

Exploring Grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry

by

Gezahegn Haile Godebo

Student Number: 214578033

A thesis submitted in fulfilment of the requirements for the degree of

Master of Education

(Mathematics Education)

In the School of Education (Edgewood campus)

University of KwaZulu-Natal

Supervisor: Dr Jayaluxmi Naidoo

Co-Supervisor: Mrs Eshara Dowlath

2018

DECLARATION

I, Gezahegn Haile Godebo, declare that

1. The research reported in this thesis, except where otherwise indicated, is my original work.
2. This thesis has not been submitted for any degree or at any other university.
3. This thesis does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.
4. This thesis does not contain other persons' writing, unless specifically acknowledged as being sourced from other researchers. Where other written sources have been quoted, then:
 - 4.1 Their words have been re-written but the general information attributed to them has been referenced.Where their exact words have been used, their writing has been placed inside quotation marks, and referenced.
5. This thesis does not contain text, graphics or tables copied and pasted from the internet, unless specifically acknowledged, and the source being detailed in the thesis and in the references.

Gezahegn Haile Godebo (student number: 214578033)

Date



11/03/19

Dr Jayaluxmi Naidoo

Date



11/03/19

Mrs Eshara Dowlath

Date



11/03/19

DEDICATION

To my wife Yemsirach T. Godebo, with love, for without her understanding, support and prayer, this would not have been possible.

ACKNOWLEDGEMENTS

I thank the Almighty God for giving me wisdom, strength, courage and determination to pursue and complete this study.

I would like to express my sincere gratitude to my supervisor **Dr. Jayaluxmi Naidoo** for her steady guidance, constructive comments and advice, insight and great patience throughout this research study.

Likewise, I would like to express my sincere gratitude and appreciation to my co-supervisor **Mrs. Eshara Dowlath** for her assistance in critical reading the last draft.

I would like to express my sincere gratitude and appreciation to **Angela Bryan** and **Leela Reddy** for their assistance in editing the last draft.

I would like to thank the following individuals for their valuable contribution to help make this study a success:

- My lovely wife Yemisrach for her genuine support, understanding and encouragement throughout the study.
- The learners and management of Thornwood Secondary school, for being cooperative and participating in the study.

Acknowledgement is expressed to my brothers, my sister, my mom and my dad for believing in me and understanding the value of education.

ABSTRACT

The GeoGebra programme is a free computer application programme that provides an algebra view, Geometry view, spreadsheet view and an input bar. This study explored how the GeoGebra programme contributed to learners' learning and understanding of Euclidean Geometry. The research focused on participants' experiences as they used the GeoGebra programme to support their understanding of Euclidean Geometry. It highlighted learners' perspectives on the role of the GeoGebra programme in supporting an exploration of Euclidean Geometry in particular and mathematical ideas in general. The focus of the study was to explore the way in which the GeoGebra programme is used, as a learning tool and mediating artefact in the learning of Euclidean Geometry in Grade 11 Mathematics. This study also aimed to explore learners' experiences and perceptions when the GeoGebra programme is used to support the learning of Grade 11 Euclidean Geometry. The main research questions that guided this study focused on how learners used the GeoGebra programme Euclidean Geometry to support their understanding and why the GeoGebra programme is used in the way that it is when learning Grade 11 Euclidean Geometry.

The study is rooted within a Constructivist view of learning and mediated learning and the approach used is a case study. The research was carried out in a public school that involved 16 learners. Data was generated by using tasks, lesson observations and interviews.

Based on a qualitative analysis of the data generated, the findings indicate that the introduction of the GeoGebra programme did have an influence on the learning practice in three dimensions, namely: (1) the GeoGebra programme provided a medium for visualisation that linked the development of mathematical ideas and concepts through computer-based learning, (2) the GeoGebra programme created an independent constructive learning environment and (3) the utilisation of the GeoGebra programme as a learning tool enhanced learners' conceptual understanding of Euclidean Geometry understanding.

TABLE OF CONTENT

Declaration.....	i
Dedication	ii
Abstract.....	iv
Table of content.....	v
List of tables and tables of figures	x
Abbreviations	xii
CHAPTER ONE	1
Introduction and background.....	1
1.1 Introduction	1
1.2 Background to the study.....	1
1.3 Rationale and motivation of the study	3
1.4 Focus of the study.....	6
1.5 Research questions	6
1.6 Significance of the study	7
1.7 Description of key terms	8
1.7.1 Euclidean Geometry	8
1.7.2 The GeoGebra programme.....	8
1.7.3 Mathematics	9
1.7.4 Learning	9
1.7.5 Traditional learning	10
1.8 The outline of the main study.....	10
CHAPTER TWO	11
Review of related literature.....	11
2.1 Introduction	11
2.2 Exploring the use of technology based tools in Mathematics education	12
2.3 Learning Mathematics through the use of technology	13

2.4 The role of technology in the learning of Mathematics	14
2.5 Euclidean Geometry content in South African Schools' Mathematics Curriculum	16
2.5.1 stage 1: visualisation	18
2.5.2 Level 2: Analysis	18
2.5.3 Level 3: Abstraction/ordering	18
2.5.4 Level 4: Deduction	19
2.5.5 Level 5: Rigour.....	19
2.6 The use of technology for Mathematics learning	23
2.6.1 Dynamic Geometry programme.....	23
2.6.2 The GeoGebra programme.....	23
2.6.3 The GeoGebra programme supports visual learning	24
2.7 Integrating educational technology into a Mathematics lesson	27
2.8 The GeoGebra programme supports discovery learning.....	29
2.9 Learners' engagement and achievement using the GeoGebra programme	29
2.10 Teachers' perceptions and attitudes about the use of technology in the classroom	31
2.11 Exploring the relationship between the use of the GeoGebra programme and learners' Mathematics achievement	33
2.12 Difficulties in use of the GeoGebra programme in Mathematics classrooms	34
2.13 Conclusion.....	34
CHAPTER THREE	37
Theoretical Framework	37
3.1 Introduction	37
3.2 Constructivism.....	37
3.3 The Constructivist perspective.....	38
3.4 The Constructivist approach.....	40
3.5 Constructivism and the use of technology in the learning of Mathematics	41
3.6 Constructivism and mathematics learning	42
3.7 Constructivism and the Curriculum Assessment Policy Statement (caps).....	44
3.8 Conclusion.....	45
CHAPTER FOUR	45

Research methodology	45
4.1 Introduction	45
4.2 The term research means	46
4.3 The research design.....	46
4.4 The research approach	47
4.5 Population and sampling	48
4.6 The brief description of the study.....	49
4.7 Exploring the researcher’s philosophical position in the study	50
4.8 Research instruments.....	52
4.8.1 Pre-task and post-task developed using Euclidean Geometry	52
4.8.2. Structured observation schedule.....	53
4.8.3 Semi-structured interviews.....	54
4.9 Data generation procedures	56
4.10 Methodological norm	57
4.10.1. Reliability	57
4.10.1.1 Reliability of the interviews	58
4.10.1.2. Reliability of observation schedule	58
4.11. Validity of the pre-task and post-task	58
4.12 Triangulation	58
4.13 Data analysis	59
4.14 Research ethics	61
4.15 Trustworthiness /credibility /validity /reliability	63
4.15.1 Credibility.....	63
4.15.2 Transferability	64
4.15.3 Dependability	65
4.16 Conclusion.....	66
CHAPTER FIVE	67
Findings, analysis and interpretation.....	67
5.2 Data generation strategies	68
5.3 The use of the geogebra programme within the class room	69

5.3.1 Incorporating the GeoGebra programme into Euclidean Geometry learning	69
5.3.2 Using the GeoGebra programme to promote learners' understanding of Euclidean Geometry	74
5.3.3 The use of the GeoGebra programme to promote learners' achievement in Euclidean Geometry	75
5.3.3.1 Exploring learner performance on the pre-task	76
5.3.3.2 Exploring learner performance on the post-task	77
5.4 Learners' experiences and perceptions of the use of the GeoGebra programme as a learning tool	86
5.4.1 Learners' attitudes towards mathematics while engaging with the GeoGebra programme	87
5.4.2. Learners' interest towards learning Euclidean Geometry with the use of the GeoGebra programme	90
5.4.3 Incorporation of the GeoGebra programme enhances learners' attention	92
5.4.3 The GeoGebra programme empowered self-regulated and independent learning	93
5.4.4 Integrating the GeoGebra programme with Euclidean Geometry activities enhances visualisation	95
5.4.5 The use of the GeoGebra programme stimulated practical learning	98
5.5 Conclusion	99
CHAPTER SIX	101
Summary of the study findings, implications, limitations and conclusion	101
6.1 Introduction	101
6.2 Summary of the study findings	101
6.3 Recommendations based on the findings	108
6.3.1 Recommendations for practice	108
6.3.2 Recommendation for educators	109
6.3.4 Recommendations for policy makers	110

6.3.4 Recommendations for possible future studies.....	110
6.4 Limitations of the study	111
6.5 Concluding remarks	111
REFERENCES.....	112
Appendix I: Confirmations of language editing.....	121
Appendix II: Turnitin report.....	124
Appendix III: Permission letter by school to conduct research	125
Appendix IV: Participants consent form.....	126
Appendix V: Parental consent form.....	128
Appendix VI: Ethical clearance.....	131

LIST OF TABLES AND TABLES OF FIGURES

LIST OF TABLES

Table	Description	Page Number
Table 2.1	Classification of teaching methods according to the learners' role in the educational process	14
Table 2.2	Content / skills coverage: Number of subtopics in NCS and CAPS	22
Table 2.3	Weighting per topic by percentage of marks	22
Table 5.4	Illustrates the processes that were followed to generate data from the participants in order to answer the research questions	69
Table 5.5	Summary of correct responses obtained during the traditional lesson	77
Table 5.6	Summary of correct responses obtained when teaching using the GeoGebra programme	79

TABLE OF FIGURES

Figure	Description	Page Number
Figure 2.1	A diagram showing the Van Hiele Theory hierarchy	18
Figure 2.2	A diagram showing a Learning Cone of experience	26
Figure 5.3	A diagram showing an activity given to learners	71
Figure 5.4	A diagram showing learners interacting with each other when solving the activity based on circle Geometry	71
Figure 5.5	A diagram showing learners verifying the perpendicular chord theorem while using the GeoGebra programme	72
Figure 5.6	Diagram showing an activity to verify the supplementary angles theorem with the GeoGebra programme.	73
Figure 5.7	A diagram showing a snapshot of the GeoGebra programme used to sketch and verify the cyclic quadrilateral ABCD.	74
Figure 5.8	A diagram showing learners' interaction in order to verify the cyclic quadrilateral theorem.	75

Figure 5.9	A diagram showing Ndelany's response to question 1 in the post-task.	81
Figure 5.10	A diagram showing Ndelany's response to question 1 in the pre-task.	82
Figure 5.11	A diagram showing Daisy's response to question 4 in the post-task.	83
Figure 5.12	A diagram showing Daisy's response to question 3 in the post-task.	84
Figure 5.13	A diagram showing Thula's response to question 5 in the pre-task.	85
Figure 5.14	A diagram showing the traditional chalk and talk class learning structure.	86
Figure 5.15	A diagram showing Thula's response to question 6 in the post-task.	87
Figure 5.16	A diagram showing learners' active learning.	90
Figure 5.17	A diagram showing learners doing the class activity within a group.	93
Figure 5.18	A diagram showing learners attentively working while using the GeoGebra programme.	94
Figure 5.19	A diagram showing learners doing the class activity independently.	95
Figure 5.20	A diagram showing the suggested approach for teaching Euclidean Geometry	97

ABBREVIATIONS

ARCS: Attention Relevance Confidence and Satisfaction

CAPS: Curriculum and Assessment Policy Statement

DOE: Department of Education

FET: Further Education and Training

NCS: National Curriculum Statement

CHAPTER ONE

INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

This study sought to explore learners' experiences, perceptions and understanding in Euclidean Geometry, when they were exposed to the GeoGebra programme-based learning environment. The major focus of this chapter will be to formulate the problem of the study by providing a description of the study background, rationale, and focus of the study. Further, the research questions that guided the study, as well as the structure of the thesis will be outlined.

1.2 BACKGROUND TO THE STUDY

Learners' poor performance in Mathematics has been a foremost challenge facing the South African basic education system (Chimuka, 2017). The Department of Basic Education experts have repeatedly reviewed "the curriculum in general and the Mathematics curriculum in particular for various reasons, but learners' mathematics poor performance in South Africa has yet to compare favourably to international standards" (Chimuka, 2017,p.1)

The South African school curriculum, in particular the secondary school Mathematics curriculum, has undertaken extensive changes since 1994. Chimuka (2017, p.65) claimed that "the democratic government of South Africa has issued several curriculum-related reforms intended to democratise education and eliminate inequalities established by the apartheid education system." Moodley (2013) stated that, since 1994, the curriculum change applied in South Africa has passed through three major phases. Moodley (2013, p.29) argued that "the first phase involved the cleansing of the curriculum of its racist and sexist elements in the immediate aftermath of the democratic elections." The second phase, according to Moodley (2013), was "the employment of outcomes-based education (OBE) through Curriculum 2005 (C2005) while the third phase involved the review of C2005, culminating in the creation of the Revised Curriculum Statement" (Moodley, 2013). So far, the education curriculum in South Africa has gone through a fourth transformation (Chimuka, 2017). The National Curriculum Statement (NCS) of 2002, revised in 2009, has been phased out to make way for the Curriculum and Assessment Policy Statement (CAPS) of 2012. "These changes, while they are to all intents and purposes desirable, have inevitably brought with them a number of pedagogical and

instructional challenges” (Chimuka, 2017, p.1). In the NCS, the Mathematics Grade 12 examination consisted of three papers (Papers 1, 2 and 3) of which Papers 1 and 2 were compulsory for all learners while Paper 3 was optional (Department of Basic Education, 2012). The “topics in Paper 3 were optional topics, hence, they were not taught in many schools because learners had not registered for Paper 3” (Chimuka, 2017, p.2.)

In the CAPS Mathematics curriculum, some of the NCS optional topics were integrated into either Paper 1 or Paper 2 so that two Papers, compulsory for all Grade 12 learners were set while Paper 3 was cancelled. Some of the topics that have been integrated into Papers 1 and 2 of the new CAPS Mathematics curriculum for the Further Education and Training band (FET) (Grades 10–12) are Euclidean Geometry, Descriptive Statistics and Interpretation, Probability, and Bivariate Data which are highly demanding topics. The inclusion of Paper 3’s content into Papers 1 and 2 was a curriculum change of great magnitude for both teachers and learners.

In South Africa, Euclidean Geometry has some impact on learners’ poor performance (Department of Basic Education, 2012). Geometry involves two (2D) and three dimension (3D) problems and also forms part of coordinate geometry and trigonometry. The DOE (2016) cited “many learners were struggling in Euclidean Geometry and that learners did not show the construction on their sketches nor did they state in which constituted a breakdown in the proof.”

The National Basic Education (2012, p.78) report cited “learners struggling in 2D and 3D, and that insufficient development in spatial perception, further showed that learners lack deeper conceptual understanding.” This is due to the traditional approach of Mathematics learning that is stimulus-response based. Stimulus-response means “frequent repetition of a little thinking based on learnt rule or procedure” (Suppes, 1969). Again when the traditional approach is used it leads to compartmentalisation and subsequently learners not being able to integrate concepts into other topics. Malan, Ndlovu & Engelbrecht (2014, p.9) claim that “the reason for poor performance of learners is linear justification, where learners cannot reverse their thinking.” According to Farrajallah (2016, p.58) Mathematics learning “requires learners to apply the principle of individualisation so that they can reflect better on concept understanding.” This study seeks to find an alternative way of curbing poor understanding of Euclidean Geometry concepts in Mathematics (Horzum & Ünlü, 2017).

Thus, learning within the GeoGebra programme may be able to empower learners' ability in Mathematics. Therefore, "integrating GeoGebra into these courses may bridge the gap between learners' understanding and Euclidean Geometry learning." Farrajallah (2016) stated that "visual media contributes to learners' Geometry achievement and facilitates their active involvement." Murni, Sariyasa, and Ardana (2017) claim that visualisation is fundamental to the discovery learning process in Geometry and deploying spatial images. Many studies reported that the incorporation of dynamic Geometry software, such as the GeoGebra programme into the learning of Euclidean systems, is more effective in inspiring learners for enquiry learning than being passive knowledge recipient only from their teacher" (Kösa, 2016; Murni et al., 2017; Williams, Charles-Ogan, & Adesope, 2017). In addition, Mathematics Education researchers "provided evidence that the effective use of GeoGebra had a positive impact on learners' conceptual understanding and performances in a wide variety of Mathematics topics including Geometry" (Samru, 2015, p19). Therefore, the aim of this study is to fill the gap in the literature and to examine the use of the GeoGebra programme on learner's Euclidean Geometry learning.

This study sought to explore Grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry. The essence was to explore the level of learning occurring in Euclidean Geometry when learners use the GeoGebra programme in the classroom. The study was conducted at Thornwood Secondary school which is a public school situated in the Pinetown District within KwaZulu-Natal, South Africa. The participants in this study were learners who were doing Mathematics in Grade 11.

1.3 RATIONALE AND MOTIVATION OF THE STUDY

The researcher has been involved with FET Mathematics teaching for more than 10 years. Through interaction with learners in the teaching and learning process, the researcher experienced that learners' perceptions were that Mathematics should be memorised. The learners' perceptions are that they should be given notes and a few examples so that they can memorise the content. The researcher in this study experienced that learners could not explain why they are doing certain algorithms in a specific way. Learners' responses in explaining their methodology are, "My teacher taught me to do it in this way."

For obscure reasons, “the teaching of Mathematics in most schools in South Africa is often done through traditional instruction where learners are positioned as passive recipients of knowledge” (Mutodi & Ngirande, 2014). According to Pfeiffer (2017) only a marginal group learn Mathematics in the current traditional teaching approach. If only the marginal groups are learning Mathematics by chalk and talk, then the usual traditional teaching approach should be changed. Gono (2016, p.176) argued that it is common for Mathematics teachers, especially from “middle primary years onwards, to demonstrate specific procedures to their learners, supplemented by repetitious practice of similarly constructed examples; the intention of which is to develop procedural fluency.” Mosia (2016) argued that the traditional ‘chalk and talk’ learning can become both tiresome and restrictive for learners. Learners must have a sense why Mathematics works in a specific way and they must be able to talk and share their ideas. Horzum and Ünlü (2017) argued that poor achieving learners can easily “lose confidence in their ability, and that they can also develop poor attitudes to learning and to school.” This may result in such learners not participating in the classroom and losing interest in Mathematics. Tatar (2013) argued that “teachers can be more effective when they provide explicit guidance accompanied with practice and feedback.” This study therefore utilised pre-designed activities with an active learning approach and with the traditional ‘chalk and talk’ approach of teaching.

The “conceptions, attitudes, and expectations of the learners regarding Mathematics and Mathematics teaching have been considered to be a very significant factor underlying their school experience and achievement” (Leikin & Zaslavsky, 2013). These conceptions determine the way learners approach Mathematics tasks, in many cases leading them into non-effective routes. According to Mosia (2016) learners have been found to hold a strong procedural and rule-oriented view of Mathematics and to assume that mathematical questions should be quickly solvable in just a few steps, the goal just being to get right answers. For them, the role of the learner is to receive mathematical knowledge and to be able to demonstrate so; the role of the teacher is to transmit this knowledge and to ascertain that learners acquired it (Mutodi & Ngirande, 2014). Such conceptions may prevent the learners from understanding that there are alternative strategies and approaches to many mathematical problems, different ways of defining concepts, and even different constructions due to different starting points” (Gono, 2016, p.89). They may approach the tasks in the mathematical class with a very narrow frame of mind that keeps them from developing personal means and build confidence in dealing with mathematical ideas.

Research by Mosese (2017), Shadaan and Eu (2014) and Perry and Steck (2015) show that Mathematics lessons can be made more stimulating if technology is introduced. Bist (2017, p.40) also supports the “contention that there is a lack of Mathematics interventions” that focus on the use of technology in the teaching and learning of Mathematics. The researcher therefore used the GeoGebra programme as a tool for teaching and learning because of the lack of adequate research done with Mathematics intervention with technology, especially with GeoGebra in South Africa.

Strimel and Grubbs (2016, P. 28) put forward the argument that in “this technologically advanced era it would seem natural to question whether ‘relatively accessible and affordable technologies can contribute towards addressing the poor quality of teaching and learning.’ Our learners are born in a technological era and enjoy the rapidly increasing availability of technology. Teachers can make use of technology to create a learning environment where learners can enjoy and explore Euclidean Geometry. The user can create with the GeoGebra programme, in less time, the same number of activities as done with pencil and paper which gives learners more time to investigate and create more activities to do more investigations. Therefore, learners who do not use technology such as the GeoGebra programme are missing out on exploring and investigating various activities. Khobo (2015, p.39) argued that “learners should be allowed to find their own levels and explore the paths leading there with as much and little guidance as each particular case requires.” Discovery for them must be enjoyable and learning by reinvention can possibly motivate learners (Strimel & Grubbs, 2016).

Learners should be exposed to a “learning environment in which they construct mathematical knowledge and have possibilities of coming to higher levels of comprehension” (Tatar, 2013). Teachers can design activities in such a way that learners can use technology to help them understand mathematical concepts better. Consequently, this study deployed a constructive active learning approach with the GeoGebra programme.

Thus, the use of a technological programme, such as GeoGebra, for the learners’ learning of Euclidean Geometry in Grade 11 Mathematics may act as a positive stimulus to learners’ learning of the concepts.

1.4 FOCUS OF THE STUDY

In this study the traditional way of learning of Euclidean Geometry was supplemented by a new approach, which is the use of the GeoGebra programme in the classroom when learning Euclidean Geometry contents. It was assumed that a new approach could bring about positive changes to the learning of Euclidean Geometry concepts. Unlike the traditional way of learning of Euclidean Geometry, the GeoGebra programme integrated teaching and learning approach was believed to provide more contributions in increasing learners' participation in whole class discussions, interactions and arguments on theory construction, and in developing conceptual understanding and problem- solving strategies (Ocal, 2017).

Given the strong correlation between traditional learning and the new GeoGebra integrated learning approach, the purpose of the study was to explore the way in which the GeoGebra programme is used, as a learning tool and mediating artefact in the learning of Euclidean Geometry in Grade 11 Mathematics. It also aims to explore learners' experiences and perceptions when the GeoGebra programme is used to support the learning of Grade 11 Euclidean Geometry learning.

The effective role of the GeoGebra programme based lessons on learners' learning experiences was assumed to be associated with the aspects of motivation, interactions and discussions, learner-centred learning, conceptual understanding and problem solving strategies (Saldana, 2013). The research questions were developed based on these aspects. Therefore, each of the research questions below represents one of the aspects.

1.5 RESEARCH QUESTIONS

Questions to be answered in the study:

1. How can the GeoGebra programme be used in the learning of Grade 11 Euclidean Geometry?
2. Why is the GeoGebra programme used in the way that it is when learning Grade 11 Euclidean Geometry?

1.6 SIGNIFICANCE OF THE STUDY

Improving Mathematics learning in secondary schools is a contemporary problem to which practical solutions are yet to be found. This study has sought to contribute in this regard by exploring alternative learning methods, especially for topics traditionally regarded as problematic for learners, such as Euclidean Geometry.

“Learning Geometry is not easy and some research found that a number of learners fail to develop adequate understanding of Geometry concepts” (Shadaan & Eu, 2014; Tsiteisia, 2014). The lack of “understanding of Geometry concepts may often discourage learners and thus it leads them to poor learning in Geometry” (Mosia, 2016, p.59).

Learners make use of the GeoGebra programme, as it is learner-friendly, will guide them to develop a Constructivist learning approach, and is likely to expose learners to explore other approaches towards Euclidean Geometry learning.

Some studies noted that the use of the GeoGebra programme for learning Euclidean Geometry assists the learners to develop their talents in conceptual understanding (Chimuka, 2017; Gweshe, 2014; Mosia, 2016; Shadaan & Eu, 2014). By using the GeoGebra programme learners begin the process of concrete learning in the classroom, which they will be able to transfer to industrial practical problem-solving in their future endeavours. Mosese (2017, p.78) argued that “using technology for learning of Geometry can result in a positive effect in today’s world, in areas such as technological systems of communication, construction, manufacturing and transportation.”

Kösa (2016, p.456) argued that there is a “growing belief among Mathematics teachers that the GeoGebra programme has the potential to transform Mathematics education.” Consequently, this study aimed to add to the list of research on the use of the GeoGebra programme when learning Euclidean Geometry, especially within the South African context.

The study may serve as a guide to Mathematics learners in finding alternative and/or supplementary ways of learning Euclidean Geometry. Since many learners are not motivated to learn Euclidean Geometry in Mathematics, this study could help the Mathematics education

community and other stakeholders within Mathematics education in South Africa (Mosia, 2016).

There is a need to understand how the use of the GeoGebra programme contributes to learning Euclidean Geometry theorems in Grade 11 Mathematics.

1.7 DESCRIPTION OF KEY TERMS

1.7.1 Euclidean Geometry

Geometry (originally from Greek word, geo = earth; metria = measure) arose as the field of knowledge dealing with spatial relationships (Luneta, 2014). Geometry is “the branch of Mathematics that is concerned with the properties and relationships of points, lines, angles, curves, surfaces, and solids” (Mukiri, 2016, p.124). Geometry was revolutionised by the Greek mathematician Euclid, who introduced mathematical rigour about the axiomatic methods still in use today. Oladosu (2014) said Euclid entered as one of the greatest of all mathematicians and he is often referred to as the father of Geometry. The standard Geometry mostly taught in school is Euclidean Geometry. Euclidean Geometry is sometimes termed to be “the Elements” from Euclid’s famous book (Mosia, 2016). Euclid based his approach upon axioms (statements) that could be accepted as truths, as a result he termed postulates. Mukiri (2016) claimed that though some of “these postulates are self-explanatory, Euclid operated upon the principle that no axiom could be accepted without proof.” Euclid included common words points and lines to cover up semantic errors. As a result he built the theory of plane Geometry that has shaped Mathematics, science and philosophy.

1.7.2 The GeoGebra programme

The “GeoGebra programme is as an open-source dynamic Mathematics programme that incorporates Geometry, algebra and calculus into a single and open-source package” (Gerry, 2017, p.53). A dynamic Geometry programme is a computer programme for interactive creation and manipulation of geometric constructions. A characteristic feature of such programmes is that they build a geometric model of objects, such as points, lines, circles, etc., together with the dependencies that may relate the objects to each other. The user can manipulate “the model by moving some of its parts, and the programme accordingly – and instantly – changes the other parts, so that the constraints are preserved” (Ozcakir, 2013, p.56).

This “free dynamic Geometry, algebra, and calculus programme was developed for both teachers and learners to make the teaching and learning of Mathematics more effective and permanent” (Shadaan and Eu, 2014, p.8). The GeoGebra programme may be defined as “an effective and important tool in establishing a relationship between Geometry and algebra concepts in school Mathematics since it proved its capability and potential in Mathematics education” (Kutluca, 2013). The GeoGebra programme may be used with learners ranging from elementary level to college level, aged from 10 to 18, beginning with simple constructions of diagrams up to complex Geometry problems. The learners can explore Mathematics alone or in groups while the teacher is a guide in the background and provides support when needed. “The learners’ results of their experiments with the GeoGebra programme constitute “the basis for discussions in the class so that teachers can have more time to concentrate on fundamental ideas and mathematical reasoning (Abu, 2013, p.21).

1.7.3 Mathematics

Mathematics is a language that makes use of symbols and notations for describing algebraic, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns, relationships in physical and social phenomena and between mathematical objects themselves (Mosese 2017). It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute to decision-making. Mathematical problem- solving enables learners to understand the world (physical, economic and social) around them and most mathematical problems are enhanced by learners thinking more creatively (Mosese, 2017).

According to Wiersum (2014) Mathematics is an important applied subject in real life that deals with the logic of shape, quantity and arrangement. This study gives insights according to Leikin and Zaslavsky (2013) for learners to obtain all the necessary skills and knowledge of Euclidean Geometry in Mathematics that will help them in the real world and as an abstract science of numbers, quantity, shape, change and other properties.

1.7.4 Learning

Learning is the act of “acquiring new, or modifying and reinforcing existing knowledge, behaviours, skills, values, or preferences and may involve synthesizing different types of information” (Shadaan & Eu, 2014, p.3).

According to Leikin and Zaslavsky (2013), learning is the act, process or experience of gaining knowledge or skills and can be done in different ways for example, a learner may learn in

groups, by doing simulation and presenting, also regarded as a process of getting knowledge and finding out about something. Learning is “the lifelong process of transforming information and experience into knowledge, skill, behaviours and attitude” (Barhoumi, 2015). In this study, learning can be done in different ways, for example, through mediating with the GeoGebra programme and through cooperating with peers and teacher while using the GeoGebra programme in the classroom.

1.7.5 Traditional learning

The “traditional learning is a learning approach in which the learners’ focus is on what the teacher says and what she/he writes on the chalkboard” (Gweshe, 2014, p.48). In most cases, the teacher speaks from the front of the class, explaining, guiding, controlling and deciding what learners must do, and occasionally writes notes, diagrams and questions on the chalkboard. “Learners are often seated in rows and are expected to pay attention and follow instructions. The traditional method of instruction is largely teacher-centred (teacher conveys what she/he knows to learners)” (Mosia, 2016, p.58)

1.8 THE OUTLINE OF THE MAIN STUDY

This study is organized into six chapters.

Chapter One starts with an introduction which discusses background to the study, rationale, focus of the study, and significance of the study, and description of key terms. The introduction also clarifies what is entailed in each chapter.

Chapter Two provides the related literature documenting previous research findings about how learners learn Euclidean Geometry concepts. The literature review includes discussions related to the technology based tools in the Mathematics education, learning Mathematics using technology, the role of technology in the learning of Mathematics, Euclidean Geometry in the Mathematics curriculum, using a dynamic programme, the GeoGebra programme for Mathematics learning, integrating technology into Mathematics lessons. Also included is how the GeoGebra programme supports discovery learning, learner engagement and achievement using the GeoGebra programme, teachers’ perceptions and attitude with the use of technology in the classroom, GeoGebra’s influence on learners’ achievement in Mathematics, and the challenges of using the GeoGebra programme in the Mathematics classroom. Chapter Two also presents types of studies and findings related to Euclidean Geometry.

Chapter Three presents the theoretical perspectives that frame the study. This includes a discussion of Constructivists' views of learning, meaning making and perspectives of Geometry and its learning.

Chapter Four describes the research method and processes. It also discusses the research design, research approach, population and sampling, the brief description of the study, the researcher's philosophical position in the study, research instruments, data generation procedures, data analysis, research ethics, and trustworthiness i.e credibility, validity & reliability that were acknowledged in the current study.

