the participant teachers were not qualified to teach mathematics; for instance, teacher B displayed insufficient content to teach mathematics. This implies that Pedagogical Content Knowledge (PCK) of such teachers may be limited as far as mathematics is concerned. Teacher E may also have had the same problem as teacher B since he did not specialize in mathematics, though a graduate teacher. This may affect teaching and learning of mathematics negatively in schools.

4.4.2 What is your major subject? (Specialization)

Teacher A: Mathematics and Science

Teacher B: General

Teacher C: N/A

Teacher D: Mathematics

Teacher E: General

Teacher F: N/A

This reveals that some teachers may be compelled to teach mathematics for one or other reason. If so, the lack of interest in teaching the subject may rub off on the learners who suffer when there is a lack of any enthusiasm of the subject; for instance, teachers B and E may not have had an interest in teaching mathematics because they were compelled to teach the subject. This may affect mathematical proficiency as far as concept/ pedagogical content is concerned.

4.4.3 For how long have you been teaching mathematics?

Teacher A: Since 1995

Teacher B: Over 15 years

Teacher C: N/A

Teacher D: Since 1989

Teacher E: Since 1991

Teacher F: N/A

Experience is good but if one does not keep oneself updated this can be a problem. Dynamics play a past in every subject, and teachers should adjust. In mathematics, the curriculum changes often and every teacher should keep abreast with the trends and innovations. The participants' answers to this question revealed that almost all teachers were experienced in teaching mathematics but some of them did not do well on the task sheet or in teaching. Possibly they were not updating themselves in order to cope with the new curriculum. It could also mean that they were not interested in the subject but were teaching it because of shortage of mathematics teachers.

4.4.4 Which grades are you teaching presently?

Teacher A: Grade 9 only

Teacher B: Grade 8 and 9

Teacher C: N/A

Teacher D: Grade 7, 8 and 9.

Teacher E: Grade 8 and 9.

Teacher F: N/A

The answers to this question confirmed that participant teachers were presently teaching mathematics in Grade 9. It also means that all tasks administered by them were within what they had to teach in their classes, thus, concept/pedagogical content knowledge, procedural fluency, adaptive reasoning, strategic competency and disposition were expected to be demonstrated by the teachers. Surprisingly, some of the answers provided by teachers on the task sheet did not reflect the fact that they taught the same grade from which the questions were taken.

4.4.5 What are some of the challenges you face as far as teaching mathematics in Grade 9 is concerned?

(a) Learners want to memorize mathematics instead to understanding it.

(b) They lack mathematical terms which makes interaction during mathematics lesson difficult.

(c) Learners' previous knowledge is full of misconceptions, thus, mathematics is a difficult subject.

(d) Most learners do not have good mathematics background.

(e) Hands on activities to teach algebra are difficult (lack of algebra teaching AIDS).

(f) Large number in a class which prevents group learning and individual attention.

(g) Inadequate teaching and learning support material.

(h) Insufficient mathematics teachers to support.

Responses to this question showed that teachers were facing numerous challenges which make teaching of mathematics difficult; however, a teacher needs to be creative enough to overcome most of these challenges; for instance, teachers who are experiencing inadequate teaching and learning support material need to be innovative in order to improvise in order to support their teaching. Some teachers may have lacked good pedagogical content knowledge and instructional practices that prevented them from improvisation. In this case, interest and enthusiasm are very necessary but are found wanting among most of the teachers in this study.

4.4.6 How are you trying to overcome these challenges?

(a) Interact with their parents (Teachers A and D).

(b) Organise extra classes (Teacher D).

(c) Improvise teaching and learning support material such as flash cards, real objects etc. (teacher D).

In trying to provide answers to this question, it was realized that some teachers did not have any idea of how to overcome some of the challenges; for instance, teachers B and E did not respond to the question. Possibly they lacked conceptual understanding. It could also be that they were not interested in teaching the subject but were compelled to teach it.

4.4.7 Learners had basic knowledge about mathematics before coming to school. Do you agree with that? Explain.

(a) Yes, since they can count and also identify some shapes before coming to school.

(b) Yes, mathematics is a daily life living.

4.4.8 How do you approach your lessons in order to build on the basic knowledge?

(a) Always give activities to the learners to find out whether they have previous knowledge about the concepts to be introduced (Teachers D).

(b) Prepare about the new concept (A and D).

(c) Find out what the learners already know and build on it (A, D and E).

(d) Clear the misconceptions about the concept (D).

In providing answers to this question, it was noted that some teachers did not have enough information about how to build on basic knowledge; for instance, teacher B provided no answer while teacher E provided a single piece of information. This could be owing to a lack of concept and instructional practices. It could also mean that they do not prepare before teaching. Teachers A and D provided two or more bits of information but that is not enough. They could have considered the questions and answers method, discussions, a quiz, etc in order to build on the basic knowledge.

4.4.9 Reality is one of the important aspects of teaching mathematics. If you agree with this, can you tell me how you make your lessons real?

(a) Always use materials around the environment to support my teaching. Example, using round houses to teach geometry. Also improvise some of the materials in the environment to support teaching (D)

(b) Using of concrete material (A)

In attempting this question, some participants such as teachers A and D, provided answers which were insufficient while others did not provide any answers. This could be through lack of concepts and adoptive reasoning. It could also mean that they lack in-service training that reminded them of certain aspects such as improvisation, use of concrete materials etc, in teaching mathematics. They could have considered using readily available materials such as shape of the classroom, windows, door, cans, playground, fields, ages and heights of the learners etc to teach some concepts such as 2D and 3D shapes, data handling, etc.

4.4.10 What are some of the strategies you use to make your lesson real?

(a) *Give project to learners on the topic under discussion (A and D).*

(b) *Give investigative type of question (D).*

In attempting to answer this question, some teachers provided inadequate answers for the question. This may be due to lack of good instructional practices. It could also be lack of interest in the subject. One could have considered using readily available materials in the teaching environment to make mathematics lesson real, as described under question 4.4.9 above, thus, giving mathematics projects and investigations to learners are not enough to make a mathematics lesson real.

4.4.11 How do you develop your learners' interest in Mathematics?

(a) *Use of concrete object during lesson (teacher A).*

(b) *Give group work and discussion (teacher D).*

The answers to this question revealed that teachers needed more information regarding their teaching experience; for instance, the answers given by teachers A and D were correct but not sufficient to help achieve the goal of developing learners' interest in mathematics. Teachers could have considered organizing mathematics competitions among learners, motivation and awards, career guidance, exhibitions in mathematics, etc.