Chapter Five discusses the in depth data analysis which provides a comprehensive discussion of the techniques used to analyse the data. Chapter Five also discusses the questions that participants were asked and the responses obtained.

Lastly, Chapter Six constitutes the discussion of the results and conclusions made based on the findings of the study. Chapter Six also discusses how the research questions were answered in the current study. The main findings are also presented. The discussion of results is presented in terms of the research questions and the theoretical framework of the study.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 INTRODUCTION

In this chapter, I review the literature in the areas of the integration of technology in Mathematics Education. More specifically, I review themes related to the integration of

technology, specifically the GeoGebra programme, in Grade 11 Mathematics. This section highlights the review of literatures in the broad area of technology in education and I consider the role it has in the classroom in today. I also highlight research about how effective the introduction of educational-based technology can be in the learning environment. Next, I review research findings on the significance and challenges associated with technology integration.

I then focus on looking at current research regarding the role that technology plays in the Grade 11 Mathematics classroom. Finally, I focus on the use of the GeoGebra programme when learning Grade 11 Euclidean Geometry in Mathematics.

2.2 EXPLORING THE USE OF TECHNOLOGY BASED TOOLS IN MATHEMATICS EDUCATION

In order to explore how technology is used in learning Mathematics and what factors contribute as barriers for the integration of technology in learners' practice, one must have appropriate knowledge about the term technology. Educational technology may comprise traditional and modern technology, such as overhead projectors and basic hand held calculators as well as newer technology, such as data projectors, electronic interactive white boards, and computer software (Jones, Torres, & Arminio, 2014). Furthermore, Vasquez (2015) stated that educational technology comprises the awareness of the latest media whereby course material can be conveyed as well as multimedia representation that ties learning styles and learners' experiences.

The Association for Education Communication and Technology in the United States of America is concerned with standardising definitions of technology (Jones et al., 2014). However, according to (Gerry, 2016, p.28) "technology is the study and ethical practice of facilitating learning and improving performance by creating, using and managing appropriate technological processes." However, Davies and Hughes (2014) claim that technology facilitates learning implying that it provides arrangement of resources and tools in such a way that learning is meaningful instead of superficial.

The South African White paper on e-Education (Lichtman, 2014), defines technology as the combination of hardware and software and communication, allowing the processing, handling and exchanging of data information and knowledge, thereby increasing what is humanly

possible. This study uses the term technology to illustrate the educational technology as the combination of hardware and software used by learners for learning Mathematics.

2.3 LEARNING MATHEMATICS THROUGH THE USE OF TECHNOLOGY

In the previous section the term technology was discussed and this section looks at the learning of Mathematics using technology. The use of technology in education endorsed by the South African Department of Education: Learning using technology is questionable. Supporting learners to a national-state curriculum goal is a vital aspect required in the teaching learning environment. It must however, be very thoughtfully selected and integrated into educational planning and management (DoE, 2004). Looking at the technologies used in the education system endorsed by the South African Department of Education, the teaching and learning approaches have been applied so far in education should be investigated to integrate technology appropriately for learning (Lichtman, 2014).

Educational methods or approaches refer to a coordination of approaches of educational cooperation between teachers to learners and learners to learners, with learners constructing new knowledge and skill while concurrently improving their cognitive skill (Cuncka and Savicka, 2012).

A learner's role in the learning process can be categorised as (a) passive, (b) active and (c) interactive (Mosia, 2016) and it is summarised in Table 2.1 together with the main teaching methods associated with each role.

Table 2.1 Classifications of teaching methods according to the learners' role in the teaching and learning process. Adapted from Cuncka & Savicka (2012, p.1488)

Approaches	Explanation	The teaching approaches Applied
Passive	Learners are seen as not co-operative educational "objects" that need to understand the learning material.	"Lectures, reading demonstrations, learners' answers, in front of the class."

Active	Learners are seen as educational “subjects” and they are exposed to tasks which give them to be investigative. The learners are seen as active participants in dialogues in the classroom.	“Creative tasks, dialogue with the teacher.”
Interactive	“All learners as well as the teacher are involved in the educational process. The teacher is only the organiser (or facilitator) of educational process who provides a qualitative educational environment.”	“Interactive educational methods such as projects, problems, discussions and games.”

A cooperative approach could be more easily adopted with the introduction of technology creating opportunities for learners, for instance making conjectures about Euclidean Geometry properties for themselves when using the GeoGebra programme. As stated by Cunka & Savicka (2012) with the arrival of technology our attention should be paid to the paradigm of a modern pedagogy-learner who is in the centre of a practical education process, and can learn independently or in a group in a suitable place, at a suitable time and speed.

2.4 THE ROLE OF TECHNOLOGY IN THE LEARNING OF MATHEMATICS

The use of educational technology programmes in the Mathematics classroom may create the appropriate learning environment. However, it is very important to find an appropriate programme which is compatible with the intended learning area. In Mathematics education, content specific technology tools include computer algebra systems, dynamic geometry environments, interactive applets, handheld computation, data collection, analysis devices, and computer-based applications (Fahlgren, 2015). These educational-technology programmes can enable learners to explore the properties of Euclidean Geometry in Mathematics. According to Ozcakir (2013) content neutral technology programmes include communication and collaboration tools and web-based digital media. These educational technology programmes create an environment for the learners to interact and to deduce their conclusions from what they had experienced during the course of their learning (Wah, 2015).

In 2013, the Van Meter Community School in Iowa implemented a one-to-one laptop programme for the learners who are in Grades 6-12. They also enacted a strong technology focus throughout the district. After the implementation of this programme, the school management had identified the critical change brought to the school with regard to learners' behaviour, creative learning and co-operation in the education process. Learners are being allowed to develop their abilities and strengths by doing activities in which they are passionate. The school had reported that due to the introduction of one-to-one laptop programme learners became independent explorers of their studies (Brinkmann & Kvale, 2014).

The implementation of one-to-one laptop at the Van Meter Community School created an opportunity for learners to browse, virtual reality programmes, Prezi software slideshows, YouTube videos, and reading and writing blogs. According to the reports from the school, one of the Grade 6 learners expressed her appreciation how she was excited and enjoyed using the laptop for her learning. The learners' excitement during their study illustrated that the implementation of educational-technology created an environment for learners' engagement towards their learning. The use of this technology created vibrant exposure for them to be investigative and self-explorative. They are using technology to interact and exchange ideas, research independently, adapt to new situations, and take ownership over their own learning (Brinkmann and Kvale, 2014).

Vasque (2015) claimed that technology has an important part to support in the learning of Geometry, particularly Euclidean Geometry. This view is also reverberated by the Department of Education, Symon and Cassell (2012) and Adams and Lawrence (2014) recommend that the subject teachers should introduce educational technologies for learners to develop some models using technology. This approach may assist them to demonstrate the knowledge they have gained through the learning process. In line with the DBE's views on technology, the current study is going to explore Grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry.

Similarly, Molnar (2016) claimed that computer programmes have graphic animation and computing capabilities that enhance imitations and visualisation. These important capabilities were tested by determining whether there was a significant difference in the average pre-task and post-task performance of learners who were learning Euclidean Geometry using a

computer programme and those who learnt without computer technology support (Mthethwa, 2015). It was further argued that the positive impact of technology in the learning environment provides learners with greater access to a massive range of information and resources, thus empowering them to become free mediator learners, able to create meaningful personalised learning experiences outside the traditional classroom (Abramovich, 2013). Similarly, technological assisted learning environments can situate learning in authentic contexts and support the construction of knowledge by providing models, coaching, and support for collaboration (Zengin & Tatar, 2017).

According to Holloway and Jefferson (2013) an integrated system of geometrical knowledge in Mathematics is a result of insights and conceptual experiences. It is my view that solving Euclidean Geometry problems while interacting with the GeoGebra programme would provide learners with such kinds of experiences, necessary for the development of an integrated system of Euclidean Geometry learning. Furthermore, as learners worked with the computer programme and with each other in connecting already existing geometric knowledge to new and to prior knowledge, their conceptual understanding and adaptive reasoning elements of Mathematical skill are expected to develop (Zengin and Tatar, 2017).

Ramani and Patadia (2012) claim that the GeoGebra programme assisted learning may be used as a complement to outdated learning. It is my belief that the GeoGebra programme may become essential in the South African township schools which are overcrowded, and teachers have time constraints for one-on-one learner attention (Pfeiffer, 2017). Therefore, this study is to explore the significant impact on learners' learning brought forward after the implementation of the GeoGebra programme in learning Euclidean Geometry.

2.5 EUCLIDEAN GEOMETRY CONTENT IN SOUTH AFRICAN SCHOOLS' MATHEMATICS CURRICULUM

The Curriculum and Assessment Policy Statement of Basic Education of South Africa (CAPS, 2010) states that Grade 11 learners who study Mathematics are expected to cover Euclidean Geometry within the last three weeks of the 2nd term (approximately 15 hours), and revise theorems and axioms learnt in the previous Grades. According to Mthethwa (2015) there are four examinable theorems as proofs and learners are required to master the remaining theorems with respect to their applications in Grade 11 and Grade 12.

Perry and Steck (2015) claim that Euclidean Geometry provides numerous benefits for those who will be joining engineering as a career. However, learners in South Africa particularly in public schools do not like Geometry related topics (Ramatlapanana, 2017). The learners lack of interest towards Euclidean Geometry learning stems from teachers' traditional 'chalk and talk' teaching approach (Gweshe 2014). The traditional teaching approach does not expose learners to an opportunity to construct their knowledge during the course of Mathematics learning (Abu, 2013). Learners are often observed writing stipulated principles and procedures regularly from the blackboard with the result that they always wanted to use the same allegory which is given by their teacher to solve geometric problems (Pfeiffer, 2017). The learners are expected to reach a single answer only using the provided procedure by their teacher. If the given algorithm is not assisting them to solve the problems, they simple give up. Ultimately many learners collide into the Geometry fence in the schools and ultimately develop lack of self-confidence to continue their Mathematics journey (Horzum & Ünlü, 2017). The Euclidean Geometry is similarly described as a strong blockage for learners who aim to continue their studies in tertiary education. (Mthethwa, 2015).

In South Africa, Euclidean Geometry has had some impact on learners' poor performance (KwaZulu-Natal Department of Basic Education, 2016). Geometric thinking needs development and understanding as suggested by the Van Hiele's levels (Leendertz, Blignaut, Blignaut, Els, & Ellis, 2013). The Van Hiele theory illustrates that learners must pass five learning stages when exposed to Euclidean Geometry learning, namely visualisation, analysis, abstraction, deduction and rigour (Chimuka, 2017) as illustrated in Figure 2.1.

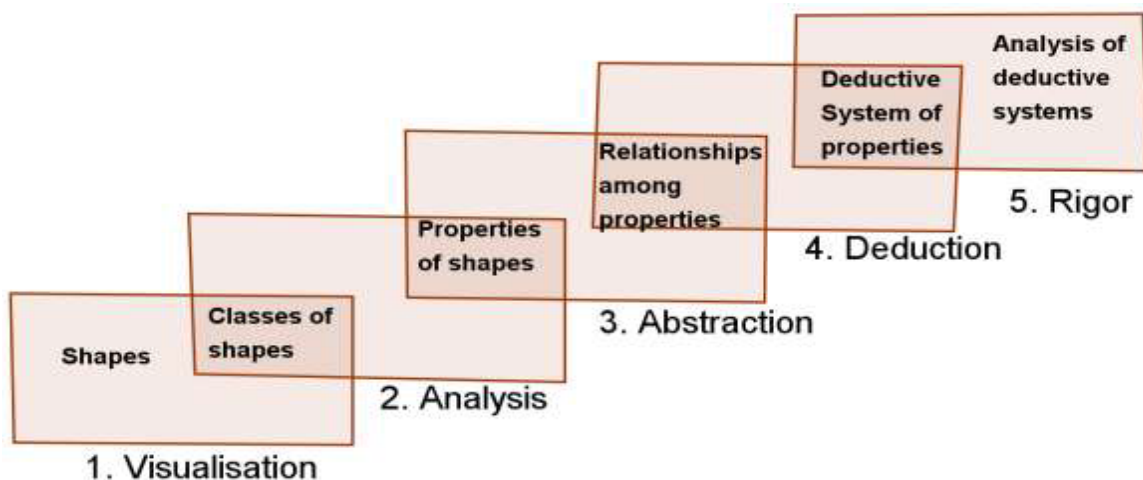


Figure 2.1: A diagram showing the Van Hiele Theory hierarchy Adapted from Alfred Chimuka 2017, p.11

2.5.1 Stage 1: Visualisation

The Van Hiele Theory claims that the visualisation stage is when learners identify the geometrical figures by viewpoint only, often by associating them to an identified geometric pattern. In this stage the characteristics of a geometrical diagram are apparent. Learners simply visualise geometrical shapes, however they cannot accurately establish the appropriate characteristics of the geometrical diagrams. During the stage of visualisation, learners can deduce knowledge from their perceptions and experiences (Chimuka, 2017).

2.5.2 Level 2: Analysis

A learner reaching this stage of analysis is capable of categorising every aspect of the particular geometrical shapes with their characteristics to differentiate one geometrical object from another. At the analysis stage, learners can deduce general principles from the properties of geometric shapes. Though learners may identify the names of the geometrical shapes, they might not establish the mathematical relationships of the geometrical figures. When describing an object, a learner working at this level might list all the properties she/he knows, but does not make connections between figures (Kekan, 2016). According to Chimuka (2017) the properties are realised as separate entities independent of one another. For instance, a particular geometric square figure can be described in the following manner: all the four sides are equal, and all the interior angles are also equal in size, but learners might not at this stage realise that a square is a typical rectangle. In Euclidean Geometry, there are two theorems considered as two separate theorems, however, the two theorems infer one another: (i) the theorem states that the angle subtended at the centre is twice that subtended at the circumference, and (ii) the theorem that states that, the angle subtended by the diameter is a right angle.

2.5.3 Level 3: Abstraction/Ordering

Hank (2016) described the term abstraction as an idea or principle considered or discussed in a purely theoretical way without reference to actual examples and instances. Kenkan (2016) states that “at these levels are able to identify relationships between properties and figures. They can describe and provide informal opinions to rationalise their thought. For instance, in Euclidean Geometry theorems, learners managed to categorise that the two theorems, the angle

subtended at the centre is twice that subtended at the circumference, and the angle subtended by the diameter is a right angle, implying one another

The key cognitive activity at this stage is ordering (sequencing) (Chimuka, 2017). Reasonable inferences and conclusions of learners' argumentations, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood. Akgul (2014) shows that at this level, logical implications and class inclusions are understood. For example, in an equilateral triangle, all sides are equal, implies that all angles are equal.

2.5.4 Level 4: Deduction

According to Chimuka (2017) deduction is the stage of processing reasons by which one concludes something from known facts or circumstances. Deduction importantly illustrates that the stage whereby, learners can synthesise proofs with an appropriate Euclidean Geometry theorem reason to illustrate their claims (Akgul, 2014; Kekana, 2016; VanHiele, 1957). According to Chimuka (2017) the higher-order Euclidean Geometry theorems or proofs in South African secondary school Mathematics can be attempted by learners operating at the deductive level, for example, proof of theorems such as: The angle between the tangent to a circle and a chord drawn from point of contact is equal to an angle in the alternate segment.

2.5.5 Level 5: Rigour

The last stage, is rigour. Learners at this stage recognise the prescribed features of deductive reasoning, i.e formulating the similarities as well as differences between geometrical concepts. For example, it is stated that in proving Euclidean Geometry theorems, learners require the whole set of skills, such as statement of what is to be proved, construction of additional lines (abstraction) and statement of implied or given theorems. The learners know how to utilise the indirect proof and proof by contra-positive methods (Kekana, 2016). The theory as propounded by the Van Hiele's theory was meant to be categorised, the implication being that a learner may not work out with appropriate understanding on one stage without having to pass through the stepwise hierarchical stages.

For this reason, the learning environment created by the teacher can provide experiences that help learners to progress from the visualisation stage to the rigour stage, hence the use of the

GeoGebra programme for Euclidean Geometry content learning may create a similar situation for learners.

In the learning of Euclidean Geometry, it has been often understood that learners still lack the cognitive and process abilities in the entire understanding of Euclidean Geometry (Akgul, 2014). Learners appear to confront a difficulty in applying the Euclidean Geometry concepts when they come to solve the geometrical problems provided for them. Therefore, the teaching approach has to expose learners to the learning environment which can manipulate and provide visual learning. This insight is supported by research (Farrajallah, 2016) whereby “learners faced challenges in studying Euclidean Geometry and many struggle to grasp the concepts and required knowledge” (Yildiz and Baltaci, 2016).

Moodley (2013) stated that a computer-generated programme can assist in Euclidean Geometry learning, because it facilitates an active interactive manipulation of geometric figures. A learner can move, relate or stretch the figure, and observe what properties stay the same. Therefore, visualisation could be core to the learning process in Euclidean Geometry and manipulating spatial images. Wilmot and Schafer (2015) claim that multiple representations enhance conceptual understanding. The GeoGebra programme may supplement good and effective learning. Farrajallah (2016) claimed that the GeoGebra programme is a powerful tool that supports visual learning in classrooms. However, to maximise the use of the GeoGebra programme, it must be well aligned with NCS and CAPS ideologies. The teacher’s role as a facilitator, should thus always guide learners and serve to help clarify misconceptions. Learners are technologically enthusiastic and seek improvement as an essential part of their lives and thrive on technology-based learning. Learners’ expectations include flexibility, self-discovery, instant feedback, collaborative learning and a digital approach that, incidentally, is highly entrenched within the GeoGebra programme (Vasque, 2015). It prepares learners for their higher education and in turn for their careers in the future (Olivier, 2014).

Various studies noted that the incorporation of dynamic Geometry computer programmes, such as “the GeoGebra programme into the learning of Euclidean Geometry, is more effective than the outdated learning in stimulating learners’ geometrical thinking skills” (Bist, 2017, p.345). According to Arbain and Shukor (2015) the GeoGebra programme improves learners’ learning of Geometry concepts as they query more and form estimations. Furthermore, Bhagat and Yen-Chang (2015) stated that the GeoGebra programme with its structural dynamism allows

learners to engage with visual representations of geometric structures and gives learners opportunities to discover constraints, abstracts as well as construct their own structures. Ozcakir (2013) also informed that visual media contributes to learners' Geometry learning and facilitates their active involvement.

Euclidean Geometry in Grade 11 Mathematics is an important section which requires rigorous attention in respect of the performance of learners. The current study aimed to investigate how the use of the GeoGebra programme works in terms of supporting learners' poor understanding of Euclidean Geometry concepts as an alternative learning platform. Vasque (2015, p.31) claims that the GeoGebra programme with its attributes in terms of visualisation may assist in improving learners' understanding of these Euclidean Geometry concepts in Mathematics.

Table 2.2 Content / skills coverage: Number of subtopics in NCS and CAPS. Adapted from (Mthethwa, 2015, p.24)

Topic Content / Skills	National Curriculum Statement				Curriculum and Assessment Policy Statement			
	Grade 10	Grade 11	Grade 12	Total	Grade 10	Grade 11	Grade 12	Total
Number of topics in Euclidean Geometry	6	1	0	7	15	11	6	32

Table 2.2 shows that the Euclidean Geometry content coverage increases in the Mathematics curriculum. There were some aspects of Euclidean Geometry in NCS, but those introduced in CAPS seem to be revised and with a higher level of demand. This means that CAPS is

significantly more demanding (DoE, 2011). Therefore, exposing learners within the GeoGebra programme might enhance learners' constructive learning to understand Euclidean Geometry concepts better.

Table 2.3 Weighting per topic by percentage of marks. Adapted from (DBE, 2011, p.10)

Topics	NCS (Percentage of marks)				CAPS (Percentage of marks)			
	Grade 10	Grade 11	Grade 12	Total	Grade 10	Grade 11	Grade 12	Total
Algebra	12.5	8.3	6.7	8.8	15.0	15.0	8.3	12.5
Euclidean Geometry	5.0	3.3	0	2.5	15.0	16.7	16.7	16.3
Trigonometry	12.5	16.7	20.0	16.9	20.0	16.7	13.3	16.3

Table 2.3 shows the three topics which are given higher emphasis in the CAPS than in the NCS which are highly demanding topics. In pacing, CAPS requires high and fast pacing as compared to NCS, and the pedagogical approach to CAPS leans more to a higher cognitive demand. According to Mthethwa (2015) the Euclidean Geometry in CAPS is deemed to be considerably more challenging than the NCS, since the CAPS content exceeds that of the NCS in both breadth and depth.

Table 2.3 clearly shows that on the content base, the subtopics of Euclidean Geometry have increased from a total of 7 in NCS to 32 in CAPS. There was an increase of 8 subtopics for both Grades 10 and 11, from 6 subtopics to 15 subtopics and 1 subtopic to 11 subtopics respectively (DBE, 2011). Subsequently, the highly demanding Euclidean Geometry section needs some sort of intervention. As a result this study proposes the utilisation of the GeoGebra programme as a learning manipulative to facilitate learners' constructive learning ultimately to conquer this highly demanding Euclidean Geometry section.

The current study seeks to address the alternative learning ways for learners to perform better in highly demanding topics, especially in Euclidean Geometry which has relatively more marks allocated (See Table 2.3). Therefore, it is very important to look at the uses of some dynamic Geometry programmes in respect to supporting Mathematics learning, in particular Euclidean

Geometry learning. In the following section the researcher will ascertain the use of the Dynamic Geometry programme on Euclidean Geometry.

2.6 THE USE OF TECHNOLOGY FOR MATHEMATICS LEARNING

2.6.1 Dynamic Geometry programme

The first programme to be developed was the Geometer Supposer by Judah Schwartz and Michal Yerushalmy (Doolittle, 2014). It contained three different programmes; triangles, quadrilaterals and circles. Supposer's key features were to formulate geometrical figures by repeating drawings exploiting different beginning points.

An interactive Geometry programme like CABRI Geometry and Geometers' Sketchpad were developed independently around the same time (Pfeiffer, 2017). The first free Dynamic Geometry programme environment is WinGeon developed by Rick Parris with two versions: one for two dimensions (2D) and the other for three dimensions (3D) (Gerry, 2016; İçel, 2013). The mentioned interactive programme options, although useful in improving the understanding of Geometry concepts, are not easily accessible to the learning community of public schools as they are not free (Kekana, 2016).

2.6.2 The GeoGebra programme

GeoGebra is a dynamic Mathematics programme that was created by a mathematician Markus Hohenwarter while doing his Master's degree at the University of Salzburg in 2002 (Giurgiules & Mion, 2015). According to Vasque (2015) the GeoGebra programme is an open source dynamic Mathematics programme which aims to merge a dynamic geometric programme and computer algebra system in order to have a single software package that accommodates for algebra, calculus, as well as Geometry. The GeoGebra programme could be downloaded from the GeoGebra website at <http://www.geogebra.org> (Ocal, 2017). The GeoGebra programme is very popular worldwide. To cater to a broader range of learners in the world the GeoGebra programme has been translated into many official languages and official GeoGebra website visitors have also greatly increased (Vasque, 2015).

The GeoGebra programme is a programme that is based on global Mathematics standards which support the curriculum. It was designed in a way that enables learners to develop a deep understanding of the mathematical theories through the practical application, and the self-

discovery of the concepts (Kosa, 2016). The programme is composed of a set of toolbars that provide an active learning feature in which the learner can easily access and construct the geometrical figures. This unique feature of the GeoGebra programme enables learners to construct the geometry figures and then to establish Euclidean Geometry proofs with appropriate reasons (Farrajallah, 2016). Exposing learners to the GeoGebra programme based Euclidean Geometry learning environment may meaningfully engage learners towards constructive learning.

2.6.3 The GeoGebra programme supports visual learning

Most of the study findings indicated that visual learning is an important form of learning as it includes five different skills; observation, recognition, interpretation, perception, and self-expression (Vasque, 2015). Exposing learners to a visual learning environment creates an opportunity for them to view and analyse the geometry concepts. The visual learning might support higher degree recall. The learners being able to recall the geometric properties may lead them to develop comprehensive understanding of Euclidean Geometry. Learners being engaged in the visual learning classroom may analyse, make conjectures, and express ideas to others by sketching or drawing images (Bist, 2017). Most importantly, the visual learning attracts learners' attention and interest. Researchers agreed that engaging learners in their learning is the hardest job to do, particularly towards Mathematics. The use of visual teaching and learning aids might assist the teaching and learning process to be the most interactive. (Bansilal, 2015; Kutluca, 2013; Yildiz & Baltaci, 2016) .

For instance, Robert Gagne exclusively illustrated how the learning goal could be achieved. His work on Conditions of Learning Theory includes “conditions of learning, association learning, five categories of learning outcomes and nine events of instruction” (Edgars, 1969). His learning theory particularly emphasises what and how to teach. (Mosia, 2016). Gagne believed that nine steps should be followed for true learning to happen. According to his nine hierarchical learning steps, the first step of learning is paying attention. In order to entice learners to be hooked to the teaching and learning process, the lessons should be supplemented with the visual learning aids. (Cole, 1985) .

The figure 2.2 clearly illustrates that learners can remember 50 percent of what they see. Learners can remember only 20 percent of what they hear. Therefore, the incorporation of the

GeoGebra programme as a learning programme into Euclidean Geometry learning might facilitate an environment for visual learning. Most importantly, learners remember 90 percent of what they do. The tasks designed for the current study mainly focused on a learner- centred approach. That means they will do the given tasks on the computer screen and they can see their work (Kutluca, 2014). This theory was developed as a result of Edgar Dale’s research in the 1960s that “learners retain more information by what they do as opposed to what they hear, read, or observe” (Anderson, 2013, p.35).

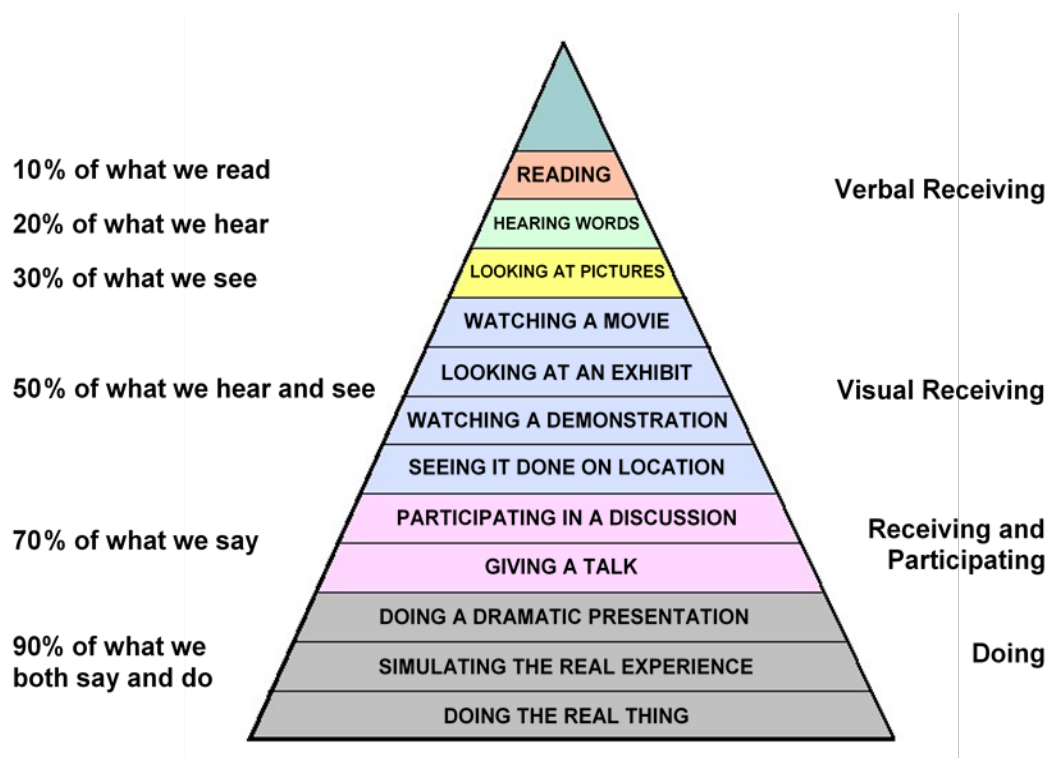


Figure 2.2: A diagram showing the Learning Cone of experience adapted from (Anderson, 2013, p.35)

Hence, exposing learners to a visual learning environment is one of the key aspects to be considered while someone designs the lesson plan. As long as the learning environment supports learners engagement within visual learning, half percentage (50%) of the lesson being taught will be remembered by the learners (Luneta, 2014). Meanwhile, using the GeoGebra programme in the Mathematics classroom inspires learners to do their tasks on the computer screen. The aim of the current study is to create an environment for learners that will assist them to understand the learning material, not to memorise it (DBE, 2011). Once the learners see visually projected geometrical diagrams on the computer screen, they can easily start to think about the properties and relationships of geometrical objects. Therefore, visual learning aids are very important tools of learning to illustrate theoretically abstract lesson contents (Farrajallah, 2016). Hence, visual learning computer programmes may encourage learners to

think critically from what they observe and may help them to develop better visual learning skill.

Farrajallah (2016) also emphasises that mental images enable learners to interact with mathematical concepts. Furthermore, Naidoo (2012) stated that the visual image is a symbolic demonstration of the visual appearance of an object. Visuals help to break down abstract Mathematics concepts leading to better understanding and comprehension and advanced mathematical skills (Kosa, 2016).

According to Shadaan and Eu (2014) visual learning based computer programmes can increase the learning potential of all learners along with their ability to acquire and communicate mathematical concepts. According to Tsitisia (2014) learners who were engaged in the visual learning medium developed a higher level of inquiry learning interest in the classroom. Some researchers also in this regard indicate that that visual learning programme, particularly the GeoGebra programme advances the concrete understanding of learners in Euclidean Geometry (Farrajallah, 2016; Khobo, 2015; Kösa, 2016; Molnar, 2016). For instance, a study was conducted at the University of Putra Malaysia with 53 participating learners who had experiences of different visual abilities. These 53 learners were grouped into two categories and were taught with different teaching approaches. Group one used the GeoGebra programme as a learning tool and the other group did not use the GeoGebra programme to learn Euclidean Geometry. This experimental study revealed that learners with high visual spatial ability understood the Euclidean Geometry concepts better than learners who have low visual spatial ability. The learners who used the GeoGebra programme as a visual learning tool achieved better in their Euclidean Geometry when compared to those who did not use the GeoGebra programme (Oladosu, 2014). Findings from various studies confirm that using the GeoGebra programme as a learning tool supports visualisation.

Some researchers indicated that the GeoGebra programme is an appropriate platform for visual learning (Sadaan & Eu, 2014; Vasques, 2015). Consequently, geometric figures can easily be displayed as visuals with the use of the GeoGebra programme. For example, a research project studied the impact of a one-to-one laptop with the GeoGebra programme in three economically different schools in California at Stanford University. In the study of Stanford University, learners were given an opportunity to spend more time on the internet to learn Mathematics by themselves. After the implementation of the research project learners were given a test to write.