4.4.12 How do you see mathematics lessons when conducted in groups?

(a) *The lesson becomes interested and learners interact with one another (A, D and E).*

(b) *It becomes productive because the learners discuss effectively and even come out with new concepts and ideas. They also understand the concept easily because they explain to each other in the group (D).*

The answers to this question were quite impressive, especially on the part of teacher D. This is a sign of having sound knowledge of instructional practices. They could also have considered promoting team work.

4.4.13 What kind of teaching support material do you normally use when teaching mathematics?

(a) *Textbooks, workbooks, instrument box and the use of chalkboard (A, B, D and E).*

(b) Real objects from the environment, improvise some of them and some are from Rhodes University and the Department of Education (D).

Attempting to answer this question, most of the teachers provided answers that were considered to pertain to normal teaching and learning materials; for instance, teachers A, B, and E mentioned materials that were always provided by the school and the Department of Basic Education. Those materials alone cannot help to promote effective teaching and learning. Teachers also need to consider the answer from teacher D on improvising. This could be owing to lack of good instructional practices and lack of interest in teaching the subject. Teacher D showed initiative, creativity and interest in teaching mathematics, based on the answers provided.

4.4.14 Do you invite resource personnel during some of your lessons to witness teaching?

Teacher A: Once in a while

Teacher B: Never did that before and it never occurs to me.

Teacher C: N/A

Teacher D: Try to find help from other colleague teachers

Teacher E: Not really

Teacher F: N/A

Answers to this question revealed that teachers taught what they were familiar with and left the rest. This happens if mathematical knowledge is lacking. Taking the task sheet into consideration, it can be deduced that most of the participant teachers lacked mathematical knowledge and they did not invite resource personnel to witness teaching and learning of mathematics; for instance, Teachers B and E's responses show that they were ignorant about requesting for assistance from another mathematics teacher. It could also be that they thought learners would loose interest in their teaching.

4.4.15 How do you help your learners to maintain positive attitudes towards mathematics?

(a) Make mathematics fun, they play with mathematics and also make lessons friendly (D).

(b) Practice mathematics daily (A and D).

Only two participant teachers were able to respond to this question. This revealed that most of the teachers did not know how to help their learners to adopt a positive attitude towards mathematics. This might be a question of low self esteem on the part of the teachers.

4.4.16 What can you say about the learners' performance?

Most of them are below average (A, B, D and E).

This answer is obvious since most teachers lacked mathematical knowledge. Teachers can only boost the learners' performance if they (teachers) try to acquire mathematical knowledge. Moreover, also mean that learners these teachers were teaching probably had a very limited mathematics background. It may also mean that teachers did not prepare well enough to meet the demands of the learners. The Pedagogical Content Knowledge (PCK) informs the teacher of what to do to help the learners.

4.4.17 What are possible causes of poor performance?

(a) Learners' attitude towards mathematics.

(b) Inadequate teaching and learning aids.

(c) The foundation of the learners in mathematics is not good.

(d) The size of the class does not allow effective learning.

These are some of the challenges that were discussed in detail in question 4.4.5 above.

4.4.18 Do you draw up a lesson plan everyday, weekly or monthly?

Teacher A: Monthly

Teacher B: Not often

Teacher C: N/A

Teacher D: Monthly

Teacher E: Not often

Teacher F: N/A

If teachers fail to plan, they plan to fail as teachers. Mathematics needs thorough preparation before classes begin to avoid confusion. Teachers' answers to this question revealed that most of the teachers did not take lesson preparation seriously. Perhaps they did not know the value of preparation; for instance, the answers provided by teachers B and E clearly showed that some teachers did not take lesson preparation seriously. This could mean that there was no proper monitoring or supervision in schools.

4.4.19 Are you comfortable with every topic in the syllabus?

Every participant responded by saying 'no' to this question

4.4.20 If no, what happens to those topics before the year ends?

Teacher A: Seek assistance from other colleagues.

Teacher B: Try to teach them somehow.

Teacher C: N/A.

Teacher D: Invite resource person.

Teacher E: Try to teach them towards the end of the year.

Teacher F: N/A.

In attempting to provide answers to this question, it was noted that some teachers did not know how to help the learners to understand the concepts which they were not comfortable; for instance, teachers B and E's answers were alarming; probably they were ignorant of how to develop and learn new concepts. They could have considered attending in-service training and not necessarily waiting for one organized by the Department of Basic Education.

4.4.21 How do you teach a new concept so that your learners understand?

(a) *Prepare in advance (A and D).*

(b) *Build on the previous knowledge (A, B, D and E).*

(c) *Ask learners to bring concrete material related to the topic (D).*

In attempting to answer this question, some teachers' answers were inadequate regarding new concepts; for instance, teachers B and E provided only one idea and that was insufficient. This could be because of lack of good instructional practices and concepts. Teacher A provided two answers which were somewhat better than A. This could also owing to lack of instructional practices. Teacher D tried to give three answers showing that the other teachers could also have provided more answers regarding methods

4.4.22 How often do you give tasks to your learners?

Quiet often but feedback takes time due to the size of the class (A, B, D and E).

The answer to this question revealed that teachers faced problem regarding feedback to learners when tasks were given. This was due to the sizes of the learners' classes. This could also reflect lack of commitment and dedication to service. They could have considered adopting a strategy of giving feedback as soon as possible by allowing learners to mark their friends' work.

4.4.23 Do you always feel satisfied with your learners' performance after giving them tasks?

Not really. Learners are not prepared to work hard. Do not read instruction (A, B, D and D).

4.4.24 If not, how do you help them improve?

We try our best to help the learners.

4.4.25 How often do you attend in-service training /workshops in mathematics?

When organised by the Department of Education

4.5 CONCLUSION

The chapter focused on data presentation and analyses. The next chapter will include the discussion, conclusion and recommendations.

CHAPTER 5

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This study aimed to explore different types of knowledge that teachers demonstrate when teaching mathematics in schools in the Mthatha District. The chapter follows up on the research questions outlined in Chapter one, conclusions and recommendations are drawn from the findings of the study.

5.2 DISCUSSION

5.2.1 Research question 1: What types of knowledge are necessary for teaching mathematics?

According to Shulman (1986), teachers need to master two types of knowledge:

"(i) Content, also known as "deep" knowledge of the subject itself, and

(ii) Knowledge of the curricular development" (Shulman, 1986, p. 4). Content knowledge encompasses the "structure of knowledge", the theories, principles and concepts of a particular discipline. Of great importance is content knowledge that deals with the teaching process, including the most useful forms of representing and communicating content and how learners' best learn the specific concepts and topics of a subject.

Related studies (Shulman, 1986; Solis, 2009) have shown that pedagogical content knowledge is essential for mathematics teachers in teaching of mathematics. This is very important in the sense that teachers with bad content knowledge in most cases find it difficult to teach some specific concepts in mathematics to learners, as they (teachers) cannot interpret some concepts and further relate them to the real life situations for better understanding of learners.

Pedagogical content knowledge, according to Solis (2009) and Kilpatrick et al (2001), has more to do with how teachers interpret some concepts in mathematics through transformation of subject-matter knowledge that supports learners' understanding in teaching of mathematics. This helps to address some fundamental elements of pedagogical content knowledge of teaching mathematics as indicated by Solis (2009, p. 2) and Kilpatrick et al (2001, p. 370) as addressed in the review related literature, thus

"(i) Knowledge of representations of subject matter (content knowledge),

(ii) Understanding of students' conceptions of the subject and the learning and teaching implications that were associated with the specific subject matter,

(iii) General pedagogical knowledge (or teaching strategies),

(iv) Curriculum knowledge,

(v) Knowledge of educational contexts,

(vi) Knowledge of the purposes of education."

"(vii) Knowledge of mathematics,

(viii) Knowledge of students and

(ix) Knowledge of instructional practices."

These elements knowledge according to review of literature in the current study shows that they (knowledge) were presented poorly in teaching mathematics.

5.2.2 Research question 2: What kinds of knowledge do the selected teachers demonstrate in their teaching?

It emerged from this study that most of the selected teachers demonstrated limited mathematical knowledge. This mathematical knowledge according to Kilpatrick et al (2001), involves conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition. Participant teachers who were unable to demonstrate the above knowledge revealed that many teachers displayed limited mathematical knowledge. For example, teacher A could not explain "symmetry" to his learners when he was teaching transformation during the observation. Teacher B also could not explain "mean" to the learners when he teaching data handling. An effective mathematics teacher is supposed to handle mathematics questions with understanding. According to Luneta (2014), teachers who are effective and knowledgeable in teaching mathematics frequently reflect on their linked mathematical knowledge bases and carefully combine them with their experience, skill and understanding when teaching mathematics, thus, an effective mathematics teacher needs to use mathematical knowledge acquired and blend it together with past experiences, in order to approach mathematics problems. Luneta (2014) explains that content knowledge of the subject is essential for a teacher to possess. This content knowledge is classified as 'common content knowledge' and 'specialized content knowledge' which is necessary for a mathematics teacher to possess in order to solve and teach mathematics effectively (Luneta, 2014). It was however evident that most of the participant teachers could not demonstrate this kind of knowledge. There are numerous factors that may link to this. Some of these have been discussed in the conclusion of this study (5.3).

It also emerged from this study that knowledge of instructional practices for participant teachers were not up to standard. The limited knowledge of mathematical practices was revealed when participant teachers were observed; for instance, during lesson observation, it was realised that most of the participant teachers lacked planning, organisation skills and so on; which were part of knowledge of instructional practices. Knowledge of instructional practices involves curriculum, tasks and tools for teaching. Teachers' performance in class revealed a lack of knowledge of instructional practices. Effective instruction in any topic requires a teacher to develop sound instructional strategies and knowledge of functional resources and activities (Luneta, 2014). It is therefore very important that teachers should be aware of the teaching environment and adapt their instructional approach or strategy to teaching various concepts by engaging in a more practical way that helps learners' understanding of the concepts under discussion.

It emerged from this study that most teachers lacked knowledge of the curriculum. This was evident when some teachers appeared to be novices regarding the mathematics topics in the curriculum; this is the situation learners finished the year without being introduced to some of these topics in the curriculum. For instance, participant teachers were asked during one-on-one interview whether they were comfortable with every topic in the syllabus (4.4.19 and 4.4.20) and their responses were "no". In this case, progression in the curriculum was disrupted since the foundation for a particular topic was not laid properly. Curriculum/teaching vision is essential in teaching because it forms part of laying a good foundation that explains how teachers approach their subjects (Khoza, 2016) . The curriculum goal can only be achieved if teachers are conversant with the curriculum vision. This is the reason why acquiring knowledge of curriculum is necessary for effective teaching and learning. Teachers are more likely to reflect on their teaching in order to improve teaching practice when they understand the curriculum visions (Khoza, 2016).

The study established that most teachers did not have sufficient mathematics teaching tools for teaching the subject. This was evident when teachers were interviewed. According to the teachers; they did not receive enough teaching and learning materials from the Department of Basic Education. Limited instructional aids in most of these schools affected teaching of mathematics. Insufficient material resources such as mathematical instrument, textbooks, manila cards and so on; discourage teachers from doing their best. It is very important that mathematics teachers are conversant with instructional materials which can support them to communicate more effectively with the learners in the process of teaching and learning. It is rare to acquire meaningful instructional knowledge without adopting a style of consciously using instructional materials. According to Schoen (2006), when one observes a great mathematics lesson, preparations that have been done prior to the lesson are not always evident, thus, prior effort and time are not often visible. In other words, teachers are expected to put in maximum effort in their closet in order to acquire knowledge of instructional practices for teaching mathematics effectively. The impact of a lesson on learners' learning is greatly influenced by what one can probably observe in lesson observation (Schoen, 2006). Materials that support teaching and learning are in position to help teachers to fully prepare for any mathematics lesson. This means that support materials for teaching such as textbooks, workbooks, instrument box, calculators and so on enable the teacher to prepare adequately for mathematics lessons. This study revealed that most schools are unaware of the importance of instructional materials and the affects the mathematical knowledge of the teachers since they (teachers) are not provided with these materials; and most of them do not make any effort to improvise some of these materials. Many mathematics teachers are no longer seeing the necessity of the instructional aids to their teaching of mathematics. If one changes an instructional component of teaching mathematics, it can affect other components directly or indirectly. This is because the entire teaching context must be considered when one is taking teaching decisions (Schoen, 2006). This implies that teaching mathematics should be approached holistically to produce results. Teachers who are capable of making detailed and explanatory plans are able to clear most misconceptions learners bring to classroom. They are able to explain difficulties that learners encounter during teaching and learning by pre-empting them.