All learners achieved better in the Mathematics test after the implementation of the research project. In addition to the work learners were doing in Mathematics, the researchers noted that “one-to-one laptop implementation increased learners’ likelihood to engage in the writing process, practice in-depth research skills, and develop multimedia skills” through explanation and production of knowledge (Vasques, 2015, p.36).

2.7 INTEGRATING EDUCATIONAL TECHNOLOGY INTO A MATHEMATICS LESSON

Technology touches almost every aspect of our daily lives; life at work, at home, and even in our spare time. Rapid developments and advancements in technology “create new opportunities for learning, however, many school systems are struggling to keep step with these changes” (Martinez, 2017, p.97). Some of the commonly used technologies in education today are laptops, tablets, interactive display boards (i.e. smart boards and Promethean boards), digital and video cameras, document cameras, the internet, and more (Khobo, 2015). There is a strong consensus in current literature and research studies pertaining to technology in education that there is a need to integrate technology into the classroom (Liu, 2013; Ljajko & Ibro, 2013; Martinez, 2017). The current generation of learners is referred to as digital natives (Prensky, 2012). Learners who are in the current technologically advanced world are supposed to develop better thinking and problem- solving skills than their ancestors who were not lucky enough to use this opportunity (Prensky, 2012).

Various recent research studies and literature written within the last 5 years have indicated that when technology is used effectively in classrooms, it can facilitate ways of learning that are much better matched to how children learn, as opposed to the resources of traditional classrooms (Bester & Brand, 2013; Strimel & Grubbs, 2016; Zengin & Tatar, 2017).

Gerry (2016) also indicated that the GeoGebra programme integration in the learning process as a technology tool in the learning environment currently has drawn academic attention. Providing a rich learning environment to promote social interaction, critical thinking skills and a holistic understanding of learners’ learning experiences has brought about the call to integrate the GeoGebra programme in the Mathematics classroom (Gono, 2016).

Classroom atmosphere must be appealing to encourage learners in learning Euclidean Geometry in Mathematics. There are several schools that have started moving to tablets instead

of textbooks in the classroom. For example, Clearwater High School in Turkey, made the switch four years ago and the results of the learners were outstanding (Yildiz and Baltac, 2016). On the other hand, the cost for the books was significantly reduced. This is because the Kindles the school use only cost about \$70 which can be less than the cost of one text book. The school found that this gave every learner the ability to go on the internet which they may not be able to do at home; easily carry around their textbooks and study wherever they were. They found that test scores of the learners rose by 18% in the first year that the Kindles were introduced. Learners were able to see their homework assignments, complete work, read their textbooks and much more right at the touch of their finger (Muhtadi, 2013). Therefore, the learning material designed aligned within the GeoGebra programme can create a learning atmosphere which is active and cooperative. Similarly, the Department of Basic Education of South Africa outlined important Principles and Standards for School Mathematics. One of the principles and standards speaks about integrating technology as one of the key aspects to advance the quality of Mathematics learning, which in turn suggests that, teachers should use technology to enhance their learners' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well-graphing, visualising and computing. (DBN, 2011). In this fast revolving technological revolution, the teaching and learning approaches and material has to be reviewed to be in the same speedy moving technology vehicle. It is the educators and the policy makers' responsibility to train and equip a technologically competent generation for the technological networked world. (Farrajallah, 2016).

Hence, the absence of highly advanced technological networking in the learning and teaching process must be seen as a current key challenge facing the education system (Abramovich, 2013). By allowing learners to solve Euclidean Geometry problems while interacting with computers, the current study strived to provide an alternative to traditional learning while at the same time contributing to the already existing empirical evidence on the GeoGebra programme assisted learning in Mathematics education.

Furthermore, the use of the computer programme, GeoGebra in the classroom tends to limit the teacher's involvement in the learning process (Aydos, 2015). The GeoGebra programme based learning may force learners to create their own solution paths, thus developing a stronger relational understanding of Euclidean Geometry concepts, as compared to learners who are

exposed to the usual traditional learning where the teacher is tempted to tell learners what to do, but not why, thus developing instrumental knowledge (Kutluca, 2013).

2.8 THE GEOGEBRA PROGRAMME SUPPORTS DISCOVERY LEARNING

The study conducted by Aydos (2015) stated that using the GeoGebra programme in Mathematics education promotes innovative learning. The current study meant to explore the impact of using a proposed computer guided discovery learning model on learners' conceptual and procedural knowledge in Mathematics. The experimental group of learners was taught by the model of computer-guided discovery learning and the control group of learners learned by the traditional learning fashion. The study seeks to find out if the computer-guided discovery learning model plays an important role in learners' theoretical and practical knowledge. The result of the study show that the computer-based innovative learning approach using the GeoGebra programme in Mathematics learning has substantial potential and provided better results in acquiring both, theoretical and practical knowledge than traditional learning experience can (Aydos, 2015).

The use of GeoGebra creates an active learning situation in which learners can construct their knowledge using the provided technology programme (Zengin & Tatar, 2017).The utilisation of the GeoGebra programme might assist learners to make connections of the patterns and properties of Euclidean Geometry concepts (Kösa, 2016). Research revealed that learners have an important ground on which to construct the geometrical figures and establish their properties using the GeoGebra programme as a learning tool (Holloway & Jefferson, 2013). Researchers found that utilisation of the GeoGebra programme enabled learners to construct, measure angles and size and deduce their conclusion about the figure they articulated (Aydos, 2015).

2.9 LEARNERS' ENGAGEMENT AND ACHIEVEMENT USING THE GEOGEBRA PROGRAMME

The findings of research indicated that the GeoGebra programme increased learners' interest in Mathematics (Stols & Kriek, 2012). This position is in alignment with the findings of Kim and Md-Ali (2017), and also on learners' perception on the GeoGebra programme in the learning of Euclidean Geometry. Learners' engagement increased towards Mathematics learning in both male and female gender when exposed to the GeoGebra programme based

lesson (Howard, 2018). The study has also shown the worth and usefulness of the GeoGebra programme in improving learners' achievement in Mathematics (Azlim, Amran, & Rusli, 2015).

The study was conducted at Ekurhuleni North district of the Gauteng province, in South Africa. This study involved 65 Grade 11 learners from two separate schools. The researcher treated one of the schools as an investigational cluster and another school as a controlled cluster for his study. The study was to investigate the effect of the GeoGebra programme assisted learning and traditional learning approach on the learners' experiences and perceptions in Euclidean Geometry. The findings of the study revealed the use of The GeoGebra programme as a learning tool had a positive impact on learners' learning experiences and their perceptions. (Gweshe, 2014).

A similar study was conducted at UMkhanyakude rural district in KwaZulu-Natal province that involved 112 learners from five different schools. The aim of the study was to determine the impact of the GeoGebra programme on Grade 11 learners' performance in Euclidean Geometry when using the GeoGebra programme as a learning tool. This study discovered that the learners who were exposed to the GeoGebra programme were highly interested in the lessons. Learners who used the GeoGebra programme performed better in the post-test than the learners who learnt in the traditional methods. The study was based at the University of Zululand. According to the researcher, there was no significant difference among the treatment and controlled groups in the pre-test however, "there was a significant difference in the post-test in which the test scores for the treatment group were higher than the control group" Methethwa (2015, p.185). The findings from a similar study show that the GeoGebra programme incorporated learning engages learners better than at any other time and increased their performance more than traditional learning approaches (Bhagat & Yen-Chang, 2015).

Networking technology within the education system might improve the culture of learning. The current sophisticated technology is believed to be fertile ground for the teaching and learning process, particularly for Mathematics education (Chowdhury & Ahmed, 2015). The study from California State Polytechnic University discovered that most learners replied to a set of questions after the post-task took place that assessed their attitudes on the use of the GeoGebra programme (Vesques, 2015). Learners' responses indicated that they liked learning within the GeoGebra programme. Some researchers who conducted their studies on the basis of learners'

interest and focus when using the GeoGebra programme for learning also agree that the GeoGebra programme enhances learners' focus towards the Mathematics lesson (Sahin & Kislal, 2016; Zengin & Tatar, 2017). Mathematics is a highly abstract subject. As a result, learners frequently experience lack of focus for an extended time interval. However, within the GeoGebra programme designed lessons, learners showed interest in the Mathematics lessons and spent more time working on them than in traditional learning approaches (Perry and Steck, 2015).

2.10 TEACHERS' PERCEPTIONS AND ATTITUDES ABOUT THE USE OF TECHNOLOGY IN THE CLASSROOM

One of the key elements that must be considered is teachers' attitudes regarding technology incorporation into the learning system. The teachers must have courage and interest to design the learning material within the appropriate educational technology programme. Designing and developing technology-based lessons might require time, passion and skill. A study conducted by Vasque revealed that teachers who were trained to prepare lessons for learners within the educational based technology showed a higher level of shift from traditional teaching methods. The Vasque's study finding indicated that teachers' lesson developing skill also improved significantly. The responses from the participant teachers indicated that the technology programmes minimise the weight of the work in the classroom. Most importantly, technology based lessons create more time intervals for learners to use in the classroom (Pfeiffer, 2017). Current educational technology programmes assist hugely to draw or design models and diagrams. As long as the technology equipment is installed in the classroom, there will not be postponement and excuses to project visual demonstration for learners (Murni et al., 2017). According to Vasque teachers reported that within technology incorporated teaching classrooms learners can communicate mathematical concepts in various methods using dynamic multiple representations and mathematical modelling (Vasque, 2015).

According to Mosia (2016) the integration of technology into the teaching and learning system had also improved teachers' Technological Pedagogical Content Knowledge. Mosia also observed during his study the lessons designed by the teacher for their lessons. He found out that the teachers did integrate their content knowledge, pedagogical knowledge, and technology knowledge. Furthermore, in the "teacher reflections almost all teachers expressed positive

views about teaching and learning Mathematics with the GeoGebra programme” (Mosia, 2016, p.78).

Another study looked at the influence of Grade 10-12 Mathematics teachers’ behavioural beliefs and perceptions towards technology based teaching. The researcher investigated the intention of integrating the GeoGebra programme into Mathematics learning. The researchers investigated the impact of teachers’ attitudes in their classrooms to develop concepts in the context of transformations, functions, or Geometry (Kriek & Stols, 2011). The teachers’ actual usage was compared with their intention to use the software (Kriek & Stols, 2011). The study took place in South Africa with two samples of teachers in two different schools. The study involved 22 teachers. The motive of the teachers’ usage determined the actual use of the GeoGebra programme was the apparent usefulness of the technology and the general technology proficiency of the teachers (Kriek and Stols, 2012). Both perceived usefulness and general technology proficiency combined determined the behavioural intention. The researchers found that teachers’ “actual behaviour” is influenced by the “perceived usefulness of the technology.” The researchers also revealed that the teachers who did not know how to use technology showed no interest in using the GeoGebra programme in their teaching (Stols & Kriek, 2012). A way to improve teachers’ use of the GeoGebra programme in their classrooms is to ensure that they have general computer proficiency and to allow them to experience the advantage of using the GeoGebra programme (Kriek & Stols, 2012).

The teachers who did not know how to use the GeoGebra programme proposed in their responses that recurrent, “drill and kill” teaching methods are the best approach of teaching (Kriek & Stols, 2012). Therefore, teachers who have the traditional, old way of teaching beliefs must be educated for the benefits of learners, because traditional learning approaches do not create an active constructive learning environment. The constructive learning ideology should be positioned in the education system to assist learners to make sense of what they learn.

The lesson plans developed by the teachers who believe that the traditional learning approach is an effective teaching style did not fit for teacher and learners to use in the technology- based learning environment. This agrees with researcher (Martinez, 2017) who suggests, if we truly anticipate enhancing teachers’ uses of technology, we must consider what teachers’ current classroom practices are rooted in, and mediated by, existing pedagogical beliefs.

2.11 EXPLORING THE RELATIONSHIP BETWEEN THE USE OF THE GEOGEBRA PROGRAMME AND LEARNERS' MATHEMATICS ACHIEVEMENT

The findings of many studies revealed that the GeoGebra programme based learning approaches enhance learners' engagement towards Mathematic learning (Malan, Ndlovu, & Engelbrecht, 2014). In the GeoGebra based Mathematics learning, particularly Euclidean Geometry learners showed enthusiasm to draw, construct and measure the dimensions of the Geometric figures (Ljajko & Ibro, 2013). Some studies also highlighted that learners exposed to the GeoGebra programme based Euclidean Geometry made them spend more time in their work (Giurgiules & Mion, 2015; Pfeiffer, 2017). Researchers also agreed that the GeoGebra programme based learning classroom creates an active and cooperative learning ground for learners to exchange their thoughts and ideas freely (Gono, 2016; Perry & Steck, 2015).

Furthermore, the findings from some studies indicated that the GeoGebra programme based Euclidean Geometry learning facilitates an environment for learners to have dialogues with learners as well as with their teacher during the course of their learning (Kekana, 2016; Yildiz & Baltaci, 2016). Therefore, it is important to infer that the GeoGebra programme used Mathematics learning classroom establishes a constructive learning environment.

The learners being able to draw and measure the dimensions of geometric figures within the GeoGebra programme assisted learners to internalise the properties and relationships of Euclidean geometry concepts. Meanwhile, many researchers used pre-test and post-test to investigate the significant impact brought due to the implementation of the GeoGebra programme as a learning tool. According to the research findings learners who were exposed to the GeoGebra programme based learning performed much better in their post-test (Aydos, 2015; Bist, 2017; Pfeiffer, 2017). Hence, the findings from various studies confirm that the GeoGebra programme based learning promoted learners better performance in Mathematics. The current study also aimed to investigate the learners' experiences and perceptions when they engaged in the GeoGebra programme based Euclidean geometry learning.

2.12 DIFFICULTIES IN USE OF THE GEOGEBRA PROGRAMME IN MATHEMATICS CLASSROOMS

It is noted that the use of the GeoGebra programme for learning in the classroom has problems like lack of learners' computer literacy. Some studies indicate the following points as drawbacks when the GeoGebra programme is used in the classroom for learning: An absence of pedagogical science of computer-assisted learning, lack of didactically usable programmes for learning and schools rely heavily on external technicians or informatics (Martinez, 2017; Williams et al., 2017). Technology must supplement Mathematics classroom experiences and enhance learning (Pfeiffer, 2017) and (Mutodi & Ngirande, 2014). Furthermore, Kul (2013) believes learners can learn from computers without an intermediate human instructor. The researcher agrees with (Mutodi & Ngirande, 2014) who firmly believe in the availability of effective teachers as facilitators. It is very important for teachers to guide learners and see that the standard of learning is maintained at a supreme level and set appropriate targets. Computers and installed programmes can require technical attention at any time, thus an experienced technician must be at hand to cater for such unforeseen circumstances (Gerry, 2016). For every complete session a teacher must assess the level of progress in terms of knowledge advancement.

GeoGebra.exe error is one which might be caused by:

Related registry files being corrupted, windows or drivers being outdated, malicious spyware or virus invasion, GeoGebra.exe file being corrupted or deleted mistakenly and improper programme installation or removal. These may lead to serious problems such as these listed by Gerry (2017) as follows:

- It takes a long time to start up / shut down the computer, open a website or launch a programme;
- Instead malicious programmes are downloaded or installed unawares;
- Annoying error messages constantly pop up on the computer;
- Blue 'screen of death' happens occasionally;
- Windows settings can be changed adversely.

2.13 CONCLUSION

Mathematics learning within the technology based environment seems productive. In this technological advanced world, most of the private and government public services are

delivered on the computer screen. The computers are used for all services since computers facilitate the best quality work. Therefore, it is very important to consider integration of educational technology into the learning classroom to enhance quality of learning. Mathematics concepts are interrelated and learners must learn the contents hierarchically. In the Mathematics learning, content where one topic must be understood before the next topic is introduced (Ozcakir, 2013). If learners miss main components of one topic, then that might hamper the learners not to move forward with her/his learning.

Alternatively the GeoGebra programme assisted learning may be an important approach of learning Mathematics which can motivate and also improve the performance of learners in Euclidean Geometry. Learners need to be engaged in the learning of Euclidean Geometry. It is important that teachers must expose learners to the environment in which learners will be able to engage to their full potential. If learners are absorbed and inspired in the learning environment, then they will more probably be effective in Euclidean Geometry. In order to make the learning occur it is important to implement technology throughout the Euclidean Geometry content in the Mathematics classrooms. Using technology in the classrooms, as suggested by different studies, can increase learners' engagement, increase motivation to learning, allow for better teacher-learner interaction, support learner collaboration, assist in the accuracy of mathematical computation and help learners not only feel more comfortable with learning Euclidean Geometry but also allow for a deeper understanding of the Euclidean Geometry concepts.

The positive effect of using the GeoGebra programme throughout the Euclidean Geometry content can assist learners' learning to higher-order thinking (discovery learning) that can help learners even beyond the classroom (Cole, 1985; Mthethwa, 2015). To this extent, the use of the GeoGebra programme within the Euclidean Geometry from lower Grades to higher Grades is required for the advancement of Euclidean Geometry learning. However, learners are enthusiastic about computers (Edgar, 1969), but evidence on the link between the GeoGebra programme assisted learning, Euclidean Geometry performance, and motivation is limited (Gagne, 1985; Shoemaker, 2013). In particular, evidence on the GeoGebra programme assisted Grade 11 Euclidean Geometry learning is scarce in South Africa. It is through research studies such as this one that the use of the GeoGebra programme assisted lessons in learning Euclidean Geometry may be determined. Therefore, this study will contribute by adding to the already

existing body of knowledge on the use of the GeoGebra programme assisted Euclidean Geometry learning versus a traditional learning approach. Finally, it is expected that teachers will continue to integrate the GeoGebra programme in better ways in the classroom to help learners be prepared for today's ever-changing technology obsessed society.

CHAPTER THREE

THEORETICAL FRAMEWORK

3.1 INTRODUCTION

The aim of the current study is to investigate the learners' experiences, perceptions and understanding of Euclidean Geometry when using the GeoGebra programme as a learning tool. The previous chapter highlighted various related studies to find out insights on what has been said about the current study. This chapter is to establish the theoretical framework in which the current study should be based. In this chapter the perceptions and beliefs about how the instructional material should be designed and provided for learners to make sense of their learning experiences will be discussed. The learners making sense of their experiences and perceptions is grounded in the viewpoint of Constructivism.

Therefore, this section highlights the Constructivist beliefs and perceptions about modern Mathematics learning. Constructivism, therefore, was adopted as a theoretical framework in the current study

3.2 CONSTRUCTIVISM

Constructivism is a “philosophical and epistemological approach, which describes learning as a change in meaning constructed from experience” (Duffy and Jonassen, 1992, p. 37). Constructivism positions learners as a central component in the teaching and learning process. Constructivism learning theory strongly believes in establishing a conducive, well equipped learning environment (Duffy & Jonassen, 1992). This type of learning environment has to comprise aspects such as, well prepared learning materials, supportive materials, and well-established classroom organisation. Once the physical learning environment has been orderly established, the next step should be engaging learners to experience and explore the learning materials. The learning environment established in the Constructivist approach may encourage learners to develop an investigative and inquiry learning habit. This investigative and inquiry learning approach may further empower learners to demonstrate the lesson by creating some models. The main concern of Constructivism in the learning process is placing learners in a central position to make meaning of what they are expected to learn. According to Constructivism, knowledge is the outcome of personal experiences and understanding (Simon, 1995).

Furthermore, Doolittle (2014, p.497) argued that Constructivism “derives from a philosophical position that human beings have no access to an objective reality, that is, a reality independent of our way of knowing it.” Simon (1995) also claims that learners create theory from their perceptions and experiences of their own understanding.

From the Constructivism viewpoint discrete explanation and understanding from experiences can be “shaped” for common consensus (Vrasidas, 2000). Hence, deploying the Constructivist learning philosophy in the Mathematics classroom might promote learners’ engagement to the Mathematics learning.

Therefore, the use of the GeoGebra programme as a learning tool in the Mathematics classroom might facilitate constructive learning. The current study, therefore, is grounded in a Constructivist perspective emphasising how learners should be exposed to the teaching and learning process and how the GeoGebra programme, as a learning tool facilitates in the learning process.

3.3 THE CONSTRUCTIVIST PERSPECTIVE

Various Mathematics studies have currently focused on Constructivist learning theory (Doolittle, 2014; Perry & Steck, 2015). It seems present educational researchers have adopted Constructivism as a vital learning theory. Reformers, policy makers, researchers and teachers, are actively involved in supporting the utilisation of Constructivist ideas for creating and implementing new curricula or activities to improve learners’ learning (Kul, 2013).

It is important to realise that Constructivism in the education system is to bring a paradigm shift in the teaching and learning approach. Constructivism focuses on creating and facilitating potential ground for learners to explore and explain their experiences by themselves. In order for learners to construct their own knowledge; the structure of lesson design and methodology of teaching have potentially necessitated changes. The Constructivists suggest the learner-centred teaching and learning approaches to be implemented in the education system. Basically, the lesson should be designed within the Constructivist view. The teachers must provide psychological and expertise support when it is needed by the learners (Gono, 2016). Paying attention to Mathematics learning, there has been much debate about whether

Mathematics is a process of invention, creating a way to define the world, or a process of discovering truth (Simon, 1995).

Constructivists place emphasis on learners making meaning of what they experience while they are learning which has important implications in Mathematics education. The Constructivists suggest that Mathematics teachers do not just impart knowledge that they have gained from their experiences. In other words, this means traditional teachers are working as if they are pouring knowledge into learners' minds. This passive transfer of knowledge from teachers to learners has been rejected by Constructivists (Amineh & Asl, 2015). Therefore, Mathematics teachers must provide learning resources for learners to study and interpret in their own way of understanding rather than telling them to copy conclusions from someone's experiences as a reality. This means individual learners can only construct her/his Mathematics knowledge from provided resources or from her/his real life scenarios. This implies that learners must not be given stipulated procedures to solve particular Mathematics problems. The learners will have to create their own procedure to find the solutions for the problem they are in which is, construction of knowledge. From "this point of view, while traditional teaching regards the individual as a sponge that absorbs knowledge, Constructivism considers the individual to be like a growing tree" (Kul, 2013, p.48). Hence, teachers who believe in Constructivist philosophy are expected to design Mathematics tasks in an unrestricted questioning method. "The Constructivist approach argues against direct teaching, suggesting it is a restrictive method, as learners are only exposed to information that is fed to them" Amineh and Asl (2015,p.13). If the learners are situated in the basis of constructive learning, they will probable create procedures and principles from their learning experience.

The concept of the Constructivist philosophy is not gaining knowledge from the teaching process. However, according to Constructivists learners should be the active participants in the learning process (Neuman, 2014). Similarly, Simon (195, p.142) describes "Constructivism as a philosophy that emphasises the active role of learners in constructing their own knowledge by building understanding and making sense of information."

In favour of Constructivist philosophy, Keller (1987, p.4) claims that "in Constructivism learners actively make sense of knowledge, connect it with previously assimilated knowledge and make it their own knowledge by constructing their own interpretations." This implies that

learners through the learning process ought to practice aspects such as hypothesising, designing models, revising, analysing, applying and drawing conclusions (Saha, Ayub, & Tarmizi, 2012).

In this study, the Constructivist philosophy of learning is implemented and the lessons and tasks are designed within the GeoGebra programme. Thus the GeoGebra programme-based teaching and learning approach is regarded as a better lesson design to attract learners' interest towards Mathematics learning. The GeoGebra programme is considered as a learning mediation tool. The tasks are designed in a way that learners can draw diagrams and analyse information within the GeoGebra programme. This way of learning may encourage learners to develop higher order thinking. Meanwhile, using computer technology in Euclidean Mathematics may create an opportunity for learners to strengthen their creativity during the course of learning. The tasks developed for this study are mainly focused on drawing, measuring geometric dimensions and analysing information. These tasks were prepared from the Euclidean Geometry content and were designed in the open-ended question approach. Due to the nature of the tasks, learners were not expected to reach single answers. The current study teaching and learning approach is regarded as a Constructivist learning approach because the tasks are open-ended for learners to draw their conclusions. In line with the current study viewpoint, Keller (2013, p.75) argued that "Constructivism gives learners an opportunity to think, make their own interpretations, construct and internalise knowledge for themselves while interacting with their surroundings." Similar reasons justify the choice of Constructivism to frame this study.

3.4 THE CONSTRUCTIVIST APPROACH

Currently, Constructivist theory is considered as a basis for reforms in Mathematics education. In some studies the Constructivists' learning approach has been repeatedly recommended for Mathematics education (Bist, 2017; Gono, 2016; Kumar & Chun-Yen, 2015; Pfeiffer, 2017). Constructivists pay attention to the learning process and the way the learners are exposed to the learning environment is most important. As a result, the current study put effort in designing Euclidean Geometry lessons within the GeoGebra programme. This has been implemented in order to create a Constructivist philosophy dominated learning classroom. Jonssen (1991, p.11) argued that "in a Constructivist-learning environment, learners use technology to manipulate data and explore relationships." This implies learners come to the classroom to challenge problems and find out solutions for their experiences. The traditional philosophy of learning

which made learners passive and simply engaged them in collecting information has been overturned (Gono, 2016). It is recommended that learners develop Euclidean Geometry knowledge from the provided tasks and resources.

This study is also embedded in the Constructivist philosophy, with particular regard to the development of Euclidean Geometry knowledge through the mediation of the GeoGebra programme.

3.5 CONSTRUCTIVISM AND THE USE OF TECHNOLOGY IN THE LEARNING OF MATHEMATICS

Technology encourages learners' self-directedness because exposing learners to a computer based learning environment might provide an opportunity for learners to be independent explorers (Amineh & Asl, 2015). The transition from the traditional method of teaching to the use of innovative methods such as technology-based teaching is encouraged and will eventually help to develop a constructivist view of learning (Ljajko & Ibro, 2013). The use of technology programmes, particularly the GeoGebra programme in education has caused the theory of learning, namely Constructivism to receive attention. Learners become more "empowered by gaining access to real information and work on authentic problems when using computers" (Amineh and Asl, 2015, p.10). Constructivism focuses on the learning process. Therefore, engaging learners within the computer based lessons may create excitement and innovative exposure for them.

It seems the traditional instructional learning philosophy which has been used so far needs substantial reform. The learners' culture of making meaning from their learning experiences might reinforce the level of understanding. Learners can internalise the Euclidean Geometry knowledge that they have gained through the process of constructive learning which will retain permanently in the mind of the learner. This encourages learners to achieve better in Mathematics education. The philosophy of learning that forms the foundation of instructional design has also changed as teachers attempt to develop new approaches to improve learner performance. Therefore, in the current study learners were expected to draw Euclidean Geometry diagrams and process information using the GeoGebra programme. Based on the diagrams projected on the computer screen learners were expected to establish geometric relationships. The computer technology in this regard assisted learners to see automatic

feedback from the computer screen. The learners' task is analysing the information reflected on the computer and drawing a conclusion. In line with the current study philosophy Wah (2015, p.149) claims that "technological tools should be placed within a learning environment to support learning. The curriculum where technology is integrated requires an appropriate learning theory as a framework in which the learner could be more creative and productive". The learner- centred environment of Constructivism empowers the teaching and learning process in order to improve the level of learning from basics to higher-order skills (Keller, 1987). Various studies also recommended the integration of educational based technology into Mathematics classrooms to support the learning process (Brito & Dias, 2017).

The tasks prepared from Euclidean Geometry for learners to practice within the GeoGebra programme engaged them in an innovative way to explore the Geometry figures. The current study approach intensively encourages learners to develop self-confidence to work independently. Bist (2017, p.342) also confirmed that "Constructivist theory and the use of technology for learning work well together as they both encourage the learner to work independently while developing their own understanding."

Related studies reviewed during the course of this study also agreed that use of the GeoGebra programme for Euclidean Geometry learning promoted constructive Mathematics learning. This implies that teachers can easily expose learners to access the intended lessons using the computer technology. According to Azizul and Din (2016) using the GeoGebra programme played an important role in terms of learners' engagement and through the process it supported learners to move their hands to practice by themselves

3.6 CONSTRUCTIVISM AND MATHEMATICS LEARNING

Maintaining learners' positive attitude towards Mathematics learning is one of the problems facing teachers in the classroom. Yet, most learners have already developed a negative perception for Mathematics education. According to researchers this deep-rooted negative attitude towards Mathematics comes from the traditional teaching and learning approach in the Mathematics classrooms. For instance, if the learner could not remember a certain stipulated procedure given by her/his teacher to solve a particular mathematics task using the procedure provided for them, learners just gave up and put the tasks aside because this task provided for the learners required a single response. Meanwhile, they have developed the habit of leaving

the tasks aside if they cannot do them in the way in which she/he was told to do. Through time learners start to hate Mathematics and simply consider Mathematics as complicated and not an achievable subject. These deep-rooted Mathematics learning problems can be overturned by changing the learners' perceptions about Mathematics learning (Horzum & Ünlü, 2017). The teachers' perceptions also have significant impact on the learners' learning. Their perception determines the way they prepare lessons for their learners. Teachers are required to develop lessons in the constructive learning approach. Placing Mathematics learning in the Constructivist perspective is the most important aspect to achieve the goal of learning (Amineh & Asl, 2015).

In the Constructivist learning context, the Mathematics teacher should expose learners to play a major role in creating Mathematics procedures and principles. Therefore, "the acquisition of mathematical knowledge becomes a learner-based activity rather than a passive activity involving the memorisation and acceptance of an independent body of truths" (Zengin and Tatar, 2017, p.76).

The tasks for the learners should expose learners to develop creative thinking and innovative learning. Similarly, Kul's (2013 p. 48) claim suggested that "learners should be engaged within activities that will require reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation."

The Mathematics teachers can play a significant role by designing the lesson which is interesting and novel which may encourage learners to develop discovery learning. "Designing tasks is a very important aspect in establishing an effective Mathematics learning environment" (Keller, 1987).

Exposing learners to a constructive learning environment may encourage higher order thinking that may lead learners to interrelate and establish mathematical relationships. Bist (2017) argued that new knowledge cannot be transmitted directly but must be constructed by the learner from prior knowledge and through the technology mediated learning. In the Constructivist learning classroom, learners are active in the learning process as they construct their own knowledge and are creative thinkers. Meanwhile, through the learning process they can develop creative problem solving skills.

Mathematics is one of the most important subjects, considering the opportunities that are associated with studying it. Wiersu (2014, p.47) also states that “Mathematics provides useful, self-enhancing and marketable skills.” Furthermore, it provides fulfilment in employment and also offers learners enriching ways of seeing and understanding the world which is an essential component for working as critical citizens in modern society (Amineh & Asl, 2015).