It emerged from this study that teachers also displayed limited knowledge of mathematics tasks. The study revealed that participant teachers were not able to demonstrate knowledge of tasks that could encourage learners to think critically. For instance, it was observed that most participant teachers asked general questions such as "What do you want to be in future?" What level do you expect in mathematics by the end of the year?" and so on. The choice of academic tasks is very necessary in shaping learners to do well in examinations. The mathematical tasks teachers choose to use in their teaching affect not only the mathematics that learners learn but also the depth and quality of that learning (Schoen, 2006). Tasks are central to learners' learning, shaping the chance to learn but also granting them the opportunity to view the subject matter (Kilpatrick et al, 2001). According to Schoen (2006), appropriate tasks should be chosen and conveyed to the learners in ways that stimulate interest, maintaining learners' engagement and leading discussions in which the importance of mathematical ideas or concepts embedded in the tasks are brought to the surface. Teachers can choose tasks that develop important aspects of mathematical proficiency (Schoen, 2006). It is important that tasks should also involve mathematical context that has the potential to attract and maintain learners' interest. A good question captures one's interest, activates mental activity and may stimulate learners' creative thinking (Schoen, 2006), thus, a task should create an opportunity for the learners to be innovative. It is very important for effective teachers to understand that teaching mathematics demands considerable preparation for effective teaching and learning and instructional materials are necessary during preparation for teaching mathematics.

5.2.3 Research Question 3: How do teachers use this knowledge in their teaching?

It emerged from the study that most of the teachers displayed limited mathematical knowledge, knowledge of instructional practices and knowledge of curriculum; thus, most of the teachers were not able to demonstrate the expected knowledge in teaching the subject. Sound pedagogical content knowledge in the district was lacking. Teaching mathematics without understanding the concepts disadvantages the learners as was noted in these schools.

It emerged from the study that knowledge of instructional practices was not fully demonstrated by the teachers; for instance, most teachers did not seem to think they needed to prepare adequately. This may have a negative effect on teachers' delivery. Preparation is part of instructional practices which should be taken seriously. Preparation could be considered as the foundation of what goes on in the classroom, thus, success or failure in class can be attributed to the way and manner in which preparation was done. Another part of knowledge of instructional practices is the use of teaching and learning material. It was evident that most of the teachers could not use teaching aids during teaching. This meant learners had to be imaginative. Other instructional practices which were not very evident were cooperative learning, administration of tasks, giving feedback to learners, use of appropriate teaching methods, etc.

The study also revealed that knowledge of curriculum was lacking among most of the teachers. It was evident during the interviews that most of the teachers were not comfortable with some topics in the curriculum. Teachers may simply not teach these topics; avoidance being the result.

5.3 CONCLUSIONS

From the results of this study, it can be deduced that teachers demonstrated limited mathematical knowledge, knowledge of instruction and knowledge of curriculum due to some of the following reasons:

5.3.1 Negative attitude towards the subject: Personal interaction with some participant teachers revealed that they were not interested in teaching mathematics but the current situation compelled them to handling the subject. Such situations were due to lack of mathematics teachers and securing their posts in their schools at any cost. According to some teachers, hardly will staff establishment affects teachers who have to teach critical subjects such as mathematics and science. Teachers who are not qualified to teach mathematics are not likely to be effective.

5.3.2 Difficult to adjust to dynamics of the subject: It was revealed that many teachers found it difficult to adjust since the system often changed. Some of the mathematics teachers found it difficult to adapt to the new curriculum since the new system presented certain concepts they may never have encountered before.

5.3.3 Lack of in-service training: It was revealed that many teachers did not attend in-service training in order to keep abreast of the new system or to upgrade themselves as far as teaching of mathematics was concerned. Absence of in-service training prevents teachers from developing their skills as professionals (Osamwonyi, 2016).

5.3.4 Lack of supervision: Proper monitoring that would assist mathematics teachers was lacking in some schools. This meant that some teachers were not motivated to work effectively for good results.

5.3.5 Inadequate preparation before class: Most teachers did not plan/prepare before going to class to teach. This has a negative impact on teaching concepts under discussion. Knowledge can only be demonstrated properly if it is well understood. This calls for adequate preparation.

5.3.6 Class size: It was noted during the observation that some classes were overcrowded. The number of learners in some classes prevented teachers from realizing their full potential as far as mathematical knowledge and knowledge of instructional practices were concerned; for instance, most of the teachers could not group their learners for cooperative learning due to lack of space in the classrooms.

5.3.7 Lack of qualified mathematics teachers: The study revealed that some teachers were not qualified to handle mathematics resulting in poor pedagogical content knowledge and lack of knowledge of instructional practices to deal with the teaching of mathematics.

5.3.8 Inadequate teaching and learning materials: Most teachers could not demonstrate their knowledge effectively due to a lack of teaching and learning materials. The study revealed that teaching aids apart from textbooks were not available in most of the schools to support teachers in teaching effectively.

5.4 RECOMMENDATIONS

Based on the findings presented in this study, the following suggestions are made in order to improve mathematical knowledge:

5.4.1 The Department of basic Education should frequently organize regular in-service training for mathematics teachers in schools. It is very important for The Department of basic Education to constantly be updating mathematics teachers through in-service training. This will enable teachers to improve upon their mathematical knowledge for effective teaching.

5.4.2 Schools should be provided with adequate teaching and learning materials by The Department of Basic Education in order to improve teachers' knowledge of instructional practices. Teachers can be motivated to improve upon their knowledge of instructional practices if they can see The Department of Basic Education doing something about the challenges they face on a daily basis in schools.

5.4.3 There is a need for teachers to upgrade themselves in order to meet curriculum demands. Teachers should therefore strive to upgrade themselves. This will assist them to acquire wider and sound mathematical knowledge for teaching. There is also a need for the Department of Basic Education to provide financial aid for prospective mathematics teachers that will motivate them to upgrade their qualifications

5.4.4 The Department of Basic Education should strengthen their supervision team to monitor mathematics teachers in schools. This will encourage mathematics teachers to prepare well and give of their best.

5.4.5 Use of resource persons: Many of the participant teachers avoided some of the topics. This is due to the fact that they were not conversant with the topics or lacked the pedagogic content knowledge for those topics. This challenge could be overcome by inviting a resource person to teach those topics for the teacher or to demonstrate methods.

5.4.6 Workshops: Schools or clusters of schools ought to organize workshops for their mathematics teachers. In this case, in-depth discussions and addressing specific issues relating to the subject of mathematics might help.

5.4.7 Employing qualified mathematics teachers: The Department of Basic Education and the schools should try, as possible, to employ qualified mathematics teachers who can teach the subject effectively. These qualified teachers should possess good PCK and five strands of Kilpatrick et al’s (2001) of mathematical proficiency. There is also a need for teachers who undergo training to be committed to teaching of the subject once they are armed with the additional skills.

REFERENCES

Andy, H. (2006). Research Glossary. Retrieved from: http://www.learnhigher.ac.uk

Asikhia, O. A. (2010). Students and Teachers' Perceptions of the Causes of Poor Academic Performance in Ogun State Secondary Schools [Nigeria]: *Implications for Counseling for National Development*. Retrieved from: http://www.eurojournals.com

Babbie, E. R. (2008). *The Basics of Social Research.* (4th ed.). United States of America. Thomas Wadsworth Publications.

Ball, D. L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 29(1), pp. 14-17. Retrieved from: http://hdl.handle.net/2027.42/65072.

Brown, G., & Atkins, M. (1989). *Effective teaching in higher education.* London: Routledge.

Centre for Development and Enterprise (CDE). (2011). Value in the Classroom: The Quantity and Quality of South Africa's Teachers. *Abridged Report on Research*. Johannesburg: Centre for Development and Enterprise.

Cobb, P. (2000). Constructivism in Social Context. In L.P. Steffe & P.W Thompson (Eds). *Radical Constructivism in Action: Building on the Pioneering Work of Ernst von Glarsersfeld* (pp.152-178), London: Routledge Falmer.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed). London: Routledge.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5th ed). London: Routledge

Cohen, L., & Manion, L. (1994). *Research Methods in Education* (4th ed.). London: Routledge.

Damon, V. (2001). Research Methods in Qualitative Research. Retrieved from: http://www.ehow.com/.

Douglas, R. G. (2002). *Design teaching strategies: A behaviour analyses systems approach.* United States of America: Maple-Vall Book Manufacturing Group.

Edwin, R. V. T., & Vanora, H. (2001). The importance of pilot studies. Retrieved from: www.http:// http://sru.soc.surrey.ac.uk/SRU35.html

Etkina, E. (2010). Physics pedagogical content knowledge: *Graduate School of Education*, 020110, 1-26. Retrieved from: http// doi.org/10.1103/PhyRevSTPER.6.020110

Gill, E. (2015). Teachers! What is your teaching style? Retrieved from: http://education.cu-portland.edu/.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. American Educational Research Association, 42(2), 371-406. Retrieved from: http://mjcoonkitt.wordpress.com/2013/11/13/hill-rowan-ball-2005-math-know-for-teach/

Jacobson, E., & Kilpatrick, J. (2015). Understanding teacher affect, knowledge, and instruction over time: an agenda for research on productive disposition for teaching mathematics. *Journal of Mathematics Teacher Education*, 18(5), 401-406. Retrieved from: http://doi: 10.1007/s10857-015-9316-9

Jaworsky, B. (1994). *Investigating Mathematics Teaching: A Constructivist Enquiry*. London: Falmer Press.

Judith, B. (2004). *Doing your Research Project. A guide for first time researchers in education and social sciences*. (3rd ed.). United Kingdom.

Khoza, S. (2016). Is teaching without understanding curriculum visions and goals a high risk? *South Africa jounal of High Education, 30*, 104-119. Retrieved from: doi:http://dx.org/10.20853/30-5-595

Kilpatrick, J., Swafford, J., & Findell, B. (Ed). (2001).*Adding it up: helping children learn mathematics.* Washington: National Academy Press.

Kim, B. (2001). Social Constructivism. Emerging Perspectives on Learning, Teaching and Technology. Retrieved from: http://projects.coe.uga.edu/ep1tt/

Krauss, S. E. (2005). Research paradigm and meaning making: A primer. *The Qualitative Report*, 10(4), 758-770. Retrieved from: http://www.nova.edu/ssss/QR/QR10-4/krauss.pdf

LeCompte, M.D., & Schensul, J.J. (1999). *Analyzing and Interpreting ethnographic data*. Book five of the Ethnographer's tookit. Walnut Creek, CA: Altamira Press.

Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, 13(2), 75-90. Retrieved from: htts://doi.org/10.12973/iejme/2698

Louis, C., Lawrence, M., & Keith, M., (2009). *Research Methodology in Education.* (6th ed.) .New York. Published by MPG Books Ltd.

Luneta, K. (2014). Foundation phase teachers' (limited) knowledge of geometry. *South Africa Journal of Childhood Education,* 71-86. Retrieved from: https//sajce.co.za>article>view

Mack, N., Cynthia, W., Kathleen, M. M., Greg, G., & Emily, N. (2011). Qualitative Research Methods: *A data collector's field guide*. Retrieved from: http://www. fhi.org/.

Maree, K. (2007). *First Step in Research.* South Africa. Pretoria: Van Schaikwyk.

Maree, K. (2010). *First Steps in Research.* Sixth Impression. Pretoria: Van Schaik Publishers.

Matthew, K. (2011). Pedagogical Content Knowledge. Retrieved from: http://mkoehler.educ.msu.edu.

Maxwell, J.A. (1992). Understanding and validity in qualitative research. *Harvard Educational Review*, 62(3), 279-301. Retrieved from: https://doi.org/10.17763/haer.62.3.8323320856251826

McMillan, J.H., & Schumacher, S. (2006). *Research in Education: Evidence Based Inquiry*.(6th ed.). London: Pearson Ed. Inc.

Miheso-O'Connor Khakasa, M., & Berger, M. (2015). Status of Teachers' Proficiency in Mathematical Knowledge for Teaching at Secondary School Level in Kenya. *International Journal of Science and Mathematics Education,* 14(S2), 419-435. Retrieved from: http:// doi:10.1007/s10763-015-9630-9

Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A new framework for teacher knowledge. *Teachers College Record* 108 (6), 1017-1054. Retrieved from: http://punya.educ.msu.edu/2008/01/12/mishra-koehler-2006/

Oppermann, E., Anders, Y., & Hachfeld, A. (2016). The influence of preschool teachers' content knowledge and mathematical ability beliefs on their sensitivity to mathematics in children's play. *Teaching and Teacher Education*, 58, 174-184. Retrieved from: http:// doi:1016/j.tate.2016.05.004

Orb, A., Eisenhauer, L., & Wynaden, D. (2001). Ethics in Qualitative Research. Retrieved from: http://www.columbia.edu/.

Orodho, J. A. (2004). *Techniques of writing research proposals and reports in education and social sciences*. Nairobi: Masola Publishers.

Osamwonyi, E. F. (2016). In-Service Education of Teachers: Overview, Problems and the way Forward. *Journal of Education and Practice, 7,* 83-87. Retrieved from: https://files.eric.ed.gov>fullt

Özkan, F. (2011). Evaluating the effect of teaching strategies and Learning styles to students' success. *Firat University.* Retrieved from: http://www.firat.edu.tr/.

Patton, M.Q. (2002). *Qualitative Research & Evaluation Methods (3rd ed.).* Thousand Oaks, Calif. :Sage.

Prawat, R.S., (1992). Teachers' Beliefs about Teaching and Learning: A Constructivist Perspective. *American Journal of Education*, 354-395. Retrieved from: https//www.journals.uchicago.edu>

Rigelman, N.R. (2007). Fostering Mathematical Thinking and Problem Solving: The teacher's role. Teaching Children Mathematics, 308-314. Retrieved from: https//eric.ed.gov>

Robson, C. (2002). *Real World Research.* (2nd ed.). Oxford: Blackwell.