Constructivists look at the insight of individual learners when constructing knowledge (Saha et al., 2012). Their view is useful when scrutinising the way in which the individual understands particular Euclidean Geometry concepts. Their focus is on “learners’ understanding and development of knowledge, specifically in Mathematics. The current study views technology as important hence, it is essential that technology is combined with Constructivism in the learning of Euclidean Geometry” (Doolittle, 2014, p.487).

3.7 CONSTRUCTIVISM AND THE CURRICULUM ASSESSMENT POLICY STATEMENT (CAPS)

The idea of the Curriculum and Assessment Policy Statement (CAPS) Grades 10-12 Mathematics policy document (DOE, 2011) is based on the principles of the National Curriculum Statement Grades R-12. One of these principles of the National Curriculum Statement is grounded in “use science and technology effectively and critically showing responsibility towards the environment and health of others.” (DOE, 2011, p. 4). In order to create an active and critical learning environment, learners should be actively engaged in problem solving given tasks, for example, when learners are required to prove Euclidean Geometry theorems, to prove that opposite angles of a cyclic quadrilateral are supplementary, a Constructivist thinking process is supposed to be applied. Furthermore, Mosia (2016, p.214) suggested that “learners should demonstrate an understanding of the world as a set of related systems by recognising that problem -solving contexts do not exist in isolation.” This implies that learners connect new knowledge with old knowledge as they construct understanding, and critique their ideas and those of others while interacting with the real world. In a Constructivist classroom, “the teacher provides learners with resources and activities that ensure they are actively involved and participate in, while constructing their own knowledge and understanding” (Kulu, 2013, p.46).

In other words, learners in the GeoGebra programme assisted classroom must solve Euclidean Geometry problems by manipulating lines, angles and shapes on the computer, measuring the angles and lines to check if their solutions are correct, and critically discussing their solutions and solution paths with their peers. Similar reasons justify the choice of Constructivism to frame the current study.

3.8 CONCLUSION

This chapter established views into Constructivism, the theoretical framework adopted in this study. This framework provides a lens through which powerful insights into development of Euclidean Geometry knowledge through the GeoGebra programme mediated learning (Yildiz & Baltaci, 2016). By examining the educational theories of Piaget the principles behind how a Constructivist classroom is different from the traditional Mathematics classroom, for the learners, and how this links to the GeoGebra aided Euclidean Geometry learning classroom is shown (Yildiz and Baltaci, 2016). The teachers' instruction informed by a Constructivist model must look, sound, think and feel different from a traditional classroom; similarly, how the learners look, sound, think and feel will differ in a Constructivist classroom. The goal in a "Constructivist classroom is to produce an inquiring and accepting atmosphere that leads to each learner reaching his or her full potential through constructing meaning, understanding and reasoning (Amineh and Asl, 2015). The next chapter presents descriptive explanations the methodology designed in the current study.

CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 INTRODUCTION

This chapter describes the current research site, the target population, the research, approach, research plane and the data collection procedures. The data capture instruments and strategies are discussed in this chapter as are the research approach and data analysis strategies. Appropriate research design has been implemented in order to establish answers for the research question presents in Chapter One. The study explores Grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry in a public school in the Pinetown district, a district in the province of KwaZulu-Natal, South Africa.

4.2 THE TERM RESEARCH MEANS

Research can be described as “a systematic and organised effort to investigate a specific problem to provide a solution” (Bertram and Christiansen, 2014, p.12). Consequently, a research finding is to add new knowledge, develop theories as well as gathering evidence to prove generalisations (Creswell, 2014).

4.3 THE RESEARCH DESIGN

The research design is a comprehensive plan to conduct the research study. This design comprises the aspects such as from whom, when, how and under what circumstances data will be obtained (Ranjit, 2013). Hence, a research design is a planned approach and structure of a study to look at answers to research questions. Neuman (2014, p.26) claims “that the aim of the research design is to plan and structure a given study in such a manner that the eventual validity of the research is exploited.” Furthermore, the research design is regarded as the procedure of situations for generating and analysing data in an approach that aims to construct meaning to the study.

This study adopted the qualitative approach. The foundation of qualitative research falls in the interpretive approach to public investigation (Yin, 2012). A qualitative research design is based on “multiple socially constructed realities and aims to understand a social phenomenon from participants' perspective” (Cohen and Morrison, 2007, p.461). The qualitative research design is a very important design when “exploring an area where little is known or where you want to have a holistic understanding of the situation, phenomenon, episode, site, group or community” (Ranjit, 2013, p. 153). This design is suitable when the study focuses on wider exploring and understanding rather than quantitatively confirming. It “provides an overview

and an in-depth understanding of a case (s), process and interactional dynamics within a unit of study” (Ranjit, 2013, p.57).

4.4 THE RESEARCH APPROACH

The current study is a qualitative exploratory case study investigation supported within the interpretive paradigm. Interpretivist models define the features of a phenomenon (Marshall & Rossman, 2014). It includes “information about participants’ needs, desires and a variety of other information that is essential in producing what is beneficial in participants’ lives” (Marshall and Rossman, 2014, p.102). The study employed case study research enquiry as it sought an in-depth understanding of the impact of exploring Euclidean Geometry learning in Mathematics when using the GeoGebra programme. It also sought to understand how learners construct their own knowledge using their Mathematics experience.

The purpose of the current study was to explore the use of the GeoGebra programme, as a learning computer programme and to explore learners’ experiences and perceptions when the GeoGebra programme is used to support the learning of Grade 11 Euclidean Geometry learning. In order to attain the current study objective, it was required to establish a complete explanation of how learners, through the engagement with the GeoGebra programme, were able to internalise Euclidean Geometry concepts. A qualitative study is to “extract and understand the phenomenon, the process, the particular attitude and worldviews of the people involved in this study, namely learners at a school” (Bryman, 2012, p.49). The interpretive paradigm is “grounded on the premise that each person's way of making sense of the world is as convincing and as worthy of respect as any other.” (Bertram & Christiansen, 2014, p.26). Furthermore, Cohen, Manion, and Morrison (2011) claimed that in a qualitative study the concern of the researcher lies in understanding how participants make meaning of a situation or phenomenon.

This study is an exploratory case study, in that, I will pursue my quest to explore or understand how the use of the GeoGebra programme affects understanding of Euclidean Geometry concepts in computer-based learning. Yin (2012, p.5) claimed that a “case study research method can be divided into exploratory, descriptive and explanatory approaches in an effort to address who, what, where, how, and why research questions.” This research method suits the aim of my study. I expect responses to the research question from the learners through their

learning experiences and perceptions. This method fits the purpose for my study as I need to produce responses to these questions from learners through their experiences with the GeoGebra programme.

This exploratory case study design is based upon the notion that the case being investigated is atypical of cases of a certain type. Therefore this single case can provide insight into the events. “It is an approach in which a particular instance or a few carefully selected cases are studied intensively” (Ranjit, 2013, p.123). In selecting a “case therefore researchers usually use purposive, judgemental or information-oriented sampling techniques” (Yin, 2012, p.88). As a result, I consider purposive sampling for the current study to conduct interviews with participant learners.

Therefore, the current case to be studied is the learners’ experiences and perceptions when learning Euclidean Geometry using the GeoGebra programme. In this study different data generation instruments have been used to generate the comprehensive information. This generated information from the participant learners will provide the most insight as to the value of the GeoGebra programme used in the understanding of Euclidean Geometry concepts. An exploratory case study approach was preferred as it favours an opportunity to study one condition in detail over a limited time interval (Neuman, 2014).

4.5 POPULATION AND SAMPLING

Population is a complete set of elements that possesses some common characteristics (Ranjit, 2013). The target population is a “subject of individuals with specific common characteristics in whom one wants to study one’s intervention, while a sample is a portion, piece, or segment that is representative of a whole” Newman, 2014, p.246). Sampling is a statistical method of obtaining representative data or observations from a total population. Sampling is the process of choosing a set of subjects with which to conduct a study (Bryman, 2012).

The target population for the current study consists of sixteen Grade 11 learners who are studying Mathematics this academic year (2017). The sixteen Grade11A learners were from the township areas of Pinetown district in KwaZulu-Natal Province in the Republic of South Africa. The Grade 11 Mathematics learners were chosen as the researcher is teaching in the school and could thus easily access the learners, which made for convenient sampling.

Purposive sampling is used to select participant learners to take part in the interview session guided by an exploratory case study approach which is adopted for the current study. “Purposive sampling is a method used when one chooses participants who are information-rich, based on the purpose of the study” Bertram and Christiansen, 2014, p.76). Three learners, ages 16-19, were randomly selected for an interview session from a Mathematics Grade 11A class. I held one-on-one interviews with the three learners directly after the GeoGebra programme -based lessons had been completed.

4.6 THE BRIEF DESCRIPTION OF THE STUDY

The study was conducted in the classroom covering a series of six lessons in two phases. During the first phase the learners were exposed to the usual traditional chalk and talk learning approach for three consecutive lessons. Thereafter, a pre-task was given for the learners. The result of the pre-task was recorded, and the scores were noted.

An arrangement was made to teach Grade 11A (16 learners) using the GeoGebra programme in the classroom. Subsequently, the intervention teaching phase using the GeoGebra programme was continued for another three consecutive lesson sessions in the classroom. Before the implementation of the GeoGebra programme, for one lesson, the learners were introduced to the GeoGebra programme in order to acquire knowledge and skills in the formation and manipulation of geometric objects. Thereafter, in two learning phases the learners were exposed to the tasks prepared from the Euclidean Geometry content. Euclidean Geometry based lessons are designed from the prescribed learners’ textbooks and teacher guides for Grade 11 distributed by the National Department of Basic Education. The current study learning content development was established based on the Department of Education work programme. The Euclidean Geometry lessons were developed into a sequence of tasks that I prepared as learning materials for Grade 11 learners. In the course of each session the learners were directed to work the tasks within the GeoGebra programme. Learners were encouraged to work independently on the computer screens. They were provided with tasks and subsequently had discussions with their classmates. I moved within the class providing support, that is giving clues on which icon to use in order to draw lines like a chord, a tangent, parallel lines, solving examples similar to questions on the tasks Chowdhury and Ahmed (2015), and where necessary demonstrating on the white board. Learners were exposed to the

GeoGebra programme based learning environment to explore and establish the properties of Euclidean Geometry concepts.

All the learners' activities were observed using an observation schedule in both the GeoGebra programme lesson and the lesson without the use of the GeoGebra programme. Field notes were also taken to analyse to what extent the learners' were inspired to learn Euclidean Geometry contents within the GeoGebra programme. The information generated using an observation schedule also helps to analyse the level of learners' participation during this study and the data could help to analyse the role of the GeoGebra programme in learners' constructive learning of activities. Bertram and Christiansen (2014, p.36) proposed that "the advantage of observation as a data generation instrument is that it is a powerful method for gaining insight into situations." As a result structured observations have been scheduled for this study. Finally all the learners (16 learners) wrote a post-task. However, the post-task was the same as the pre-task, that is, the same task was administered twice.

On the final day, three learners, of ages 16-19, were interviewed from Grade 11A, who were purposively selected from the group. The interview discussions were also voice-recorded for analysis. I held one-on-one interviews with the selected three learners to interrogate them about their learning experiences and their perceptions about Euclidean Geometry learning within the GeoGebra programme.

4.7 EXPLORING THE RESEARCHER'S PHILOSOPHICAL POSITION IN THE STUDY

In this study, the constructivist approach of learning was adopted. Participant learners were exposed to the learning environment which is equipped with necessary resources. The intention was to engage learners to Euclidean Geometry learning using the GeoGebra programme. In this type of learning arrangement learners were expected to be involved actively in the learning process to develop Euclidean Geometry knowledge by themselves. I tried to evaluate the learners' strategies that they used for solving Euclidean Geometry problems in both the GeoGebra programme lesson and the lesson without the use of the GeoGebra programme. The use of computers in the classroom created an opportunity for learners to use more time to practice their tasks. This learning classroom also assisted learners to have dialogue with their peers as well as with the teachers which maintains the Constructivist approach of learning, as

compared to the traditional 'chalk and talk' approach. I tried to facilitate, guide and support the learners, while allowing them to explore, discover and formulate conjectures on their own. "With appropriate adult help, children can often perform tasks that they are incapable of completing on their own" (Vygotsky, 1978, p.53).

Learners' engagement within the GeoGebra programme is an important learning approach to create a constructive learning classroom. I used a projector to explain to the learners what the GeoGebra programme is and how they are supposed to use it. The learners were told to use computers only for Euclidean Geometry learning and were not allowed to draw or create unnecessary diagrams during this study. The essence was to use the extended time with learners working with the intended tasks. The GeoGebra programme was installed for learners to practice Euclidean Geometry within the programme. They were subsequently exposed to access the GeoGebra programme. They were then asked to engage within the GeoGebra programme, dragging each of the variable sliders to visualise the resulting effect. "This explorative exercise was intended for self-learning, self-discovery, visualising, conjecturing, evaluating, and testing and for internalization to take place" (Bhagat and Yen-Chang, 2015, p.81). Besides, the use of the GeoGebra programme, learners were also involved in peer discussions to ascertain deeper meaning of their understanding and meaning-making (Bhagat & Yen-Chang, 2015). The learners' reaction was also video recorded while learners were interacting with the GeoGebra programme. All the lessons accompanied by activities were handed to each learner. The activities were collected at the end of each lesson.

According to the Attention, Relevance, Confidence, and Satisfaction (ARCS) theory of motivation postulated by John Keller, "a learner's motivation can be stimulated through instruction by creating conditions that will arouse the learner's desire to be interested in order to achieve her/his goal" (Keller, 1978, p.45). Curriculum developers have not considered establishing resources which inspire learners. (Keller, 1987). Therefore, I used the computer technology as a motivational resource to inspire and engage learners to innovative methods of learning. Establishing "a learning environment that will stimulate and sustain learners' motivation" is an important aspect of the education system (Keller, 2013, p.194). Letting learners in the classroom be actively engaged in a problem-based task using the GeoGebra programme inspired learners for discovery learning. Vygotsky (1978, p.68) stated that "when scaffolding, a teacher supports the learner by arranging a task such that it can be done successfully by the learner." Even though, the lessons had been carefully established, the

learners were briefed before they started their work within the GeoGebra programme because learners were using the GeoGebra programme for the first time. The learners were provided with tasks designed from Euclidean Geometry and I clarified how to access the computer programme. They were inspired to work the tasks within the GeoGebra programme. I tried to establish a learning environment which seems to be a Constructive learning classroom. It was expected that learners construct their Euclidean Geometry Knowledge using the GeoGebra programme as a learning tool. “Knowledge can be constructed from previous experience; so new learning should be based on learners’ informal and previous knowledge, resulting in reinforcing or adaptation of that knowledge” (Keller, 2013; Perry & Steck, 2015; Wah, 2015). This is best stated by Piaget: “A truth learnt is only a half-truth; the whole truth is reconquered, reconstructed and rediscovered by the pupil himself/herself” (Amineh & Asl, 2015).

For the current study my philosophical position is Constructivist philosophy (epistemology) of Euclidean Geometry learning. “Epistemology- an area of philosophy concerned with the creation of knowledge; focuses on how we know what we know or what are the most valid ways to reach truth” (Neuman, 2014, p.95).

4.8 RESEARCH INSTRUMENTS

For this study five types of data generation instruments were used to generate information from the target research population (i.e. learners). These research data generation instruments were:

-

- Pre-task,
- Observation schedule during first phase of lesson sessions,
- Post-task,
- Observation schedule during the course of the second intervention phase of lesson sessions and
- Semi-structured interviews.

4.8.1 Pre-task and post-task developed using Euclidean Geometry

The task consisted of question items which are drawn from six theorems that deal with Euclidean Geometry in Grade 11 (DBE, 2011). The item questions were open-ended type of questions which prepared learners to construct their solutions. For these question items learners

were not expected to reach a single answer or response as these tasks were designed to position learners in the Constructivist learning approach.

Learners were not given any procedure or principles to solve the tasks. However, they were told to use their own strategies to construct their mathematical solutions. This approach is to demonstrate Constructivist perspectives in the Mathematics classroom.

All the items were drawn from the text books and past Grade 11 question papers. In addition to items 2, 3, 4, 5 and 8 learners were expected to demonstrate Constructivist skills to solve the problems.

The tasks were used to compare the learners' performance of the traditional 'chalk and talk' against the GeoGebra programme used learning. The performance was measured in terms of task achievement of learners in both learning contexts. "The same task was used for pre-task and post-task so as to determine if there was any improvement in terms of learners' understanding of the concepts in the given tasks, and also to ensure that all the conditions were similar except for the interventions" (Bertram and Chritiansen, 2014, p.79). In particular, a pre-task was used to determine the performance status of learners before the GeoGebra programme implementation, and after introduction of administering the GeoGebra programme, the post-task was administered, which was aimed to explore the effect of each technology-based learning approach on the learners' presentation.

4.8.2. Structured observation schedule

Bertram & Chritiansen (2014, p.78) define "observation as the orderly and coherent writing down of the format of events of participants, with no need for communication with respondents." The interpretative research seeks to understand the meaning of the phenomenon of the participant learners. Deciding what to observe is important as many items can be the subject of observations. However, I consider the following items as key areas which should be included in the observation schedule for the current study:

- The characteristics of individual learners, including their gestures, verbal and nonverbal behaviour;
- The interactions between or among learners (peer discussion, participation);
- The "actions" taking place, while the lesson is in progress ; willingness to interact with the GeoGebra programme and

- The physical surroundings, including visual and audio cues.

I wrote down what is observed in order to use the information to interpret the meaning of the participant learners' actions and speech. Observation entails jotting down notes, making comments, exploring situations, actions as well as making notes of the particular circumstances in which the learning has taken place. Cohen, Manion & Morrison (2011, p.398) claim that "what is noticed through seeing and hearing, is in actual fact the researcher's side or story of what is there." There is no need to depend on learners to give their own views about the GeoGebra programme used for learning Holloway and Jefferson, 2013).

According to Rajit (2013, p.134), the "different kinds of observation that can be employed in qualitative research, are complete observer, observer as participant, participant as observer, and complete participant." I decided to become an observer as participant. Bapir (2012) maintained that the researcher succeeds in becoming involved in the process but focuses on observing the process. To some extent, I was also a participant observer, because I was present to hear the perspectives while teaching and facilitating the GeoGebra programme supported learning. I was also a non-participant observer, which means that I simply observed by monitoring, listening and recording (Bryman, 2012).

I consider observation an important means to witness closely the research phenomenon which is one way to collect primary data. Rajit (2013, p.134) claimed that "observation is a purposeful, systematic and selective way of watching and listening to an interaction or phenomenon as it takes place." Therefore, a structured observation schedule was used to observe learners' attitude towards the GeoGebra programme itself as well as attitudes towards the GeoGebra programme supported learning, learners' participation and their explanation in their work book. It is also appropriate in situations where "full and/or accurate information cannot be elicited by questioning, because respondents either are not co-operative or are unaware of the answers because it is difficult for them to detach themselves from the interaction" (Bertram & Christiansen 2014, p.84). This would help to determine whether the learners were coping or not coping with using the GeoGebra programme for learning Euclidean Geometry concepts.

4.8.3 Semi-structured interviews

Yen (2012, p.139) states that "interviews are regarded as the most widespread method of collecting data, using an interpretive paradigm in a qualitative inquiry." An "interview is

described by some researchers as an exchange of opinions between two or more learners on a subject of common interest, conducted by one person who wishes to obtain information” from the other person (Lichtman, 2014, p.103). The purpose of using interviews as a data gathering method for this study is not to make generalisations, but to allow me to explore the perceptions and experiences of the learners (Bertram & Christiansen, 2014). Many qualitative researchers consider interviews as a major data collection strategy (Satu Elo, 2014). The objective of using a semi-structured interview here is to generate primary information from the participant learners, according to Bertram & Christiansen (2014, p.83) who claimed that “interviewing is a good method to use for gaining in- depth data from small number of learners.” In addition, the tangible advantage of interviews is that the researcher is physically present with the interviewees, therefore any misapprehensions may be speedily clarified (Brinkmann & Kvale, 2014).

Researchers could employ different types of interviews, namely structured interviews, semi-structured interviews and unstructured interviews. This study used semi-structured interviews which often take the form of a conversation with the intention that the researcher explores the participant learners’ view (Marshall & Rossman, 2014).

Semi-structured interviews were “the best interview type in this study as they are well suited for the exploration of the perceptions and opinions of learners regarding the GeoGebra programme and allow inquisitiveness for more information and explanation of responses.” (Lichtman, 2014, p.37)

A standardised interview schedule provided exactly the same wording and sequence of questions for each respondent to be certain that differences in the answers were due to differences among the respondents rather than in the questions asked (Guest, Namey, & Mitchell, 2013). Due to the same wording in the questions asked reliability and validity rested on passing on resemblance meaning to questions (Jones et al., 2014). Questions in the interview schedule were meant to address the learners’ experiences and perceptions about the use of the GeoGebra programme when learning Euclidean Geometry. These interview questions were also established in line with the research objectives and the research questions of the study, for example, asking how a learner would advise her/his teacher to use the GeoGebra programme or traditional ‘chalk and talk’ approach in a way that would make learners perform better in Euclidean Geometry. This question is related to the objective of using the GeoGebra

programme to learning Euclidean Geometry and the research question of how the GeoGebra programme is used to learn Euclidean Geometry. The participant learners were exposed to the question: Would you like your future Mathematics lessons to be taught using the GeoGebra programme? Provide reasons for your response. These type of interview questions were aimed to probe the effect of the GeoGebra programme- based learning on the motivation levels of learners in Euclidean Geometry.

Consequently, with purposively selected three learners, individual one-on-one interviews were conducted. Purposive sampling is used to select an interview participant. This sampling strategy contributed to probe learners who performed fairly well in the post-task; one learner who performed averagely, and one learner who performed poorly in the post-task. Interviews were semi-structured to allow the study to probe learners' responses where it was necessary, for instance, probing a learner how she/he would like to be taught Euclidean Geometry and then probing the learner to describe why, after giving a response. It is very important to ask more questions to obtain more detailed information if the respondent has not given sufficient detail by other means (Bertram & Chritiansen, 2014). For all interview participant the same interview questions were used. Interviews were conducted to explore and file the result of the GeoGebra programme versus traditional 'chalk and talk' used learning on learners' performance, and motivation levels towards the Euclidean Geometry concept. During the interview discussion all participants were video-recorded for analysis purposes. This interview information captured aims to establish correlation with the data generated using different strategies too.

4.9 DATA GENERATION PROCEDURES

This study conducted in the classroom consisted of a series of six lessons in two phases. During the first phase I taught for three consecutive lessons in a usual traditional 'chalk and talk' approach. Thereafter, the pre-task was administered for the participant learners. The data gleaned from the pre-task and post-task design were quantitative in nature, though analysed qualitatively. The pre-task was carried out in the week that proceeded the intervention week. The pre-task was marked and the results recorded and the scores kept aside.

Subsequently, the intervention using the GeoGebra programme was carried out for three consecutive days. The lesson content was established into a series of tasks that I prepared as

learning materials for this study. In the course of each session the learners were guided to work with the electronic GeoGebra programme under the directions of the tasks provided for them. Meanwhile, these two phases of lessons were observed using a structured observation schedule. A copy of an observation schedule is attached as an Appendix, at the end of the study.

After the implementation of the GeoGebra programme, a post-task and interviews with the learners were administered in order to evaluate the intervention. The post-task was provided after the use of the GeoGebra programme for learning Euclidean Geometry. In fact, the participants were given the same questions that they took in the pre-task. The scoring is presented in the next chapter.

I set semi-structured interview questions with the aim of allowing learners to respond to them in a more detailed and descriptive manner.

Finally, learners were interviewed one-on-one inside the school premises in a quiet science laboratory classroom. A set of semi-structured questions was used for guidance and the learners were encouraged to share their individual experiences regarding the use of the GeoGebra programme in the learning of Euclidean Geometry in particular. Each interview was video recorded for later transcription.

The use of a one-on-one interview helped in understanding how learners used the GeoGebra programme, and how they felt about the use of the GeoGebra programme in the learning of Euclidean Geometry.

4.10 METHODOLOGICAL NORM

According to (Kumar, 2013), there are two important questions which need to be asked when it comes to a study, firstly, “to what extent the result will be appropriate and meaningful (Validity)? Secondly to what extent will the results be free from errors? This is reliability.”

4.10.1. Reliability

Creswell (2014) explains that the consistency of a data collection tool as the degree to which the instrument measures accurately and consistently what it was intended to measure.

4.10.1.1 Reliability of the interviews

To ensure the reliability in the interview data collected, interview questions were approved by the study supervisor and two Mathematics teachers who have experience of using the GeoGebra programme for teaching Mathematics. Changes were made in the way questions were initially structured in order for them to elicit the necessary responses. Similar wording was used in order to be certain that all the participants understood the questions in the same way without any bias. I read the words carefully to ensure that no participant was given unfair advantage over another participant. No follow up questions were asked to ensure consistency in the questioning. The same video-tape was used for all participants in order to ensure fairness in the quality of the instrument used. All interviews were conducted at the end of the lessons in the school science laboratory to avoid noise and disturbance of any kind.

Participants were free to answer the questions as the interview was one - on - one without other people in the laboratory who might make the participant uneasy or anxious.

4.10.1.2. Reliability of observation schedule

In order to ensure the reliability during the lesson observation, the same video tape was used with all the learners. This measure ensured that no data was distorted due to the use of selective memory. I remained uninvolved in some cases to avoid interfering with the study avoiding any element of bias. I focused on my role as observer on all lessons observed.

4.11. VALIDITY OF THE PRE-TASK AND POST-TASK

The task was approved by a Grade 12 Mathematics teacher in order to check whether it was appropriate for the learners' level. The study supervisor also made input before the task could be administrated to the learners. A Grade 11 learners CAPS (Curriculum Assessment policy Statement of Department of Basic Education of South Africa) text book was used for questions to make certain that the questions were consistent and applicable to the level of the learners.

4.12 TRIANGULATION

Triangulation is a systematics strategy to crosscheck the reliability and validity of the research data using different data collection tools. Triangulation is a “method used by qualitative researchers to check and establish validity in their studies by analysing a research question from multiple perspectives” (Bertram and Christiansen, 2014, p.68). Furthermore, Creswell (2014) claims that “triangulation is a process using different data collection methods to cross

validate data and the findings.” Several data collection methods were used for this study, namely, pre-task and post-task, observation and interviews, thus ensuring triangulation.

The theory of triangulation entails the employment of different data generation tools to investigate the particular problem identified by the researcher (Neuman, 2014). I was looking for information on the use of the GeoGebra programme when learning Grade 11 Euclidean Geometry, to gain insight into how computers can be upgraded in township schools, to assist learners in achieving the required results. The “benefits of triangulation include increasing confidence in research data, creating innovative ways of understanding a phenomenon, revealing unique findings, challenging or integrating theories and providing a clearer understanding of the problem” (Neuman, 2014, p.218).

4.13 DATA ANALYSIS

Analysis of data comprises aspects such as: interpreting, defining, describing and drawing conclusions and establishing suggestions and recommendations (Miles, Huberman, & Saldana, 2013). According to Creswell (2014, p.102) “qualitative data analysis refers to assessing data that approximates or characterises but does not measure the attributes, characteristics, properties and so on of the subject in question.”

For this study qualitative data analysis was employed in order to interrogate more in-depth of the study. The purpose was to describe a situation and gain insight on the use of GeoGebra in Euclidean Geometry, theoretically. For this study an in-depth explanation was needed from a small sample, intended to then draw out patterns from assessment of explanatory explanations and individual responses, to then gauge concepts and insights. As a result, semi-structured interview questions with no pre-determined response categories and questions were kept broader, contextual and flexible to explore the learners’ experiences and perceptions in depth. The current study applied the following qualitative data analysis underlying assumptions as suggested by (Neuman, 2014,p.76):

- “Data analysis is determined by both the research objectives and multiple readings and interpretations of the raw data. Therefore the findings are derived from both the research objectives outlined by the researcher and findings arising directly from the analysis of the raw data;

- The primary mode of analysis is the development of categories from the raw data into a model or framework that captures key themes and processes judged to be important by the researcher;
- The researcher's findings result from multiple interpretations made from the raw data, by researchers who code the data. Inevitably, the findings are shaped by the assumptions and experiences of the researchers conducting the research and carrying out the data analyses. In order for the findings to be usable, the researcher (data analyst) must make decisions about what is more important and less important within the data;
- The trustworthiness of findings can be assessed by a range of techniques such as (1) independent replication of the research, (2) comparison with findings from previous research, (3) triangulation within a study, (4) feedback from participants in the research, and (5) feedback from users of the research findings.”

In the current study as researcher, I was concerned primarily with process, meaning how participants make sense of their experiences in the learning of Euclidean Geometry using a traditional approach as opposed to the GeoGebra programme supported approach.

Based on interpretive frameworks, the current study used the following philosophical assumptions as suggested by Creswell (2014) and Thomas (2011), in shaping the direction of the current study:

- **“Ontological assumptions (The nature of reality):** This relates to the nature of reality and its characteristics. The current study embraced the notion of multiple realities and reported on these multiple realities by exploring multiple forms of evidence from different individuals' perspectives and experiences,
- **Epistemological assumptions (How researchers know what they know):** Hence I was very meticulous with participants during the study. Thus an attempt was made to obtain subjective evidence based on individual points of view from participants was successful,
- **Methodology (The methods applied in the research):** Inductive, emerging approaches were adopted, while being shaped by the researcher's experience during

data analysis and collection” (Creswell, 2014, p.73). The next chapter (Chapter Five) presents the data analysis of this study in detail.

4.14 RESEARCH ETHICS

Bertram and Chritiansen (2014) stated that researchers are compelled to ensure that their research complies with ethical standards to protect the participants from unfair criticism that may arise from participating in the research, whilst (Neuman, 2014, p.107) writes that “ethical dilemmas can be resolved through the protection of the participants’ confidentiality and abstaining from deception or involvement with deviants.” The ethics of research is concerned with the degree of trustworthiness with regard to integrating and placing the information obtained from the sources (Creswell, 2014).

A permission letter from the school principal was requested for the research (see, Appendix II) and the school principal signed a gate keeper permission letter for the research study. The permission letter was submitted to the University of KwaZulu-Natal through my Supervisor’s office and an ethical clearance certificate was received from the University of KwaZulu-Natal research office (Appendix VI). The informed consent of each participant learner was presented in writing (see Appendix IV). The participant learners’ parent signed the informed consent to approve that their children would be involved in this study (see appendix V).

The participants were given a workshop and asked to read and sign the consent form. The benefits, rights, risks and dangers involved as a result of their involvement in the research were clearly clarified. Participants were further asked that should any unforeseen circumstances occur, they are free to withdraw from participation. Participants voluntarily agreed to be part of the study. The data obtained during this study was treated with confidentiality and all materials used were kept but will be destroyed immediately after the awarding of research. I used English as medium of communication in order to avoid discrimination due to language barrier. As a researcher, I clarified the objectives of the research to the principal and school management team of the participating school.