Schoen, H. L. (2006). *Teaching Mathematics through Problem Solving Grade 6-12.* Iowa: Library of Congress Cataloging-in-Publication Data.

Schreiner, E. (2015). Math & Teaching Methods. Retrieved from: http://www.ehow.com/info

Shay, S. (2019, January). Matric Pass Rate Poses Significant Challenge for Universities. *University of Cape Town News*. Retrieved from: www.news.uct.ac.za>article>

Shepherd, D.L. (2012). The Impact of Subject Knowledge on Learner Performance in South Africa: *A within-pupil across-subject approach.*VrijeUniversiteit, Amsterdam, Netherlands. University of Stellenbosch, South Africa.

Shuhua, A. (2000). *Mathematics Teachers' beliefs and their impact on the teachingpractice of teaching in China.* Washington DC: Educational Resources Information Centre (ERIC)

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14. Retrieved from: www.fisica.uniud.it>pdf>s

Sibuyi, C.D. (2012). Effective Teachers' Pedagogical content Knowledge in Teaching Quadratic Functions in Mathematics. Unpublished master's thesis, University of Pretoria, Pretoria. Retrieved from: https//respository.up.ac.za>d

Solis, A. (2009). What Matters Most in the Professional Learning of Content Teachers in Classrooms with Diverse Student Populations. Retrieved from: http://www.idra.org.

Stacey, K. (1989). What Is Mathematics Thinking? And Why Is It Important? *University of Melbourne*, Australia. 147-164.

Vygotsky, L.S. (1978). *Mind in society:* the development of higher psychological processes, Cambridge, MA: Harvard University Press.

White, C.J., (2005). *Research: A practical guide. (1st ed.).* Pretoria, Published by Ithuthuko Investment.

APPENDIX 1

TASK SHEET

SECTION A

1.1 If the answer to a sum is ¾, what could the numbers be? Explain your answers.----------------------------

----

1.2 You need to explain to your class how to do the following: multiply 5^9 by using a calculator. What will you say?---

----

1.3 What is the difference between $x^2 \div x^3$ and $x^3 \div x^2$? ---

----

1.4 Which is bigger x^2 or x^3. Explain your answer.---

1.5 If one angle of a triangle equal to 45^0, is it possible to find other angle? If so, give example. -----------

----

1.6 Assuming a learner in your class answers $\sqrt{49}$ as -7. Is the learner correct or wrong? Explain.--------

----

1.7 The perimeter of a rectangular plot of land is 29,5m, if the length is increased by 2m and the breadth is reduced by 1m, the area of the plot remains unchanged. Show this.-----------------------------
----

1.8 If the perimeter of a square is 24 cm, what is the length of each side? Explain------------------------
----

1.9 If the area of a triangle is 5,635cm^2 what could the height be? Explain---------------------------------
----

1.10 If the area of the trapezium is 39cm^{2,} what could the height be? Explain -----------------------------
----

1.11 Find the remainder when x^2-x+1 is divided by x+1 How do you explain this?-------------------------
----

1.12 Is $12x^3y^2z^5$ a factor of $24x^4y^5z^6$? How do you know?--
----

1.13 Is $3p^2$ a factor of $6p^4$?Explain your answer.---

1.14 If the x- intercept is 4 , what could the y-intercept be ?Explain .--

----

1.15 The diameter of the object is 7cm. The height of the object is 5,5cm. identify the geometric Object with explanation.--

----

1.16 Use the interquartile to determine if there are any outliers in the following data Series 5; 20;6;5;7;8;15 --

----

1.17 Solve for x without using a calculator, show the calculation steps. Type equation here.

$x = (\sqrt{8} + \sqrt{2})$. Explain your answer.--

$x = \sqrt{\frac{1}{\sqrt{x}}} = 3$ --

1.18 The circumference of a circle is 52 cm. Calculate the area of the circle correct to 2 decimal Places.--
----

1.19 given a=22+b determine whether

A. a is greater than b
B. B. b is greater than a
C. C) b= 18
D. D. cannot tell which is greater.

1.20 If p and q are both positive integers ($p \neq 0; q \neq 0$), how many pairs(p;q)satisfy $4p+5q=150$?

SECTION B

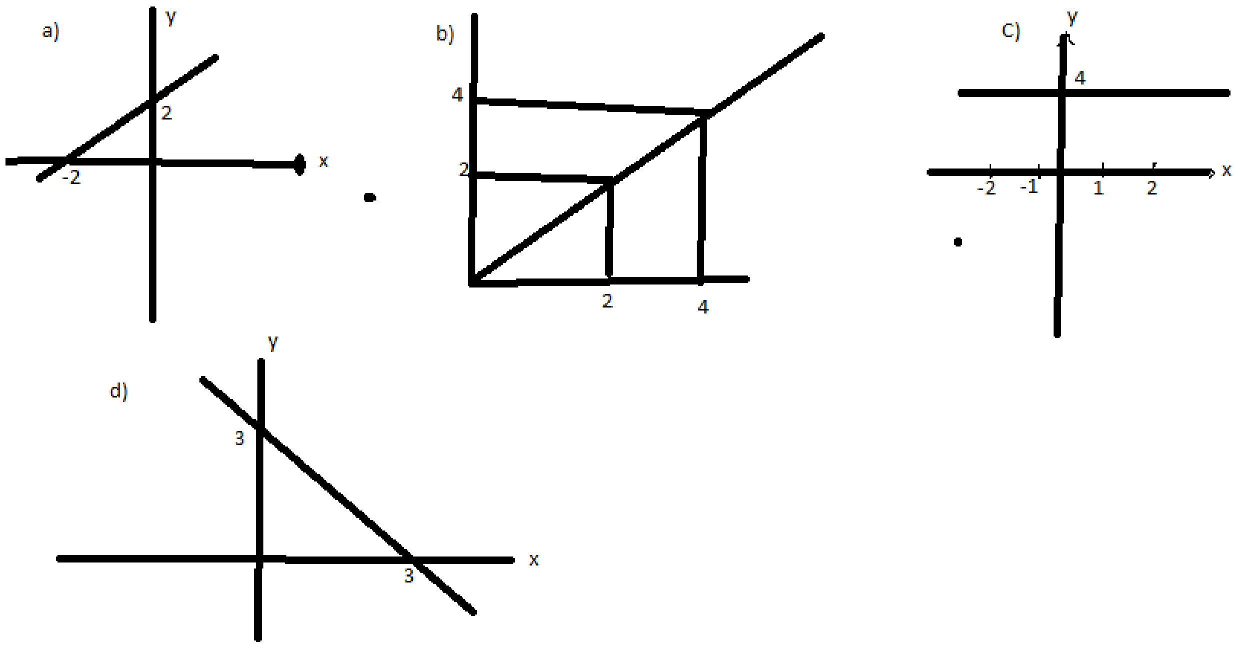
METHODS IN TEACHING

2.1 Suppose you want to introduce Pythagoras Theorem to Grade 9 class. Describe one activity you would use as an initial introduction to Pythagoras Theorem.---
----

2.2 Explain how you would help your Grade 9 children to understand the names of the side of the right angled triangle relative to the angle under the consideration. ---
----

2.3 The Volume V of a cylindrical bin (open at one end) of radius r and height h is $\mathbf{v=\pi r^2 h}$ .Describe how you would lead Grade 9 learners to discover for them selves that the volume of a right cone having radius and height as the cylinder is $\frac{1}{3}\mathbf{\pi r^2 h}$**.** --
----

2.4 Explain how you would help Grade 9 learners to find the gradient and the equation of each of the following lines.


a)
y
2
-2
x
b)
4
2
2
4
C)
y
4
-2
-1
1
2
x
d)
y
3
3
x

APPENDIX 2

OBSERVATIONAL SCHEDULE

NAME OF TEACHER :

LESSON TOPIC :

GRADE :

DATE :

OBSERVER :

Evaluation scale

VE: Very Evident

E: Evident

SE: Slightly Evident

NE: No Evidence

A. Teaching for Thinking

Part 1 : **Questioning**

	VE	E	SE	NE
(a) The teacher's questioning style encourages confidence in the learners				
(b) The teacher's questioning style encourages the learners to participate in the learning				
(c) The teacher's questions are clear.				
(d) The teacher's questions are relevant.				
(e) The teacher's questioning show continuity.				
(f) The teacher's questioning style encourages the learner's understanding.				
(g) The teacher's questioning style encourages the learners to transfer what they have learnt.				
Part 2: **Teacher feedback**	VE	E	SE	NE
(a) The teacher displays good listening skills				

(b) The teacher uses feedback to focus the learners' attention on the topic (concept)				
(c) Feedback from the teacher is used to build the learner's self esteem				
(d) The teacher uses the learners' answers to elaborate on lesson issues.				
Part 3**: Cooperative learning**	VE	E	SE	NE
(a) Effective use is made of pairs				
(b) The teacher organized the seating of the learners appropriately.				
(c) The instructions are clear				
(d) Learners are given the opportunity during cooperative learning to use their ideas				
(e) The teacher plays a supportive role when the learners are working in groups.				
(f) The teacher mediates feedback from the group effectively				
(g) The teacher makes sure that the learners have a record of work they have done together as a group or in pairs.				
Part 4: The role of language in learning	VE	E	SE	NE
(a) The teacher creates time for the learners to talk about the topic				
(b) Learners are encouraged to talk in group				
(c) Learners are encouraged to use their own words (mathematical				

term)				
(d) There are opportunities for the learners to explain and elaborate on their own answers.				
(e) There are opportunities for the learner to interpret the answers of others.				
Part 5: **The use of learning AIDS**	VE	E	SE	NE
(a) The teacher uses concrete learning AIDS.				
(b) The teacher makes use of the textbook.				
(c) The concrete AIDS helps the learners to understand the concepts.				
(d) The textbook helps the learners to understand the concepts				
(e) The teacher takes the learners through concrete experiences to symbolic and then abstract experiences.				
SECTION B**: LEARNER BEHAVIOUR**	VE	E	SE	NE
(a) The learners are confident about answering questions.				
(b) The learners participate actively in the learning process.				
(c) The learners are willing to answer questions.				
(d) There is evidence that the learner can work effectively in pairs and/or groups.				
SECTION C: **TEACHER/LEARNER RELATIONSHIP**	VE	E	SE	NE
(a) The teacher establishes a warm atmosphere..				
(b) The teacher is open and flexible.				
(c) Praise and encouragement is used appropriately				

APPENDIX 3

Questions during One- on-One- Interview

1. Please, can you give me your full name?
2. What is your highest qualification?
3. What is your major subject?(specialization)
4. For how long have you been teaching mathematics?
5. Which grades are you teaching presently?
6. What are some of the challenges you face as far as teaching mathematics in grade 9 is concerned?
7. How are you trying to overcome these challenges?
8. Learners had basic knowledge about mathematics before coming to school. Do you agree with that? Explain.
9. How do you approach your lessons in order to build on the basic knowledge?
10. Do you think learning mathematics will be meaningful, if it is not built on the previous knowledge? Explain
11. How do your learners respond to mathematics lessons when you are using real teaching material from the environment?
12. Reality is one of the important aspects of teaching mathematics. If you agree with this, can you tell me how you make your lessons real?
13. What are some of the strategies you use to make your lesson real?
14. How do you develop your learners' interest in mathematics?
15. Which language do you normally use when teaching mathematics and why?
16. How do you see mathematics lesson when conducted in groups?
17. Which types of mathematics questions do you give your learners in order for them to interact with the environment?
18. Do you think mathematics can be learned in isolation without social interaction?
19. What kind of teaching support material do you normally use when teaching mathematics?
20. Do you invite resource personnel during some of your lessons?
21. How often do you allow your learners to discuss in groups during mathematics lessons?
22. What are some of your observations during group discussion lessons?

23. Do you send your learners out during some of your mathematics lessons to explore the environment?
24. Give me some of the topics you treated with them this year which compelled you to send them outside the classroom?
25. What do you like about the lessons which normally make you send your learners outside the classroom?
26. Describe one of your interesting lessons and what it so interesting?
27. How do you help your learners to maintain positive attitude towards mathematics?
28. What are the grades you taught Mathematics?
29. Which school are you presently teaching?
30. What can you say about the learners' performance?
31. What are possible causes?
32. How do you prepare yourself before any Mathematics lesson?
33. Do you prepare lesson plan everyday, weekly or monthly?
34. Are you comfortable with every topic in the syllabus?
35. If no, what are the topics you are not comfortable with?
    - How do you help the learners with such topics?
36. What strategies do you use during Mathematics lessons?
37. How do you teach a new concept so that your learners understand?
38. What are your questioning styles?
39. How often do you give tasks to your learners
40. Do you always satisfy with your learners' performance after giving them tasks? If no, how do you make them improve?
41. How often do you attend in-service training /workshop in mathematics?
42. Do you give feedback to your learners regularly?