I assured the participants that anything discussed during the study would be kept confidential and would not be used for purposes other than this study. The real names of the participants and the names of the schools would be kept anonymous in order to protect their identity from

unnecessary criticism or ridicule. The learners' identity will not be exposed in the research report.

The responses were double checked by the participants before being transcribed to the study in order to avoid possible misinterpretation and misuse. I allowed each participant learner to review their responses before they were finalised to ensure that my transcriptions were in accordance with what the participants had responded during the semi-structured interview. I clearly explained that there would be no rewards or payments due to them after they had participated in this particular research, however, the researcher committed to show them the results of the study when finalised.

The following principles guided this research as given by Creswell (2014) and Wassenaar and Mamotte (2012);

- The principle of Beneficence: this study has provided a valuable contribution towards people's wellbeing. The results of the study "could assist the schools of the Pinetown District in the KwaZulu-Natal Province of the Republic of South Africa." Feedback of the results is expected to be given to the Thornwood Secondary School.
- The principle of Non-Maleficence: research must not cause harm to the participants. In this study there was no anticipated harm that could be caused to the participants.
- The principle of Autonomy: "research must respect and protect the rights and dignity of other participants" (Creswell, 2014, p.79). All participants were consulted and made aware in this regard, in writing that they had the right not to participate.
- The principle of Justice: the benefits and risks of research must be fairly distributed. The research was conducted and planned in such a way that no risk was anticipated (Creswell, 2014).

The objective of the study was clarified in the consent form (see Appendix IV and V). The participants were assured that the information received from them will be used only for the current study and they were told that their names would be kept strictly confidential. "Ethics has to do with the application of moral principles to prevent harming or wronging others, to promote the good, to be fair" (Kumar, 2013, p.219). More importantly I was conducting educational research which is according to Marshall and Rossman (2014), educational research

which focuses primarily on human beings and as such the researcher is ethically responsible to protect the rights and welfare of participants in the study.

4.15 TRUSTWORTHINESS /CREDIBILITY /VALIDITY /RELIABILITY

An important issue is the validation of data within a qualitative research approach. Rajit (2013, p.172) discusses the “difficulties of using conventional constructs such as validity, reliability, generalizability and objectivity”. Instead, he suggests using the closely corresponding constructs of credibility, dependability, transferability and conformability while discussing the trustworthiness of the research. It is important to ensure the study’s trustworthiness by maintaining transferability, credibility, dependability, and conformability. “Trustworthiness is a demonstration that the evidence for the results reported is sound and the argument made based on such results is equally strong (Creswell, 2014, p. 81).

4.15.1 Credibility

According to Brinkmann and Kvale (2014), the credibility of a study refers to the capacity of the researcher to consider the difficulties that are present in the research process and to deal with themes or categories that are not easy to describe. Bapir (2012) suggests that the researcher should be involved over an extended period at the location of the study, should carry out continuous observation to discern universal qualities, and should also carry out triangulation. According to Brinkmann and Kvale (2014, p. 107), “triangulation was first adopted in the social sciences, to impart the notion that to verify information, a variety of data sources are required.” When triangulation was first used in qualitative research, it implied the verification of information. This meant that numerous data sources resulted in gaining a better understanding of the events that were being investigated (Jones et al., 2014) . Some researchers are in favour of triangulation, because they feel that depending on one method only may lead to prejudice or an inaccurate opinion about a certain part of the research investigation on the part of the researcher (Guest et al., 2013).

Lichtman (2014) claimed that the researcher may use some procedures to diminish the degree to which their own standpoints may encroach or infringe. In ensuring credibility (confidence in the truth of the findings) the researcher used the following techniques as in line with (Kumar, 2013, p.172):

“Extended times spent with participant learners to explore their learning experiences and perceptions. As a researcher I spent sufficient time in the settings, in order to learn and understand the culture or phenomenon of interest. Thus I was on the sites long enough to become oriented to the prevailing circumstances so that the context is appreciated and understood. Prolonged engagement with the respondents over an extended period provides evidence of a scrupulous research investigation. In this study, a conscious effort was made to spend time with participant learners at school.

- Persistent observation for provision of depth of the study,
- Triangulation to facilitate deeper understanding of the study situation and
- Built trust with the respondents.”

4.15.2 Transferability

Transferability concerns whether the conclusions of this study are transferable to other contexts (Yin, 2012). Though it is very problematic to maintain transferability primarily for a qualitative study strategy, however, it can be achieved by stating a detailed description of the study procedure. Therefore, to ensure transferability of this study, I described the study procedure extensively and thoroughly for others to follow and replicate in the future. The submission and publishing of papers in journals and presentations at seminars and conferences can create opportunities for other researchers to review the study outside the local research context.

In quantitative research, there appears to be the belief that results must be generalised. In qualitative research, the results of an investigation will not be generalised, but will be applicable only to the background and circumstances of a particular study (Miles et al., 2013). Qualitative researchers, however, try to yield the probability that the outcomes may be transferable to other contexts, so that people may benefit from the wisdom gained during the research study. The possibility that the results may be applicable, can be analysed in terms of the prospect that another situation would be very similar for the results to be pertinent (Symon & Cassell, 2012). Qualitative researchers strive to give in-depth descriptions or accounts of the “context” of the study, as well as of the participants, so that other individuals can evaluate if the research can be applied to their own context (Kumar, 2013, p.172). However, the fact that “thick and rich descriptions” are generally used in qualitative research, illustrates the significance of the trustworthiness of a study (Yin, 2012, p.99).

Transferability alludes to the degree to which the results can be administered to other participants or different/distinct “contexts” (Symon & Cassell, 2012, p.107). All observations are specified by the particular contexts in which they transpire. The qualitative researcher contends that learning secured from a particular context will not automatically be related to different contexts, or even similar contexts, at some other time. In qualitative research, transferability is demonstrated by other observers reading the study.

Qualitative researchers use purposive sampling. They are indicating that they strive to select samples that will provide the exact information, as well as samples that could provide the greatest amount of information about the participants and about their contexts (Yin, 2012, p. 79). As a result, three learners, purposively selected were interviewed.

In this study, the context of the school that participated in the research process was described in full. Complete details of all the respondents of this study were also provided to permit other scholars to make an assessment in terms of applying the results of this study to their own situations. In this manner, other scholars may gain awareness of the circumstances surrounding the research findings and the respondents. To ensure transferability, purposive sampling was deployed by selecting samples that would produce the greatest volume of information about the respondents and their unique circumstances.

4.15.3 Dependability

The “stability” of research information is referred to as dependability (Kumar, 2013, p.172). Dependability can be dealt with, as pointed out by (Kumar, 2013), by using more than one method so that the weak points of the one method are neutralised by the strong points of another. Creswell (2012, p.158) further suggests that an “audit trail should be initiated, which implies that an outside auditor could be used to scrutinise the data collection methods and interpretation.”

The trustworthiness of a study rests on the degree to which other audiences are enabled to determine if the research processes are appropriate for the objectives of the study (Marshall & Rossman, 2014). The dependability of research findings can be obtained through an “inquiry audit”, which means that aspects of the entire research procedure are made accessible to both participants and other observers. Certain aspects, such as the formulation of the research

questions and the gathering and analysis of data, are examples of particulars regarding the research process that are procurable (Kumar, 2013).

Particulars of the full research findings are made available to other scholars and also by using “the inquiry audit” (Numan, 2014, p.219). For this purpose, complete information regarding the total study procedures is at the disposal of other audiences as well as to respondents. The issue of dependability in this study was addressed by utilising various data gathering methods, as mentioned by (Creswell, 2014). During my visits to the schools, especially for the interviews, the participants were invited to examine my observation notes as well as the transcriptions of the interviews.

The last principle for confirming the trustworthiness of a research finding is confirmability. Unlua, Dokmeb, and Tufekcic (2015) claim that the confirmability (objectivity) of a study concerns the extent to which the research findings are uninfluenced by the researcher. As Creswell (2014, p.148) points out, “no research is ever free from influence of those who conduct it.” Qualitative data consists of interpretations of text or images. One aspect of this has been taken into account in cases where the responses from all the learners have been reported in order to provide an overall picture which is not involved in filtering of evidence. Another aspect concerns the usage of the interpretative frameworks in the analysis of data. Suggested explanations by the author have been discussed with colleagues to check for rival explanations (Creswell, 2014).

4.16 CONCLUSION

This chapter has described how the current study can be conducted. In this section all the aspects regarding the target population, sampling, data collection strategies, research approach adopted, data analysis strategies and research ethics have been discussed in detail. Different data generation instruments are used to establish higher degree of reliable findings in this study. The next chapter will discuss the findings obtained from the data generation instruments. The data generated from the instruments such as pre- task, post-task, observation and interview during the course of this study will be discussed in the next chapter to draw the conclusion of the study.

CHAPTER FIVE

FINDINGS, ANALYSIS AND INTERPRETATION

5.1. INTRODUCTION

In the previous chapter, the research methodology of the current study was discussed. This chapter presents the analysis of data that was generated from Grade 11 learners who participated in this study. Data were generated using pre-tasks and post-tasks, observation schedules and semi-structured interview schedules. This chapter has been organised and divided into several sections. The first section provides the outline of data generation strategies. The second section focuses on answering research question one: How can the GeoGebra programme be used in the learning of Grade 11 Euclidean Geometry? The third section, addresses research question two: Why is the GeoGebra programme used in the way that it is when learning Grade 11 Euclidean Geometry? The last section addresses the conclusion and summary of this chapter.

5.2 DATA GENERATION STRATEGIES

Table 5.4 illustrates the strategies that were used to generate data from the participants in order to answer the research questions. As clarified in Chapter Four, pseudonyms were used to protect the learners' identity.

Table 5.4 describes the processes of data generation strategies

The two main research questions are:	Participants	Data generation methods
1. How can the GeoGebra programme be used in the learning of Grade 11 Euclidean Geometry?	Pseudonyms : Daisy Karen Nelly	Observation schedule , Pre-task and post-task and Semi-structured interview schedules
2. Why is the GeoGebra programme used in the way that it is when learning Grade 11 Euclidean Geometry?	Pseudonyms: 1. Bongani 9. Mxosis 2. Daisy 10. Ndelany 3. Dovela 11. Nelly 4. Fanu 12. Sibub 5. Ikona 13. Siphumela 6. Karen 14. Siya 7.Lindo 15.Thula 8. Londi 16. Zimela	Observation schedule Pre-task and post-task Semi-structured interview schedules

The current study focused on exploring Grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry. Data were generated from Grade 11 Mathematics learners. These learners conducted an experiment using the pre-designed class activities for this study and their performance on class activities is discussed. Two research questions were taken into consideration when the questions for the class activity, pre-task, post-task, observation and interview schedules were designed.

5.3 THE USE OF THE GEOGEBRA PROGRAMME WITHIN THE CLASS ROOM

5.3.1 Incorporating the GeoGebra programme into Euclidean Geometry learning

Learners were exposed to the GeoGebra programme and used Euclidean Geometry learning during the course of this study. However, the usual traditional talk and chalk teaching and learning approach was also investigated in order to understand how learning occurred in these two different learning approaches. Subsequently, the learners' perceptions and experiences towards Euclidean Geometry learning and their achievement was explored on two different occasions on Euclidean Geometry. The purpose of this section is to present and analyse how learners performed before and after being exposed to teaching with the GeoGebra programme. The section starts with the content analysis of data obtained during the class observation. This section particularly focused on how learners used the GeoGebra programme within the classroom. Constructivism was adopted as a theoretical framework in the current study. Most importantly this section focused on how learners construct their responses of the tasks provided for them based on the GeoGebra programme.

Learners were given the pre-designed class activities after they familiarised themselves with the GeoGebra programme. Figure 5.3 shows how learners were exposed to verify the perpendicular bisector theorem with the GeoGebra programme. This was particularly addressing the research question 1: How can the GeoGebra programme be used in the learning of Grade 11 Euclidean Geometry?

Class activity -1

In the circle with centre C, $CD \perp AB$, $CD = 4$ and $AB = 10$ cm. Determine the value of x .

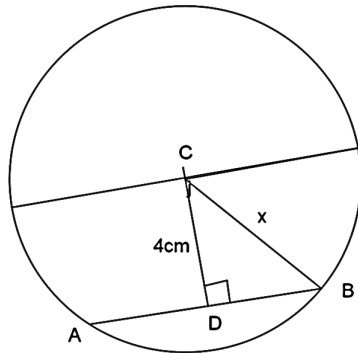


figure 1

Figure 5.3: A diagram showing an activity given to learners

Figure 5.3 shows the activity which the learners were expected to solve while using the GeoGebra programme in order to verify the theorem. The learners had to measure the sizes of the different angles in the diagram with the angle-measuring tool in the designed GeoGebra programme, subsequently; learners had to complete the activity. Figure 5.4 shows how learners interacted with each other to complete the activity. .



Figure 5.4: A diagram showing learners interacting with each other when solving the activity based on circle geometry

Figure 5.4 shows how learners explained things to each other. The learners were discussing what they could deduce from the diagram (see Figure 5.4). All learners were required to verify the perpendicular bisector theorem with the GeoGebra programme. Figure 5.5 shows how learners verified the perpendicular chord theorem by using the GeoGebra programme.

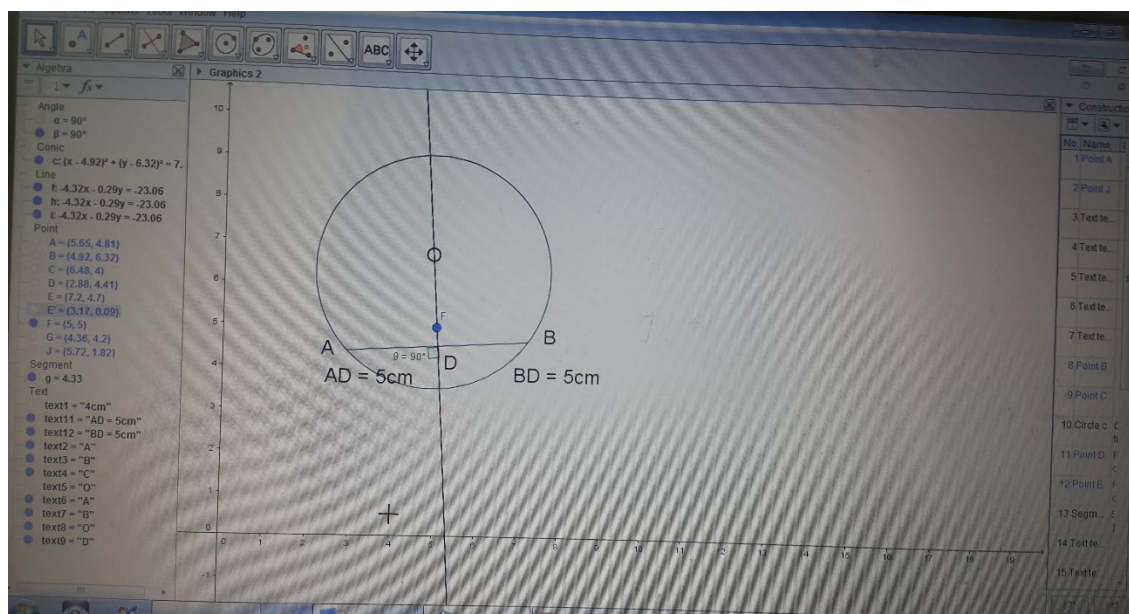


Figure 5.5: A diagram showing learners verifying the perpendicular chord theorem while using the GeoGebra programme.

This activity was the first lesson in which the learners used the GeoGebra programme. This activity required learners to construct a circle and then to apply different constructions. The learners did not just construct the circle in the GeoGebra programme, but had to explore the properties of Euclidean Geometry. It was also expected for learners to complete questions in the class activity. The learners therefore had an opportunity to generalise and make conclusions of rules of the perpendicular bisector theorem on the basis of experience obtained when using the GeoGebra programme. They made deductions from special cases. Figure 5.5 shows how the learners had to measure the length of the chord, each half of the chord and sizes of the angles with the GeoGebra programme.

The key aspect in Figure 5.5 was how GeoGebra shaped the thinking of the learners and how it helped them to understand and visualise the theorem. The key process here was how the learners used GeoGebra on her/his own as a tool to test the conjectures and to validate her/his answers. They tested and validated their answers by the dragging of point A, B, C or D when using the GeoGebra programme (see Figure 5.5). The learners acquired physical knowledge because they explored the properties of Euclidean Geometry while using the GeoGebra programme. The learners made a conjecture on the basis of insight, numerical investigation and measurement (Murni et al., 2017; Tatar, 2013). The GeoGebra programme provided the learners with visual images that contributed to their growing mathematical understanding

(Pfeiffer, 2017; Shadaan & Eu, 2014). This shows how the learners constructed their own understanding of the theorem that states that the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. They acquired logical-mathematical knowledge by showing understanding that the segment is only perpendicular if it is drawn from the centre of the circle to the midpoint of the chord. Logical-mathematical knowledge means knowledge acquired through series trials (Hank, 2016). The learners' interaction and discussion with each other, and with the teacher, show how they negotiated meaning for the mentioned theorem. Although the theorem was not proved at this stage the learners' discussions moved them from informal to formal mathematical reasoning. This shows how the learners moved to higher order thinking.

Another class activity posed for the learners was to verify that the sum of the two opposite angles of a cyclic quadrilateral are supplementary. Figure 5.6 shows how the learners had to measure the sizes of the different angles in the diagram with the angle-measuring tool in the designed GeoGebra programme.

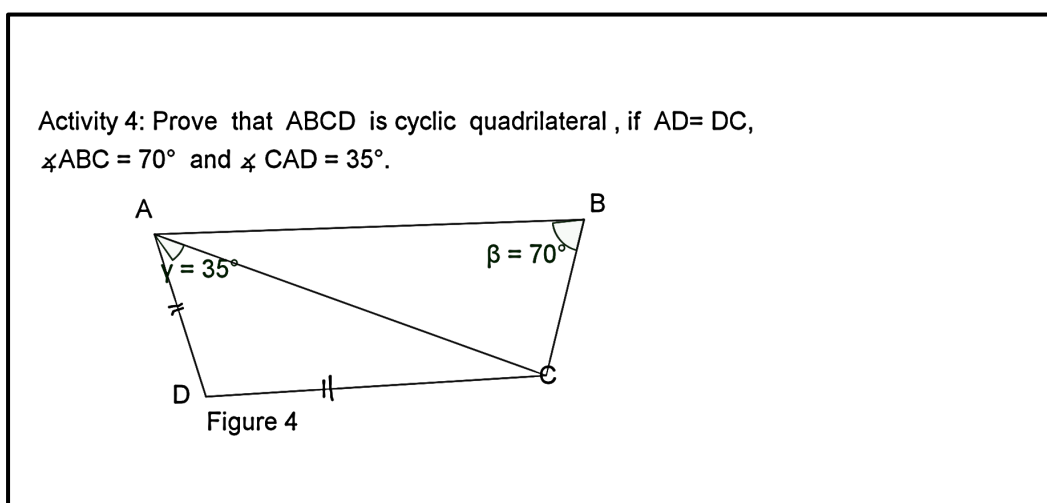


Figure 5.6: A diagram showing an activity to verify the supplementary angles theorem with the GeoGebra programme.

This was the pre-designed activity given in the classroom to verify the supplementary theorem using the GeoGebra programme. The supplementary theorem posits that “the opposite angles of a cyclic quadrilateral when added are equal to 180° , meaning they are supplementary.” This activity covers the two research questions. The learners drew quadrilateral ABCD with

diagonal AC using the GeoGebra programme. They measured $\angle ADC$ and found that the angle measured 110° , learners then measured $\angle ABC$. This practical application of using the GeoGebra programme to justify proofs and theorems in Euclidean Geometry addresses research question 1: How can the GeoGebra programme be used in the learning of Grade 11 Euclidean Geometry?

All learners were required to verify that for any quadrilateral, if the sum of their interior opposite angles does not add up to 180° , the quadrilateral automatically cannot be cyclic as explored by a learner. In quadrilateral ABCD, $\angle A = 70^\circ$ opposite to $\angle C = 110^\circ$. If the points A, B, C & D lie on the circle, the sum of the two opposite angles must be equal to 180° . Figure 5.7 shows how learners used the GeoGebra programme to verify the cyclic quadrilateral theorem

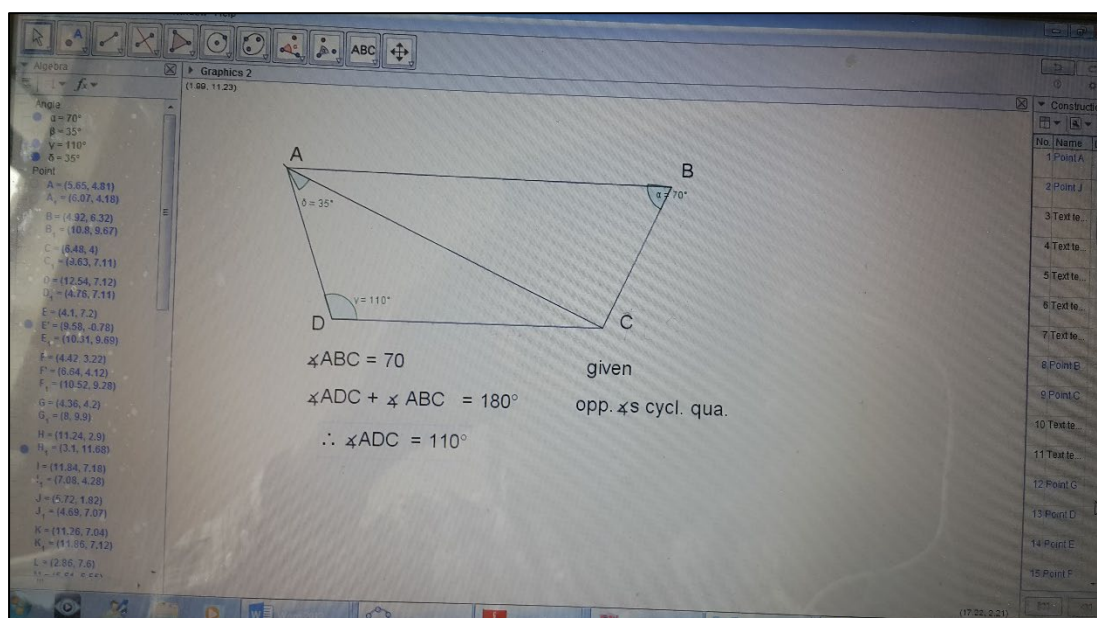


Figure 5.7 A diagram showing a snapshot of the GeoGebra programme used to sketch and verify the cyclic quadrilateral ABCD.

Figure 5.7 shows that learners used the GeoGebra programme to draw the quadrilateral ABCD. Thereafter, they measured all four angles using the GeoGebra programme. Learners had measured $\angle A = 70^\circ$ and $\angle C = 110^\circ$ and when added up these two angles, the sum of angles A and C was equal to 180° , therefore $\angle A$ and $\angle C$ are supplementary. Additionally, learners measured the other two angles, $\angle B = 70^\circ$ and $\angle D = 110^\circ$. When they added these two angles together, the sum was also 180° . Consequently, points A, B, C, & D are cyclic (meaning these points lie on the circle).

Figure 5.8 was captured during the class observation which shows how learners were interacting with each other to verify the cyclic quadrilateral theorem using the GeoGebra programme.

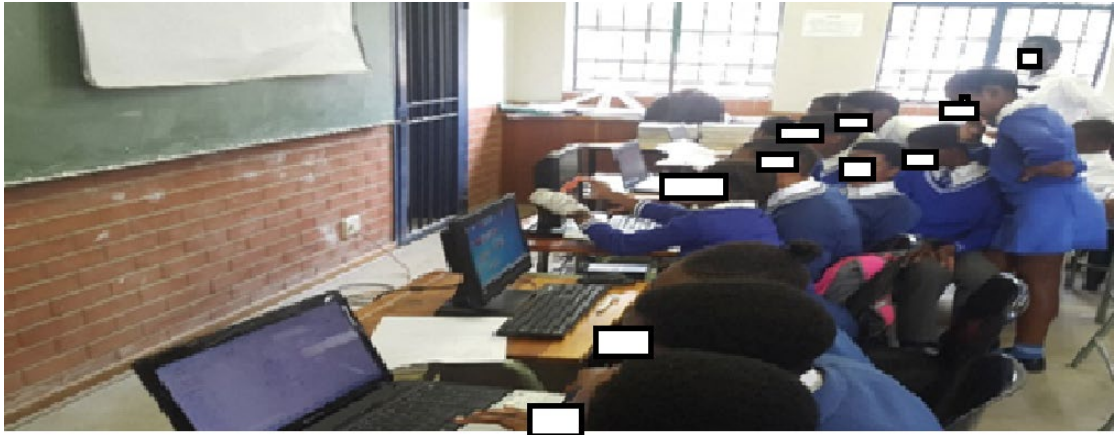


Figure 5.8 Learners' interaction in order to verify the cyclic quadrilateral theorem.

Figure 5.8 shows how the learners were dedicated to prove the cyclic quadrilateral theorem in the classroom. The learners were discussing what they could deduce from the diagram. Learners on the right of the picture and learners pointing at the screen were working together and learners on the left of the figure showed their answers.

5.3.2 Using the GeoGebra programme to promote learners' understanding of Euclidean Geometry

All the learners performed better in post-task than in the pre-task. The result from the pre-task and post-task give the overview of the learners' generic conceptual understanding as a result of the class room intervention. There was a significant difference between the pre-task and post-task responses. All learners were able to write constructive arguments when they responded to the post-task. This shows that learners had developed a better understanding of the Euclidean Geometry concepts.

This was an indicator that the use of the GeoGebra programme as a learning tool made problem-solving more efficient because it provided immediate feedback visually on the computer screen. Therefore, learners could either verify or reject the tasks posed to them. This is evident in the subsequent interview excerpt after the implementation of the GeoGebra programme: Nelly stated that:

“...there was a fascinating moment during the GeoGebra assisted learning because I could see circles, triangles and their measurements at the same time on the computer screen. Once I drew the diagram, it just displays everything on the computer screen so that I could see it...”

This claim positively affirmed the better performance the learner achieved in the post-task. The learner alluded to the fact that the GeoGebra programme assisted to verify and justify the Euclidean Geometry theorems on the computer screen. Furthermore, Nelly was fascinated because the GeoGebra programme produces quick feedback and allowed her to see the outcomes on the computer screen.

The learners' improvement in the post-task indicated that the GeoGebra programme created an important learning environment which is in line with the Conditions of Learning Theory framework which addresses the questions, “what is learning?” and “how does learning take place?” (Gagne, 1985). As part of Gagne's nine events of instruction, the first event of instruction is gaining attention particularly, through visual learning. Certainly, visualisations are an important aspect in that we remember 20 percent of what we hear, 50 percent of what we see and hear, and 90 percent of what we do as a task (Edgar Dale's Cone of Experience, 1969) (See, section 2.5.3).

The current study using the GeoGebra programme as a learning tool shows that learners may retain more knowledge when they do the tasks by themselves.

The study conducted by Kim and Md-Ali (2017) also confirms that the use of the GeoGebra programme enables learners' logical reasoning skills.

Therefore, the programme providing visualisations created an opportunity for learners to see, examine and understand the properties and relationships of the geometric figures.

5.3.3 The use of the GeoGebra programme to promote learners' achievement in Euclidean Geometry

Learners were assessed on two different occasions on Euclidean Geometry. The purpose of this section is to present and analyse how learners performed in the pre-task and post-task. The responses from the learners disclose how learners acquired mathematical knowledge through the designed activities within the GeoGebra programme. The learners' explanations expose how the GeoGebra programme provided an opportunity for learners to understand mathematical concepts.

5.3.3.1 Exploring learner performance on the pre-task

Sixteen learners participated in completing the pre-task after being taught using ‘chalk and talk’. The pre-task was implemented directly after the Euclidean Geometry lesson was taught using the traditional ‘chalk and talk’ method. The pre-task was conducted to assess the levels of learning during the course of traditional teaching and learning in terms of knowledge and understanding. The outcomes of the pre-task were used as a baseline to establish genuine answers for the two research questions. A detailed analysis of the pre-task is presented in Table 5.5

Table 5.5 illustrates the learners’ correct responses on each question that was asked. 100% is a percentage which shows that participants’ responses were all correct. While the learners were proving the Euclidean Geometry theorems using the paper and pencil method, their baseline performance was, as follows: In activity 1; correct responses were 75%, in activity 2; 56%, in activity 3; 13%, in activity 4; 94%, in activity 5; 88% and in activity 6, correct responses were 0%. Some named angles using two letters and some discussed angles which did not exist. While justifying their arguments, the learners provided inappropriate reasons.

Table 5.5 Summary of correct responses obtained during the traditional lesson

Class activity during the lessons	Activity based questions	Percentage of correctly completed activities
Activity 1	<ul style="list-style-type: none">• Draw a circle using the traditional pencil and paper method.• Determine the relationship between the perpendicular line drawn from the centre of the same circle and the chord.	75%,
Activity 2	<ul style="list-style-type: none">• Determine the relationship between the angle at the centre of a circle and the angle at the circumference.	56%
Activity 3	<ul style="list-style-type: none">• Prove that the opposite angles of a cyclic quadrilateral are supplementary.	13%

Activity 4	<ul style="list-style-type: none"> Investigate the relationship between the opposite angles of a cyclic quadrilateral. 	94%
Activity 5	<ul style="list-style-type: none"> Investigate the nature of tangent lines from exterior points on a circle. 	88%
Activity 6	<ul style="list-style-type: none"> Investigate the relationship between the angle between a tangent to a circle and a chord drawn at the point of contact to the angle which the chord subtends in the alternate segment. 	0%

5.3.3.2 Exploring learner performance on the post-task

The pre-task and post-task were administered for the same group of sixteen learners. The post-task was completed directly after the GeoGebra programme was used to teach the Euclidean Geometry lesson. The same question items were administered for both the pre-task and the post-task to ensure similar conditions. A detailed analysis of the post-task is presented in Table 5.6

Table 5.6 illustrates the learners' correct responses on each question that was asked. 100% is a percentage which shows that participants' responses were all correct. While the learners were proving the angle at the centre theorem using the paper and pencil method, their baseline performance was, as follows: In Question 1; correct responses were 50%, in Question 2.1; 52%, in Question 2.2; 60%, in Question 3.1; 51%, in Question 3.2; 50% and in Question 3.3, correct responses were 50%. Most of the learners responded to the task correctly; however, they did not support all the steps with appropriate reasons. While justifying their arguments, most of the learners provided appropriate reasons.

Table 5.6 Summary of correct responses obtained when teaching using the GeoGebra programme.

Class activity during the lessons	Activity based questions	Percentage of correctly completed activities
-----------------------------------	--------------------------	--

Activity 1	Constructing circles using the GeoGebra programme. <ul style="list-style-type: none"> Determining the relationship between perpendicular line from the centre of the same circle and the chord. 	100%
Activity 2	Construct figure 2 using the GeoGebra programme and label the figure. <ul style="list-style-type: none"> Determine the relationship between the angle at the centre of a circle and the angle at the circumference. 	100%
Activity 3	Explore the relationship between the angles subtended by a chord of the same circle on the same side of the chord and the angle subtended by arcs of equal length.	94%
Activity 4	Prove that the opposite angles of a cyclic quadrilateral are supplementary.	100%
Activity 5	Investigate the relationship between the two opposite angles of cyclic quadrilateral by determining their sums.	31%
Activity 6	Explore the nature of tangent lines drawn from exterior points.	94%
Activity 7	Explore the nature of the angle between a tangent to a circle and a chord drawn at the point of contact to the angle which the chord subtends in the alternate segment.	62%

Table 5.6 indicated that learners performed comparably better in the post-task than in the pre-task. During the post-task learners supported their claims with appropriate reasons. It seems that learners had developed the appropriate reasoning skill when working with the GeoGebra programme. There were significant differences in learner achievement between the pre-task and post-task. This shows that the learners understood the Euclidean Geometry better when they used the GeoGebra programme. The possible reasons for this finding could be that the

GeoGebra programme enabled learners to check the correctness of their methods and the accuracy of their work. Since the GeoGebra programme is dynamic, learners had opportunities of re-examining their work.

In addition, the production of good-quality sketches requires competence in technical drawing skills, which not all learners possessed. The GeoGebra programme generated sketches which were neat and accurate. Learners could use these visuals to construct, verify and justify the theorems. The GeoGebra programme allowed learners the opportunity for instantaneous exploration. Consequently, this improved the learning process in terms of speed and quality (Horzum & Ünlü, 2017). When learners learn using the GeoGebra programme they spend less time drawing diagrams (sketches) and making calculations; this allows them more time to explore the characteristics of different Euclidean Geometry theorems. All these factors could have contributed to the better achievement on the post-task. Each of the participants was required to complete the task. Figure 5.9 shows learner Ndelany's response to question 1 in the post-task.

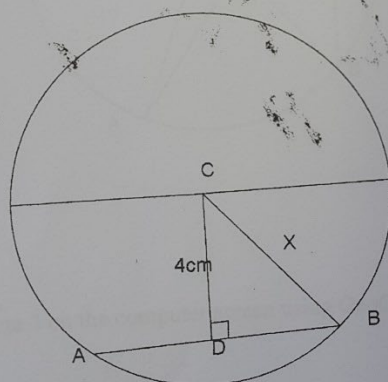
APPENDIX –IV: GeoGebra programme used during learning sessions. (Intervention)

Theorem 1; Perpendicular line from centre bisects chord.

Activity1: Constructing circles using GeoGebra programme

In the circle with centre C, $CD \perp AB$ $CD = 4$ cm and $AB = 10$ cm. Determine the value of x .

Learner name: Ndelany



$$a^2 + b^2 = c^2 \quad (\text{pythagoras})$$

$$x^2 = (4\text{cm})^2 + (5\text{cm})^2$$

$$= \sqrt{41\text{cm}^2}$$

$$= 6,4\text{cm}$$

V.Good!
Achieved!

Figure 1

Figure 5.9: A diagram showing Ndelany's response to question 1 in the post-task.

In the pre-task, participant learner, Ndelany used the theorem of Pythagoras without proving $\hat{B} = 90^\circ$. Whereas, in the post-task he showed $CB \perp AB$ from the GeoGebra programme based activities. He was able to support his claim with reasons in the post-task. This entails that the learner seemingly gained reasoning skills because he started using mathematical reasons to justify arguments. However, he could not show that $AD = DB = 5$ cm, nor could he provide sufficient reasons to conclude his answer. He was only able to follow simple deductions between each consecutive steps and arrived at the correct response.

Figure 5.9 indicates that Ndelany responded correctly to this particular task item. It was evident that he understood the concept after using the GeoGebra programme as a learning tool. Through the use of the GeoGebra programme, all participants were actively involved in measuring the sizes of angles and the length of lines AD & BD using the GeoGebra programme. These practical learning skills animated the lesson for the learners and assisted them in figuring out the relationship between the perpendicular bisector and the chord. This contributed to the learner's better achievement in the post-task. Figure 5.10 shows Ndelany's response to question 1 in the pre-task

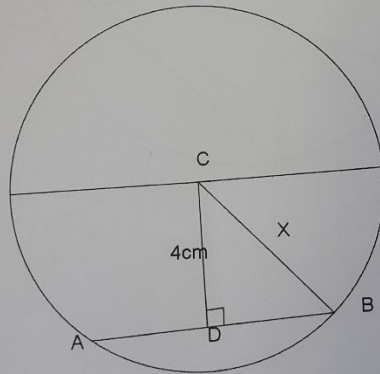
APPENDIX – 1: Non GeoGebra programme assisted learning sessions.

Theorem 1; Perpendicular line from centre bisects chord.

Activity: 1

Learner Name : Ndelany

In the circle with centre C, $CD \perp AB$, $CD = 4$ cm and $AB = 10$ cm. Determine the value of x .



$$\begin{aligned}
 r^2 &= x^2 + y^2 \\
 &= (4\text{cm})^2 + (5\text{cm})^2 \\
 &= \sqrt{16 + 25} \\
 &= \sqrt{41} \\
 \therefore x &= 6.4\text{ cm} \rightarrow \text{Achieved!}
 \end{aligned}$$

(???)
No Reason

Figure -1

Figure 5.10: A diagram showing Ndelany's response to question 1 in the pre-task

It seems this learner assumed that $\hat{D} = 90^\circ$, because in the diagram it appears as if $OB \perp AB$ and that is why he used the theorem of Pythagoras. He judged the sketch by its appearance and compared angles visually to determine the size of angles and assumed $CD \perp AB$. The new concept of circle Geometry was the theorem that states that the line arising from the centre of a circle to the chord of a circle is a perpendicular bisector of the chord. Ndelany used a correct formula (Pythagoras theorem) to determine the relationship between the perpendicular bisector and the chord. The learner correctly calculated the value of $x = 6.4\text{cm}$. However, Ndelany did not write any reason why he was using the Pythagoras theorem to solve the value of x . Furthermore, Ndelany used the theorem of Pythagoras yet, he did not mention that $AD = DB = 5$ cm.

Similarly, another task item which consisted of constructing a quadrilateral, diagonals and labelling all the measurements was posed to the learners. The essence of the task was to determine the relationship between the opposite angles of a cyclic quadrilateral. All learners made an effort to find the relationship to verify and justify the cyclic quadrilateral theorem. The learners found this activity challenging, but the GeoGebra programme provided an opportunity to think about both the mathematical properties of the figure that they were

expected to construct, and how to use the tools in GeoGebra to construct and measure all dimensions of the figures (See Table 5.6). It seems important to note that these learners could not prove this theorem appropriately while using the paper and pencil method. This happened despite the different teaching strategies that I used to demonstrate the proof. Daisy is one of the participant learners who completed the post-task with correct responses after the implementation of the GeoGebra programme (See Figure 5.11).

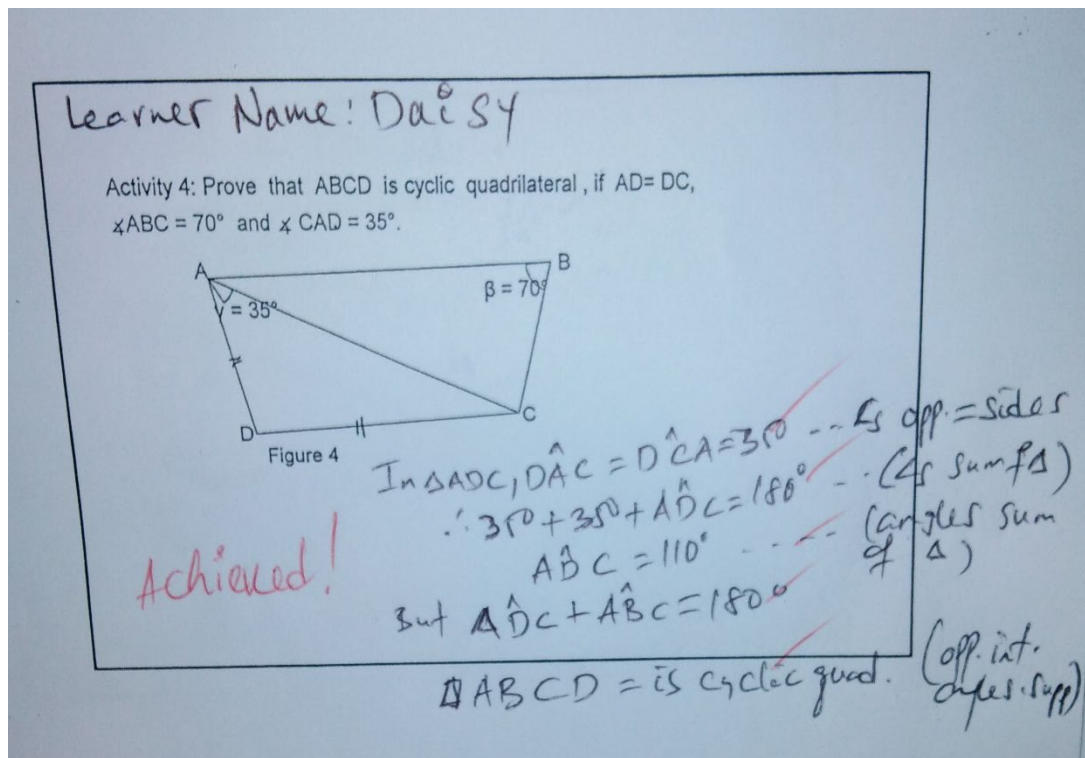


Figure 5.11: A diagram showing Daisy's response to question 4 in the post-task.

In the pre-task Daisy did not explain how the sum of the two opposite angles of the cyclic quadrilateral are supplementary. Whereas, in the post-task she presented all the steps with the necessary and sufficient reasons to conclude that the sum of the two opposite angles of the cyclic quadrilateral is 180° . Figure 5.12 shows that Daisy's response to questions in the pre-task after the traditional lesson was completed.

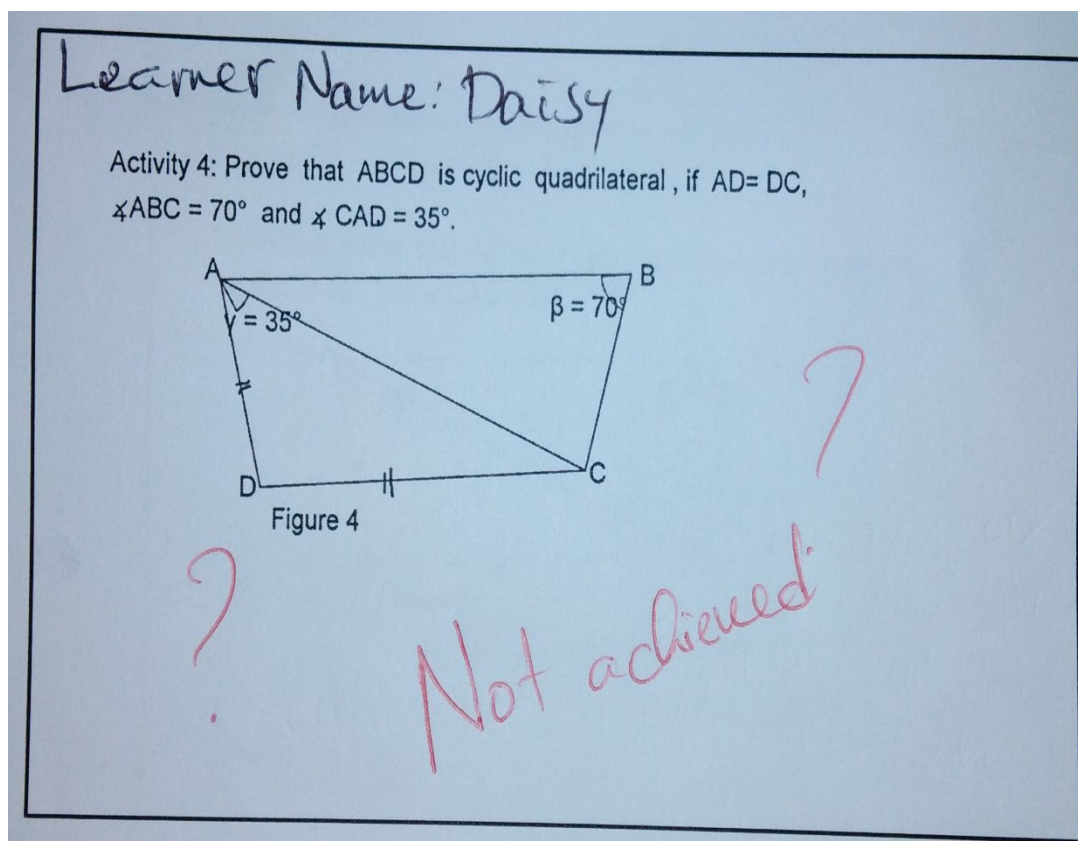


Figure 5 12: A diagram showing Daisy's response to question 3 in the post-task.

This exhibits that in the traditional learning environment learners do not necessarily make sense of their learning and they are always busy writing notes to prepare themselves for tests and exams. However, during the traditional 'chalk and talk' learning approach, learners are not capable of responding to class tasks because perhaps they did not understand the concept of the lesson. Similarly, learners were also required to complete the pre-task based on the tangent-chord theorem after they had completed the traditional chalk and talk lesson. Figure 5.13 shows Thula's response to question 5 in the pre-task.

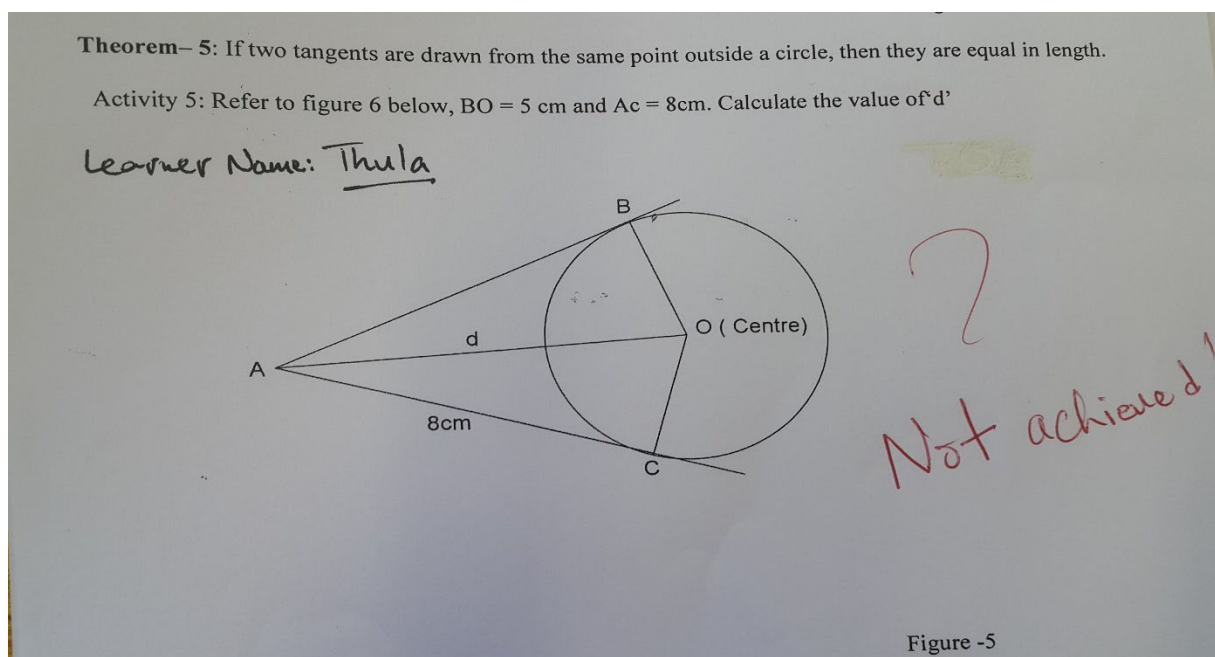


Figure 5.13: A diagram showing Thula's response to question 5 in the pre-task.

According to my class observation the learners were writing notes from the chalkboard. Despite learners writing notes, it was not possible to have broad class discussions at the end of each lesson due to time constraints.

The classroom environment and organisation essentially subscribes to a teacher-centred situation whereby the teacher uses most of the time to cover the content. These all contributed to the learners losing their interest toward the lesson. Karen was one of the learners who contributed during the interview questioning. This is what Karen said:

"... before I was not interested in learning Mathematics, every time I have got negative thought and I got discouraged when I fail to pass maths..."

From all the contributions I garnered that what made the learning not interesting and not effective as learners put it, was the fact that they were not involved in the lesson and they did not engage due to the structure of the lesson design. Figure 5.14 shows an overview of the observed traditional learning classroom structure.

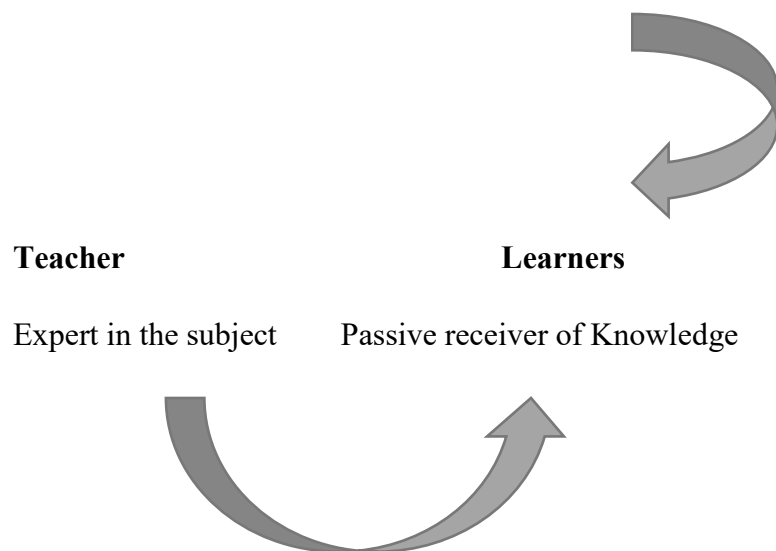


Figure 5.14: A diagram showing the traditional chalk and talk class learning structure

In this learning environment learners are compelled to write notes without make sufficient sense of the lesson (Amineh & Asl, 2015). During the class observation it was noted that learners spend less time on practice which leads them to become less active in the classroom. However, the learning environment should provide liberal opportunities for dialogue and the classroom should be seen as a community of discourse engaged in activity, reflection, and conversation (Amineh & Asl, 2015). As a result, learners did not achieve their learning goal.

However, Thula also completed the post-task after he had been exposed to the GeoGebra programme used lessons. Thula's response had improved as illustrated in Figure 5.15.

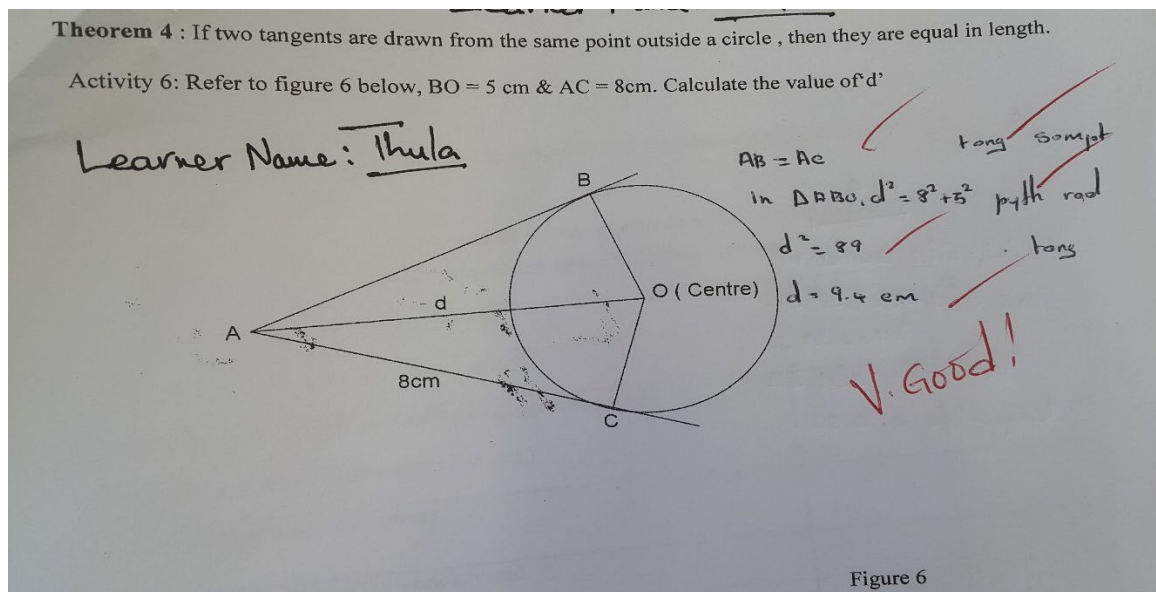


Figure 5.15: A diagram showing Thula's response to question 6 in the post-task

In the pre-task, Thula could not explain why $AB = AC$. In the post-task he showed all the steps with the necessary and sufficient reasons to conclude that $AO = 9.4\text{ cm}$. In the post-task, he had moved up to higher level of understanding. He began to apply reasons to justify his arguments and did not merely make assumptions.

From all these contributions I garnered that what made the post-task easier for the learners was the fact that they were involved in the lesson and they did the work on their own. From a cognitive perspective, the GeoGebra programme offered the opportunity for learners to investigate the Geometry concepts under study. The GeoGebra programme made it possible for learners to realise the externalisation of the hidden ideas embedded in the formation of those concepts, and thereby made them accessible for all learners. Horzum and Ünlü (2017) claimed that by using the GeoGebra programme, learners may improve their achievement by concretising and visualising subjects and promote permanent knowledge retention.

5.4 LEARNERS' EXPERIENCES AND PERCEPTIONS OF THE USE OF THE GEOGEBRA PROGRAMME AS A LEARNING TOOL

An attempt has been made to closely examine learners' perceptions regarding the GeoGebra programme assisted Euclidean Geometry learning. Following the application, all the sixteen participant learners stated that the GeoGebra programme contributed to learning the Geometry concepts. Some of elements that had an impact on the learners' experiences and perception

were extracted from the data obtained. These elements are: learners' attitude, interest, concentration or attention, self- regulation, visualisation and practicality

5.4.1 Learners' attitudes towards mathematics while engaging with the GeoGebra programme

The data obtained from classroom observations and from the interviews after the implementation of the GeoGebra programme revealed that all the learners showed an interest to learn and were absorbed in the lesson.

This section addresses the research question focussing on why the GeoGebra programme is used in the way that it is when learning Grade 11 Euclidean Geometry.

Every learner's view of Mathematics is a compound of knowledge, beliefs, conceptions, attitudes, and feelings. Literature suggests that attitudes and beliefs are interlinked. Mutodi and Ngirande (2014) claimed that attitudes may influence the formation of new beliefs. The current study was designed to foster learner interest and get learners interested in what they were learning by using a computer programme. Compatible with a Constructivist perspective, learners' attitude is considered as true (Kul, 2013), and can be a determining factor of knowledge production (Mutodi & Ngirande, 2014). The degree to which learners construct their own knowledge depends on the level of interest and belief toward the task they are tasked with accomplishing. It is imperative to prove the extent to which the GeoGebra programme changed the learners' attitude towards Mathematics learning.

Before the lessons in which the GeoGebra programme was incorporated learners had a negative attitude towards Mathematics as captured by Nelly's contribution on what learners' views of Euclidean Geometry were.

Nelly argued that:

"I must say that the GeoGebra programme opened my eyes and assisted me to understand Euclidean Geometry because I used to think maths is difficult, tricky and challenging subject and far to believe it."

This indicates that Nelly appeared more in favour of using the GeoGebra programme to explore mathematical concepts, and is more likely to prefer a computer based learning environment rather than the traditional learning environment. Although, it is not easy to change

one's attitude about Mathematics through involvement with a short term project based on the use of the GeoGebra programme however, in the short period of the study, rich mathematical discussion and engagements occurred. Self-directed and exploratory mathematical activities in the GeoGebra programme provided an opportunity for learners to explore geometrical relationships. As a result learners, developed constructive learning experiences and attitude which encompasses a new way of thinking about Mathematics.

Learners considered the GeoGebra programme as a tool for providing dynamic and multiple representations of both algebraic expressions and geometrical figures that facilitate a visual learning of abstract mathematical concepts. They came to believe that visualisation and manipulation have a positive effect on their mathematical understanding. The view that the GeoGebra programme proved visual and fascinating was confirmed by Daisy.

During the interview Daisy posited that:

"...there was fascinating moment during the GeoGebra assisted learning because I could see circles, triangles and their measurements at the same time on the computer screen. Once I draw diagram, it just display everything on the computer screen so that I could see it."

Her comment suggests that the graphical features of the GeoGebra programme appeared to have appealed to the Euclidean Geometry learning and encouraged her to take an interest in exploring visual effects with it, and she became interested in understanding the geometrical relationships embedded in the visual patterns. With the same perception, during the same interview, Karen concurred with the view posited by Daisy.

Karen strengthened this perception when she said that:

"...the moment I was exited was constructing triangles, circles, etc and measuring the size of angles on the computer screen."

Karen's remark unpacks the use that the GeoGebra programme contributed in making the work attractive and enjoyable which stimulated the learners' interest. In line with the traditional view of Mathematics teaching and learning, the learners felt that more practice and examples could provide better understanding. This type of view seems to reveal the fact that the mathematical activities within the Geiger programme provides a link between learners' preference for visual learning and the request of the task oriented learning system for more practice in a given time interval.

Engaging and building positive learners' attitude towards Mathematics is an important aspect in the teaching and learning process (Mutodi & Ngirande, 2014). Learning attitudes associated with the use of the GeoGebra programme increased the learners' engagement in classroom activities. The observations during the course of this study showed that learners engaged deeply and strongly in classroom activities. Figure 5.16 shows learners were actively interacting within the class room.



Figure 5.16: A diagram showing learners' active learning

Figure 5.16 revealed that the dynamic Geometry environment provided an alternative learning environment which appealed to the learners. That is, some features of the GeoGebra programme such as the construction protocol, dragging tool and investigations with the slider were well liked by the learners. All learners were pleased with these powerful features and used them extensively throughout all the lessons. A good example could be indicated by Daisy's extensive use of the dragging tool to explore dynamic geometrical objects: Daisy went further to describe her experiences during the lesson:

"... The GeoGebra programme helped me to be confident and it makes me too sure about my sums because it gives accurate value on the computer screen..."

This shows that using the GeoGebra programme enabled the learner's confidence in Mathematics. It is because, the GeoGebra programme automatically produced the outcomes of the work completed on the computer and it is easy for them to find out the geometric relationship.

Evidence from similar studies also confirmed that the use of the GeoGebra programme in the classroom impacts positively on learners' attitudes towards their engagement with their Mathematics work, particularly in Euclidean Geometry learning. Learners become more involved in the activities showing a higher level of interest (Martinez, 2017; Murni et al., 2017).

The findings of the interview and observation show that all the learners who participated in this study enjoyed the Euclidean Geometry lessons that were taught using the GeoGebra programme. Also, based on the lesson observations, the learners appeared more motivated while learning the geometric concepts. This was obvious from their positive attitudes while working with the learning materials. The learners' behaviour throughout the intervention reflected that they liked studying Geometry with the use of computers. From the learners' discussions and interactions during the lessons it was noticeable that the GeoGebra programme - based learning raised the learners' interest and enthusiasm toward Geometry as they interacted more with the learning materials. Similarly, Perry and Steck (2015) claimed that technology may have a positive impact on learners' levels of concentration and interest which assists in reinforcing learning.

5.4.2. Learners' interest towards learning Euclidean Geometry with the use of the GeoGebra programme

Learner sentiments as captured in Daisy's interview affirm that the GeoGebra programme incorporated in the Euclidean Geometry lesson was fascinating and interesting.

According to Daisy using the GeoGebra programme:

"... Was fascinating and interesting ... because I could see circles, triangles and their measurements at the same time on the computer screen. Once I draw diagram, it just display everything on the computer screen ..."

It is evident that the use of the GeoGebra programme as a new approach was an attractive way to learn Euclidean Geometry because learners were given the opportunity to experience hands-on learning of Geometry. The GeoGebra programme supports the execution of tasks, produces accurate and real data, helps generate patterns more quickly and allows learners to work at their own pace.

With the same insight, in that same interview, Nelly concurred with the view posited by Daisy.

Nelly went on further to explain why she thought the lesson was interesting. Nelly elaborated that:

“The moment I was interested and excited was constructing triangles, circles, and measuring the size of angles on the computer screen.”

It was evident that the simulation, presentation of shapes, insertion of pictures, and colourful displays had not only assisted the learners’ understanding of geometric shapes and their relationships but also had improved learners’ interests to apply, explore and create new learning.

Learners were more confident to assert control of their own learning without the continuous help of the teacher. Learners sought the teacher’s help when they experienced a challenge in the learning process.

Confirming this perspective Daisy claimed that:

“...the GeoGebra programme helped me to be confident and it makes me too sure about my sums because it gives accurate value on the computer screen. And also give me opportunity to call my teacher to assist me when I got stuck.”

Based on the data from the class observations, learners were more deeply involved in the activities which became more attractive and enjoyable. Generally, learners responded positively to the use of the GeoGebra programme: they engaged well with lessons, their behaviour was good and their attitudes to learning were very good. Figure 5.17 shows how learners were absorbed in their group activity in the class room.



Figure 5.17: A diagram showing learners doing the class activity within a group

5.4.3 Incorporation of the GeoGebra programme enhances learners' attention

The observation data also shows that the use of the GeoGebra programme provided a rich mathematical learning environment in which learners were highly focused in their activities. It appears to have the potential to facilitate peer interaction, as well as to focus that interaction on learning. All the learners indicated that they would like future Mathematics lessons to be taught using the GeoGebra programme because it assisted them to focus and take responsibility for their learning. These sentiments are exemplified in the excerpt that follows:

Karen confirms that:

“... Even though, the Euclidean geometry lessons are tricky and it looks difficult it was understandable, when I use the GeoGebra programme. The GeoGebra programme helps me to pay attention and taking responsibility for my learning.”

It can be inferred from Karen's statement that the GeoGebra programme offers automatic functions that provide the opportunity for learners to draw, measure, calculate, etc. with great ease. However, extracting geometrical properties and relationships requires prominent attention. Figure 5.18 shows that the learners pay excessive attention towards their learning.



Figure 5.18: A diagram showing learners attentively working while using the GeoGebra programme

All these contributions by learners were affirming the extent at which the learners focus on what they are doing in the class room. It was interesting to note that all learners showed interest in the class activities and spent more time engaging with the GeoGebra programme. Learners often used their teacher as a mentor but as they became more familiar with the GeoGebra programme they reduced the frequency of seeking help from their teacher. Learners worked in groups and spent a short time getting familiar with the environment, asking for help from the teacher and then started to use strategies that became more sophisticated and sought help from their peers. This clearly shows that learners were paying attention and taking responsibility for their study.

5.4.3 The GeoGebra programme empowered self-regulated and independent learning.

Independence refers to an ability to think and work with implications for reaching solutions (Martinez, 2017). Thus, it does not refer to work in the absence of specific direction. During the lesson observations, it was apparent that the incorporation of the GeoGebra programme within the classroom allows more time for learners to work on their lessons and increases the amount of activities the learners do. Figure 5.19 shows that the learner seated in the middle named Ndelany seems very independent. It was observed during the class observation that Ndelany only engages in discussion with his peers when he finishes the given task.

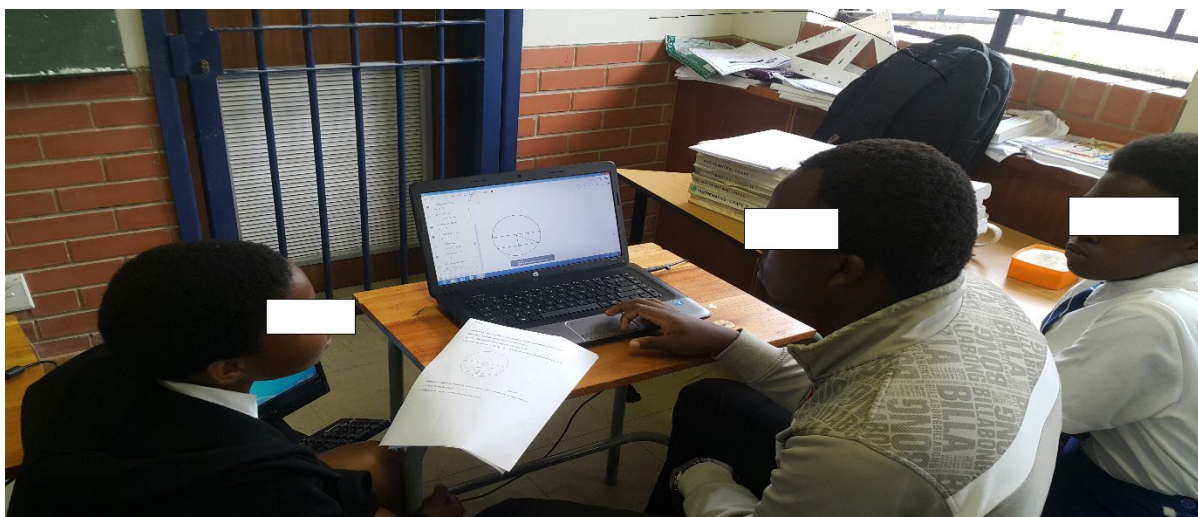


Figure 5.19: A diagram showing learners doing the class activity independently

Figure 5.19 clearly shows that this learner is working at his own pace. It is virtually impossible to have passive learners while engaging with in the GeoGebra programme in the class room. Many of the learners worked independently and later started discussing or consulting with a peer, or with the teacher. Other similar studies also confirm that the GeoGebra programme changes passive learners to generate independent explorers (Belgheis & Kamalludeen, 2018; Pfeiffer, 2017).

Confirming this perspective Daisy stated that:

“... The GeoGebra programme helped me to be confident and it makes me too sure about my sums because it gives accurate value on the computer screen. And also give me opportunity to call my teacher to assist me when I got stuck.”

What I could infer from Daisy’s statement was that the incorporation of the GeoGebra programme enabled learners to utilise their independent skills and knowledge to solve new problems. Thus, the systematic use of the GeoGebra programme may equip learners with such competency under the setting of Constructivism learning. It seems that the GeoGebra programme assisted her to develop constructive solutions using confidence and self-motivation that unlocked her potential. According to Daisy’s remark learning using the GeoGebra programme made the learners more confident to assert control of their own learning without the constant need of the teacher. Learners sought the teacher’s help especially when they experienced difficulty during the course of the lesson.

During the class observation, learners showed independence in such a way that they controlled the available time to think and work effectively.

Concurring with this view Karen stated that:

“...the GeoGebra programme helps me to pay attention and taking responsibility for my learning.”

This means that she appeared to show a propensity to taking care of her learning by using the GeoGebra programme.

It can then be gleaned that the GeoGebra programme has provided an opportunity for every learner to unlock their individual potential through active participation in the lesson which is a key element of Constructivist learning.

The finding from the study conducted by Bist (2017) also confirms that the use of the GeoGebra programme in geometric learning promotes independent, self-regulated learning and active learning that are key characteristics of cognitive Constructivist teaching. This study also adopted Constructivism as a study framework.

5.4.4 Integrating the GeoGebra programme with Euclidean Geometry activities enhances visualisation

It was my assumption that the role of the computer in Mathematics might be seen by learners as entertainment rather than as a pedagogical or conceptual learning tool in their Mathematics learning. However, after the completion of some lessons, it was evident that learners may foster their competency with the GeoGebra programme and develop a critical perspective towards the use of computers within a geometrical context.

Considerable attention was given to create a constructive learning classroom set up in order to align with the lessons within the current study theoretical framework. Figure 5.20 gives an overview of a proposed approach of learning within the Euclidean Geometry classroom in the current study.

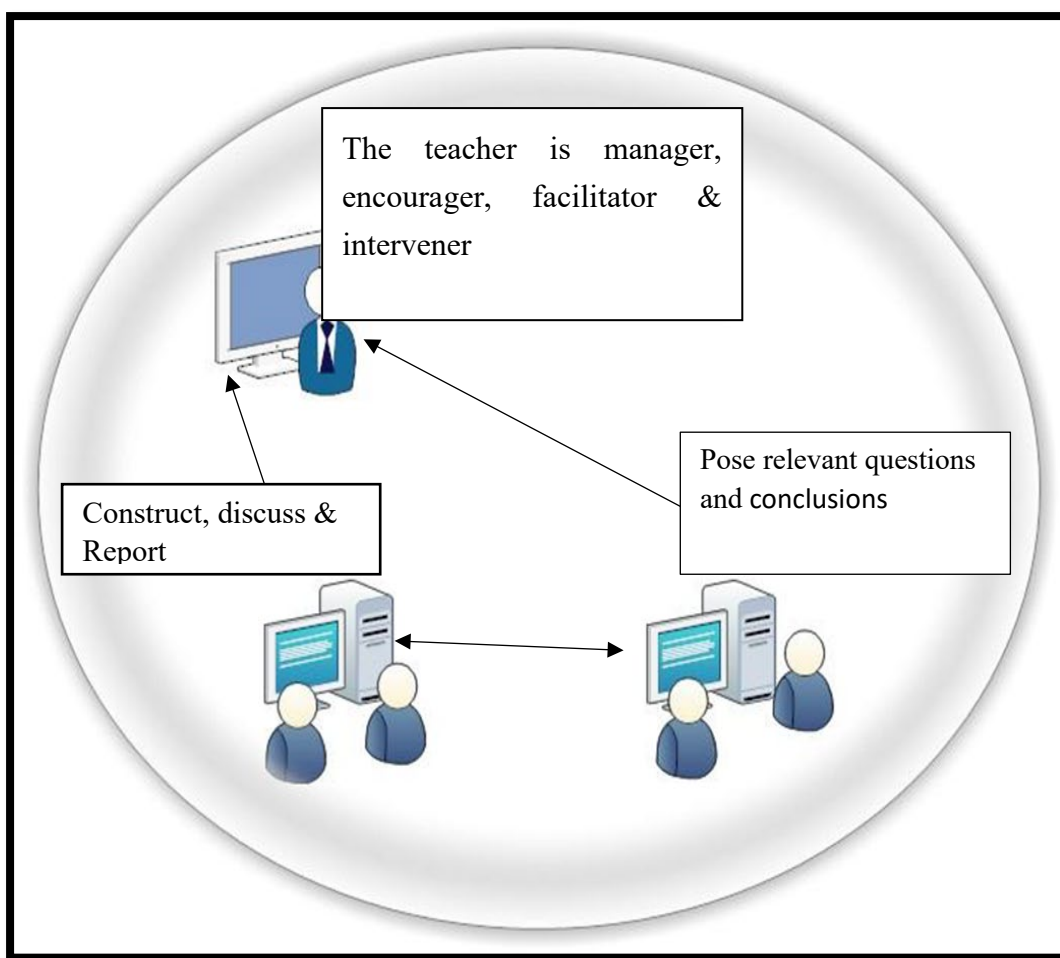


Figure 5.20: A diagram showing the suggested approach for teaching Euclidean Geometry

Figure 5.20 shows that an introduction of the GeoGebra programme into Mathematics classrooms modifies the organisation of the classroom. This approach favours learners playing a vital role in the learning process.

All the sixteen participants seemed to have developed an awareness of the potential of the GeoGebra programme in Mathematics. They considered the GeoGebra programme as a tool for providing dynamic and multiple representations of both algebraic and geometric expressions. It provides the visualisation of abstract Mathematics concepts. The learners came to believe that visualisation and manipulation have a positive effect on mathematical understanding. When exposed to further probing, all the learners were excited and were fascinated during the lessons.

According to Daisy, she had a:

“... fascinating moment during the GeoGebra used learning becauseonce I draw diagram, it just display everything on the computer screen so that I could see it.”

Her comment suggests that the graphical aspects of the GeoGebra programme appeared to have appealed to the learners and encouraged them to take an interest in exploring the visual effects of the programme, and they became interested in understanding the mathematical relationships embedded within the visual patterns.

It is a belief shared by all participant learners that the GeoGebra programme serves as a demonstration tool and provided for them better visual and dynamic representations of mathematical concepts. For Instance, Daisy stated that:

“... once I draw diagram, it just display everything on the computer screen so that I could see it.”

This entails that if learners are able to observe mathematical relationships with the help of visualisation, they are likely to learn Mathematics better and faster. Therefore, the GeoGebra programme-based environment is relatively active in supporting a better understanding of mathematical ideas.

All the learners seemed to appreciate Euclidean Geometry learning if it involves the GeoGebra programme as opposed to the traditional ‘chalk and talk’ approach. The GeoGebra programme provided an alternative means to interact with each mathematical activity which appealed to the participating learners. That is, some features of the GeoGebra programme, such as the construction protocol, dragging tool and investigations with the slider were viewed as powerful ideas of the GeoGebra programme. All learners were pleased with these powerful features and used them extensively throughout all the lessons.

After the GeoGebra programme was used during the lessons, Daisy’s response was probed further on how the GeoGebra programme assisted her to solve the task which was posed in the classroom. Daisy claimed that:

“...the GeoGebra programme helped me to be confident and it makes me too sure about my sums because it gives accurate value on the computer screen. And also give me opportunity to call my teacher to assist me when I got stuck”

Daisy's statement illustrated that the dynamic Geometry learning environment provides immediate feedback about learners' actions that might enable them to reflect on their conceptualisation. Communication with the GeoGebra programme provides useful feedback encouraging learners to search their mistakes by looking at a construction protocol. This view was shared by many participants, for example Nelly and Karen stated that working with the GeoGebra programme could be considered as a mathematical thinking activity in which one uses imagination and mathematical reasoning.

The participants' accounts above revealed the belief that learners could discover mathematical ideas and connect multiple representations of these ideas dynamically in the GeoGebra programme environment. In this sense, learners came to acknowledge pre- designed Geometry activities which are aligned to the GeoGebra programme as an opportunity for them to discover Mathematics using the GeoGebra programme as a learning tool.

It is my finding that the use of the GeoGebra programme engages learners in the method of entertainment and in the learning and teaching process learners reveal that the GeoGebra programme is a powerful learning tool. Learners were able to differentiate between drawings and geometric objects which are the cues for visual thinking under the designed constructive framework, to transfer it to the required sector and to learn more complex subject matter. It means that the GeoGebra programme enables learners to use existing cognitive and visual skills to develop efficiency, experiences, autonomy and hence confidence in their construction (Bist, 2017). It seems incorporating the GeoGebra programme within the classroom helps learners to make the connection between visual and symbolic representations by visualising on the computer screen.

5.4.5 The use of the GeoGebra programme stimulated practical learning

It was overwhelming for me to witness all learners completely absorbed in the lesson and class activities using the GeoGebra programme. When asked how the use of the GeoGebra programme facilitated Euclidean Geometry learning, Daisy stated that the GeoGebra programme assisted her to practice Euclidean Geometry concepts.

Daisy said that:

"... It helped me to practice measuring the size of angles and it also facilitate to prove Euclidean Geometry problems with right reasoning..."

It seems that the GeoGebra programme empowered learners' reasoning skills and establishing of ideas. This refers to the formation and consolidation of ideas by the learners while justifying their decisions and conjectures. Writing constructive responses with appropriate geometric reasoning is one of the difficult aspects of Euclidean Geometry learning which might be considered the most difficult phase in the Geometry class. However, the use of the GeoGebra programme enhances the learners' ability to write proofs step by step with appropriate reasons. Learners reinforce their learning and establish mathematical ideas while working on the computer, exploring and discovering relationships, discussing and reasoning about their results and participating in whole-class discussion. This category is linked with pattern generation in facilitating the classroom activity but focuses more on the identification of ideas from the patterns generated rather than on the access to pattern generation. When exposed to further probing, how the GeoGebra programme assisted in Euclidean Geometry learning, Nelly exposed the reasons. In her response she described that the GeoGebra programme helps her to practise.

According to Nelly the GeoGebra programme:

"...helps to practice Euclidean geometry learning with reasons...."

Thus, Nelly seemingly regarded the use of the GeoGebra programme as a new approach and an attractive way to learn Mathematics because the GeoGebra programme created an opportunity to experience hands-on learning of Mathematics using a technology based tool.

5.5 CONCLUSION

In this chapter, data analysis methods, study results and a discussion of the findings are presented. This study aimed to explore how learners used the GeoGebra programme in Mathematics, particularly for Euclidean Geometry learning in the classroom. The study also aimed to explore learners' experiences and perceptions towards the Euclidean Geometry lesson when they were exposed to the GeoGebra programme used learning. In order to address the two research question of the current study, data were generated using a pre-task, a post-task, observation schedule and a semi-structured interview schedule.

Findings from this study are consistent with the findings of several related studies on the use of the GeoGebra programme in teaching and learning Geometry. In consideration of these assessments, the teaching of Euclidean Geometry with materials, which were prepared with the GeoGebra programme, is more successful than the traditional ‘chalk and talk’ method.

Accordingly, to integrate educational technology within Mathematics lessons fosters improved academic achievements by enhancing understanding and justification. This is due largely to the practicality and appeal of the GeoGebra programme in terms of engaging learners toward Mathematics learning. Particularly, the visual component increased learners’ attention, interest, and self-regulation in Mathematics lessons.

Therefore, the proposed Constructivist approach that employed the use of the GeoGebra programme as a learning tool proved to provide a better learning environment than the traditional ‘chalk and talk’ approach.

The next chapter discusses the summary and findings of the study. Chapter Six also focuses on the recommendations, limitations and conclusion of the study.

CHAPTER SIX

SUMMARY OF THE STUDY FINDINGS, IMPLICATIONS, LIMITATIONS AND CONCLUSION

6.1 INTRODUCTION

Chapter Five dealt with the presentation and discussion of this study's findings. This chapter presents a summary of the results of the current study, implication of these results and limitations and conclusion. Recommendations are also suggested for possible further studies within Mathematics education.

6.2 SUMMARY OF THE STUDY FINDINGS

This section summarises the combined results of the different data sources (the pre-task and post-task, classroom observations and interviews) to answer the research questions. All the data generation tools designed based on the main research question.

How can the GeoGebra programme be used in the learning of Grade 11 Euclidean Geometry?

The study question seeks an insight on how the GeoGebra programme may be incorporated within the learning of Euclidean Geometry. In order to address this research question, learners were exposed to the pre-designed Euclidean Geometry activities. The tasks were designed considering the appropriateness of the tasks within the GeoGebra programme. During the GeoGebra programme teaching and learning process necessary information was captured using the designed data generation instruments. These generated data have been analysed and some important aspects emanated which were discussed and summarised in this section in accordance with the main research questions.

According to the findings of this study the GeoGebra programme can be used for Euclidean Geometry learning as a learning tool. Learners mainly used the GeoGebra programme to draw figures and to measure the dimensions of the Euclidean Geometry figures. The learners used the results of their measurements to verify and justify Euclidean Geometry axioms and theorems.

The interpretive interview result of this study also indicated that learners used the GeoGebra programme to sketch the geometric figures. Then they continue to measure and label, explore

Geometry properties, formulate conjectures, check facts and even build proofs with appropriate reasons. Evidently, during the interview discussion when the learners asked about how the GeoGebra programme assisted the Euclidean Geometry learning, Karen one of the interview respondent learners, stated that:

“... the GeoGebra programme was used to measure angles ... because it helped me to practice measuring the size of angles and it also facilitated to prove Euclidean Geometry problems”

Nelly also reiterated,

“... the GeoGebra programme helped me to be confident and it makes me too sure about my sums because it gives accurate value on the computer screen when I draw figures. And also give me opportunity to call my teacher to assist me when I got stuck.”

So, it seemed useful to let learners investigate practically by themselves, using the GeoGebra programme.

The response from learners indicate that they used the GeoGebra programme as a learning aid to sketch, measure, label, verify and justify properties and relationships within Euclidean Geometry. According to the findings from this study the GeoGebra may be a suitable teaching aid. The GeoGebra programme helps learners to understand concepts in Euclidean Geometry (Murni et al., 2017).

The analysed results of the interviews also revealed that the learners considered the GeoGebra programme as an important learning aid to explore and visualise the Euclidean Geometry concepts. Learners also indicated that it is important for them to see the relevance of Mathematics and where it fits into the real world. The learners used the GeoGebra programme so that they could explore, visualise and see the relevance of Mathematics in order to help them understand concepts better.

The intervention was employed to answer the research question by providing effective assistance through the use of the GeoGebra programme, helping justify the relevant Geometry proofs and theorems. For instance, the class task was for all learners to justify some of the properties of the cyclic quadrilateral (see section 5.3.1). During the course of this study the entire teaching and learning process was observed on the basis of how the learners used the

GeoGebra programme during the learning of Euclidean Geometry. Observation data was captured and analysed. The analysed observation results revealed that learners used the point tool for plotting four points, the text tool to label points, the line segment tool for joining labelled points, the distance tool to measure the line segment, the angle tool to measure all interior angles and the pen tool to indicate and dot the key properties of a cyclic quadrilateral.

The observation results also revealed that learners became accustomed to sketch, label and measure geometric figures using the GeoGebra programme before justifying and verifying Geometry proofs and theorems. Thereafter, learners discussed their work amongst themselves as well as with their teacher to deduce their conclusions.

The findings of the observations also emphasise that the pre-designed active arrangements are important processes to be considered when using the GeoGebra programme as a teaching and learning tool for Mathematics. It was noticed that the pre-designed activities were used as a guideline which provided clear information for learners to explore the properties of Euclidean Geometry within the GeoGebra programme. The teacher moved around in the classroom while learners were working on the computer activities and assisted by looking, answering questions, providing help or explaining challenging concepts. Learners worked individually or in pairs with computers. Some important aspects emanated from the classroom observations, which included that learners were intensively involved in sketching and measuring geometric figures within the GeoGebra programme. It was also noticed that learners regularly sought support when they were challenged by the task and the teacher offered support frequently. However, the teacher support gradually decreased as the learners worked independently. This is in line with the view of Amineh and Asl (2015) on how teachers who are facilitators in a constructive learning environment first provide support and help for learners, which is gradually decreased as learners begin to learn independently.

Furthermore, according to the observation results of this study the GeoGebra programme provided learners with an interactive environment in which they can quickly and easily, create manipulatives as well as measure and analyse digital representations of key concepts from Euclidean Geometry. The GeoGebra programme allows learners to drag points on geometric objects and to quickly be able to make and test conjectures and generalisations about properties.

The findings from the analysed pre-task and post-task results also indicated that learners used the GeoGebra programme to sketch geometric figures and to measure dimensions of the geometric figures before justifying and verifying the geometric properties. As a result, learners performed better in the post-task (see section 5.3.3.2). The improved achievement in the post-task entails that the exploratory learning approach using the GeoGebra programme has also improved the cognitive abilities which resulted in sound reasoning capabilities (see section 5.3.2). The understanding of the Euclidean Geometry theorems was attained as the learners explored the properties and relationships of the Euclidean Geometry theorems within the GeoGebra programme. Findings from the post-task analysis results also revealed that learners used the GeoGebra programme to verify and justify the Euclidean Geometry theorems and they started to synthesise and evaluate the steps in Geometry theorems by supporting these steps with appropriate reasons (see section 5.3.1). This was in line with Shrestha (2017,p.153) “who contends that a simple drawing of mathematical objects and figures is not for the building of a comprehensive understanding of basic mathematical concepts”, and Ozcakir (2013) “found that using GeoGebra motivates and helps learners learn at a higher level, while exploring and conjecturing as they draw and measure.”

Thus, the GeoGebra programme enables the learning of Euclidean Geometry much easier for learners. The GeoGebra programme assisted the processes of sketching, measuring, verifying and justifying through different tasks. The automatic feedback of the GeoGebra programme facilitates the process of verifying and justifying of the theorems. From the first session participants used as many sketches as they could, to establish common characteristics of the Euclidean Geometry figures.

Sliders enabled learners to dynamically manipulate representations of Euclidean Geometry activities and observe how the properties of the Euclidean Geometry changed, leading to formulation of formal descriptions.

While engaging with the GeoGebra programme the study also aimed to explore learners’ perceptions and experiences guided by research question two.

Why is the GeoGebra programme used in the way that it is when learning Grade 11 Euclidean Geometry?

Learners had numerous perceptions about the use of the GeoGebra programme which all denoted learners' positive attitude towards its use. Bist (2017) noted that the learners had overwhelmingly positive attitudes towards the use of the GeoGebra programme.

This study sought to explore whether the integration of the GeoGebra programme had any significant impact on learners' perception, experiences and understanding of Euclidean Geometry. To accomplish this purpose this study had to explore learners' perceptions before and after the integration of the GeoGebra programme within the lessons which focussed on Euclidean Geometry content. Changes in learners' perceptions after the implementation of the GeoGebra programme were regarded as indicative of the impact of the GeoGebra programme on Euclidean Geometry learning. After interviewing learners with respect to their perceptions about the use of the GeoGebra programme when learning Euclidean Geometry, the study also attempted to establish which aspects of using the GeoGebra programme had the most significant impact on learners' perceptions.

Interpretatively analysed results showed that learners' perception of Euclidean Geometry before the lessons within which the GeoGebra programme was incorporated were that, Euclidean Geometry was difficult and a complex section of Mathematics. After the use of the GeoGebra programme learners' perceptions were that Euclidean Geometry was not as difficult as they had indicated in the beginning. The learners regarded the GeoGebra programme as an effective learning tool because it contributed to making the work more attractive, enjoyable, interesting and stimulating. The lesson observations also indicated that learners were motivated and their concentration was noticeably better than usual.

The role the GeoGebra programme played on the learning of Euclidean Geometry was observed during the course of this study. The findings from this study demonstrated that the GeoGebra programme assisted in the execution of tasks and provided the opportunity for learners to work at their own pace. The analysed results indicated that the GeoGebra programme has the potential to improve learners' learning experiences; the GeoGebra programme enables the effective implementation of a constructivist model of learning. The findings of the present study demonstrated that the use of the GeoGebra programme as a learning tool has a mediating role for learners' strengthening of geometrical reasoning skills as well as construction of knowledge. The GeoGebra programme contributes to learners' use of their mathematical knowledge and stimulates them into making their thinking visible. The

understanding of geometrical concepts and relationships was also achieved through the visual manipulation of objects provided by the GeoGebra programme. This was affirmed by the learners' improved understanding and achievement obtained through the post-task. Accordingly, to integrate educational technology into Mathematics lessons fosters improved academic achievements by enhancing understanding and justification. This is due largely to the practicality and appeal of the GeoGebra programme in terms of engaging more senses of learners, such as visual. Particularly, the visual component increased learners' attention in Mathematics lessons, meaning that the various abstract concepts associated with the subject became easier to focus on and grasp.

The findings of this study also indicated that the use of the GeoGebra programme provided Constructivist Mathematics learning environments in which learners are engaged within classroom activities. It appears to have the potential to make learners pay attention when learning Euclidean Geometry. According to the results of this study, the learners appeared to become more independent and self-regulated while learning. Evidence from similar studies also shows that teaching with the GeoGebra programme handed over a high degree of independence to learners who worked on their own and at their own pace helping them to reinforce learning (Farrajallah, 2016; Kim & Md-Ali, 2017). Moreover, the GeoGebra programme may provide an environment in which the learner is in control and in a position to investigate and control their learning" (Bist, 2017). Furthermore, "learners' autonomy over their learning fosters learners' interaction and increases their self-esteem and confidence" (Bester & Brand, 2013).

Results from the interviews indicated that learners felt confident to explore Euclidean Geometry within the GeoGebra programme. Research done by Khobo (2015) suggests that small discoveries made by learners may strengthen self-confidence, and may go a long way toward the learners enjoying Mathematics. These results of this study's findings revealed that learning within the GeoGebra programme opens up windows for learners to learn Mathematics in new ways, making learning more joyous.

Thus, exposing learners to the GeoGebra programme integrated learning approach has changed learners' attitude positively because its use provides an opportunity for learners to become more committed to the tasks given to them. The incorporation of the GeoGebra programme enabled learners to feel more in control (Chimuka, 2017). It was also observed during this study

that learners became deeply engrossed in their class activities. The results from interviews indicated that the lesson was attractive and more enjoyable. Generally, learners responded positively to the use of the GeoGebra programme: they engaged well with lessons, their concentration was high and their attitudes to learning were positive. These results also demonstrated increases in learners' enjoyment, interest and confidence, as a result of this new approach.

The changes in the learners' perception about Mathematics learning were evident in the analysed interview responses. Through the course, the learners seemed to have found a way to construct their knowledge. This view reflects the Constructivist views that concentrate on the learners' own construction of Euclidean Geometry knowledge through active contribution in Mathematics learning (Howard, 2018).

According to the findings from the classroom observations of this study, learners preferred to use the GeoGebra programme as a learning tool than the usual traditional 'chalk and talk' learning. The responses of the learners from the interviews also showed how they acknowledged and how they could visualise certain concepts within the GeoGebra programme (see section 5.4.4). The results from the observations showed how learners interacted with each other, with the teacher and with the GeoGebra programme (see section 5.4.1). The observations also showed how the GeoGebra programme provided learners with an opportunity to acquire physical and geometrical knowledge whilst working within the GeoGebra programme. The responses from the post-task were triangulated with the findings of the observation and the interviews. The results indicated that they wanted the teacher to use the GeoGebra programme as a teaching tool, or that they also wished to work with it themselves. This confirmed the need for different styles of learning approaches (Pfeiffer, 2017). Learners during the investigation were beginners in the use of the GeoGebra programme who prefer prepared activities (Ramatlapanana, 2017). The results thus confirm that learners do not necessarily need to know the GeoGebra programme before they can effectively use it to explore, learn, conceptualise, conjecture, etc. This research used pre-designed activities for the more complex activities, such as the investigation of the Euclidean Geometry construction and justifying their relationships.

Learners preferred using the GeoGebra programme, because they could see how they could benefit by it. The findings showed that the learners' diverse preferences must be considered when using the GeoGebra programme, although it will not be practicable to consider each one's

preferences. The results thus suggest that in order to keep learners interested and motivated to learn, technology based tools should be used rather than an over-reliance on the traditional ‘chalk and talk’ methods.

The research has revealed that the GeoGebra programme is a useful mediating computer programme to develop solutions for complex geometry problems. It was found that working within the GeoGebra programme has helped learners visualise mathematical concepts. The learners were found to be using the GeoGebra programme as a supporting tool to learn Euclidean Geometry in Mathematics. Many researchers (Shadaan & Eu, 2014; Zengin & Tatar, 2017 ; Zilinskiene & Demirbilek, 2015) reported that there are several factors that affect the learning process. The appropriate use of the GeoGebra programme was found to be one important factor in enhancing the ability to learn.

6.3 RECOMMENDATIONS BASED ON THE FINDINGS

After careful analysis and interpretation of the results from this study I elicited some recommendations that may be useful for practice, Mathematics educators, policy makers and further studies. These implications are laid out as follows: section (6.3.1) Recommendations for practice, section (6.3.2) Recommendations for educators, section (6.3.3) Recommendations for policy makers and section (6.3.4) Recommendations for future studies.

6.3.1 Recommendations for practice

This study has some implications for Mathematics teaching and learning. Considering the findings of this study it is important that the GeoGebra programme may be incorporated as a learning tool in Mathematics education. The incorporation of the GeoGebra programme in this study, brought about positive changes in learners’ experiences and perceptions of Mathematics, particularly Euclidean Geometry. The GeoGebra programme could be therefore considered as an alternative approach for the improvement of enthusiasm amongst learners of Mathematics, if used appropriately (Pfeiffer, 2017; Shadaan & Eu, 2014). The GeoGebra programme if used effectively may assist learners to develop a positive attitude towards Mathematics and increase interest in the learning of Mathematics. If the GeoGebra programme is used together with specifically designed activities, it may enable learners to explore geometrical concepts while appreciating the benefits of constructing their own knowledge based on what they visualised which may enhance learners’ persistence when working with problems in Mathematics.

The learners' engagement and their attitudes toward using the GeoGebra programme in the classroom is a vital aspect (Perry & Steck, 2015). These study findings revealed that all the participant learners showed positive attitudes toward the use of the GeoGebra programme. When the learners were interrogated with regard to future use of the GeoGebra programme within Mathematics lessons all learners expressed the desire to implement the GeoGebra programme in every Mathematics lesson.

As indicated by learners' responses during the interviews in relation to the use of the GeoGebra programme, they appeared to be receptive to use the GeoGebra programme to support their exploration of Euclidean Geometry concepts in Mathematics.

6.3.2 Recommendation for educators

The findings of this study indicated using the GeoGebra programme in the teaching of Euclidean Geometry had an important effect on learners' performance. The GeoGebra programme when used as a meditation learning tool in the learning process provided visual representations that enhanced understanding. An important finding of this study was that the use of the GeoGebra programme had a positive impact on changing the learners' perception about the nature of Mathematics, particularly Euclidean Geometry. Hence if used properly the GeoGebra programme could yield similar results in other sections within Mathematics. This study may provide educators with an example of this application to make them aware of the positive influence of the GeoGebra programme on learners' understanding of Mathematics. Considering this finding Mathematics educators are recommended to use the GeoGebra programme in which learners make their own discoveries as this assists learners in developing inquiring minds and confidence in the handling of mathematical concepts. To facilitate constructive learning within the classroom, activities must be structured within the GeoGebra programme. Learners need to be exposed to opportunities which allow them to explore multiple possibilities. As they investigate connections and conceptual conceptions, learners will gain intellectual affirmation and also their sense of competency will be improved. Therefore, it is recommended that Mathematics educators design teaching contexts aligning with the GeoGebra programme to create an effective learning environment. This will assist learners' in achieving an improved Mathematics performance.

6.3.4 Recommendations for policy makers

The findings of the current study positively correlate with the other similar studies' findings. The findings from this study indicated that learners' use of the GeoGebra programme for Euclidean Geometry enhanced learners innovative learning. After the implementation of the GeoGebra programme learners performed better in the Euclidean Geometry tasks. Similar studies conducted in the South African context also established positive findings with regard to the use of the GeoGebra programme for Euclidean Geometry learning (Gweshe, 2014; Mosese, 2017; Mosia, 2016; Pfeiffer, 2017). Therefore, this implies that the policy makers and curriculum developer should consider the integration of suitable selected technology programmes into Mathematics contents. In order to make the work simpler for teachers and learners the Mathematics work programme should be designed in accordance with a technology programme. For example the curriculum designers may introduce the GeoGebra programme based Euclidean Geometry activities in textbooks. Additionally, Mathematics or science stream learners should be offered a basic computer science course as a subject to empower their computer sciences skill.

Furthermore, the curriculum designers and Mathematics text books developers should include Geometry activities useable in the GeoGebra programme. In other words, the Mathematics particularly Euclidean Geometry activities based on the GeoGebra programme should be included in Mathematics textbooks for secondary school learners.

6.3.4 Recommendations for possible future studies

Future studies should consider the investigation based on the integration of technologies for Mathematics learning. The current study findings indicated that learning Euclidean Geometry within the GeoGebra programme engaged learners to be active participants of the teaching and learning process. As a result, learners' performance improved in Euclidean Geometry. The scope of target population and the time interval used to conduct this study was limited. Therefore, this study recommends that further studies be carried out in order to continue investigation in a broader scope of population and wider range of time interval. The current study was based only on the Euclidean Geometry content. Hence the findings of the current study cannot be generalised to the other sections of Mathematics. It is recommended for further studies to be conducted using the same lesson strategy with different Grades.

For instance, longitudinal research studies may be conducted in order to examine the long-term effects of the GeoGebra programme on learners' Mathematics achievement, geometric thinking and attitude towards Mathematics and the use of technology in teaching and learning. Therefore, the outcome of using the GeoGebra programme may be explored with a group of learners ranging from secondary school to tertiary education. As a result, this study recommends similar research studies to be conducted across all the Grade levels in order to understand the incorporation of the GeoGebra programme on learners' Mathematics performance, improvement in geometric thinking and developing positive perceptions towards Mathematics learning.

6.4 LIMITATIONS OF THE STUDY

This study adds to literature on the use of new technologies by investigating learners' experiences of using the GeoGebra programme to learn Euclidean Geometry. The sample for the study was Grade 11 Mathematics learners at the same school. Consequently, the scope of this study is limited; the validity of this study's results should be tested further by conducting a large size investigation using larger samples than this one.

Furthermore, the study occurred over a very short period and was restricted to particular curriculum requirements. The results could be of further benefit if the study had been conducted in more places, with more than one researcher and more participants. Although the study cannot be generalised to an overall population because of the small number of research participants, the findings of this study concur with previous similar studies on the use of the GeoGebra programme (Kumar & Chun-Yen, 2015; Martinez, 2017; Ocal, 2017) .

6.5 CONCLUDING REMARKS

The current study has established that learning Euclidean Geometry within the Geogebra programme enhances learners' performance, learners' reasoning skills and stimulates learners' interest to learn Euclidean Geometry. Based on the results of this study, I recommend teachers to expose learners to the GeoGebra programme based Euclidean Geometry learning. Learners' attitude is the key determinant of learner achievement (Belgheis & Kamalludeen, 2018; Bist, 2017); hence any teaching and learning method that changes learners' attitude positively, will assist in alleviating learners' poor achievement in Euclidean Geometry in particular and weak performance in Mathematics in general.

The learners' interest and attitude play a determinant role in their learning. With regard to learners' interest and perceptions the current study found that Mathematics learning within the GeoGebra programme enhances learners' interest and engagement towards Mathematics learning. This implies that the integration of the GeoGebra programme into the Mathematics curriculum can deter learners from a persistent Mathematics hatred problem.

This investigative Mathematics learning exposure created an important opportunity for learners to make sense of what they experience. Learners being exposed to Euclidean Geometry content which is designed within the GeoGebra programme encourages learners to think critically and assisted learners to establish mathematical relationships. Through this constructive learning process, learners internalise their experiences and can establish the properties of Euclidean Geometry.

REFERENCES

Abramovich, S. (2013). Computers in Mathematics Education: An Introduction. *Computers in The Schools*, 30(2), 4-11.

- Abu, M. (2013). Improving the Level of Geometric Thinking Secondary School Students Using the GeoMetry Learning Video Based on Van Hiele's Theory. . *International Journal of Educational & Research in Education*, 2(1), 16-22.
- Adams, K., & Lawrence, E. K. (2014). *Research methods, statistics, and applications*. London: SAGE.
- Akgul, M. B. (2014). *The effect of using dynamic geometry software on eight grades students' achievement in transformation geometry , geometric thinking and attitudes toward mathematics and technology* (Masters Degree Dissertation), Middle East Technical University Middle East.
- Amineh, R. J., & Asl, H. D. (2015). Review of Constructivism and Social Constructivism. *Journal of Social Sciences , Literature and Languages*, 1(1), 9-16.
- Arbain, N., & Shukor, N. A. (2015). The effects of GeoGebra on students achievement. *Procedia - Social and Behavioral Sciences*, 172(5), 208 -214.
- Aydos, M. (2015). *The impact of teaching mathematics with GeoGebra on the conceptual understanding of limits and continuity: The case of turkish gifted and talented students*. (Masters Degree Dissertation), Ihsan Dogramatic Bilkent University Ankara.
- Azizul, S. M. J., & Din, R. (2016). Teaching and Learning Geometry Using Geogebra Software via MOOC. *Journal of Personalized Learning*, 2(1), 40-51.
- Azlim, M., Amran, M., & Rusli, M. (2015). Utilization of Educational Technology to Enhance Teaching Practices: Case Study of Community College in Malaysia. . *Procedia - Social and Behavioral Sciences*, 3(1), 1793-1797.
- Bansilal, S. (2015). Exploring Student Teachers' Perceptions' of The Influence of Technology in Learning Mathematics South African Journal of Education, 35(4), 1-8.
- Bapir, M. A. (2012). *Validity and Reliability in Qualitative Research* University of Warwick: University of Warwick press
- Barhoumi, C. (2015). The Effectiveness of WhatsApp Mobile Learning Activities Guided by Activity Theory on Students' Knowledge Management. *Contemporary Educational Technology*, 6(3), 221-238.
- Belgheis, S., & Kamalludeen, R. (2018). The Intention to Use GeoGebra in the Teaching of Mathematics Among Malaysian Teachers. *Malaysian Online Journal of Educational Technology*, 6 (1), 109 - 115.

- Bertram, C., & Christiansen, I. (2014). *Understanding Research*. Pretoria, South Africa: Van Schaik Publishers.
- Bester, G., & Brand, L. (2013). The Effect of Technology on Learner Attention and Achievement in The Classroom. *South African Journal of Education*, 33(2), 1-15.
- Bhagat, K. K., & Yen-Chang, C. (2015). Incorporating GeoGebra into Geometry learning-A lesson from India. *Eurasia Journal of Mathematics, Science & Technology Education*, 11(1), 77-86.
- Bist, P. R. (2017). Use of GeoGebra in Geometric Construction. *Imperial Journal of Interdisciplinary Research*, 3(9), 337 - 347.
- Brinkmann, S., & Kvale, S. (2014). *InterViews: learning the craft of qualitative research interviewing* (3rd ed.). Los Angeles, Calif: SAGE.
- Brito, R., & Dias, P. (2017). Digital Technologies , Learning And School: Practices. *The online journal of new horizons in education*, 7(1), 221.
- Bryman, A. (2012). *Social Research Methods* (4th ed.). London: Oxford University Press.
- Chimuka, A. (2017). *The Effect of Integration of GeoGebra Software in the Teaching of Circle geometry on Grade 11 Students'*. (Masters Degree Dissertation), University of South Africa, Pretoria.
- Chowdhury, B. & Ahmed, I. (2015). Issue of quality in a qualitative research:.. *Innovative Issues and Approaches in Social Sciences*, 8(1), 1-23.
- Cole, M. (1985). *The zone of proximal development: Where culture and cognition create each other*. In J.V Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* New York: Cambridge University press.
- Creswell, W. J. (2014). *Qualitative , Qualitative, and mixed methods approaches*. Los Angeles, USA: SAGE Publications Ltd.
- Davies, M., & Hughes, N. (2014). *Doing a successful research project: using qualitative or quantitative methods* (2nd ed.). Houndmills, Basingstoke, Hampshire: Palgrave Macmillan.
- DBE. (2011). *Curriculum and Assessment Policy Statement Grades 10-12. Mathematics*. Pretoria, South Africa.: Government Printing Works.
- DoE. (2011). *Curriculum and assessment policy statement (CAPS) Grade 10-12: Mathematics*. Pretoria: National Education Department.
- DOE. (2016). *Matric Results Analysis Diagnostic Report (Grade 12)* (pp. 446). Pretoria: Department of Basic Education

- Doolittle, P. E. (2014). Complex constructivism: A theoretical model of complexity and cognition. *International Journal of Teaching and Learning in Higher Education*, 26(3), 485-498.
- Duffy, T., & Jonassen, D. (1992). *Constructivism and the technology of instruction: A conversation*. Hillsdale, NY: Lawrence Elbaum Associates.
- Edgar, D. (1969). *Theory of Audio- Visual Methods in teaching (3rd ed.)*. New York: (3 ed.). New York: Holt, Rinehart & winston.
- Farrajallah, A.-A. (2016). The Impact of the Employment of Geogebra Software in Acquiring Some Visual Thinking Skills and On the Academic Achievement among 8th Grade Students. *Journal of Mathematics (IOSR-JM)*, 12(2), 53-64.
- Gagne, R. M. (1985). *The conditions of learning and theory of instruction (4 ed.)*. New York: Holt, Rinehart & winston.
- Gerry, S. (2016). *Constructing dynamic triangles together: The development of mathematical group cognition*. Cambridge, UK: Cambridge University Press
- Giurgiules, M., & Mion, M. (2015). Learning Geoetry Throught Mimesis and Digital Construct. *Practical Application of Science*, 3(9), 45-50.
- Gono, E. N. (2016). *The contributions of Interactive Dynamic Mathematics software in probing understanding of mathematical concepts: Case study on the use GeoGebra in learning the concept of modulus functions*. (Doctoral Degree Dissertation), University college of London England,London.
- Guest, G., Namey, E. E., & Mitchell, M. L. (2013). *Collecting qualitative data: A field manual for applied research* Thousand Oaks, Calif: SAGE.
- Gweshe, L. C. (2014). *The effect of using a Computer Assisted Instruction on teaching Circle Geometry in Grade 11*. (Masters Degree Dissertation), University of South Africa, Pretoria, South Africa.
- Hank, P. (Ed.) (2016) Merriam Webster's advanced learners' dictionary. : Retrived 19 October,2016 [http:// www . merriam-webster.com/dictionary/learning](http://www.merriam-webster.com/dictionary/learning).
- Holloway, W., & Jefferson, T. (2013). *Doing qualitative research differently: a psychosocial approach*. (2nd ed.). London: SAGE.
- Horzum, T., & Ünlü, M. (2017). Pre-Service mathematics teachers' view about GeoGebra and its use *Acta Didactica Napocensia*, 10(3), 77- 90.
- Howard, A. J. (2018). *A Descriptive Study of High School Mathematics :Teachers' Perceptions of Flipped-Mastery Learning in a High School Math Class*. (Doctoral Degree Dissertation Unpublished), Concordia University, Portland.

- İçel, R. (2013). *Bilgisayar destekli öğretimin matematik başarısına etkisi: GeoGebra örneği*. (Masters Degree Dissertation Unpublished), Selçuk University, , Selçuk,Turkey.
- Jones, S., Torres, V., & Arminio, J. (2014). *Negotiating the complexities of qualitative research in higher education: fundamental elements and issues* (2nd ed.). New York: N.Y. : Routledge.
- Kekana, G. R. (2016). *Using GeoGebra in transformation geometry : an investigation based on the Van Hiele model*. (Masters Degree Dissertation), University of Pretoria, Pretoria.
- Keller, J. M. (1987). Strategies for stimulating the motivation to learn. *Performance and Instruction Journal*, 2(1), 1-7.
- Keller, J. M. (2013). *Motivation design for learning and performance: The ARCS model approach*. New York: NY: Springer.
- Khobo, R. J. (2015). *The effect of Using Computer For the Teaching and Learning of Mathematics to Grade 10 Learners at Secondary School*. (Masters Degree Dissertation), University of South Africa Pretoria
- Kim, K. M., & Md-Ali, R. (2017). GeoGebra: Towards Realizing 21st Century Learning in Mathematics Education. *Malaysian Journal of Learning and Instruction*, 93(115), 93-116.
- Kösa, T. (2016). Effects of Using Dynamic Mathematics Software on Preservice Mathematics Teachers' Spatial Visualization Skills: The Case of Spatial Analytic Geometry. *Academics Journal*, 11(7), 449-458.
- Kul, U. (2013). *Professional development of Turkish primary mathematics teacher within a computer - integrated learning environment : An exploratory of change in beliefs* (Doctoral Degree Dissertation Unpublished), University of Leicester, Turkey
- Kumar, C. K., & Chun-Yen, B. (2015). Incorporating GeoGebra into Geometry learning-A lesson from India. *International Society of Educational Research*, 11(1), 77-86.
- Kutluca, T. (2013). The Effect of Geometry Instruction with Dynamic Geometry Software; GeoGebra on Van Hiele geometry Understanding Levels of Students. *Educational Research and Reviews*, 8(3), 1509-1518.
- Kutluca, T. (2014). The effect of geometry instruction with dynamic geometry software; GeoGebra on Van Hiele geometry understanding levels of students. *Academic Journals*, 8(17), 1510 - 1518.

- Leendertz, V., Blignaut, A., Blignaut, H., Els, C., & Ellis, S. (2013). Technological Pedagogical Content Knowledge in South African Mathematics Classrooms. *A secondary Analysis of SITES 2006 data. Pythagoras*, 34(2), 1-9.
- Leikin, R., & Zaslavsky, O. (2013). Cooperative Learning in Mathematics. *The Mathematics Teacher*, 92(3), 240-246.
- Lichtman, M. (2014). *Qualitative research for the social sciences*. Los Angeles: Calif. : SAGE.
- Liu, N. (2013). iPad infuse creativity in solid geometry teaching. *The Turkish Online Journal of Educational Technology - TOJET*, 12(2), 177-192.
- Ljajko, E., & Ibro, V. (2013). Development of Ideas on a GeoGebra-aided Mathematics Instruction. *Online Submission*, 3(3), 1-7.
- Luneta, K. (2014). Foundation phase teachers' (limited) knowledge of geometry. *South African of Childhood Education, University of Johannesburg, RSA.*, 4(3), 71-86.
- Malan, S., Ndlovu, M., & Engelbrecht, P. (2014). Introducing problem-based learning (PBL) into a foundation programme to develop self-directed learning skills. *South African Journal of Education*, 34(1), 1-16.
- Marshall, C., & Rossman, G. B. (2014). *Designing Qualitative Research* (6th. ed. ed.). New York: Sage publications.
- Martinez, A. R. (2017). *The Effects of Using GeoGebra on Student Achievement in Secondary Mathematics*. (Masters Degree Dissertation), California State University, Monterey Bay.
- Miles, M., Huberman, A., & Saldana, J. (2013). *Qualitative data analysis* (3rd ed.). London: Sage.
- Molnar, P. (2016). Solving Linear optimization world problems by using GeoGebra. *ICTE*, 5(2), 16-28.
- Moodley, G. (2013). *Implementation of the curriculum and assessment policy statement: Challenges and implications for teaching and learning* (Masters Degree Dissertation), University of South Africa, Pretoria, South Africa.
- Mosese, N. M. (2017). *Evaluating the effectiveness of the use of information and communication technology in the teaching and learning of trigonometry function in grade 12* (Masters Degree Dissertation), University of South africa, Pertoria.
- Mosia, M. (2016). *Improving teachers' technological pedagogical content knowledge for teaching Euclidean Geometry using integrated information comminication technologies software*. (Doctoral Degree Dissertation), University of the Free State, Bloemfontein

- Mthethwa, M. (2015). *Application of GeoGebra programme on Euclidean Geometry in rural high schools- grade 11 learners*. (Maters Degree Dissertation), University of Zululand, Zululand.
- Muhtadi, M. A. (2013). The Impact of Technology on Education. *Al-Nasser University Journal*, 4(2), 1-8.
- Mukiri, M. I. (2016). *Feasibility of using GeoGebra in the teaching and learning of Geometry concepts in secondary schools in Kajiado county*. (Doctoral Degree Dissertation), Kenyatta University, Kenya
- Murni, V., Sariyasa, S., & Ardana, M. (2017). *GeoGebra Assist Discovery Learning Model for Problem Solving Ability and Attitude toward Mathematics*. Paper presented at the International Conference on Mathematics and Science Education (ICMScE), Universitas Pendidikan Ganesha, Indonesia.
- Mutodi, P., & Ngirande, H. (2014). The Influence of Students' Perceptions on Mathematics Performance. A Case of a Selected High School in South Africa. *Mediterranean Journal of Social Sciences*, 5 (3), 431- 445.
- Naidoo, J. (2012). Teacher reflection: The use of visual tools in mathematics classrooms. *Pythagoras*, 33(1), 1-9.
- Neuman, W. L. (2014). *Social Research Methods: Qualitative and Quantitative Approaches* (7th ed.). London: Pearson Education Limited.
- Ocal, M. F. (2017). The Effect of Geogebra on Students' Conceptual and Procedural Knowledge: The Case of Applications of Derivative. *Higher Education Studies*, 7(2), 67- 78.
- Oladosu, L. O. (2014). *Secondary School Students' Meaning and Learning of Circle Geometry*. (Doctoral Degree Dissertation), University of Calgary
- Ozcakir, B. (2013). *The effects of Mathematics instruction supported by danamic geometry activities on seventh grade students' achievement in area of quadrilaterals'*. (Masters of Science in Mathematics), Middle East Technical University, Turkey.
- Perry, D., & Steck, A. (2015). Increasing Student Engagement, Self-efficacy, and Metacognitive Self-regulation in the High School Geometry Classroom: Do iPads help? *Computers in The Schools*, 32(2), 122-143.
- Pfeiffer, C. (2017). *A Study of development of mathematical knowledge in a GeoGebra focused learning environment*. (Doctoral Degree Dissertation Academic), Stellenbosch University.

- Ramatlapana, K. A. (2017). *Prospective mathematics teachers' technological pedagogical content knowledge of Geometry in GeoGebra - based environment* (Doctoral Degree Dissertation), University of the Witwatersrand, Johannesburg, South Africa.
- Ranjit, K. (2013). *Research Methodology* (3rd ed.). London: SAGE Publications Ltd.
- Saha, R., Ayub, A., & Tarmizi, R. (2012). The effects of GeoGebra on mathematics achievement: Enlightening coordinate geometry learning. *Procedia-Social and Behavioral Sciences*, 8(3), 686 - 693.
- Sahin, M., & Kislal, T. (2016). An analysis of University Students' Attitudes Towards Personalized Learning Environments. *The Turkish Online Journal of Educational Technology*, 15(1), 1-10.
- Satu Elo, M. K., Outi Kanste, Tarja Pölkki & Kati. (2014). Qualitative Content Analysis: A Focus on Trustworthiness. *Open*
- Shadaan, P., & Eu, L. (2014). Effectiveness of Using Geogebra on Students' Understanding in Learning Circles. *The Malaysian Online Journal of Educational Technology*, 1(4), 1-11.
- Shoemaker, T. L. (2013). *The effect of computer-aided instruction on attitude and achievement of fifth grade math students. Doctoral Dissertation*. (Doctoral Dissertation), Liberty University, Lynchburg, VA.
- Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145.
- Stols, G., & Kriek, J. (2012). Why don't all maths teachers use dynamic geometry software in their classrooms? . *Australasian Journal of Educational Technology*, 27 (1), 137 -151.
- Strimel, G., & Grubbs, M. E. (2016). Positioning Technology and Engineering Education as a Key Force in STEM Education. *Journal of Technology Education*, 27(2), 21-36.
- Suppes, P. (1969). Stimulus-Response Theory of Finite Automata. *Journal of Mathematical Psychology*, 6(3), 327-355.
- Symon, G., & Cassell, C. (2012). *Qualitative organizational research: core methods and current challenges*. . Los Angeles: SAGE.
- Tatar, E. (2013). The Effect of Dynamic Software on Prospective Mathematics Teachers' Perceptions Regarding Information and Communication Technology. *Australian Journal of Teacher Education*, 38(12), 1-17.
- Thomas, G. (2011). A topology for the case study in social science following a review of definition, discourse and structure. *Qualitative Inquiry*, 511- 521.

- Tsiteisia, B. Y. (2014). *The effects of the use of computers on students' learning of trigonometry in mathematics in selected senior high schools in ho municipality*. (Masters Degree Dissertation), University of Cape Coast.
- Unlua, Z. K., Dokmeb, I., & Tufekcic, A. (2015). An Action Research on Teaching Science ThroughTtechnology Supported Inquiry - Based Learning: a Pilot Study. *Procedia - Social and Behavioral Sciences*, 186(8), 46-52.
- VanHiele, M. P. (1957). *The didactics of geometry in lower class of the secondary school*. . Utrecht: University of Utrech.
- Vasque, D. E. (2015). *Enhancing students achievement using GeoGebra in a technology rich environment* (Masters Degree Dissertation), California State Polytechnic University, USA.
- Vrasidas, C. (2000). Constructivism Versus Objectivism: Implications for Interaction, Course Design, and Evaluation in Distance Education. *International Journal of Educational Telecommunications*, 6(4), 339-362.
- Wah, L. K. (2015). The Effects of Instruction Using The ARCS Models and GeoGebra on Upper Secondary Students' Motivation and Achievement in Learning Combined Trasformation. *Asia Pacific Journal of Educators and Education*, 30(5), 141-158.
- Wassenaar, D. R., & Mamotte, N. (2012). *Ethical issues and ethics reviews in social science research*. New York, London: SAGE Publications.
- Wiersum, G. (2014). Teaching and Learning Mathematics through Games and Activities. *Acta Electrotechnica et Informatica*, 12 (3), 47-61.
- Williams, C., Charles-Ogan, G., & Adesope, R. (2017). The GeoGebra Interative Software and Senior Secondary School Three (SSS3) Students' interest and acievement in mathematics. *International Journal of Mathematics and Statistics Studies*, 5(1), 1-8.
- Yildiz, A., & Baltaci, S. (2016). Reflections From The Analytic Geometry Courses Based on Contextual Teaching and Learning Through GeoGebra Programme. *The online Journal of New Horizons in Education*, 6(4), 155- 166.
- Yin, R. K. (2012). *Qualitative Research*. New York, United States of America: The Guilford Press.
- Zengin, Y., & Tatar, E. (2017). Integrating Dynamic Mathematics Software into Cooperative Learning Environments in Mathematics. *Educational Technology & Society*, 20(2), 74-88.

Zilinskiene, I., & Demirbilek, M. (2015). Use of GeoGebra in Primary Math Education in Lithuania: An Exploratory Study from Teachers' Perspective. *Informatics In Education*, 14(1), 127 - 142.

Appendix I: Confirmations of language editing

Angela Bryan & Associates

6 Martin Crescent

Westville
3629

Date: 11 March 2019

To whom it may concern

This is to certify that the Master's Dissertation : Exploring Grade 11 Learners' Use of the GeoGebra Programme When Learning Euclidean Geometry written by Gezahegn Haile Godebo has been edited by me for language.

Please contact me should you require any further information.

Kind Regards

Angela Bryan

angelakirbybryan@gmail.com

0832983312

To: Mr G.H. Godebo
From: Mrs Leela Reddy
Subject: Language Editing Service

Date: 02 December 2018

Dear G.H Godebo

Exploring Grade 11 Learners' use of the GeoGebra programme when learning Euclidean Geometry

The above Med thesis has been edited.

While I have corrected spelling and language errors (tense, concord and word choice) chapters 1 to 6 (pages 1-112). I have not edited any quotations. I have also not checked that the sources indicated in your reference list are all contained in the body of the document or that these sources are cited correctly.

The corrected document has been returned to you.

Sincerely



Leela Reddy (Mrs)
Language Editor

Appendix II: Turnitin report

Exploring Grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry

ORIGINALITY REPORT

15%

SIMILARITY INDEX

13%

INTERNET SOURCES

4%

PUBLICATIONS

9%

STUDENT PAPERS

PRIMARY SOURCES

1	Submitted to Georgia Southwestern State University Student Paper	<1%
2	resources.careered.com Internet Source	<1%
3	mojet.net Internet Source	<1%
4	www.acci.org.za Internet Source	<1%
5	www.sociology.kpi.ua Internet Source	<1%
6	www.ascilite.org.au Internet Source	<1%
7	Submitted to Middle East Technical University Student Paper	<1%
8	ulir.ul.ie Internet Source	<1%

Appendix III: Permission letter by school to conduct research

THORNWOOD SECONDARY SCHOOL

Site 422, Thornwood, P.O. Box 1013, Nagina, 3604

Tel. / Fax: 031 – 706 0222

Email: thornwoodsecondary@outlook.com



29 June 2017

Re: SCHOOL PERMISSION TO CONDUCT RESEARCH

The purpose of this letter is to grant Mr. Gezahegn Haile Godebo, Masters Student at UKZN, Edgewood Campus, permission to conduct research at Thornwood Secondary School. The research titled, “Exploring Grade 11 learners’ use of GeoGebra programme when learning Euclidean Geometry” involves grade 11 learners who are studying mathematics at Thornwood secondary school. Researcher, Mr Gezahegn Haile Godebo has informed me of the design of the research as well as the target population.

We support this effort and will provide any assistance necessary for the successful completion of this research.

Sincerely!

M.Tshabalala (Principal)

Date

Appendix IV: Participants consent form

THORNWOOD SECONDARY SCHOOL

Site 422, Thornwood, P.O. Box 1013, Nagina, 3604

Tel. / Fax: 031 – 706 0222

Email: thornwoodsecondary@outlook.com



Date: 21/06/2017

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,

PARTICIPANT CONSENT–RESEARCH STUDY

My name is GEZAHEGN HAILE GODEBO I am a Masters of Education in Mathematics candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in conducting research in your School to explore the effect of using GeoGebra programme in learning Grade 11 Euclidean Geometry. My study aims to explore the role of computer software called GeoGebra programme as a learning tool in high school mathematics. The study also aims to explore whether learners' interaction with this computer software enhances their understanding of certain mathematical concepts. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person, but reported only as a population member opinion.
- The interview may last for about 45 minutes to 1 hour.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalized for taking such an action.

- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment		
Photographic equipment		
Video equipment		

I can be contacted at:

Email: nahomg777@gmail.com

Cell: 0717085543

My supervisor is Dr J Naidoo (Ph.D) who is located at the School of Education, Edgewood campus of the University of KwaZulu-Natal.

Contact details: email: Naidooj2@ukzn.ac.za

Phone number: 0312601127

You may also contact the Research Office through:

Ms. P Ximba (HSSREC Research Office)

Tel: 031 260 3587

Email: ximbap@ukzn.ac.za

Thank you for your contribution to this research.

DECLARATION

I..... (full names of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT

DATE

.....

.....

Appendix V: Parental consent form

THORNWOOD SECONDARY SCHOOL

Site 422, Thornwood, P.O. Box 1013, Nagina, 3604

Tel. / Fax: 031 – 706 0222

Email: thornwoodsecondary@outlook.com



Date: 21/06/2017

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,

Dear Participant

PARENTS CONSENT – RESEARCH STUDY

My name is GEZAHEGN HAILE GODEBO I am a Masters of Education in Mathematics candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in conducting research in your School to explore the effect of using GeoGebra programme in learning Grade 11 Euclidean Geometry. My study aims to explore the role of computer software called GeoGebra programme as a learning tool in high school mathematics. The study also aims to explore whether learners' interaction with this computer programme enhances their understanding of certain mathematical concepts. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person, but reported only as a population member opinion.
- The interview may last for about 45 minutes to 1 hour.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalized for taking such an action.

- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment		
Photographic equipment		
Video equipment		

I can be contacted at:

Email: nahomg777@gmail.com

Cell: 0717085543

My supervisor is Dr J Naidoo (Ph.D.) who is located at the School of Education, Edgewood campus of the University of KwaZulu-Natal.

Contact details: email: Naidooj2@ukzn.ac.za

Phone number: 0312601127

You may also contact the Research Office through:

Ms. P Ximba (HSSREC Research Office)

Tel: 031 260 3587

Email: ximbap@ukzn.ac.za)

Thank you for your contribution to this research.

DECLARATION

I..... (full names of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my child participating in the research project.

I understand that my child is at liberty to withdraw from the project at any time, should he/she so desire.

SIGNATURE OF PARTICIPANT (LEARNER)

DATE

.....

.....

SIGNATURE OF PARENT (If participant is a minor)

DATE

.....

.....

Appendix VI: Ethical clearance



05 December 2017

Mr Gedebo H Gedebo 214578033
School of Education – Mathematics
Edgewood Campus

Dear Mr Gedebo

Protocol reference number: HSS/1937/017M

Project title: Exploring grade 11 learners' use of the GeoGebra programme when learning Euclidean Geometry.

Expedited Approval

In response to your application dated 18 October 2017, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

Please note: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

Dr Shamila Naidoo (Deputy Chair)

/ps/

cc: Supervisor: Dr Jayalaxmi Naidoo
cc: Academic Leader Research: Dr SR Kooze
cc: School Administrator: Ms T Khumalo and Ms P Ncayiyana

Humanities & Social Sciences Research Ethics Committee

Dr Shenuka Singh (Chair)

Westville Campus, Gwelo Mkhaki Building

Postal Address: Private Bag X54001, Durban 4000

Telephone: +27 (0) 31 260 3537/3534/557 Facsimile: +27 (0) 31 260 4809 Email: shenuka.singh@ukzn.ac.za / joeyman@ukzn.ac.za / mb.mphahle@ukzn.ac.za

Website: www.ukzn.ac.za



1990 - 2010



101 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses

Edgewood

Howard College

Medical School

Pietermaritzburg

Westville