APPENDIX 4

16 Jizana Street

North Crest

Mthatha

5100

01 December, 2014

The District Manager

Department of Education

Mthatha

Dear Sir/Madam

RE: REQUEST TO CONDUCT A RESEARCH PROJECT

I hereby request for a permission to conduct a research project in six selected schools in circuit 3 of Mthatha Education District. My focus will be “Exploring the different types of knowledge for mathematics teaching in selected schools in Mthatha.” Six mathematics teachers from different schools in circuit 3 of Mthatha District will be participants of the research.

This research is a requirement towards the completion of my study of Master’s Degree in Mathematics Education at University of Kwazulu -Natal (Edgewood).

Tuition time will not be affected by conducting my research in selected schools.

Confidentiality, anonymity and privacy will be maintained as much as possible in the data/information obtained from participants. Participants will be allowed to withdraw at any time without penalty or victimisation and will be protected from any form of abuse.

Your co-operation will be highly appreciated.

Yours faithfully

Senoo G.K (Reseacher)

Cell number: 0762873930

APPENDIX 5

EASTERN CAPE DEPARTMENT OF BASIC EDUCATION
RESEARCH REQUEST FORM

This form (and all other relevant documentation where available) may be completed and forwarded electronically to Dr. Annetia S Heckroodt (babalwa.pamla@edu.ecprov.gov.za or bernetia@iafrica.com). The last 2 pages of this document must however contain the original signatures of both the researcher and his/her supervisor or promoter or in the case of a private organization the funder or project manager. These pages may therefore be faxed or hand delivered. Please mark the facsimile for the Attention: Director: Strategic Planning Policy Research and Secretariat Services. The facsimile number is: 086 5410 389

1. PARTICULARS OF THE RESEARCHER

1.1	Details of the Researcher
Surname and Initials:	SENOO G.K
First Name/s:	GODSWAY (KOFI)
Title (Prof / Dr / Mr / Mrs / Ms):	MR
Student Number (if relevant):	214571806
ID Number:	H22 78230
Gender (Male/Female):	MALE

1.2 Private Contact Details	
Home Address	Postal Address (if different)
16 JIZANA STREET	P.O. BOX 1439
NORTH CREST	MTHATHA
MTHATHA	
Postal Code: 5100	*Postal Code:* 5099

2. PURPOSE AND DETAILS OF THE PROPOSED RESEARCH

2.1 Purpose of the Research (Place cross where appropriate)	
Undergraduate Study – Self	
Postgraduate Study – Self	
Post-Doctoral Study	X
Private Company/Agency – Commissioned by Provincial and/or National Government Department/s	
Private Research by Independent Researcher	
Non-Governmental Organisation	

7

APPENDIX 6

Province of the
EASTERN CAPE
EDUCATION

STRATEGIC PLANNING POLICY RESEARCH AND SECRETARIAT SERVICES
Steve Vukile Tshwete Complex • Zone 6 • Zwelitsha • Eastern Cape
Private Bag X0032 • Bhisho • 5605 • REPUBLIC OF SOUTH AFRICA
Tel: +27 (0)40 608 4773/4035/4537 • Fax: +27 (0)40 608 4574 • Website: www.ecdoe.gov.za

Enquiries: B Pamla   Email: babalwa.pamla@edu.ecprov.gov.za   Date: 24 June 2015

Mr. Godsway Kofi Senoo

P.O. Box 1439

Mthatha

5099

Dear Mr. Senoo

PERMISSION TO UNDERTAKE A MASTERS STUDY: EXPLORING THE DIFFERENT TYPES OF KNOWLEDGE'S FOR MATHEMATICS TEACHING IN SELECTED SCHOOLS IN MTHATHA

1. Thank you for your application to conduct research.

2. Your application to conduct the above mentioned research at six selected schools under the jurisdiction of Mthatha District of the Eastern Cape Department of Education (ECDoE) is hereby approved based on the following conditions:

    a. there will be no financial implications for the Department;

    b. institutions and respondents must not be identifiable in any way from the results of the investigation;

    c. you present a copy of the written approval letter of the Eastern Cape Department of Education (ECDoE) to the Cluster and District Directors before any research is undertaken at any institutions within that particular district;

    d. you will make all the arrangements concerning your research;

    e. the research may not be conducted during official contact time, as educators' programmes should not be interrupted;

    f. should you wish to extend the period of research after approval has been granted, an application to do this must be directed to Chief Director: Strategic Management Monitoring and Evaluation;

    g. the research may not be conducted during the fourth school term, except in cases where a special well motivated request is received;

h. your research will be limited to those schools or institutions for which approval has been granted, should changes be effected written permission must be obtained from the Chief Director: Strategic Management Monitoring and Evaluation;

i. you present the Department with a copy of your final paper/report/dissertation/thesis free of charge in hard copy and electronic format. This must be accompanied by a separate synopsis (maximum 2 – 3 typed pages) of the most important findings and recommendations if it does not already contain a synopsis.

j. you present the findings to the Research Committee and/or Senior Management of the Department when and/or where necessary.

k. you are requested to provide the above to the Chief Director: Strategic Management Monitoring and Evaluation upon completion of your research.

l. you comply with all the requirements as completed in the Terms and Conditions to conduct Research in the ECDoE document duly completed by you.

m. you comply with your ethical undertaking (commitment form).

n. You submit on a six monthly basis, from the date of permission of the research, concise reports to the Chief Director: Strategic Management Monitoring and Evaluation.

3. The Department reserves a right to withdraw the permission should there not be compliance to the approval letter and contract signed in the Terms and Conditions to conduct Research in the ECDoE.

4. The Department will publish the completed Research on its website.

5. The Department wishes you well in your undertaking. You can contact the Director, Ms. NY Kanjana on the numbers indicated in the letterhead or email nelisakanjana@gmail.com should you need any assistance.

NY KANJANA
DIRECTOR: STRATEGIC PLANNING POLICY RESEARCH & SECRETARIAT SERVICES

FOR SUPERINTENDENT-GENERAL: EDUCATION

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

12 October 2015

Mr Godsway Kofi Senoo 214571808
School of Education
Edgewood Campus

Dear Mr Senoo

Protocol reference number: HSS/0901/015M
Project title: Exploring different types of knowledge's for Mathematics teaching in selected schools in Mthatha

Full Approval – Expedited Application

In response to your application received on 10 July 2015, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Shenuka Singh(Chair)
Humanities & Social Sciences Research Ethics Committee

/pm

cc Supervisor: Dr V Mudaly
cc. Academic Leader: Professor P Morojele
cc. School Administrstor: MsT Khumalo

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za
Website: www.ukzn.ac.za

1910 - 2010
100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville