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Reverse Mortgage:

A Neural Network Approach for Pricing and Risk Assessment

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Abstract

Population aging and low precautionary savings rates has put European public social systems under strain. As a result, home-ownership among seniors as viable mean of income stream enhancement and welfare for seniors has been boldly encouraged by governments. Thus, equity release instruments for pensioners have been proposed by the market. These products are mostly encompassed in North America where the elderly are less reluctant to express their desire to transform housing into wealth. Still, southern European countries present large home-ownership rates and an aging low income population that may well unlock future demand.

Whilst housing is a highly illiquid asset and emotional attachment as well as inconvenience of moving barriers may occur, in recent literature relatively new approaches to monetize homes have undergone major developments.

Particularly, this study will be mainly concerned with the risk and profitability analysis of reverse mortgage schemes through actuarial and deep learning techniques in the attempt to conceive a framework that fully encompasses the valuation needs of companies willing to commercialize home equity based products.

Keywords: retirement, reverse mortgage, pricing, risk assessment, long short term memory neural networks

CONTENTS

MCD Monte Carlo Dropout. 29–33, 39, 40, 44, 48

MLP Multilayer Perceptron. 21, 23, 26

NAV Net Asset Value. 36, 38, 39, 41–45, 48

NNEG Non Negative Equity Guarantee. 16–18, 37, 38, 40–42

OLB Outstanding Loan Balance. 42

PAYG Pay As You Go. 6

PVI Prestito Vitalizio Ipotecario. 18–21, 29, 33, 35–37, 39, 48

RM Reverse Mortgage. 5, 9–19, 32, 42, 47

RNN Recursive Neural Network. 26, 27

SVD Support Vector Decomposition. 29

TVaR Tail Value at Risk. 44–46

VAR Vector Auto-Regressive. 15, 32

VaR Value at Risk. 43–46

INTRODUCTION

The aging dynamic of the population ushers new sustainable sources of long term care and health funding for the elderly. How the funding should be aligned to the resources available is a large topic of debate. Most people lack of competence and experience to manage their assets and governments are not willing to increase sovereign debt to address retirement thereafter. Furthermore, traditional annuity products have tarnished their reputation due to low flexibility and returns. These developments are promoting markets and individuals to assess alternative ways to provide for retirement. Thereby, Equity Release Scheme (ERS) have a particularly relevant role to play. Unlike other instruments, ERS allow pensioners to access the wealth within their house without the need to move out. Among these, in recent years Reverse Mortgage (RM) have progressively gained ground. To address RM risk management and profit assessment needs various techniques have been designed. These leave much room for innovation and application fields which will be discuss throughout this study.

1.1   Background and Problem Identification

The past three centuries have witnessed a combination of socio-economic factors in highly industrialized countries that have lead to a period of low natality and high longevity rates. The early stages of this process, commonly addressed to as demographic transaction, have mostly affected child mortality rates. Thus, producing overpopulation concerns.
However, the traditional high fertility reproductive behavior has substantially changed (Coale, 1989). This has contributed to a drastic decline in birth rates. And, improved health conditions have provoked a dramatic increase in life expectancy rates that is still set to persist and even intensify in the decades to come. Kontis et al. (2017) project an increase in the average years of life to 2030 for 35 industrialized countries with a probability of 65% for women and of 85% for men.
Carone and Costello (2006) notably identify four demographic developments occurring across EU member states: fertility rates shrinking below natural replacement rates; increase in the old age dependency ratio, i.e. the proportion of nonworking older population to the total population; growing life expectancy at birth; high net migration inflows that cannot offset declining natality

and population aging.

One of the most significant socioeconomic advancements to ever happen to mankind is increased longevity. This process initiated no longer than two centuries ago in the then industrialized economies and has since spread around the globe and across all socioeconomic groups (Ayuso, Bravo, & Holzmann, 2017b, 2017a, 2019b).

Although longevity per se has a positive influence on growth, rising elderly dependency rates negatively impacts savings and investments and provokes labour and productivity losses (H. Li, Zhang, & Zhang, 2007; Leff, 1971). Longer lives and population ageing create challenges for all societal institutions, public and private, particularly those providing retirement income, health care, and long-term care services. This puts too much pressure on retirement systems (Bryant, 2004) that no longer benefit from taxes on younger working population. Consequently, excessive public spending generated by increased pension liabilities is corroding continental Europe's traditional Pay As You Go (PAYG) public pension systems (Cipriani, 2014; Nerlich & Schroth, 2018) and promoting fiscally driven public pension reforms. The decreasing generosity of public health care systems and of public annuities, with deep adequacy and poverty concerns in several countries and within certain groups of population (e.g. women, less-educated classes and migrants), the reduction in the traditional family support at old-age as a result of sinking fertility rates, urbanisation, migration and increasing retirement age, all have increased the need for additional private savings to cover the old age income gap and to avoid relying on state-managed social transfers to reverse the risks of poverty (Ayuso, Bravo, & Holzmann, 2019c; Bravo & Holzmann, 2014; European Commission, 2018).

Concern over prohibitive costs and deteriorating solvency ratios of conventional pension schemes have produced a gradual shift from Defined Benefit (DB) pension plans to Defined Contribution (DC) pension plans (Broadbent, Palumbo, & Woodman, 2006; Schmäl, 2007). Although private pensions are becoming more widespread, the coverage rates are still small and the contribution amounts insignificant in most cases (HFCS, 2016). Most DC scheme members have not contributed enough to receive even a modest income stream in retirement. The consequence of this is giving full responsibility to individuals for their future retirement income streams which is failing to provide steady income replacement rates for pension funding (Bodie, Marcus, & Merton, 1989).

This combines with major controversies arising from the traditional life cycle model as famously designed by Brumberg and Modigliani (1954). The model introduced the idea of rational consumption and savings patterns. Hence, in response to future income risk, the willingness to save more in the present occurs, i.e. precautionary saving.

Still, empirical evidence has shown serious deviations from the life cycle-model mostly due to the lack of experience in managing retirement saving assets. Yilmazer and Scharff (2014) show how household savings don't increase with increasing health risk. Similary, Fernández-Villaverde and Krueger (2007) and Scholz, Seshadri, and Khitatrakun (2006) have proven that household expenditure increases until late in life as there is no willingness to engage in precautionary saving during working years producing poor wealth accumulation for retirement. And this can imply again low standards of life for the elderly (Hubbard, Skinner, & Zeldes, 1994).

The build-up and management of retirement income will be different for different individuals and there is no guarantee that individuals optimize consumption over their active and retirement period as predicted by the life-cycle hypothesis. Indeed, empirical studies suggest that the actual wealth accumulation, preservation and decumulation behaviour before and after retirement is often in conflict with life-cycle predictions, particularly when analysed considering the differentiated approach across the three main tiers of the population (Holzmann, Ayuso, Alaminos, & Bravo, 2019): (i) The lowest tier that typically does little saving and, as result, will have no capacity to dissave after retirement; (ii) The top tier that continues the accumulation of financial and non-financial asset after retirement and shows no sign of dissaving; (iii) The middle tier that seems to be the only one showing sign of life-cycle saving and dissaving, particularly those with no longevity insurance (public or private life annuity), but faces a number of constraints (e.g. illiquid housing wealth assets, pension taxation, undeveloped financial and insurance markets).

Despite fierce encouragement from governments to increase responsibility to manage personal funding, in Europe many still rely on public support to finance their retirement (Towers Watson, 2013). The effect of insufficient income planning and the decline of benefits arising from pension schemes along with increased longevity produce a high risk of living longer in poor health conditions and without cash.

This will undeniably impact how accumulated assets and the risks involved (longevity, investment and inflation) will be managed (Antolin, 2009). An obvious question rises: how to ensure an adequate, secure, stable and predictable lifelong income stream for all retirees that will allow them to maintain a target standard of living for however long the individual lives. In other words, how to guarantee funding and service streams that cover expenditure needs of a lifestyle towards which individuals aspire in retirement. To fund for longer lives, pensioners will ultimately rely on a retirement wallet combining income and service flows from state, employer, social institutions, family, own savings, housing wealth, continued labour income and insurance sources. The weights will be determined by both personal and institutional circumstances (Bravo, 2019).

Markets responses to finance care and provide for seniors have been numerous. The lifetime annuity is the classical product used to protect against longevity although, in the past few years provision for this instrument has collapsed. Moreover, early death risk draws many people towards less expensive term annuities that might be coupled with a differed annuity as a form of decumulation. However, their return performance is poor and doesn't offer room for flexibility to address different needs that could materialize during retirement.

Long term care cost insurance has been designed to protect against contingent costs that might occur in old age. Still, forbidding long term care insurance prices have been hindering demand (Kenny et al., 2017). On the other hand, particularly in the UK the flexibility of drawn-down pensions seems more appealing for retirement and long term care (Rashbrooke, 2018). However, draw-down income partially remains subject to the risk of living far longer than the expected life span (Mayhew, Smith, & Wright, 2018).

An alternative source for retirement consumption is housing. European countries have been thriving in home equity. In the Euro area alone, the household's wealth (excluding pension wealth, i.e. the present value of all future expected pension benefits) is primarily held in the form of

real estate assets, which represent 82.2% of total assets owned by households (85.1% in Spain, 88% in Portugal) with the remaining assets (17.8%) being financial. The largest component of real assets is the Household Main Residence (HMR), representing 60.2% of total real assets, followed by other real estate property (22.3%). On average more than 70% of EU residence live in owner-occupied accommodation (Hennecke, Murro, Neuberger, & Palmisano, 2017).

Home-ownership is higher in poorer countries and the proportion of home owners by age band has been steadily increasing with each successive generation. Evidence-based observations indicate that homeowners are generally wealthier than their non-home owning counterparts, and this conclusion is valid both across the income or net wealth distribution and across countries.

Personal pensions and private home-ownership are the two main assets individuals have to finance and supplement their retirement consumption in an asset-based approach to welfare. As such, individuals accept greater responsibility for their own welfare needs. This involves both long-term saving and investment decisions over the life cycle. Investment and saving decisions are motivated by potentially competing objectives and generate different options and outcomes at old-age. Home wealth provides a stream of housing services starting at time of house acquisition and represents wealth which could be liquidated in old age if needed. The asset serves both consumption and investment functions, which are assessed differently by different households based on personal preferences.

In such situation, the house is seen as a tool against poverty that can cut expenses and increase resources and consumption (Sass, 2017). Moreover Moscarola, D'Addio, Fornero, and Rossi (2015) show how Italy and Spain would greatly benefit from poverty reduction as a result of accessing home-ownership equity. Furthermore, according to French, McKillop, and Sharma (2018)'s estimates, releasing housing equity can boost the economy in the UK by 30%. This combines with the findings of Nakajima and Telyukova (2011) that illustrate how home-ownership is a key component in retirement saving over the life-cycle. Consequently, the home becomes central to asset backed welfare approaches, where home-ownership is envisioned as a mean of individual welfare (Ronald, Lennartz, & Kadi, 2017). As a matter of fact, Mayhew, Rickayzen, and Smith (2017) have examined the benefits of employing home equity to provide for long term care services. On top of this, governments are seeing housing assets as a way to offset public benefits' decline (Prabhakar, 2018).

The increasing need of private savings for old-age is potentially in conflict with savings for home-ownership. In many cases, an average household repays annually in mortgage capital a substantially higher amount than that saved for retirement purposes. Together with taxation, the resources required for paying for a home act as a strong disincentive to funded social security and welfare. However, there is still no clear indication that allows us to conclude whether households owning property and repaying mortgages save more than renters or to what extent homeowners with a mortgage substitute any financial savings with mortgage payments or see them as complements. If paying off a mortgage is perceived as equivalent to saving for retirement there is no apparent trade-off. However, the existence of liquidity constraints and the need to align and integrate the objectives and incentives for both investment and saving decisions might become troublesome in practice.

One possible manner of mitigating this potential conflict implies unlocking the wealth senior individuals have accumulated in their homes to help fund retirement and long term care. Home-ownership wealth could be collected by selling or renting out the home or through a series of different ERS. Some of these mechanisms involve selling the dwelling and moving or even being permitted to live in the house up until death (Towers Watson, 2013). In others accessing the wealth accumulated in the form of housing is possible while also being able to continue living in the same home up until death or move out to a residential care home (Ayuso, Bravo, & Holzmann, 2019a). The later form of ERS is the most customary among the two and will be the subject of examination of the present dissertation. Among English speaking countries it is commonly referred to as Lifetime Mortgage in the UK and as RM in Canada, USA and Australia. This contract consists of a lien on a real estate asset that pledges the repayment of a loan (mortgage). However unlike common housing mortgages where the loan helps the borrower purchase a home, a RM contract acts in 'reverse': it allows the borrower and owner of the estate to exploit his unencumbered property rights and access a loan that will finance old age income needs.

In this study we will employ the term RM to refer to a medium-long term mortgage against the value of a residential estate where the lender is a financial intermediary, e.g bank or insurance company. The borrower, who will typically be 60 or older, is entitled to a lump sum or a stream of payments equivalent to a predefined percentage of the value of the estate. The value of the loan accumulates in time at a predefined interest. However the outstanding loan can never be higher than the property value. The repayment of the loan can occur upon death of the borrower or loan refinancing, typically move-out or selling the property.

90% of RMs have been marketed in the USA since the late '80s, where they are administered by the Federal Housing Administration (FHA) (Shan, 2011). These institutionalized RMs are known as Home Equity Conversion Mortgages (HECM) and can only be accessed once reached a minimum age of 62 years old (Warshawsky, 2018). These have been largely used by low income households to increase consumption in retirement (Makoto & A., 2017). And, only recently Equity Release instruments have been introduced in the German, Spanish and Italian financial market. Particularly, in Italy although the offer has been thriving, RMs have shown no exceptional success in terms of demand (Fondazione Cariplo, 2017; Moscarola et al., 2015). Nevertheless, recent results argue that RMs can considerably cut poverty (Baldini & Beltrametti, 2015) and benefit life-cycle investment planing among Italian households (Crespi & Mascia, 2014).

At European level the first overview over ERS as of 2007 has been assessed in three reports by Reifner, Clerc-Renaud, Pérez Carrillo, Tiffe, and Knobloch (2009a, 2009b, 2009c). Whereas, a more recent comparative analysis of the impact of ERS on present and future generations of seniors has been provided by Mudrazija and Butrica (2017).

Motivated by this, the present study will focus on the empirical risk valuation and profitability profile of Italian RM instruments in the attempt to create a solid model that could enhance pricing and risk management accuracy and favor marketability.

1.2   Study Objectives and Relevance

In the previous lines, the key role that home equity based products play in alleviating poverty among elderly has been outlined. To date, the use of RM has been very limited within European countries. Still, the combination of life expectancy lengthening, downsizing of public resources and high home-ownership rates pave the way to market expansion as RM instruments emerge as an appealing mean to access housing wealth.

The most common equity release products incorporate a clause that prevents the loan to be higher than the value of the property. This guarantee bears substantial risks that makes equity release schemes difficult to price (Huang, Wang, & Miao, 2011). Thus, appropriately evaluating the risks and profitability it incorporates is not an option but a regulation wise mandatory operation for any institution that desires to market and generate profit margins from RM products. As a consequence, a fledgling large body of research over RM risk measurement and pricing has proliferated. Still, methodological controversies have arisen and left plenty of room for advancement which will be exploited throughout this study. Carefully examining the limitation these techniques bear and proposing innovative improvements implies providing companies with a reliable risk and profit assessment system that would aid the industry in developing sustainable and lucrative products as well as comply with regulation. Hence, throughout this dissertation empirical and theoretical methodologies will be devoted to provide a result for the following research questions:

1. **Employ diverse approaches to assess the risks within RM instruments and the determinants of profitability**.
   The present dissertation will employ cross disciplinary approaches, e.g. actuarial, financial and deep learning methodologies, to integrate and potentially improve state-of-the-art RM risk and profit valuation.

2. **Introduce artificial intelligence to jointly evaluate house property pricing, interest rate and inflation risk with specific application to RM products.**
   Since the '90s there has been a plethora of research and applications of Artificial Intelligence for real estate price prediction and financial management of assets, particularly Neural Network systems. As such, this research will attempt to extend these findings to house pricing, interest rate and inflation risk computation and possibly introduce innovation within the RM risk assessment framework.

3. **Asses exposure to loss and produce a final aggregated risk measure.**
   The ultimate objective is to perform a product-specific risk analysis and conduct several sensitivity analysis. In this regard, a final aggregated risk measure is required. This should account for all contingent factors.

4. **Determine the value of the loan and profitability of the lender.**
   The options inherited in different RM instruments generate pay-off structures. These can be assessed through pricing theory to explain risk adjustments that maximize loan providers' profit.

Eventually, to produce knowledge outside the research environment, this study will be transformed into practice when applied to real world products, i.e. Italian commercialized RMs. This will allow to introduce a final objective:

5. **Investigate over the Italian state-of-the-art RM market.**
   In Italy the RM loan program has been very recently introduced and it presents several peculiarities. Therefore, we will need to explore the current Italian market and adapt the resulting risk and profitability valuation framework accordingly.

1.3   Structure of the Paper

This paper consists of six chapters among which the above introduction and a final chapter where the main conclusions of the study are drawn. Within the introductory chapter we have presented a general overview over the primary topic and the objectives of research.

In the following chapter, we will explore literature concerning the equity release market and various valuation approaches. A number of valuation frameworks illustrated below will specifically deal with reverse mortgages, while others will introduce unexplored methods in this area.

Chapter three will focus on providing an overview of the reverse mortgage market conditions within the first section. Whereas section two will be entirely dedicated to presenting the current Italian reverse mortgage offer, demand and regulation.

In Chapter four we will illustrate the different techniques employed to determine a future value for all identified stochastic variables. These will be subsequently used in Chapter five to generate cash flow projections of various reverse mortgage schemes. As such, this chapter will first present the quantitative features of all contract structures under study and subsequently provide the final results and considerations in terms of net pay-off and risk measure valuation.

The final remarks will be drawn in the last chapter of this paper. Here we will illustrate a final outline of the present investigation and introduce further recommendations on research advancements on the subject of this study.

LITERATURE REVIEW

A strand of literature has been concerned with equity release products and particularly with the reverse mortgage market and valuation techniques. Understanding market conditions through related literature will then allow for the design of proper risk and profitability modelling. Many of these studies will be outlined throughout this chapter. These will supplement and complement the present research at a further stage of investigation.

2.1 Equity Release Market

Different interrelated forces affect the Equity release market. A study on this topic is provided by Sulaiman, Ishaq, and Ghani (2018). The authors provide an outlook on all different factors and sub-factors influencing the RM market and identify four major components: institutional, economic, socio-cultural and behavioral. Moreover, they show that many households rich in home equity might encounter various obstacles when attempting to access house value. Terry (2007) investigates over practical solution and provides guidance on how to make equity release easily accessible.

A broad view of the international equity release market is presented by Haurin and Moulton (2017), Gwizdala (2015) and Fondazione Cariplo (2017). While as previously mentioned, an EU specific study has been conducted in three paper studies by Reifner et al. (2009a, 2009b, 2009c) and more recently by Hennecke et al. (2017). Additionally, with particular regard to insurance companies, Towers Watson (2013) explores the potential of the equity release market in the EU. These studies provide a wide overview over several country-specific equity release markets. Whereas, existing studies conducted in key ERS markets, i.e. North America, Australia, UK and Ireland, will be illustrated below.

North America

ERS have been particularly popular in North America with the United States market as one of the most developed ERS market worldwide. A historical overview on the US RM market is provided by Huan and Mahoney (2002). The HECM is the widest equity release program in the US accounting for 90% of the market (Gwizdala, 2015). This type of reverse mortgages along

with other mechanisms designed to allow for equity release are illustrated in Kaul (2017) and Warshawsky (2018)'s studies. These explore and exhaustively present the market distribution and endorsement in recent years. While Michelangeli (2008)'s research shows how the decision to apply for a reverse mortgage in the US affects household gains and losses and life-cycle consumption. Also, the convenience of RM in the US is furthermore investigated by Nakajima (2012). Shan (2011) shades light on the determinants of the US RM market expansion.

Interestingly, the US has also experienced the development of the institutionalized HECM secondary markets. A key role in bringing liquidity to HECM lenders has been played by the government sponsored Federal National Mortgage Association, a.k.a. Fannie Mae (Begley, B. Fout, LaCour-Little, & Mota, 2017).

In Canada RM programs are solely available through a regulated banking institution named HomEquity Bank. This Canadian RM is known as Canadian Home Income Plan and shares many features with the HECM scheme (Fondazione Cariplo, 2017).

Australia

Equity release products have gained a certain recognition in the Australian financial services market over the past decades. The RM contract has received most attention among all equity extraction products and it takes mostly the form of a lump sum loan (Haffner, Ong, & Wood, 2013).

Furthermore, Brownfield (2014) cites equity release instruments as the fourth retirement income pillar for senior Australians and gives a broad overview of Australian equity release schemes among which the RM is the most common. An important study (Australian Actuaries Institute, 2016) discusses the costs and biases encountered in the Australian market and suggest reforms to unlock equity release benefits and enhance product expansion. Additionally, Jefferson, Austen, Ong, Haffner, and Wood (2017) analyze the barriers and reluctant behaviors that Australians might encounter when purchasing an ERS.

United Kingdom and Ireland

The UK has one of the most sophisticated and largest equity release market among European countries (Gwizdala, 2015). With around 40 suppliers, equity release products in the UK mainly take two forms: Lifetime Mortgage (a.k.a. Reverse Mortgage) and Home Reversion Plan. These products must comply with specific regulation required by the Equity Release Council (Fondazione Cariplo, 2017).

A recent empirical assessment over UK demand and supply components has been run by French et al. (2018) which sheds light on equity withdrawal behavior among British seniors.

With respect to Northern Ireland, Fiona Boyle Associates (2010) have presented an overview of various equity release forms and their availability.

Ireland also plays an important role in the European equity release market. The market rules applied in Ireland present similar features as the ones applied in the UK market (Gwizdala, 2015).

2.2 Valuation of Reverse Mortgages

Growing literature has been concerned with risk valuation and pricing of reverse mortgage schemes. This section will explore different studies related to RM risk identification and product pricing. These will be then used as background and support for the subsequent theoretical modelling phase.

2.2.1 Risk Valuation Literature

Kumar, Divakaruni, and Sri Venkata (2008) identify the main risks impacting RMs: Crossover Risk, Longevity Risk, Anti-selection, Moral Hazard and Litigation. In this paper the Crossover risk is split into four different components: Occupancy Risk, Mobility Risk, Interest Rate Risk and Home Appreciation Risk. On the other hand, C. C. Lee, Chen, and So De Shyu (2015) have a different definition of Crossover Risk: Crossover Risk represents the risk of heaving a value of the outstanding balance greater than the value of the property. In such case, the Crossover Risk would comprise Longevity Risk. Moreover L. Wang, Valdez, and Piggott (2008) consider Crossover Risk as the result of Occupancy, Longevity, Interest Rate, House Price, Maintenance and Expenses Risk. On the other hand Alai, Chen, Wanhee Cho, Hanewald, and Sherris (2014) identify the risks that might cause the loan balance to exceed the property value as being: Termination Risk, Interest Risk and House Depreciation Risk. In such case, Termination Risk would comprise Mobility and Longevity Risk.

The following lines will present literature that explores RM Crossover Risk, Termination Risk, Longevity Risk, House Price Risk and Interest Rate Risk individually.

Crossover Risk Literature

The Crossover Risk identifies the risk that the loan's outstanding balance at termination will be higher than the collateral value, since the lender can only recover the proceeds of the sale of the house. An increase in the crossover risk can be determined by an increase in lifespan, an increase in the interest rate at which the loan interest accumulates and through house value depreciation (H. Chen, Cox, & S Wang, 2009). Multiple pricing models for crossover risk have been proposed, for instance, by Huang et al. (2011), Chinloy and Megbolugbe (1994) and H. Sun (2015).

Termination Risk Literature

The termination risk implies that the borrower may unexpectedly end the contract. Ji, Hardy, and Li (2012) have assumed that the termination of a reverse mortgage contract could be triggered by three different factors. The first termination state is attributed to death. The remaining states occur when the home is sold and the loan paid back due to either entrance into a long-term care facility as a consequence of health deterioration or move out for other non-health related reasons, i.e. refinancing and downsizing.

Various multistate models for termination risk have been studied and empirically applied by Szymanoski, Enriquez, and DiVenti (2007), D. Cho, Hanewald, and Sherris (2015) and Sherris and Sun (2010). And, in particularly Ji (2011) developed a solid Markovian multiple state termination

risk model that jointly encompasses all drivers that might lead to unexpected contract cessation.

Longevity Risk Literature

The guarantee implied by equity release contracts inherit longevity risk (Ji et al., 2012). This is even more evident when the capital disposal is in the form of an annuity (Y. T. Lee, Wang, & Huang, 2012). Y. Chen and Wu (2014) also observe that the interest and principal payment of the loan can get much higher than the value of the property when the borrower's life expectancy unexpectedly increases determining the risk of delayed termination. Therefore, reverse mortgages are longevity-dependent products and assessing longevity risks becomes crucial for RM lenders (Yang, 2011).

Numerous techniques can be applied to provide an estimation of population longevity (see, for instance, Booth and Tickle (2008), Cairns, Blake, and Dowd (2008), Mitchell, Brockett, and Muthuraman (2013)). In particular, the effects of longevity risk models on reverse mortgage evaluation are analyzed within different studies, e.g. Y. T. Lee et al. (2012), J. L. Wang, Hsieh, and Chiu (2011), D. Cho et al. (2015) and Sherris and Sun (2010).

House Price Risk Literature

The value of the collateral might be subject to depreciation. And when the RM provider sells the property, it might have a lower price than anticipated. Thus, the lender has to account for the risk of real estate prices not conforming to expectation. To generate a risk measure for house price risk and evaluate profitability, a predictive model is required.

Hanewald and Sherris (2011) analyze different house pricing evaluation frameworks. Typically, in the evaluation of equity release programs, two types of models are most widely used: the Vector Auto-Regressive (VAR) methodology and its extensions (D. W. Cho, Hanewald, & Sherris, 2013; Shao, Sherris, & Hanewald, 2012) and Brownian-based techniques (J. L. Wang et al., 2011; Sherris & Sun, 2010). Alternatively, C. C. Lee et al. (2015) employs a Poisson based model to explain house price risk and simultaneously interest rate risk.

Moreover, hitherto there appears to be no literature over Artificial Intelligence or Machine Learning with specific application to RM valuation. Nonetheless, research over these methodologies applied to property price prediction has been available since the '90s, e.g. Wiśniewski (2017), Chaphalkar and Sandbhor (2013) and Visit, Christopher, and Lee (2004). Not to mention the impressive advancement made in technological applications, i.e. mobile application that produce real estate price prediction (Aaron & Deisenroth, 2015; Borde, Rane, Shende, & Shetty, 2017).

Interest Rate Risk Literature

Variation in interest rates generate additional uncertainty for RM providers (H. Chen et al., 2009). As such, measuring interest rate risk is required when releasing RM instruments into the market. Along real estate valuation the VAR techniques can also model other economic variables among which interest rate. Another popular model in reverse mortgage risk valuation is the Cox-Ingersoll-Ross (CIR) model (Boehm & Ehrhardt, 1994; Y. T. Lee, Kung, & Liu, 2018; J. L. Wang et al., 2011).

Literature over stochastic interest rate risk measurement models is extremely vast. Among others, Huang et al. (2011) uses the log-normal model to describe the interest rate risk when evaluating the crossover risk. And, with specific application to RM valuation, Zheng and Xikun (2016) have proposed a Poisson distribution based model.

Neural networks have also been applied in the field of interest rate modelling (Abid & Ben Salah, 2002). Although, neural networks have been mostly designed for a broader usage in risk management and financial forecasting (Maciel & Ballini, 2010; Lazo, Pacheco, & Vellasco, 2002; Toulson, 1996; W. Sun, Rachev, Chen, & Fabozzi, 2008; Bahrammirzaee, 2010), these studies can be effectively adapted to generate predictions for interest rate time series.

No-Negative-Equity-Guarantee Literature

Either for regulatory or commercial reasons, most RM contracts are sold with a Non Negative Equity Guarantee (NNEG), a.k.a. non-recourse clause in the US RM market. This clause protects the borrower from owing to the lender more than the property value (Y. T. Lee et al., 2018). Consequently, to recover the loan the RM provider cannot claim more assets than the value of the collateral (Y. T. Lee et al., 2012). The structure of the guarantee resembles a European put or call option. As a result, many studies have used the Black & Scholes formula for valuation purposes (H. Chen et al., 2009; Ji, 2011; J. S.-H. Li, Hardy, & Tan, 2010). Others preferred stochastic discount factors that would reflect house price, interest rate, inflation and rental risk to compute the present value of cash flows (Alai et al., 2014; D. W. Cho et al., 2013; Shao et al., 2012). While C. W. Wang, Huang, and Lee (2014) chose a close formula approach to model the Loan to Value (LTV) ratio from which the NNEG could be consequently derived. And, Andrews and Oberoi (2015) computed the loan's initial value as dependent on an NNEG charge that would be determined through simulation. Other risk-neutral valuation approaches have been developed by Kogure, Li, and Kamiya (2014) and Kim and Li (2017).

THE REVERSE MORTGAGE MARKET

To provide valuable contribution, the quantitative framework has to be put into practice to generate suitable outputs to be then used in RM lending policies. As such, this chapter will attempt to preliminary explore the main features of the largest RM markets and subsequently it will provide an outline of the reverse mortgage regulation and market in Italy. This will grant critical information when designing a structure for the ultimate modelling process.

3.1 Worldwide Market

The RM market has been having particular success in Anglo-Saxon countries. Specifically in the US, the RM program started in 1963 in Oregon with mortgage loans aimed at poverty reduction among seniors. And, in 1979 the Reverse Annuity Mortgage scheme has been also introduced in California. Still, the largest RM plan began in the '80s with the federal HECM. Further consolidation was achieved in the 2000s when the RM market reached a rate of 2% of mortgage loan plans. The minimum age to access these programs in the US is set at 62. The payment of the loan can be done through a lump sum, periodical installments, in the form of line of credit or a combination of all the above. The amount of the loan will depend on the entire value of the property or a percentage of it. The loan, service commission, expenses and accumulated interest are reimbursed at termination, e.g. death of the borrower. To protect heirs, the loan balance cannot exceed the market value of the property, i.e. NNEG. Moreover, the RM programs are classified into two different types: conforming and non-conforming. The conforming RM, among which the HECM, can only be offered by institutions authorized by the FHA and cannot exceed a predefined fixed sum. The second RM category has no regulatory maximum amount and can only be delivered by private companies with no federal guarantee.

The Canadian market shares similar features with the US market. Whilst the RM products are less popular among elderly Canadians, the demand has exceed initial estimates and is predicted to grow at a rate of 25-30% (HomeEquity Bank, 2015). The federally regulated HomEquity Bank is the sole national RM lender that supplies a form of Equity Release called Canadian Home Income Plan (CHIP). This contract provides a loan of up to 50% of the home value to over 55 years old seniors without any credit scoring or income requirements. Typically, to provide longevity

and house pricing risk coverage the interest rate, which is annually updated, is above ordinary mortgage rates.

The Equity Release plan in the UK is not particularly new. The first product was introduced in the '60 but due to lack of transparency, consumer trust and overwhelming complexity there has been no particular market expansion. As a consequence, in the '90s equity release providers founded the Safe Home Income Plans, today Equity Release Council, to promote qualitative market standards to safeguard consumers. Among these standards, the NNEG and the possibility of living in the property up until death were introduced. Furthermore, in 2004 a government authority, the Financial Services Authority, also started regulating and controlling the equity release market. This has positively impacted the market and has considerably raised demand. And, starting from the same year insurance companies began marketing equity release products. Thereafter, insurance companies have been progressively substituted banks as the main providers.

The North American RM equivalent in the UK is referred to as Lifetime Mortgage. The minimum age is set by the Equity Release Council at 55 and it allows a home owner to take a loan in the form of lump sum, periodical installments and draw-down income against a mortgage on the home. When the homeowner has a spouse, the contract is considered terminated only if both spouses are deceased. Interest-Only Mortgage and Fixed-Repayment Mortgage are two different forms of Lifetime Mortgage. An Interest-Only Mortgage is provided against monthly payments of the accrued interest. While the Fixed-Repayment Mortgage does not accrue any interest whatsoever in exchange of a contracted higher loan repayment at termination.

In the early '90 the Australian government started subsidizing a form of equity release. Nevertheless, only in the 2000s RM contracts took off and still today represent the most common form of ERS. Among these, several schemes have been specifically designed to aid funding costs traditionally covered by the government, e.g. South Australia's Seniors Rate Postponement Scheme. In 2014 the estimated RM market size was around \$3.66 billion AUD with a 1.5% penetration rate and four active loan providers, the majority of which are banks.

Within continental Europe an instrument similar to the Anglo-Saxon RM, Hipoteca Inversa, has been regulated with Law 41\2007 (Sánchez, 2009). In Spain the loan can only be provide to over 65 or severely disabled seniors. Also in this case, the NNEG is mandatory. Typically, the Hipoteca Inversa is complemented by a lifetime annuity that can only be provided by an insurance company.

In France a type of RM, prêt viager hypothécaire, has been introduced in 2008. The minimum age is set at 65 and can only be provided by banks. To date, only one institution, Crédit Foncier, operates on the French RM market.

3.2   Italian Market

RM products have only been adopted by financial institutions in Italy in 2005 and go under the name of Prestito Vitalizio Ipotecario (PVI). To date, these products didn't achieve any particular success. Still, the current demographic, retirement system and housing conditions pave the way to future demand expansion.

3.2.1 Regulation

PVI plans have been originally regulated in Italy through Law n. 248\2005 and Legislative Decree 20/09 /2005 n. 203. This legislation has been revisited in 2015 with Law n. 44 \2015 and its implementation regulation D.M. 22/12/2015, n. 226. This legal framework dictates several mandatory common rules for all institutions that are willing to commercialize RM plans in Italy. The implementation regulation has three Articles. The first two articles define the contractual class and frames the general standards. While the third article draws the rules for unilateral termination rights for the loan provider when the value of the house suffers severe depreciation. The definition provided by the 2015 legislation is based on the Anglo-Saxon RM market experience: medium - long term loan against a mortgage on residential housing provided by banks and other authorized financial intermediaries. The law imposes a minimum age for loan access of 60 years old[1] and the interest and expenses have to be capitalized annually. The contract has to be signed by both spouses if they are both over 60 and legally married for at least 5 years even when only one spouse is the legal owner of the estate. The loan repayment can consist of a final repayment at termination or periodical repayments of interests or of the entire loan's outstanding balance. The termination of the contract can occur: upon death of both spouses[2]; when property or housing rights are transferred to a third party; when the property has been significantly modified without the lender's approval; when the estate has been intentionally or with serious misconduct damaged by the borrower; when family members other than the signatories take up residency in the home; when the property undergoes enforcement proceedings for more than 20% of its value. Upon death of both spouses the heir will still inherit the property. If in the subsequent 12 months the outstanding loan, interest and expense balance will not be repaid, the lender can recover the proceeds from the property. In these 12 month the heir can himself sell the property and repay the loan.

To protect consumers, the non-negative equity guarantee is mandatory. And for this reason, the contract has to include simplified estimated funding scenarios that comprise every expense due at contract date for a number of years equal to the difference between 85 and the age of the youngest signatory at contract data with a minimum of a 15 year projection[3]. When the interest rate is variable the scenarios have to be at least two: a base line scenario at current interest rates and a worst case scenario with a 300 points rise in interest rates.

3.2.2 Product Offer

Despite Reverse Mortgages being envisioned as a suitable tool to alleviate poverty in Italy, in 2017 the amount of total loans were just around a few hundred million euros. The first two PVI providers were EUVIS and Monte dei Paschi di Siena. Along several major banks today PVIs have also been commercialized by smaller financial service institutions. Among these, it is possible to find such products offered by Unicredit, Deutsche Bank, Intesa Sanpaolo, BNL, Monte dei Paschi di Siena, Barclays, Banca Popolare Sondrio and Imprebanca.

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[1] In 2005 this minimum age was set at 65.

[2] When both spouses are signatories.

[3] In formulae: max{85-age youngest spouse, 15}.

The main PVI contracts are typically supplied to seniors that have a minimum age of 60 or 65 and a maximum age of 85, 90 or 100. The products commonly have a minimum and a maximum amount for the initial loan. And frequently, this loan presents itself as an upfront sum. Still, Monte dei Paschi di Siena offers the borrowers the option to receive an annual sum for a maximum of 20 years. This annual installment considers a fixed interest rate for its computation. While the interest rate accumulating over the loan amount can be fixed or variable, typically linked to IRS or $EURIBOR_{3M}$ rates.

In most contracts the reimbursement is due at termination. However, some products provide an option for the borrower to pay annually or monthly the interest accrued. A penalty might be due if the homeowner decides to repay the loan before a fixed term. The amount of the loan is normally a percentage of the value of the house, i.e. LTV. This percentage varies from contract to contract and generally tends to increase with the age of the borrower at contract stipulation date.

FOUR

RISK DRIVERS

In the previous lines we have seen that the valuation of PVI contracts depend on several risk factor: mortality, refinancing, interest rate, inflation and house prices. In the following lines we will attempt to model termination risk, which will simultaneously shape mortality and refinancing risk, thus providing also for correlation modelling. Separately, we will build a model that encompasses interest rate, inflation, property pricing risk and their inter-dependencies. This is a consequence of the common actuarial assumption of independence between demographic and economic contingent factors. The only interaction among these risk drivers occurs within catastrophic risk, which for the purpose of this investigation will be ignored.

4.1 Neural Networks

To develop a model predicting time-dependent risk factors we will employ deep learning algorithms, specifically Artificial Neural Networks (ANN).

Artificial Neural Networks were originally designed by McCulloch and Pitts (1943) and Wiener (1948) to mimic the functioning of neurons within the human brain. Still, the very first attempt to use this architecture for classification machine learning purposes was made by Rosenblatt (1958) paving the way to the introduction of the Widrow - Hoff ADALINE algorithm (Widrow & Hoff, 1960).

Werbos (1974) solved the problem of Multilayer Perceptron (MLP) propagation conceived by Minsky and Papert (1969). This led a decade later to new and robust architectures for both classification and regression (Rumelhart, Hinton, & Williams, 1988; Widrow & Lehr, 1990).

To briefly present the intuition behind ANNs, we will employ the framework provided by Kriesel (2007).

The simplest neural network version is referred to as Single Neuron Perceptron. Its structure can be visualized below:
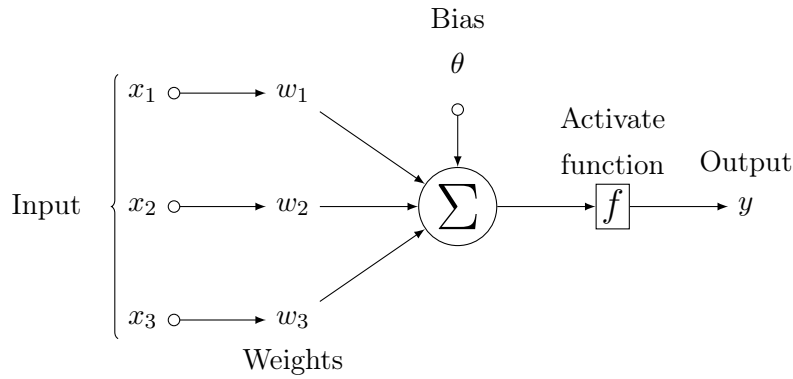
Figure 4.1: Single Neuron Perceptron

This model applies an activation function, $f()$, to the the sum of the weighted input vector, $\mathbf{x} = \{x_1, x_2, x_3\}$, and a bias term, θ, to create a final output, y.

In mathematical terms, an output neuron will be given by:

$$y = f\left(\sum_{i=1}^{3} w_i x_i + \theta\right). \tag{4.1}$$

The ANN is characterized by a learning phase where it "learns" the problem by modifying the weights w_i of its synapses through the above learning rule.

The architecture can be transformed to capture a more complex configuration of the inter-relation between multiple inputs and also their relationship with a single output. Figure 4.2 illustrates the case in which the structure of the neural network has one layer of multiple neurons between the input layer and output layer, i.e. hidden layer with more than one neuron.
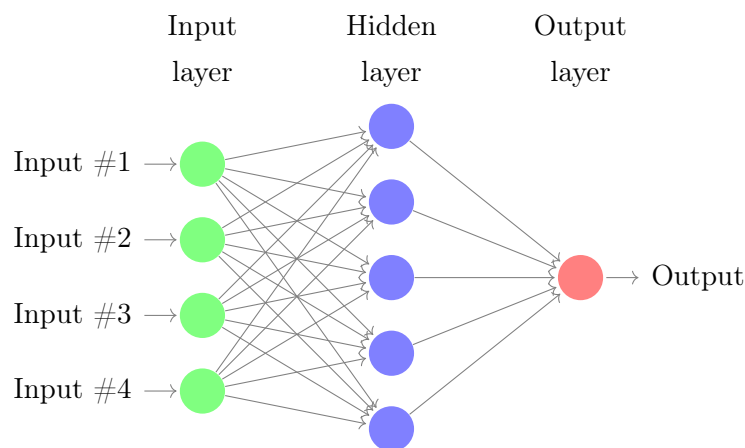

Figure 4.2: Neural Network with One Hidden Layer

The mathematical equivalent would be:

$$y = f(\mathbf{WX}), \tag{4.2}$$

where $\mathbf{W}$ and where $\mathbf{X}$ represent the weight and input matrices respectively. The weights w_{ij} are updated for each hidden neuron based on error back-propagation. The number of neurons in the

hidden layer and the number of hidden layers mainly depend on the complexity of the problem and nature of the data (Kabir & Ahsan Akhtar Hasin, 2013).

Some problems might require to deliver n different regression outputs starting from an input vector of cardinality m. In other words, we will need to transform an m-dimensional input space into an n-dimensional output space:

$$\mathcal{R}^m -> \mathcal{R}^n, \tag{4.3}$$

Figure 4.3 illustrates a neural network with 4 input neurons, one single hidden layer with 5 neurons and 3 output neurons.
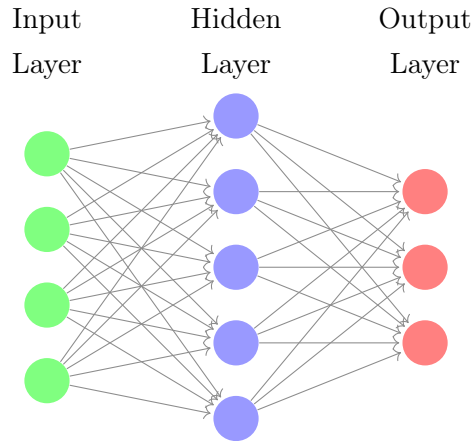

Figure 4.3: Multiple Output Neural Network with One Hidden Layer

In this case, the generic output y_k will be the result of (Bishop, 1995):

$$y_k = \tilde{f}\left[\sum_{j=1}^{t} \beta_{kj} f\left(\sum_{i=1}^{m} w_{ji} x_i\right)\right], \tag{4.4}$$

with w_{ji} as the weight between input i and hidden neuron j and β_{kj} as the weight between hidden neuron j and the output neuron k.

For MLPs (see Figure 4.4) back-propagation is the most notable learning rule (Le Cun, 1986; Parker, 1985). This technique propagates backwards the errors of the output neurons to the hidden neurons. This is particularly useful when the correct solution for the neurons is unknown, therefore we cannot directly calculate the error.

The back-propagation method is defined in two steps: one forward step and one backward step. Within the forward step, the outputs of the network are calculated propagating the inputs through the neurons of the network, while the weights of the connections remain unchanged.

The backward step consists of a two sided modification of the weights of each connection as follows:

- For the output neurons, the weights will be modified through the Delta Rule. More specifically, the weights of the connection between the last hidden layer and the network's output layer will be iteratively re-calibrated by comparing the calculated output with the desired output and generating an error through an error function, e.g, mean squared error. The pro-

cess will terminate when the error function will be lower than a predefined value (Widrow & Hoff, 1960).

- For the hidden neurons, the remaining weights will be re-determined by propagating backwards the above-mentioned error assuming that the error attributed to one neuron is the sum all errors in all subsequent directly connected neurons.

The weight between a generic neuron i and a generic neuron j are at each stage updated through:

$$w_{ij} = w_{ij} + \Delta w_{ij}, \tag{4.5}$$

with Δw_{ij} depending on the error function E:

$$\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}}. \tag{4.6}$$

The above equation implies that the algorithm moves towards a solution by means of the gradient descent method where a specified learning rate, η, is employed as movement length (Ranganathan, Nakai, & Schonbach, 2018). In practical terms, the weights are continuously modified in the negative direction of the gradient of the error. The algorithm will stop and the weights will no longer be modified when the minimum of the error function is reached, i. e. the derivative of E is zero.
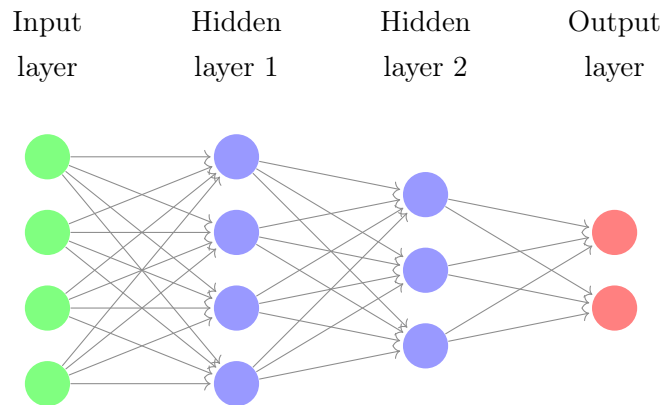

Figure 4.4: Feed-Forward Neural Networks
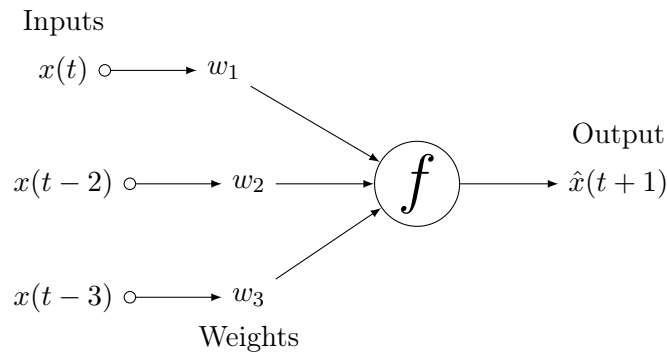
4.1.1 Recursive Neural Networks

Figure 4.5: Single Neuron Perceptron with Time Dependent Input Data

The ANN results inadequate to forecast time sequential data sets, where each element in the input vector will depend on the time it was measured, as illustrated in Figure 4.5. This means that the input at time t will have a certain dependency to the input observed at time $t-1$.

Up until now, we have examined neural networks where the information flows in one single direction, from the input to the output, i.e. feed forward neural networks. Still, it is possible to build neural networks where the information moves in one direction but also in the opposite direction generating loops which allows for information to persist. These are referred to as Recursive (or Ciclic) Neural Networks and were discovered by Hopfield (1982), Rumelhart, McClelland, and PDP Research Group (1986). Recursive neural networks result particularly effective when dealing with time series, provided their ability to learn sequentially (Gamboa, 2017).

Figure 4.6 and Figure 4.7 illustrate the transition of information within the structure of a vanilla Recursive Neural Network and within a generic Recursive Neural Network architecture respectively.
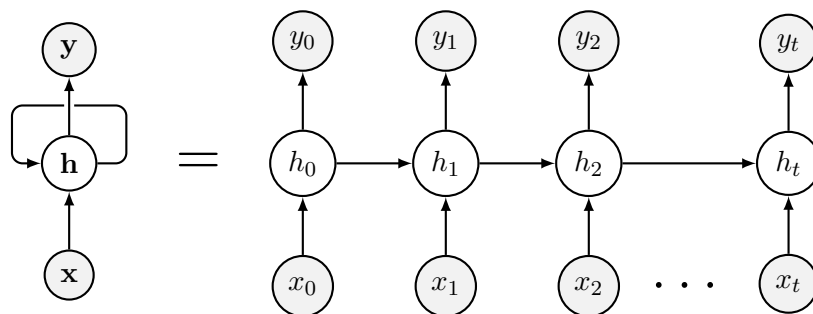

Figure 4.6: Unfolded Recurrent Neural Network with one input, one output, one hidden layer and one recurrent layer

In the above figure, a network $\mathbf{h}$ captures the information from a sequential input $\mathbf{x}$ and generates an output $\mathbf{y}$. The network $\mathbf{h}$ consists of a collection of states $\{h_0, h_1, h_2, \ldots, h_t\}$. These initially

read each input incrementally and subsequently the network constantly loops all over again while keeping memory of all the processed information.

Each state h can contain different neurons within different layers. Figure 4.7 shows an example of Recursive Neural Network (RNN) where all states hold three interconnected layers, each with n different neurons. Moreover, the final layer of a certain state at time t will be connected with the first layer of the subsequent state at time $t + 1$.

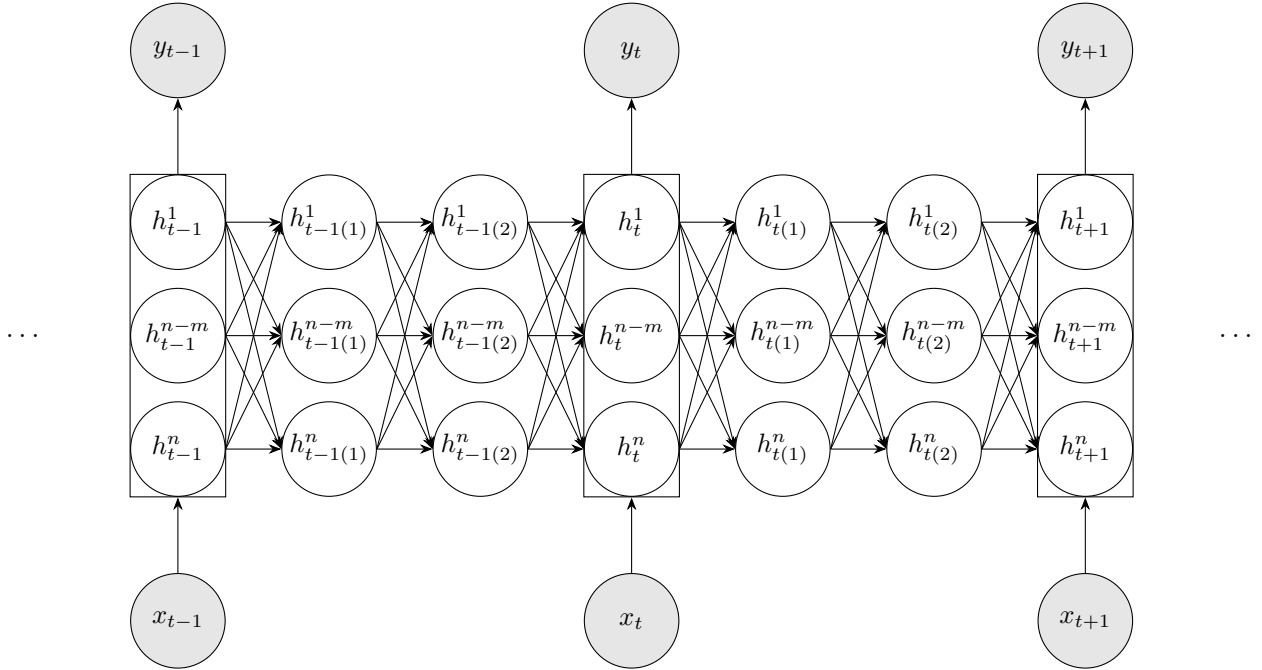

Figure 4.7: Unfold Recurrent Neural Network with multiple inputs, outputs and layers

The output at current time step, y_t, of a vanilla RNN, provided the input at current time step, x_t, and the hidden state, h, at time step $t - 1$, can be defined as (Finsveen, 2018):

$$y_t = f(L[x_t, h_{t-1}]), \tag{4.7}$$

$$L[x_t, h_{t-1}] = \mathbf{W}_1 x_t + \mathbf{W}_2 h_{t-1} + \epsilon. \tag{4.8}$$

Where L is a linear transformation function, f is typically a non-linear function, $\mathbf{W}_1$ and $\mathbf{W}_2$ are weight matrices and ϵ is the remaining bias.

While for a MLP RNN dynamic, given layer l and time t, the state transition from the previous to the current neuron can be defined as a function (Zaremba, Sutskever, & Vinyals, 2014):

$$RNN : h_t^{l-1}, h_{t-1}^l \to h_t^l, \tag{4.9}$$

with:

$$h_t^l = f(L[h_t^{l-1}, h_{t-1}^l]). \tag{4.10}$$

4.1.2 Long-Short Term Memory Neural Networks

One problem that frequently arises when using RNN for time series forecasting is their "forgetfulness". They are unable to record long term dependencies and typically retain unnecessary

information (Bengio, Simard, & Frasconi, 1994; PetnehÁzi, 2018). A variant of RNN, the Long Short Term Memory (LSTM) network, was introduced by Hochreiter and Schmidhuber (1997) to memorize long term patterns when necessary, recall what it needs to be recalled and learn only what needs to be learned.

A LSTM layer can be composed by one or more LSTM units. Within each unit the LSTM structure consists of a new hidden state that will act as the memory of the networks, i.e. cell. The memory of the cell can be altered through a single neural network layer called gate. The weights of these networks need to be learned during the training phase.

The gates of a LSTM will enable the process of retaining the necessary memory from the current input, forgetting unnecessary patterns and generating a current output based on the present and past memory. Each LSTM unit will receive three inputs: the previous short term output, the long term memory cell state generated by the previous unit and the current input. Subsequently the forget gate, output gate and input gate along with additional gates that perform regularisation will process the information and deliver a long term memory cell and a short term memory output.

Formalizing, an LSTM unit receives at a generic time t a cell c_{t-1} and the short term memory output h_{t-1} from the previous LSTM unit and the current input x_t. Provided the parameters $\mathbf{W}_1$, $\mathbf{W}_2$ and ϵ_f, the forget gate will generate an output f_t through the sigmoid function, σ:

$$f_t = \sigma(\mathbf{W}_{1,f} x_t + \mathbf{W}_{2,f} h_{t-1} + \epsilon_f), \tag{4.11}$$

and this defines the unnecessary memory to be erased (Goel, Melnyk, Oza, Matthews, & Banerjee, 2016).

At this point we can determine what memory needs to be added to the new cell state c_t as follows:

$$i_t = \sigma(\mathbf{W}_{1,i} x_t + \mathbf{W}_{2,i} h_{t-1} + \epsilon_i), \tag{4.12}$$

$$\bar{c}_t = \tanh(\mathbf{W}_{1,c} x_t + \mathbf{W}_{2,c} h_{t-1} + \epsilon_c), \tag{4.13}$$

$$c_t = i_t \odot \bar{c}_t + f_t \odot c_{t-1}, \tag{4.14}$$

where $\odot$ is the Hadamard element-wise product and $\bar{c}_t$ represents a candidate state.

The last missing element is the output of the cell state at time t, h_t, which can also take the form of the final output of the LSTM structure, y_t. This is defined through the output gate as below:

$$o_t = \sigma(\mathbf{W}_{1,o} x_t + \mathbf{W}_{2,o} h_{t-1} + \epsilon_o), \tag{4.15}$$

$$h_t = o_t \odot \tanh(c_{t-1}). \tag{4.16}$$

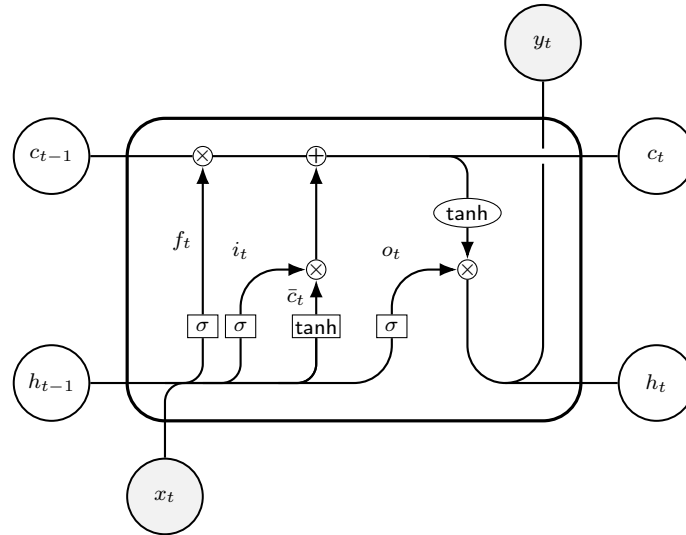Figure 4.8 illustrates the structure of a single LSTM unit.

Figure 4.8: LSTM unit diagram

4.1.3 Monte Carlo Dropout

The typical actuarial *modus operandi* implies predictive uncertainty estimates. In deep learning there is no clear way to determine Bayesian estimates. Gal and Ghahramani (2016) proposed to train a model with a variational dropout method to extract Monte Carlo samples. This will allow for inference on the posterior distribution of the neural network weights provided a prior distribution of the weights.

Given Bayesian theory, we can determine a posterior probability of an observed vector of weights of a generic neural network, ω, given data $\mathbf{X}$, as (Shahroudi, 2019):

$$p(\omega|\mathbf{X}) = \frac{p(\omega|\mathbf{X})p(\omega)}{p(\mathbf{X})}, \tag{4.17}$$

where $p(\omega|\mathbf{X})$ is the probability of $\mathbf{X}$ conditional to the current weights, $p(\omega)$ is the prior distribution of the weights and $p(\mathbf{X})$ is the probability related to the input data.

The probability of observing one single point, $\bar{x}$, provided all inputs and parameters of the model can be determined through:

$$p(\bar{x}|\mathbf{X}, \omega) = \int p(\mathbf{X}|\omega)p(\omega|\mathbf{X})d\omega. \tag{4.18}$$

Let $\mathbf{X}$ and $\mathbf{Y}$ be two separate data sets, the prediction of a point $\bar{y}$ given a new data point $\bar{x}$ will be determined as below:

$$p(\bar{y}|\bar{x}, \mathbf{X}, \mathbf{Y}, \omega) = \int p(\bar{y}, \bar{x}|\omega)p(\omega|\mathbf{X}, \mathbf{Y})d\omega. \tag{4.19}$$

Typically, the prior distribution of the weights is assumed to be distributed as a standard normal random variable for each layer and the bias vector is presumed to take the form of a point estimate.

The dropout (at training time) is a process that randomly ignores neurons within a specified set and it generally serves as a regularisation tool to prevent over-fitting. However, Gal and

Ghahramani (2016) performed a dropout process in the form of variational inference, i.e. at test time, to approximate the prediction. Consequently, the predictions at test time will no longer be deterministic. But, the output will depend on which neurons were ignored. Thus, by performing dropout at test time and assuming a Normal distribution for the weights, we can approximate $p(\bar{y}|\bar{x}, \mathbf{X}, \mathbf{Y}, \omega)$ as an average of the results, i.e. Monte Carlo Dropout (MCD):

$$p(\bar{y}|\bar{x}, \mathbf{X}, \mathbf{Y}, \omega) \approx \frac{1}{T} \sum_{t=1}^{T} p(\bar{y}|\bar{x}, \hat{\omega}_t), \tag{4.20}$$

where $\hat{\omega}_t$ represents the estimated weight vector.

4.2 Termination Risk

Termination Risk is a risk related to the uncertainty of the PVI contract term. Different situations might cause unexpected termination, e.g. downsizing, refinancing and significant intentional damage to the property.

However, this product has been designed to finance old age income streams and thus to cease upon death of both signatory spouses. For such reason and provided the available information, we will consider longevity risk as the sole termination risk driver. This will require a model to fit and predict mortality. A particular stochastic technique has been dominating mortality modelling: the Lee Carter Mortality Model (LCMM) (R. D. Lee & Carter, 1992). This will be used to determine Italian mortality components.

The necessary input mortality data for the Italian population is retrieved form the Human Mortality Database (2019). Additionally, we will adopt and expand the model through the LSTM approach developed by Richman and Wüthrich (2018), Nigri, Levantesi, Marino, Scognamiglio, and Perla (2019). The framework will be employed to forecast the time dependent parameter of the LCMM. All these methodologies will be again expanded by adding MCD.

4.2.1 The Lee Carter Mortality Model

The LCMM is the current mortality forecasting standard model. It designs a structure for the central mortality rate, $m_{x,t}$. This is the proportion of deaths on the exposed to death recorded for age x during year t. Its architecture was derived from averaging log mortality rates and applying Support Vector Decomposition (SVD) to the residuals to achieve the following form:

$$\log(m_{x,t}) = \alpha_x + \beta_x k_t + \epsilon_{x,t}, \tag{4.21}$$

where α_x, k_t and β_x are the parameters of the model which can be interpreted as the logarithm of the geometric mean of empirical mortality rates, the time trend of mortality and the age-specific mortality deviations from the overall trend, respectively. Whereas, $\epsilon_{x,t}$ represents the random effects of age and time which in traditional time series models is formulated as a series of independent normally distributed random variables.

Additionally, to achieve unique solutions the following constraints are necessary:

$$\sum_x b_x^2 = 1 \quad \text{and} \quad \sum_t k_t = 1. \tag{4.22}$$

R. D. Lee and Carter (1992) estimate parameters α_x, β_x and k_t through a two stage least square error approach. First α_x was estimated as:

$$\hat{\alpha} = \frac{1}{T} \sum_{i=1}^{T} \log(m_{x,t}) \tag{4.23}$$

and $\hat{\beta}_x$ and $\hat{k}_t$ were computed through a singular value decomposition of matrix $[\log(m_{x,t}) - \hat{\alpha_x}]$. If used alone, this procedure doesn't guarantee that the observed number of deaths equals the fitted number of deaths. Hence, a second estimation stage was introduced to fit k_t such that:

$$d_t = \sum_x \exp\{(\alpha_x + \beta_x k_t) \times E_{x,t}\}, \tag{4.24}$$

where d_t is the number of deaths at time t and $E_{x,t}$ is the population of age x exposed to death at time t.

The estimates of α_x, β_x and k_t using LCMM with the two-step procedure on yearly Italian mortality rates observed between 1872 and 2014 for ages between 0 and 110 are illustrated in Figure 4.9.
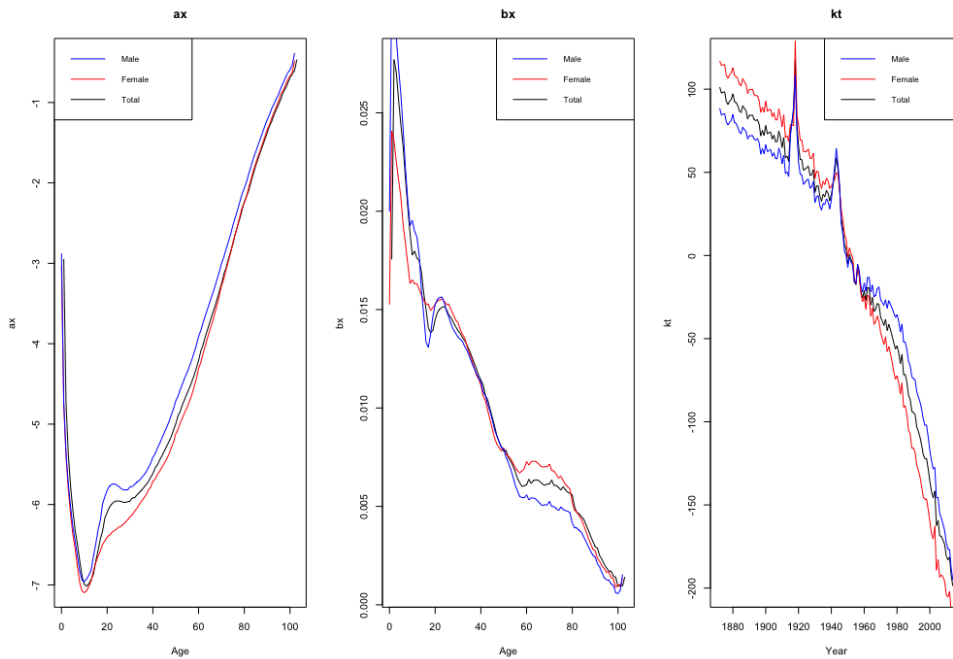
Figure 4.9: LCMM parameter estimates

The estimated k_t will serve as time series base for the forecasting technique outlined in the following section.

4.2.2 Single-Output LSTM Modelling

Provided the above theoretical framework, we've created a MCD LSTM model to forecast future values of k_t and consequently mortality rates separated by male, female and total population. To this end, first the univariate LSTM parameters have been calibrated with a number of neurons and training epochs depending on the gender modelled. The data set has been divided into an

80% training set and a 20% test or validation set. These two data sets were used to tune all hyper-parameters of the model, i.e. number of layers, number of neurons per layer and tuning epochs.

For this purpose, the error in terms of a pre-defined loss function produced by the LSTM model applied to the training set has been compared with the validation set error function. This allowed for over and under-fitting diagnosis. A higher loss function originating from the test set is typically characteristic of an under-fitted model. While, a lower test set loss function is symptom of an over-fitted network.

The validation set's error path would indicate whether capacity could be improved by re-calibrating neurons, hidden layers or training epochs. As such, when the validation set would produce a flat higher error amount, the number of hidden layers and/or neurons were increase. Alternatively, if the error function would present a lower value than the training set and a flat tendency, this would suggest an under-fitting diagnosis which could be improved by reducing hidden layers and/or neurons. If the error trend was decreasing for higher test set loss amounts or increasing for lower error values, the number of training periods would need to be raised[1].

Subsequently, we've generated 1000 random simulations using MCD modelling for the first future steps. These random simulations were used to generate each one path for 60 time steps which were then used in formula 4.21 to generate 1000 simulations for mortality rates for a given age. From these we've extracted the average value which would ultimately define the final prediction for the next 60 years. Figure 4.10 displays mortality for the input dates along with the predicted mortality for a population aged 65.
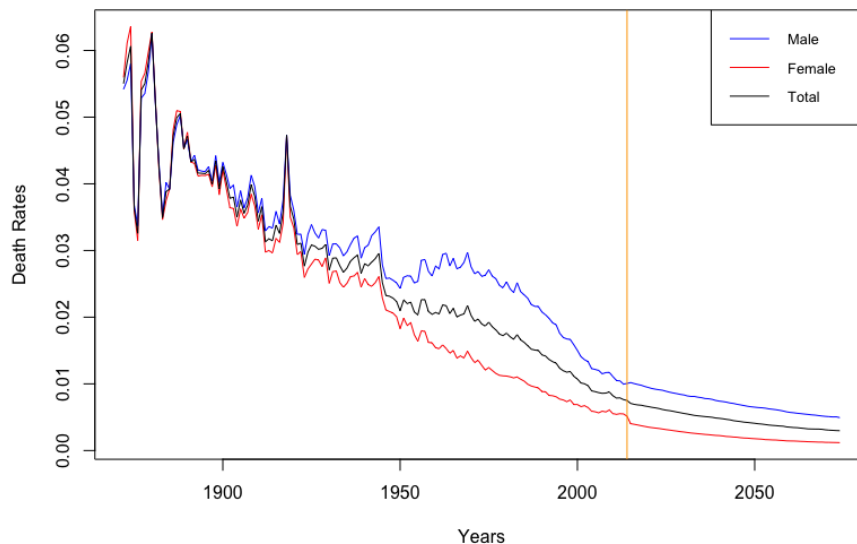

Figure 4.10: Mortality predictions for a population aged 65.

While the 1000 future mortality simulations are shown in Figure 4.11 separately by female and male population.

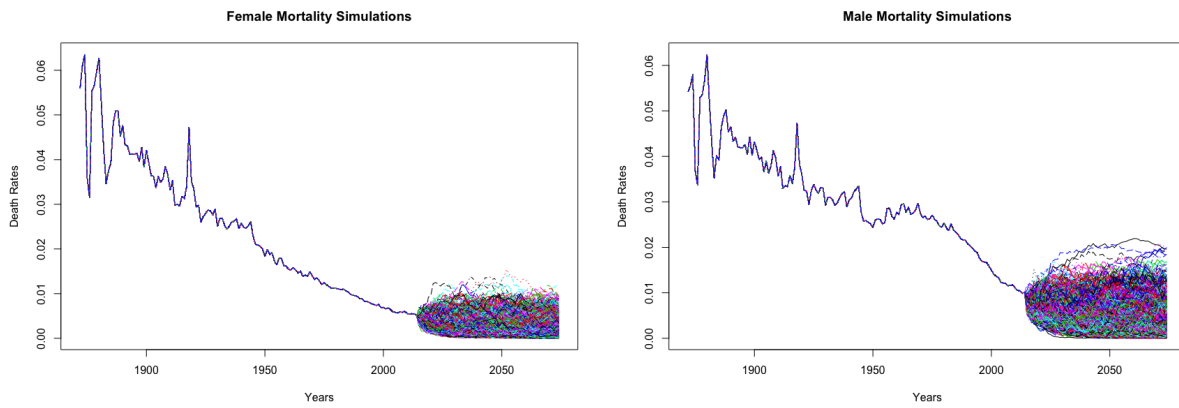---

[1]The optimal parameters are shown in Appendix A.

Figure 4.11: Mortality simulations for females and males both aged 65 displayed on the left side and on the right side respectively.

4.3   Financial Risk

Along with termination risk any RM contract presents a second class of risks: Financial Risks. These originate from the uncertainty within interest rates, mortgage rates and property appreciation or depreciation rates and, when the annuity is indexed to prices, i.e. inflation rates.

It has been proven that all these variables are correlated (Alai et al., 2014; D. W. Cho et al., 2013). In a VAR perspective that generates stochastic scenarios, these variables would be modelled together to take advantage of the their correlation when predicting future values. The VAR analysis would typically include also Gross Domestic Product (GDP) in the modelling phase to generate a more accurate prediction, provided its high correlation with the above-mentioned variables. Thus, we will employ all these variables to produce our financial risk model.

However, Verstyuk (2018) has shown that an LSTM approach outperforms the VAR technique when modelling economic variables. Therefore, a MCD LSTM perspective appears more suitable for our risk assessment problem.

The economic data set used to estimate the parameters of the predictive model is composed by:

1. House Price Index (HPI), quarter availability between Q2 2010 and Q1 2019 (ISTAT, 2019b);

2. Euro Area Interest Rate (IR) with maturity 30 years, daily availability between September 6, 2004 and June 20, 2019 (European Cntral Bank, 2019);

3. Consumer Price Index (CPI) for the entire Italian territory and products, monthly availability between January 1997 and June 2019 (ISTAT, 2019a);

4. GDP variation, annual availability between 2002 and 2019 (ISTAT, 2019c);

5. Loan Rate (LR), monthly availability between January 1995 and April 2019 (Banca d'Italia, 2019).

Provided the availability of HPI, the input data consists of quarterly variables for the periods between Q2 2010 and Q1 2019. Additionally, for homogeneity purposes the HPI, CPI and GDP

series were transformed into compounded variation rates.

The multi-variate predictive model built on an 80% training set and a 20% validation set, has three LSTM layers on top of each other, i.e. stacked LSTM, and one output layer. The hyper-parameter calibration process followed the same steps mentioned in the previous section. Depending on the loss function path and value, neurons, layers and training epochs have been increased or reduced accordingly[2].

In the simulation stage we've adopted again a similar procedure to the above mortality modelling. First, 1000 scenarios were generated for the initial future step with the MCD technique. These were used to produce each one path of 200 time steps, i.e 50 years.

Figure 4.12 shows present and future predicted values for HPI, GDP, CPI, LR and IR calculated as an average of all generated scenarios. These results show constant predictions of future developments. This is a consequence of insufficient data availability. The model assigns much higher importance to few newly observed data points and doesn't allow to perfectly capture financial variability. This is additionally confirmed when observing the 1000 simulations for each input variable illustrated in Figure 4.13 where all simulations appear confined within close boundaries. To improve performance more observations should be added.

The predictions obtained for mortality and financial risks will be used to generate pay-offs and risk assessments of various PVI contract structures. This process will be outlined in the following chapter.
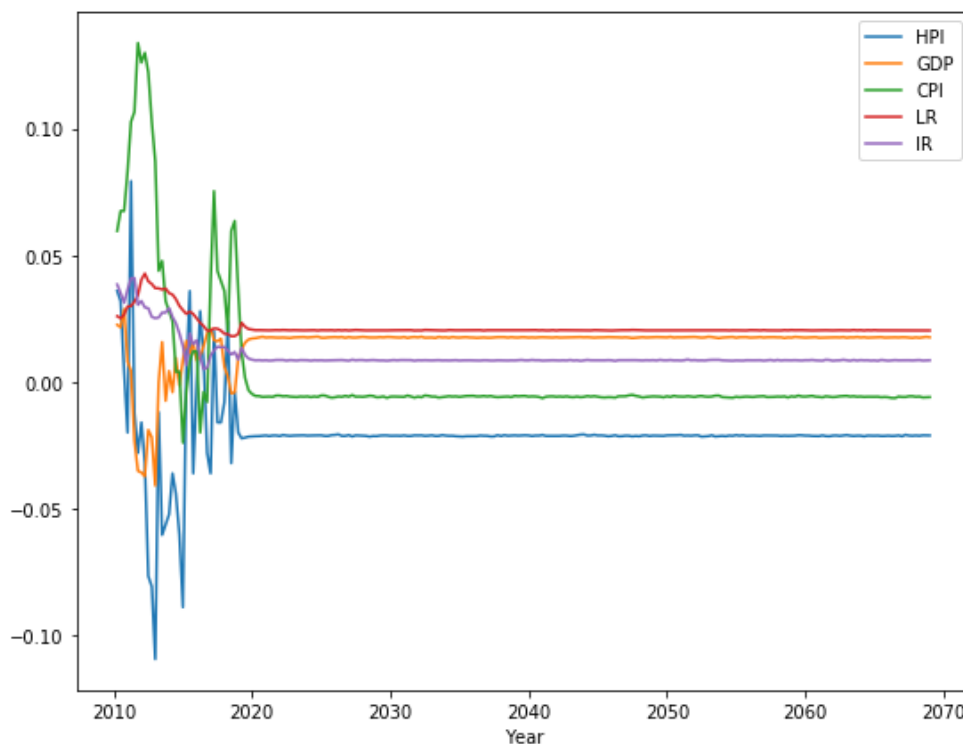

Figure 4.12: HPI, GDP, CPI, LR and IR predictions for 50 years.
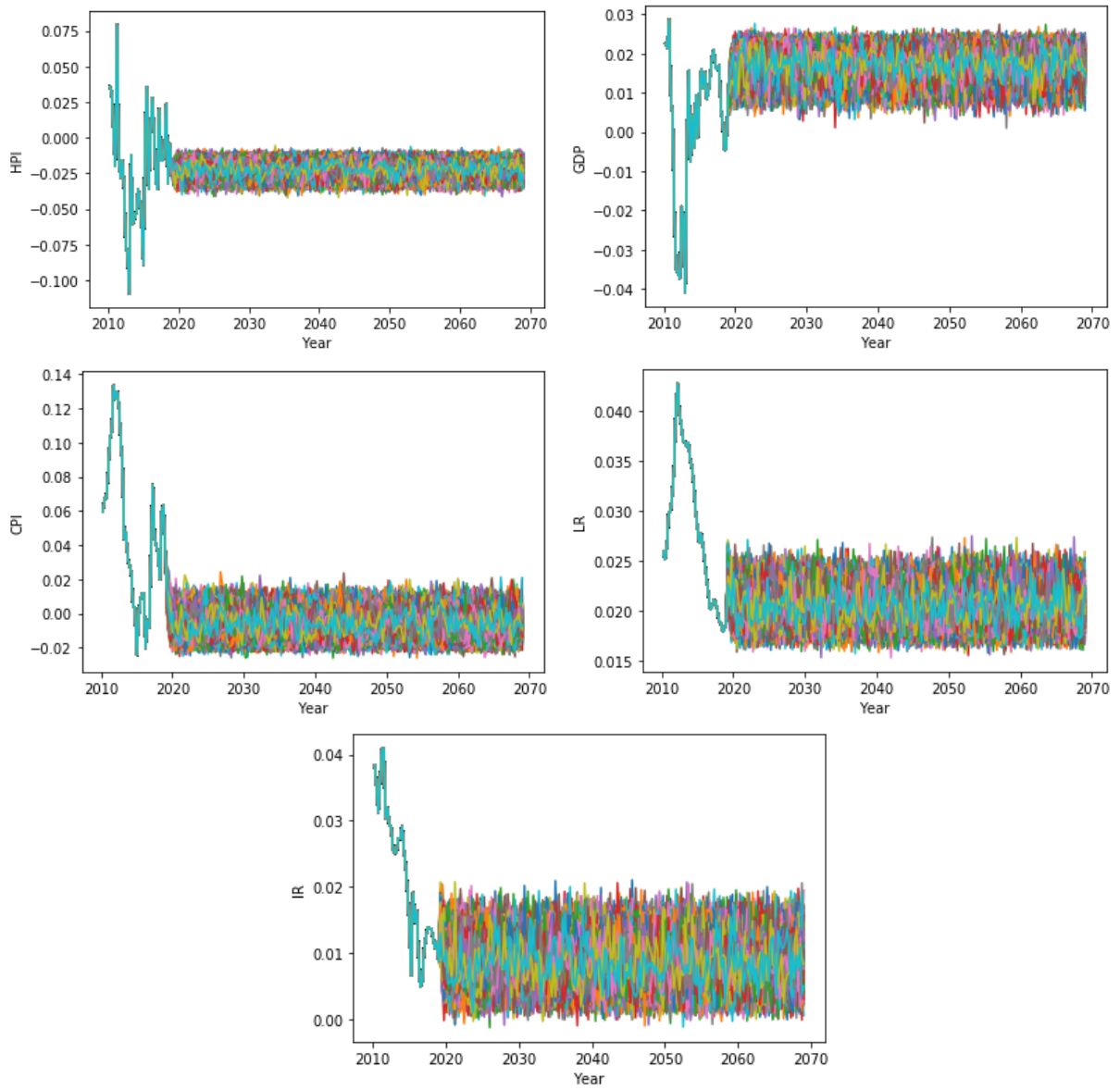
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[2]More details are illustrated in Appendix A.

Figure 4.13: Model simulations for HPI, GDP, CPI, LR and IR.

PRICING AND RISK ASSESSMENT

The loan payment of the PVI might take different forms. This will influence the determinants of the present value of all future cash flows originating from the contract and thus the pricing formula and risk profile. In this chapter we will first introduce a pay-off framework for various PVI schemes and subsequently we will assess the riskiness enclosed within these different structures.

5.1 Pricing and Pay-off Assessment

This section will examine the drivers of the main PVI schemes: the lump sum structure and the income stream structure. These will be then employed to generate a risk premium added to a mortgage rate that generates the borrower's total debt.

5.1.1 Lump Sum Structure

In a lump sum structure the borrower will receive a single sum determined as a proportion of the value of the property at contract initiation. The architecture of this product will be explored in the following lines using the notation and framework provided by Mottura (2016) and Castellani, De Felice, and Moriconi (2006).
Let:

- x be the age of the borrower at contract stipulation date;

- y be the age of the borrower's spouse at contract stipulation date;

- I_0 be the initial value of the property;

- α be the LTV percentage;

- S be the financing cost;

- $\bar{s}$ be the stochastic term of the contract;

- $m_{\bar{s}}$ be the outstanding balance at termination determined through annual capitalization;

- $I_{\bar{s}}$ be the value of the property at termination;

- β be the transaction cost for selling the house.

The balance at termination, $m_{\bar{s}}$, can be determined by compounding the value of the loan through a contractual interest rate $i + \mu$:

$$m_{\bar{s}} = \alpha I_0 (1 + i + \mu)^{\bar{s}} \tag{5.1}$$

where i is the base risk-free rate and μ is the risk adjusted premium (D. W. Cho et al., 2013). Ma and Deng (2006) calculated the maximum LTV ratio as follows:

$$\alpha = \frac{\prod_{k=1}^{T_{\bar{x}y}}(1 + g_k)}{\prod_{k=1}^{T_{\bar{x}y}}(1 + i_k^*)}, \tag{5.2}$$

with $T_{\bar{x}y}$ is the last survivor joint life expectancy of the borrower and spouse, g_k is the property appreciation rate at time k and i_k^* is the interest rate through which the lender can raise money on the market, i.e. LR.

By considering only one termination risk factor for an individual of age x with a spouse of age y, i.e. last survivor joint mortality, the capital at a generic termination date, n, can be expressed as follows:

$$Y_n = \begin{cases} min\{m_n, (1-\beta)I_n\} \text{ in case of death of both spouses} \\ 0 \text{ otherwise} \end{cases} \tag{5.3}$$

Thus, the the pay-off of the PVI capital at termination will be determined as below:

$$Y_{\bar{s}} = min\{m_{\bar{s}}, (1-\beta)I_{\bar{s}}\}. \tag{5.4}$$

Consequently, we can represent this by using the put decomposition:

$$Y_{\bar{s}} = m_{\bar{s}} - max\{m_{\bar{s}} - (1-\beta)I_{\bar{s}}, 0\}, \tag{5.5}$$

which implies that the borrower has to stipulate a loan with the PVI provider with capital at termination $m_{\bar{s}}$ without providing any collateral. This has to be combined with the purchase of a put option from the same PVI provider on the value of the property with stochastic term and strike.

Alternatively, the decomposition can be done in terms of a call option by selling the property and buying a call option on the value of the property with stochastic term and strike:

$$Y_{\bar{s}} = (1-\beta)I_{\bar{s}} - max\{(1-\beta)I_{\bar{s}} - m_{\bar{s}}, 0\}, \tag{5.6}$$

The call option can also be read as the right of the heirs to purchase the property at price $m_{\bar{s}}$. With this formulation we can produce a first structure for the Net Asset Value (NAV) at time 0 as follows (Mottura, 2016):

$$NAV(0) = V(0, Y_{\bar{s}}) - V(0, S_{\bar{s}}) = V(0, Y_{\bar{s}} - \alpha I_0), \tag{5.7}$$

where $V(0, Y_{\bar{s}})$ is the value in 0 of the amount payable by the borrower at termination. This can be expressed in actuarial terms as:

$$NAV(0) = \sum_{k=1}^{\omega - min\{x,y\}} V(0, Y_k - \alpha I_0)_{k-1|1}q_{\bar{x}y}, \tag{5.8}$$

with ω as the extreme age and:

$$Y_k = \begin{cases} min\{m_k, (1-\beta)I_k\} \text{ with probability } {}_{k-1|1}q_{\bar{x}y} \\ 0 \text{ otherwise} \end{cases} \tag{5.9}$$

with ${}_{k-1|1}q_{\bar{x}y}$ as the forward death probability of the borrower and spouse and I_k as:

$$I_k = I_0 \prod_{t=0}^{k}(1+g_t) \tag{5.10}$$

When using the put decomposition the resulting equation can be written in terms of the NNEG as:

$$NAV(0) = V(0, NNEG_{\bar{s}} - \alpha I_0) =$$
$$= \sum_{k=1}^{\omega-min\{x,y\}} (NNEG_k - \alpha I_0) \times_{k-1|1} q_{\bar{x}y} \times v_{[0,k]}, \tag{5.11}$$

where $v_{[0,k]}$ is the discount factor:

$$v_{[0,k]} = \prod_{t=1}^{k}(1+i_t^*)^{-1} \tag{5.12}$$

and the NNEG:

$$NNEG_{\bar{s}} = m_{\bar{s}} - max\{m_{\bar{s}} - (1-\beta)I_{\bar{s}}, 0\}, \tag{5.13}$$

$$NNEG_k = m_k - max\{m_k - (1-\beta)I_k, 0\} \tag{5.14}$$

5.1.2 Income Stream Structure

We have seen that PVI contracts may provide the option for the borrower to receive an income stream up until termination rather than a lump sum. This payment can take different forms: it can be fixed or variable and dependent on a pre-defined price index, typically CPI.

Assuming an annual payment and a joint life model, the present actuarial value of the fixed income stream with premium P^1, is determined as (D. W. Cho et al., 2013):

$$P^1 \sum_{k=0}^{\omega-min\{x,y\}-1} \left[\frac{{}_kp_{\bar{x}y}}{\prod_{t=0}^{k}(1+i_t^*)} \right] \tag{5.15}$$

where ${}_kp_{\bar{x}y}$ is the probability of at least one of two lives of age x and y to survive k years.

While, the expected value of a CPI indexed income stream provided up until termination is given by:

$$P^2 \sum_{k=0}^{\omega-min\{x,y\}-1} \left\{ \frac{[\prod_{t=1}^{t=k}(1+c_t)]_kp_{\bar{x}y}}{\prod_{t=0}^{k}(1+i_t^*)} \right\}, \tag{5.16}$$

where c_t is the inflation rate between a generic year $t-1$ and t.

To determine payment P^1 and payment P^2 we need to set the above expected values equal to the lump-sum amount, hence:

$$P^1 = \frac{\alpha I_0}{\sum_{k=0}^{\omega-min\{x,y\}-1} \left[\frac{{}_kp_{\bar{x}y}}{\prod_{t=0}^{k}(1+i_t^*)} \right]} \tag{5.17}$$

and:

$$P^2 = \frac{\alpha I_0}{\sum_{k=0}^{\omega-min\{x,y\}-1}\left\{\frac{[\prod_{t=1}^{t=k}(1+c_t)]_k p_{\bar{x}\bar{y}}}{\prod_{t=0}^{k}(1+i_t^*)}\right\}}. \tag{5.18}$$

Provided the above amounts we can compute the accumulated loan outstanding balance for the above contracts at time k in terms of a fixed contractual rate as follows:

$$m_k^1 = kP^1 \sum_{t=0}^{k}(1+i+\mu)^t, \tag{5.19}$$

$$m_k^2 = kP^2 \sum_{t=0}^{t}\prod_{z=1}^{k}(1+i+\mu+c_z). \tag{5.20}$$

In the case of an income stream the total liability at time k will depend on the number of payments made up until time k. Therefore, we can determine the total liability S as:

$$S_k^1 = kP^1 \tag{5.21}$$

and

$$S_k^2 = kP^2. \tag{5.22}$$

Provided these values, the NAVs can be calculated as below:

$$NAV(0)^1 = \sum_{k=1}^{\omega-min\{x,y\}} {}_{k-1|1}q_{\bar{x}\bar{y}}v_{[0,k]}\left\{min[m_k^1,(1-\beta)I_k]-S_k^1\right\} \tag{5.23}$$

$$NAV(0)^2 = \sum_{k=1}^{\omega-min\{x,y\}} {}_{k-1|1}q_{\bar{x}\bar{y}}v_{[0,k]}\left\{min[m_k^2,(1-\beta)I_k]-S_k^2\right\} \tag{5.24}$$

5.1.3 Mortgage Risk Premium and Fixed Rate

To price a reverse mortgage with either fixed or variable contractual accumulation rate we need to determine the risk premium μ.

This can be done by assuming that the non-recourse clause is entirely charged to the borrower in the form of annual premium. Consequently, to calculate μ the expected present value of the annual premium charged, π, is set equal to the expected present value of the NNEG:

$$V(0, NNEG) = V(0, \pi), \tag{5.25}$$

In this case, we consider as NNEG only the put component (D. W. Cho et al., 2013). As such, the actuarial present value at time 0 of the NNEG will be:

$$V(0, NNEG) = \sum_{k=1}^{\omega-min\{x,y\}} {}_{k-1|1}q_{\bar{x}\bar{y}}\mathbb{E}\left\{0, max[m_k^* - (1-\beta)I_k, 0]\right\}, \tag{5.26}$$

where m_k^* is the accumulated mortgage amount at risk-free rate i.

While, the expected value of the premiums assume the form of:

$$V(0, \pi) = \mu \sum_{k=1}^{\omega-min\{x,y\}} {}_k p_{\bar{x}\bar{y}}v_{[0,k]}m_k^*. \tag{5.27}$$

The results will clearly differ depending on the payment structure of the loan.

5.2   Cash Flow Simulation

In this section we will present the results of the above PVI valuation framework. To this purpose, we will assume the following base case scenario:

- A female borrower of age 60;

- A male spouse of age 63, provided the average three year age gap between male and female spouses in Italy (Bonarini, 2017);

- An initial value of the property equal to the average value of a home in Italy: €168932 (Agenzia delle entrate, 2017);

- 6% property selling transaction costs as in D. W. Cho et al. (2013);

- 01-01-2019 as stipulation date;

- A 0.88% base risk-free rate in line with the observed 30 year euro yield at stipulation date.

This information will be used to determine risk premiums and present values of lump sum and income stream PVI contracts.

5.2.1   Risk Premium

To calculate the total NAV we first need to obtain the risk premium, μ. This will require the present value of a put option. Typically, we can value a put option either through the Black and Scholes formula (Black & Scholes, 1973) or through Monte Carlo simulation of the underlying asset. In this study we will use Monte Carlo simulation, specifically we will employ HPI variation predictions originating from the multi-output neural network built in the previous chapter. These applied to the current property value are illustrated in Figure 5.1. We can easily remark the dropping value of housing properties which might undermine the final NAV of the lender.
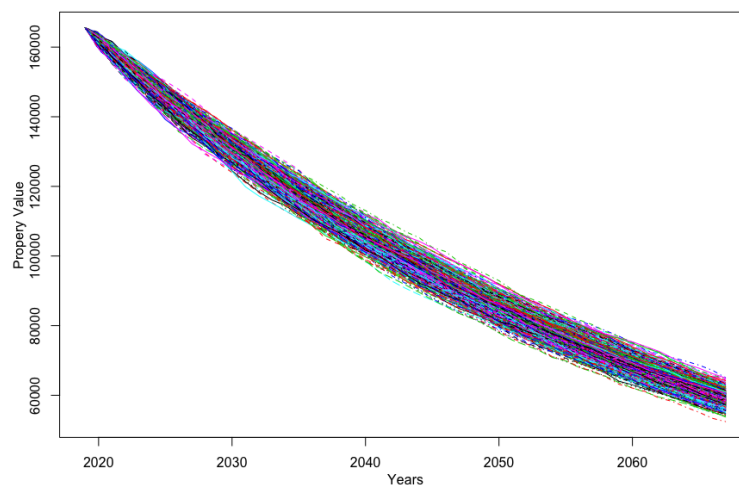

Figure 5.1: Simulations of the property value using MCD simulations of house price variation.

For comparison purposes, we have also ran 1000 Geometric Brownian Motion (GBM) simulations on the value of the property with parameters estimated on historic HPI values. Again, we can observe falling estimated prices. However, the variability produced by the GBM paths is lower.
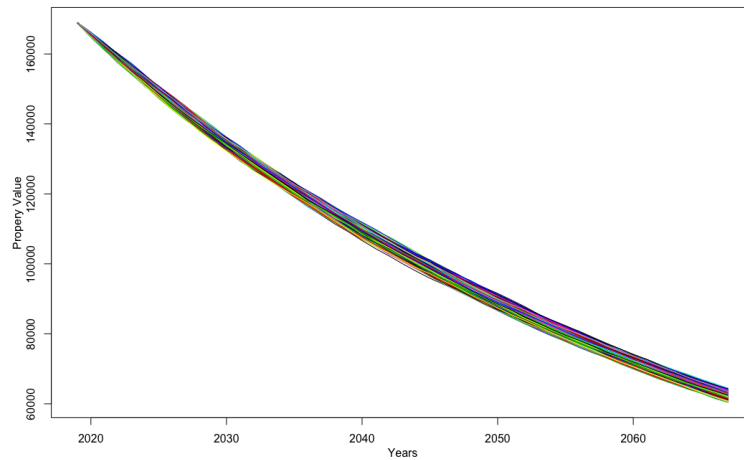

Figure 5.2: Simulations of the property value assuming a GBM for house value variation.

Once obtained the simulations we can calculate the average of the resulting put amounts and apply the discount factor to determine the expected present value of the option. With this amount we can calculate the risk premium.

The values of the risk premiums obtained for a lump sum contract and simple, CPI indexed and floored indexed[1] annuity contracts are reported in Table 5.1. For the sake of comparison, we illustrate both MCD and GBM results.

	Lump Sum	Simple Annuity	Indexed Annuity	Floored Indexed Annuity
MCD	0.0002704107	0.02202597	0.02030348	0.02193752
GBM	0.0001423631	0.02202535	0.02030286	0.0219369

Table 5.1: Risk Premium Results.

The value of the lump sum risk premium originating from the MCD model is almost double the value of the GBM risk premium for the same contract. This is a consequence of the higher house price variability captured by the MCD technique. The income stream contracts do not show the same result since the value of the NNEG is mostly driven by the outstanding balance and less by the value of the property. Therefore the variability of house prices has a smaller impact on annuity-type contracts.

Once determined the risk premium we can generate cash flow projections for lump sum and income stream contracts. These will be outlined in the following section.

5.2.2 Lump Sum Reverse Mortgage

Provided a joint life lump sum contract, in Figure 5.3 we can observe a growing outstanding loan balance and a falling property value trend. Within the current falling prices scenario, around 40

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[1]Indexed annuity with compounded CPI over time floored at 0.

years from inception the value of the outstanding balance will overtake the estimated present value of the property.

On the right side of Figure 5.3 the lender's positive and negative cash flows are illustrated. Although property value drops, the present value of the NNEG always remains above the present value of the liability with a peak at 32 years from contract stipulation date. This generates positive net asset values for the lender throughout the contract duration.
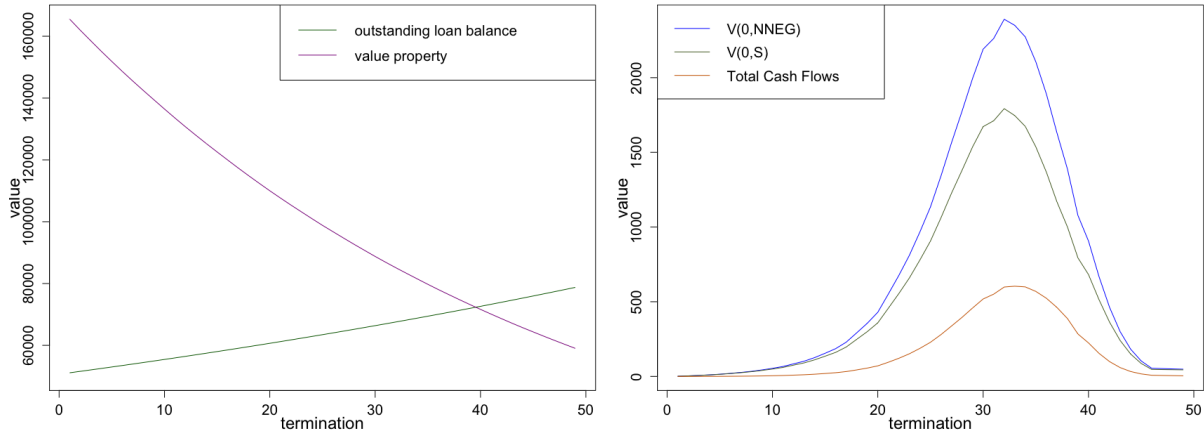
Figure 5.3: Loan balance and property value from valuation date to maximum life span on the left and present value of NNEG, costs and NAV from valuation date to maximum life span on the right.

Table 5.2 shows the resulting LTV, total value of assets and liabilities and the total NAV of a lump sum contract within the base case assumptions.

LTV	V(0,NNEG)	V(0,S)	NAV
0.2999367	35453.25	27185.83	8267.424

Table 5.2: Lump Sum Base Case Results.

The base parameters can be altered to examine potential contractual scenarios. To this end, the age of the lender has been modified to 65, and the initial value of the property has been shifted once down to €150000 and a second time up to €190000. Table 5.3 provides the results for these scenarios.

	LTV	V(0,NNEG)	V(0,S)	NAV	Risk Premium
x=65	0.3692248	46010.86	37077.89	8932.977	0.0005499383
I_0=150000	0.2999367	31480.05	24139.15	7340.904	0.0002704107
I_0=190000	0.2999367	39874.73	30576.25	9298.478	0.0002704107

Table 5.3: Alternative Scenarios Lump Sum Results.

Altering the age implies a different life expectancy. Hence, the LTV is clearly affected by the age shift, which will impact the initial lump sum amount. This along with new probabilities applied

to the actuarial present value generate higher values for all amounts analyzed in Table 5.3, thus higher profitability.

When reasoning in terms initial property value these considerations hold true: lower property values negatively impact the initial loan which is reflected on lower values for the risk premium, value of liabilities and value of assets. This negatively affects profitability. The exact opposite reasoning can be applied to higher property values at contract initiation.

5.2.3 Income Stream Reverse Mortgage

Figure 5.4 depicts projections for the previously-mentioned income stream contracts: simple annuity, CPI indexed income streams and floored CPI indexation. We can easily noticed that floored indexed income stream cash flows resemble simple annuity cash flows. This is a result of the dropping CPI trend.

On the left side of Figure 5.4 we can observe that the Outstanding Loan Balance (OLB) overtakes the property value earlier than observed for a lump sum contract. However this doesn't undermine the total NAV which is higher than the lump sum NAV for all income stream contracts. The additional profitability within these contracts is generated by larger values of the NNEG particularly, in the early years of the contract when the outstanding value of income stream contracts increases at a higher pace than the outstanding value of lump sum contracts.

The net projected cash flows are mostly positive with a peak after 28 years from contract stipulation date. Still, contrarily to the lump sum structure, net cash flows fall under zero after 37 years from initiation without jeopardizing the total asset value.
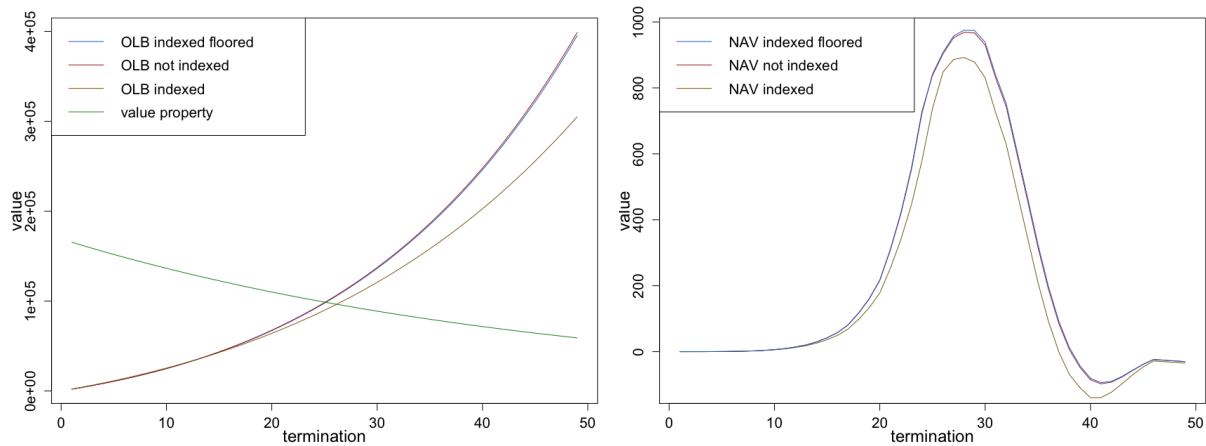
Figure 5.4: Loan balance for income stream contracts from valuation date to maximum life span on the left and Present value of the NAV for income stream contracts from valuation date to maximum life span on the right.

Table 5.4 illustrates the results obtained for the three income stream RM contracts in terms of premium, total present value of assets, total present value of liabilities and the net asset value. We can observe that the floored indexed income stream contract presents the highest NAV, thus the most profitable among the three.

	Premium	V(0,NNEG)	V(0,S)	NAV
Simple Annuity	1841.912	41404.58	30424.33	10980.25
Indexed Annuity	1942.32	40998.67	32082.85	8915.8298
Floored Indexed Annuity	1833.422	41366.78	30284.1	11082.68

Table 5.4: Income Stream Results.

Table 5.5 contains the results for income stream contracts using the lump sum alternative scenarios. The observations done for the lump sum architecture remain valid for income stream contracts. A positive shift in the lender's age generates higher values for all amounts considered. The same is true for higher initial home values. Contrarily, lower property values at stipulation date result in lower amounts for all variables and contractual forms.

$x = 65$	Premium	V(0,NNEG)	V(0,S)	NAV	Risk Premium
Simple Annuity	2542.422	50459.04	38850.6	11608.44	0.0267058
Indexed Annuity	2648.12	50079.65	40465.78	9613.8628	0.025021
Floored Indexed Annuity	2529.198	50396.72	38648.55	11748.17	0.02658335
I_0=150000	Premium	V(0,NNEG)	V(0,S)	NAV	Risk Premium
Simple Annuity	1635.491	36764.21	27014.71	9749.502	0.02202408
Indexed Annuity	1724.6463	36403.75	28487.36	7916.384	0.02030159
Floored Indexed Annuity	1627.953	36730.66	26890.2	9840.457	0.02193564
I_0=190000	Premium	V(0,NNEG)	V(0,S)	NAV	Risk Premium
Simple Annuity	2071.622	12349.85	334218.63	12349.85	0.02202762
Indexed Annuity	2184.552	46112.03	36083.99	10028.03	0.02030515
Floored Indexed Annuity	2062.074	46525.98	34060.92	12465.06	0.02193918

Table 5.5: Alternative Scenarios Income Stream Results.

5.3   Risk Analysis

In the previous section all contracts have been evaluated though the best estimate lens. As such, we have used an average value of all risk drivers' projections and delivered the final NAV. However to properly evaluate any contract we require an analysis of deviations from the average value and analyse risks. Specifically our concern will be assessing negative deviations from best estimates that potentially generate loss.

The assessment of risk is typically condensed in specific risk measures. The most commonly used risk measures are the Value at Risk (VaR) and its variants. When evaluating loss, this measure provides the total loss amount on a given time horizon with a pre-defined probability. In terms of solvency requirements this amount will represents the capital that the lender needs to set aside for compliance approval.

Using the notation provided by Jorion (2006), given a time horizon and probability α the VaR

can be computed as:

$$VaR_\alpha(X) = -inf\{ x : F_X(x) \geq \alpha \} = F_X^\leftarrow(1-\alpha), \tag{5.28}$$

where $F_X(x)$ is the cumulative distribution of a random variable X and $F_X^\leftarrow(1-\alpha)$ is the $1-\alpha$ quantile of the cumulative distribution.

This measure might not be enough to capture tail events. Therefore, in this study we will also adopt one version of the VaR: the Tail Value at Risk (TVaR). This measure quantifies the expected value of a random variable once an event outside a pre-defined probability has occurred. To formalize:

$$TVaR_\alpha(X) = \mathbb{E}[-X|X < -VaR_\alpha(X)] \tag{5.29}$$

Assuming normality, $\mathbb{E}[X|X < VaR_\alpha(X)]$ can be computed by averaging all ordered amounts under $VaR_\alpha(X)$.

To calculate both VaR and TVaR we require a distribution. This can be achieved by modelling stochastic factors through simulation which will allow for the computation of averages and quantiles, thus best estimates and risk measures.

To this end, the MCD simulations will be employed to determine the VaR for the two previously examined risk factors with probability 99.7% for time horizon $x - \omega$. This will generate two loss amounts for each risk measure: one amount will be the result of financial risk and the second will account for mortality exposure.

Under the assumption of independence and normality, the total VaR of two generic risks, X and Y, can be determined as:

$$VaR_\alpha(X + Y) = \sqrt{VaR_\alpha(X)^2 + VaR_\alpha(Y)^2} \tag{5.30}$$

Similarly, the total TVaR will be the result of:

$$TVaR_\alpha(X + Y) = \sqrt{TVaR_\alpha(X)^2 + TVaR_\alpha(Y)^2} \tag{5.31}$$

5.3.1 Results

Given the above formalization, Table 5.6 illustrates VaR and TVaR results with a probability of 99.7% on time horizon $\omega - x$ in terms of negative deviation from the NAV for all examined contacts and scenarios. Both measures are calculated separately for financial and longevity risk and subsequently aggregated by assuming normality and independence.

To determine financial risk, house price, loan rate and CPI simulations have been inputted in the NAV formulae to produce a NAV distribution and calculate the $1 - \alpha$ quantile. This will be then subtracted from the original NAV best estimate to determine the final amount at risk.

A similar approach has been used to compute longevity risk measures. However, just one variable impacts the amount exposed to longevity risk. Thus, there is no need to evaluate joint effects of more than one variable. This implies that we can directly calculate the $1 - 99.7\%$ percentile on simulated longevity rates which will be then used for cash flow projections to compute a value for the total loss. Since the main concern for the lender is that the borrower might live longer than expected, the $1 - 99.7\%$ quantile needs to consider mortality improvements, i.e. lower death

probabilities.

In all scenarios risk measures VaR and TVaR show that a lump sum contract is less risky from the lender's perspective than income stream contracts when accounting for both financial and mortality risk factors. Income stream contracts show higher levels of severity, particularly for mortality risk which is always positioned above the best estimate NAV for all annuity-type contracts.

Longevity appears as the major factor of risk irrespective of contract class or specification. The longer the borrower lives the more the debt raises. This produces higher chances that the value of the property drastically falls under the value of the total accumulated debt. The effect is even more dramatic for income stream contracts where the debt raises not only through the compounding effect but mostly due to the increase in the number of income payments.

Financial risk has a relatively low impact compared to longevity, particularly for contracts with a stream of payments, where unlike longevity it doesn't erode the average NAV. While, VaRs for lump sums under different scenarios appear more similar.

When reasoning in terms of scenarios, the most profitable scheme, $x = 65$, is also the most exposed to risk. And, the less risk capital demanding scenario, $I_0 = 150000$ produces the lowest best estimate. As for the contacts themselves, the commitment that generates higher losses is the simple annuity. However, the indexed annuity floored at 0 appears as the most lucrative.

Base Case	Lump Sum	Simple Annuity	Indexed	Floored Indexed
$VaR_{99.7}^{fin}$	611.5571	2505.908	2337.282	2497.274
$VaR_{99.7}^{long}$	894.2531	12379.84	12192.05	12379.84
$VaR_{99.7}$	1083.37	12630.92	12414.06	12576.96
$TVaR_{99.7}^{fin}$	685.2351	2830.81	2706.049	2821.208
$TVaR_{99.7}^{long}$	1229.475	13809.69	13647.54	13753.03
$TVaR_{99.7}$	1407.535	14096.85	13913.24	14039.41
$x{=}65$	Lump Sum	Simple Annuity	Indexed	Floored Indexed
$VaR_{99.7}^{fin}$	985.536	2740.208	2764.973	2722.146
$VaR_{99.7}^{long}$	1902.718	15338.67	15233.62	15249.42
$VaR_{99.7}$	2142.806	15581.51	15482.52	15490.47
$TVaR_{99.7}^{fin}$	1123.474	3124.583	3003.862	3108.888
$TVaR_{99.7}^{long}$	2363.663	17104.99	17026.66	17010.22
$TVaR_{99.7}$	2617.078	17388.03	17289.6	17291.99
$I_0{=}150000$	Lump Sum	Simple Annuity	Indexed	Floored Indexed
$VaR_{99.7}^{fin}$	543.0206	2225.017	2075.276	2217.351
$VaR_{99.7}^{long}$	794.0352	10992.25	10825.45	10944.93
$VaR_{99.7}$	961.9581	11215.18	11022.58	11167.28
$TVaR_{99.7}^{fin}$	608.4416	2513.51	2402.721	2504.984
$TVaR_{99.7}^{long}$	1091.689	12261.86	12117.83	12211.54
$TVaR_{99.7}$	1249.794	12516.83	12353.74	12465.82
$I_0{=}190000$	Lump Sum	Simple Annuity	Indexed	Floored Indexed
$VaR_{99.7}^{fin}$	687.8261	2818.489	2628.849	2808.778
$VaR_{99.7}^{long}$	1005.778	13923.99	13712.83	13864.04
$VaR_{99.7}$	1218.48	14206.38	13962.54	14145.7
$TVaR_{99.7}^{fin}$	770.6927	3183.909	3043.599	3173.109
$TVaR_{99.7}^{long}$	1382.806	15532.16	15349.85	15468.43
$TVaR_{99.7}$	1583.073	15855.14	15648.69	15790.53

Table 5.6: VaR and TVaR calculated for all examined contracts.

CONCLUSION

Seniors are faced with low precautionary saving rates and the lengthening of their life expectancy. Funding old age income has become a matter of concern for social systems and saving services providers across the globe. Releasing housing equity in various forms to provide for retirement is a popular practice in North America and Australia. One particular class of home-ownership equity release known as reverse mortgage has been recently introduced in Europe with immediate little success but high long term potential provided the favorable market conditions outlined in Chapter One.

The principal objective of this investigation has been to provide an analytical setup to value long term commitments that responds RM suppliers' requirements and with particular focus on the existing Italian market. This has been achieved by initially exploring worldwide product offer to create a proper valuation framework.

In such framework we have identified several stochastic variables that needed modelling before projecting cash flows. These variables have been grouped into two independent processes which concerned financial uncertainty on one side and mortality on the other. Both risk drivers have been modelled through a neural network approach. The eventual output has been used to establish what affects profits, losses and exposure to risk of RM products from a lender's perspective.

A reverse mortgage loan payment can take different forms. This translates into different pay-out structures. As a result, in this study we have investigated particularly over lump sum and income stream schemes.

In terms of profitability our findings show that at current Italian market conditions annuity products outperform lump sum structures. Among income stream payment schemes the CPI indexed annuity with a compounded floor at zero appears as the most lucrative. Additionally, this study also that when the signatory's age and the initial value of the property increase this positively impacts the final net asset value. We have also seen that as a result of sinking projected house prices, the outstanding loan balance will overtake the property value if the borrower and spouse survive a certain age.

When assessing negative deviations from the estimated net asset value, we have found that longevity risk prevails over financial exposure. Risk measures show major potential losses particularly for income stream payment schemes due to mortality improvements. When reasoning

in terms of different scenarios, the contractual cases that generate more profit are also a higher source of exposure to value deterioration.

6.1 Further Advancements

In our study we have assumed that termination only occurs upon death of the borrower. However, PVI contracts also include different termination clauses. Particularly, the borrower might decide to repay the loan. Ji (2011) assumes that the borrower would wish to terminate the contract when willing to move into long term care for serious health issues. This would allow for the introduction of a multi-state termination model which we leave for further exploration.

When discussing MCD, Gal, Islam, and Ghahramani (2017) argue that this valuation procedure can be interpreted as an ensemble of neural networks, where many neural networks are used jointly to learn a problem. However, a neural network ensemble does not typically only refer to altering dropout rates, but might include various neural networks classes, with a different number of neurons and layers. Averaging the weighted value of many models at hand typically produces a more accurate result (Zhou, Wu, & Tang, 2002).

Additionally, we have used MCD just for one period ahead. Alternatively the drop-out can be randomly picked n times at each time step generating a nested monte-carlo prediction. This can be subsequently evaluated on the validation data set and verify whether there have been substantial accuracy improvements.

One major drawback in using neural networks is the difficulty of explaining the contribution of each input to the total result. Arras et al. (2019) proposed the use of a Layer-wise Relevance Propagation (LRP) to determine what drives the estimation of an LSTM network. This technique can become convenient when willing to understand the behaviour of our neural network model adopted to estimate risk when market conditions vary.

The Italian house market differs across the country. The average variation amounts of housing value per region stretched from -11.4% to +4.5% between 2013 and 2014 (Agenzia delle entrate, 2017). Rather than using an overall index for the whole country, a more correct approach would be estimating HPI variations considering geographical circumstances.

When formulating cash flows contracts, contractual cases where the entire or part of the outstanding balance could be re-payed at specific periods in time were not contemplated. Typically, the accumulated interest on the loan could be paid by the borrower in installments. As such, further research could investigate the impact on the lender's net pay-off and risks of loan repayment contractual clauses.

To value risk premiums D. W. Cho et al. (2013) used stochastic risk discount factors adjusted to financial risk to account for deviations from the expected value of house value and interest rates. Moreover, in the present study we have observed a severe negative impact on risk capital due to longevity risk. Therefore, a more appropriate methodology of estimating risk premium would consider additional mortality volatility.

For Mergers & Aquisitions purposes a lender might require pay-off and risk analysis at portfolio level. This could be achieved by considering a portfolio composed by PVI products with various contractual specifications and providing a total portfolio NAV and an overall risk measure.

Aaron, N. & Deisenroth, M. (2015). *Machine Learning for a London Housing Price Prediction Mobile Application.* Imperial College London. Individual Project: Final Report. Retrieved May 25, 2018, from http://www.doc.ic.ac.uk/~mpd37/theses/2015_beng_aaron-ng.pdf

Abid, F. & Ben Salah, M. (2002). Estimating term structure of interest rates: Neural network vs one factor parametric models. *University of Sfax, Business and Economics (Working Paper).*

Agenzia delle entrate. (2017). Gli immobili in Italia. Retrieved September 30, 2019, from https://www1.finanze.gov.it/finanze3/immobili/contenuti/immobili_2017.pdf

Alai, D., Chen, H., Wanhee Cho, D., Hanewald, K., & Sherris, M. (2014). Developing equity release markets: Risk analysis for reverse mortgages and home reversions. *North American Actuarial Journal, 18*(1), 217–266.

Andrews, D. & Oberoi, J. (2015). Home equity release for long-term care financing: An improved market structure and pricing approach. *Annals of Actuarial Science, 9*(01). doi:10.1017/S1748499514000268

Antolin, P. (2009). Ageing and the payout phase of pensions, annuities and financial markets. *OECD Journal: Financial Market Trends*, (2), 1–19.

Arras, L., Arjona-Medina, J., Widrich, M., Montavon, G., Gillhofer, M., Müller, K.-R., . . . Samek, W. (2019). Explaining and Interpreting LSTMs. In *Explainable AI: Interpreting, Explaining and Visualizing Deep Learning* (pp. 211–238). Springer International Publishing.

Australian Actuaries Institute. (2016). Unlocking housing wealth options to meet retirement needs. Retrieved October 30, 2018, from https://actuaries.asn.au/Library/Miscellaneous/2016/AIHOUSINGGPwebLres.pdf

Ayuso, M., Bravo, J. M., & Holzmann, R. (2017a). Addressing longevity heterogeneity in pension scheme design. *Journal of Finance and Economics, 6*(1), 1–21.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2017b). On the heterogeneity in longevity among socioeconomic groups: Scope, trends, and implications for earnings-related pension schemes. *Global Journal of Human Social Sciences - Economics, 17*(1), 31–57.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2019a). Addressing the life expectancy gap in pension policy. Preprint submitted for publication in the Journal of Pension Economics and Finance.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2019b). Getting life expectancy estimates right for pension policy period versus cohort approach. *Journal of Pension Economics and Finance*. 1st revision Submitted.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2019c). Making use of home equity: The potential of housing wealth to enhance retirement security. IZA Discussion Paper Series No. 12656, September. Retrieved from https://ideas.repec.org/p/iza/izadps/dp12656.html

Bahrammirzaee, A. (2010). A comparative survey of artificial intelligence applications in finance: Artificial neural networks, expert system and hybrid intelligent systems. *Neural Computing and Applications*, *19*(8), 1165–1195. doi:10.1007/s00521-010-0362-z

Baldini, M. & Beltrametti, L. (2015). Prestito vitalizio ipotecario e povertã degli anziani: Un esercizio di microsimulazione. In *La casa, il benessere e le disuguaglianze* (pp. 73–87). Egea.

Banca d'Italia. (2019). Tassi d'interesse armonizzati - prestiti per acquisto abitazioni - flussi. Retrieved June 9, 2019, from http://dati.istat.it/Index.aspx?DataSetCode=DCCN_PILT#

Begley, J., B. Fout, H., LaCour-Little, M., & Mota, N. (2017). Home equity conversion mortgages: The secondary market investor experience. *SSRN Electronic Journal*. doi:10.2139/ssrn.3055881

Bengio, Y., Simard, P., & Frasconi, P. (1994). Learning long-term dependencies with gradient descent is difficult. *Trans. Neur. Netw.* *5*(2), 157–166. doi:10.1109/72.279181

Bishop, C. M. (1995). *Neural networks for pattern recognition*. New York, NY, USA: Oxford University Press, Inc.

Black, F. & Scholes, M. S. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, *81*(3), 637–654. doi:10.1086/260062

Bodie, Z., Marcus, A. J., & Merton, R. C. (1989). Defined benefit versus defined contribution pension plans: What are the real tradeoffs. In *Pensions in the U.S. Economy* (pp. 139–162). Chicago: University of Chicago Press.

Boehm, T. P. & Ehrhardt, M. C. (1994). Reverse mortgages and interest rate risk. *Real Estate Economics*, *22*(2). doi:10.1111/1540-6229.00639

Bonarini, F. (2017). L'età degli sposi, la fecondità e la mortalità. Retrieved from http://www.neodemos.info/articoli/eta-degli-sposi-la-fecondita-e-la-mortalita/f

Booth, H. & Tickle, L. (2008). Mortality modelling and forecasting: A review of methods. *Annals of Actuarial Science*, *3*(1). doi:10.1017/S1748499500000440

Borde, S., Rane, A., Shende, G., & Shetty, S. (2017). Real estate investment advising using machine learning. *International Research Journal of Engineering and Technology (IRJET)*, *4*(3), 1821–1825.

Bravo, J. M. (2019). Funding for longer lives: Retirement wallet and risk-sharing annuities. *EKONOMIAZ Basque Economic Review*, N°96(II-2019), 266–289.

Bravo, J. M. & Holzmann, R. (2014). The payout phase of funded pensions plans: Risks and payment options. OECD PISA Financial Education Program N°6/2014.

Broadbent, J., Palumbo, M., & Woodman, E. (2006). *The shift from defined benefit to defined contribution pension plans - implications for asset allocation and risk management.* Retrieved May 22, 2018, from https://www.bis.org/publ/wgpapers/cgfs27broadbent3.pdf

Brownfield, C. (2014). *The Fourth Pillar: The Role of Home Equity Release in Retirement Funding.* Actuaries Institute 2014 Financial Services Forum. Institute of Actuaries of Australia. Retrieved June 30, 2018, from https://actuaries.asn.au/Library/Events/FSF/2014/BrownfieldFourthPillarPaper140505.pdf

Brumberg, R. & Modigliani, F. (1954). Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data. In K. Kurihara (Ed.), *Post-keynesian economics* (pp. 388–436). New Brunswick, NJ: Rutgers University Press.

Bryant, R. (2004). *Demographic pressures on public pension systems and government budgets in open economies.* ESRI Discussion paper series 109 (Tokyo Economic and Social Research Institute). Retrieved May 15, 2018, from https://www.brookings.edu/wp-content/uploads/2016/06/20040922.pdf

Cairns, A. J. G., Blake, D., & Dowd, K. (2008). Modelling and management of mortality risk: A review. *Scandinavian Actuarial Journal, 2008*(2). doi:10.1080/03461230802173608

Carone, G. & Costello, D. (2006). Can europe afford to grow old? *Finance and Development, 43.*

Castellani, G., De Felice, M., & Moriconi, F. (2006). Manuale di finanza. vol. 3: Modelli stocastici e contratti derivati. In I. Mulino (Ed.).

Chaphalkar, N. & Sandbhor, S. (2013). Use of artificial intelligence in real property valuation. *International Journal of Engineering and Technology, 5*(3), 2334–2337.

Chen, H., Cox, S., & S Wang, S. (2009). Is the hecm program sustainable? evidence from pricing mortgage insurance premiums and non-recourse provisions using conditional esscher transform. *Insurance: Mathematics and Economics, 46*(2).

Chen, Y. & Wu, Q. (2014). Research on risk of implementing housing reverse mortgages in china. *International Journal of Business and Social Science, 5*(9), 78–84.

Chinloy, P. & Megbolugbe, I. F. (1994). Reverse Mortgages: Contracting and Crossover Risk. *Real Estate Economics, 22*(2), 367–386.

Cho, D. W., Hanewald, K., & Sherris, M. (2013). Risk management and payout design of reverse mortgages. *SSRN Electronic Journal.* doi:10.2139/ssrn.2233688

Cho, D., Hanewald, K., & Sherris, M. (2015). Risk analysis for reverse mortgages with different payout designs. *Asia-Pacific Journal of Risk and Insurance, 9*, 1–33. doi:10.1515/apjri-2014-0012

Cipriani, G. (2014). Population aging and PAYG pensions in the OLG model. *Journal of Population Economics, 17*(1), 251–256. doi:10.1007/s00148-013-0465-9

Coale, A. J. (1989). Demographic transition. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Social economics* (pp. 16–23). London: Palgrave Macmillan UK. doi:10.1007/978-1-349-19806-1_4

Crespi, F. & Mascia, D. V. (2014). L'utilizzo del reverse mortgage nell'ambito del personal financial planning: un'occasione mancata (o ancora da sfruttare) per il mercato italiano? *Il Risparmio Review, 3*, 59–101.

European Cntral Bank. (2019). Euro Area Yield Curves. Retrieved June 14, 2019, from https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html

European Commission. (2018). *Pension adequacy report 2018: Current and future income adequacy in old age in the EU (Volume 1).* Publications Office of the European Union.

Fernández-Villaverde, J. & Krueger, D. (2007). Consumption over the life cycle: Facts from consumer expenditure survey data. *The Review of Economics and Statistics*, *89*(3), 552–565. doi:10.1162/rest.89.3.552

Finsveen, L. (2018). *Time-series predictions with recurrent neural networks* (Master's thesis, Norway, the Netherlands).

Fiona Boyle Associates. (2010). *Assessment of the potential of equity release for older owner-occupiers (scoping study).* Northern Ireland Housing Executive. Retrieved November 20, 2018, from https://touch.nihe.gov.uk/assessment_of_the_potential_of_equity_release_for_older_owner-occupiers.pdf

Fondazione Cariplo. (2017). *House Rich Cash Poor: Come rendere liquida la ricchezza rappresentata dalla casa di abitazione.* Quanderni dell'osservatorio. Retrieved May 22, 2018, from http://www.fondazionecariplo.it/it/strategia/osservatorio/quaderni/house-rich-cash-poor-quaderno-n-26.html

French, D., McKillop, D., & Sharma, T. (2018). *Housing equity withdrawal in later life: Is a home more than asset-based welfare?* Program on the Global Demography of Aging at Harvard University. Retrieved May 20, 2018, from https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1288/2018/04/154_Housing-Equity-Withdrawal-in-Later-Life.pdf

Gal, Y. & Ghahramani, Z. (2016). Dropout as a bayesian approximation: Appendix. Retrieved August 25, 2019, from https://arxiv.org/pdf/1506.02157.pdf

Gal, Y., Islam, R., & Ghahramani, Z. (2017). Deep bayesian active learning with image data. *ArXiv, abs/1703.02910.*

Gamboa, J. C. B. (2017). Deep learning for time-series analysis.

Goel, H., Melnyk, I., Oza, N. C., Matthews, B., & Banerjee, A. (2016). Multivariate Aviation Time Series Modeling : VARs vs. LSTMs.

Gwizdala, J. (2015). Equity release schemes on selected housing loan markets across the world. *Financial Investment and Insurance: Global Trends and the Polish Market*, (381), 72–85. doi:10.15611/pn.2015.381.06

Haffner, M. E. A., Ong, R., & Wood, G. A. (2013). Mortgage equity withdrawal in Australia: Recent trends, institutional settings and perspectives. European Network for Housing Research (ENHR) Conference. Retrieved November 3, 2018, from https://melbourneinstitute.unimelb.edu.au/assets/documents/hilda-bibliography/conference-papers-lectures/2013/Haffner-Ong-Wood-ENHR2013-Paper.pdf

Hanewald, K. & Sherris, M. (2011). House price risk models for banking and insurance applications. *SSRN Electronic Journal.* UNSW Australian School of Business Research Paper No. 2011ACTL11.

Haurin, D. & Moulton, S. (2017). International perspectives on homeownership and home equity extraction by senior households. *SSRN Electronic Journal.*

Hennecke, P., Murro, P., Neuberger, D., & Palmisano, F. (2017). *Pensions and housing wealth: Quantitative data on market conditions for equity release schemes in the EU* (Thuenen-Series of Applied Economic Theory No. 146). University of Rostock, Institute of Economics. Retrieved April 27, 2018, from https://ideas.repec.org/p/zbw/roswps/146.html

HFCS. (2016). HFCS_Statistical_Tables_Wave2.zip. available from: ECB (2017): Household finance and consumption network. Retrieved, from https://www.ecb.europa.eu/pub/economic-research/research-networks/html/researcher_hfcn.en.html

Hochreiter, S. & Schmidhuber, J. (1997). Long short-term memory. *Neural Comput. 9*(8), 1735–1780. doi:10.1162/neco.1997.9.8.1735

Holzmann, R., Ayuso, M., Alaminos, E., & Bravo, J. M. (2019). Life cycle saving and dissaving revisited across three-tiered income groups: Starting hypotheses, refinement through literature review, and ideas for empirical testing. IZA Discussion Paper No. 12655.

HomeEquity Bank. (2015). Home equity bank reports record annual originations. Retrieved January 5, 2019, from https://www.homeequitybank.ca/homequity-bank-reports-record-annual-originations/

Hopfield, J. J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences*, *79*(8), 2554–2558. doi:10.1073/pnas.79.8.2554

Huan, C. & Mahoney, J. (2002). Equity release mortgages. *Housing Finance International*, *16*(4).

Huang, H.-C., Wang, C.-W., & Miao, Y.-C. (2011). Securitisation of crossover risk in reverse mortgages. *The Geneva Papers on Risk and Insurance - Issues and Practice*, *36*(4), 622–647. doi:10.1057/gpp.2011.23

Hubbard, R., Skinner, J., & Zeldes, S. (1994). Expanding the life-cycle model: Precautionary saving and public policy. *American Economic Review*, *84*(2), 174–79. Retrieved from https://EconPapers.repec.org/RePEc:aea:aecrev:v:84:y:1994:i:2:p:174-79

Human Mortality Database. (2019). Mortality. Data retrieved from Human Mortality Database. Retrieved from http://www.mortality.org

ISTAT. (2019a). Indice prezzi al consumo per l'intera colletività. Retrieved June 9, 2019, from http://dati.istat.it/Index.aspx?DataSetCode=DCSP_IPAB#

ISTAT. (2019b). Indice prezzi delle abitazioni. Retrieved June 9, 2019, from http://dati.istat.it/Index.aspx?DataSetCode=DCSP_IPAB#

ISTAT. (2019c). Prodotto Interno Lordo. Retrieved January 9, 2019, from http://dati.istat.it/Index.aspx?DataSetCode=DCCN_PILT#

Jefferson, T., Austen, S., Ong, R., Haffner, M. E. A., & Wood, G. A. (2017). Housing Equity Withdrawal: Perceptions of Obstacles among Older Australian Home Owners and associated Service Providers. *Journal of Social Policy*, *46*(3), 623–642. doi:10.1017/S0047279417000058

Ji, M. (2011). *Markovian approaches to joint-life mortality with applications in risk management* (Master's thesis).

Ji, M., Hardy, M., & Li, J. S.-H. (2012). A Semi-Markov Multiple State Model for Reverse Mortgage Terminations. *Annals of Actuarial Science*, *6*(2), 235–257. doi:10.1017/S1748499512000061

Jorion, P. (2006). *Value at risk 3rd editiom* (McGraw-Hill, Ed.).

Kabir, G. & Ahsan Akhtar Hasin, M. (2013). Comparative analysis of artificial neural networks and neuro-fuzzy models for multicriteria demand forecasting. *International Journal of Fuzzy System Applications (IJFSA)*, *3*, 1–24. doi:10.4018/ijfsa.2013010101

Kaul, L., Karen Goodman. (2017). *Seniors' access to home equity.* Housing Financing Policy Center. Urban Institute. Retrieved November 15, 2018, from https://www.urban.org/sites/default/files/publication/88556/seniors_access_to_home_equity.pdf

Kenny, T., Barnfield, J., Daly, L., Dunn, A., Passey, D., Rickayzen, B., & Teow, A. (2017). The future of social care funding: Who pays? *British Actuarial Journal*, *22*(1), 10–44. doi:10.1017/S135732171600012X

Kim, J. H. & Li, J. S. (2017). Risk-neutral valuation of the non-recourse protection in reverse mortgages: A case study for Korea. *Emerging Markets Review*, *30*(100), 133–154. doi:https://doi.org/10.1016/j.ememar.2016.10.002

Kogure, A., Li, J., & Kamiya, S. (2014). A bayesian multivariate risk-neutral method for pricing reverse mortgages. *North American Actuarial Journal*, *18*(1), 242–257. doi:10.1080/10920277.2013.872983

Kontis, V., Bennett, J. E., Mathers, C. D., Li, G., Foreman, K., & Ezzati, P. M. (2017). Future life expectancy in 35 industrialised countries: Projections with a bayesian model ensemble. *The Lancet*, *389*(10076), 1323–1335. doi:https://doi.org/10.1016/S0140-6736(16)32381-9

Kriesel, D. (2007). *A brief introduction to neural networks.* Retrieved November 5, 2018, from http://www.dkriesel.com/science/neural_networks

Kumar, P. M., Divakaruni, R. K., & Sri Venkata, M. (2008). *Reverse mortgages - features risks.* 10th Global Conference of Actuaries. Retrieved November 20, 2018, from https://scholar.google.com/scholar?client=safari&rls=en&oe=UTF-8&um=1&ie=UTF-8&lr&q=related:zwkmqjnqVRmEUM:scholar.google.com/

Lazo, J., Pacheco, M., & Vellasco, M. (2002). Portfolio selection and management using a hybrid intelligent and statistical systems. (pp. 221–238). doi:10.1007/978-1-4615-0835-9_10

Le Cun, Y. (1986). Learning process in an asymmetric threshold network. In E. Bienenstock, F. F. Soulié, & G. Weisbuch (Eds.), *Disordered systems and biological organization* (pp. 233–240). Berlin, Heidelberg: Springer Berlin Heidelberg.

Lee, C. C., Chen, K. S., & So De Shyu, D. (2015). A study on the main determination of mortgage risk: Evidence from reverse mortgage markets. *International Journal of Financial Research*, *6*(2), 84–94. doi:10.5430/ijfr.v6n2p84

Lee, R. D. & Carter, L. R. (1992). Modeling and Forecasting U. S. Mortality. *Journal of the American Statistical Association*, *87*(419), 659–671. Retrieved from http://www.jstor.org/stable/2290201

Lee, Y. T., Kung, K. L., & Liu, I. C. (2018). Profitability and risk profile of reverse mortgages: A cross-system and cross-plan comparison. *Insurance: Mathematics and Economics*, *78*,

255–266. Longevity risk and capital markets: The 201516 update. doi:https://doi.org/10.1016/j.insmatheco.2017.09.019

Lee, Y. T., Wang, C.-W., & Huang, H.-C. (2012). On the valuation of reverse mortgages with regular tenure payments. *Insurance: Mathematics and Economics*, *51*(2), 430–441. doi:https://doi.org/10.1016/j.insmatheco.2012.06.008

Leff, N. H. (1971). Dependency rates and savings rates: Reply. *The American Economic Review*, *61*(3), 476–480. Retrieved from http://www.jstor.org/stable/1813449

Li, H., Zhang, J., & Zhang, J. (2007). Effects of longevity and dependency rates on saving and growth: Evidence from a panel of cross countries. *Journal of Development Economics*, *84*(1), 138–154. doi:https://doi.org/10.1016/j.jdeveco.2006.10.002

Li, J. S.-H., Hardy, M. R., & Tan, K. S. (2010). On pricing and hedging the no-negative-equity guarantee in equity release mechanisms. *Journal of Risk and Insurance*, *77*(2). doi:10.1111/j.1539-6975.2009.01344.x

Ma, S. & Deng, Y. (2006). Insurance Premium Structure of Reverse Mortgage Loans in Korea.

Maciel, L. & Ballini, R. (2010). Neural networks applied to stock market forecasting: An empirical analysis. *Learning and Nonlinear Models*, *8*, 3–22. doi:10.21528/LNLM-vol8-no1-art1

Makoto, N. & A., T. I. (2017). Reverse mortgage loans: A quantitative analysis. *The Journal of Finance*, *72*(2), 911–950. doi:10.1111/jofi.12489

Mayhew, L., Rickayzen, B., & Smith, D. (2017). *Flexible and affordable methods of paying for long term care insurance* (No. 11-15). International Longevity Centre. Retrieved May 15, 2018, from https://ideas.repec.org/p/fip/fedpwp/11-15.html

Mayhew, L., Smith, D., & Wright, D. (2018). The effect of longevity drift and investment volatility on income sufficiency in retirement. *Insurance: Mathematics and Economics*, *78*, 201–211. Longevity risk and capital markets: The 201516 update. doi:https://doi.org/10.1016/j.insmatheco.2017.09.013

McCulloch, W. S. & Pitts, W. (1943). A logical calculus of the ideas immanent in nervous activity. *The bulletin of mathematical biophysics*, *5*(4), 115–133. doi:10.1007/BF02478259

Michelangeli, v. (2008). *Does it pay to get a reverse mortgage?* Congressional Budget Office.

Minsky, M. & Papert, S. (1969). *Perceptrons: An introduction to computational geometry.* Cambridge, MA, USA: MIT Press.

Mitchell, D., Brockett, R., Patrick; Mendoza-Arriaga, & Muthuraman, K. (2013). Modeling and forecasting mortality rates. *Insurance: Mathematics and Economics*, *52*(2). doi:10.1016/j.insmatheco.2013.01.002

Moscarola, F., D'Addio, A. C., Fornero, E., & Rossi, M. (2015). Reverse mortgage: A tool to reduce old age poverty without sacrificing social inclusion. In *Ageing in Europe - supporting policies for an inclusive society* (pp. 235–244). Genthiner Str. 13, Berlin: de Gruyter.

Mottura, C. (2016). *Sulla valutazione economica del Prestito Ipotecario Vitalizio (PIV).* Università degli studi Roma Tre. Collana del Dipartimento di Studi Aziendali. Retrieved May 21, 2018, from http://disa.uniroma3.it/wp-content/uploads/2016/11/WP2-1.pdf

Mudrazija, S. & Butrica, B. B. (2017). Homeownership, Social Insurance and Old-Age Security in the United States and Europe. *Innovation in Aging*, *1*, 183–184.

Nakajima, M. (2012). Everything you always wanted to know about reverse mortgages but were afraid to ask. *Business Review*, 19–31. Retrieved May 30, 2018, from https://www. philadelphiafed.org/-/media/research-and-data/publications/business-review/2012/q1/ brq112_everything-you-always-wanted-to-know-about-reverse-mortgages.pdf

Nakajima, M. & Telyukova, I. A. (2011). *Home equity withdrawal in retirement* (Working Papers No. 11-15). Federal Reserve Bank of Philadelphia. Retrieved May 6, 2018, from https: //ideas.repec.org/p/fip/fedpwp/11-15.html

Nerlich, C. & Schroth, J. (2018). The economic impact of population ageing and pension reforms. *ECB Economic Bulletin*, (2). Retrieved May 26, 2018, from https://www.ecb.europa.eu/ pub/pdf/other/ebart201802_02.en.pdf

Nigri, A., Levantesi, S., Marino, M., Scognamiglio, S., & Perla, F. (2019). A Deep Learning Integrated Lee Carter Model. *Risks*, *7*(1), 1–16. Retrieved from https://ideas.repec.org/a/ gam/jrisks/v7y2019i1p33-d214560.html

Parker, D. (1985). *Learning-logic: Casting the cortex of the human brain in silicon.* Technical report: Center for Computational Research in Economics and Management Science. Massachusetts Institute of Technology, Center for Computational Research in Economics and Management Science. Retrieved from https://books.google.it/books?id=2kS9GwAACAAJ

Petneházi, G. (2018). Recurrent neural networks for time series forecasting.

Prabhakar, R. (2018). A House Divided: Asset-Based Welfare and Housing Asset-Based Welfare. *International Journal of Housing Policy*, (Early Access). Retrieved April 28, 2018, from http://oro.open.ac.uk/53657/

Ranganathan, S., Nakai, K., & Schonbach, C. (2018). *Encyclopedia of bioinformatics and computational biology: Abc of bioinformatics* (1st). NLD: Elsevier Science Publishers B. V.

Rashbrooke, G. (2018). Decumulation 101: The basics of drawing down capital in retirement. *Policy Quarterly*, *10*(3). Retrieved May 20, 2018, from https://ojs.victoria.ac.nz/pq/ article/view/4498

Reifner, U., Clerc-Renaud, S., Pérez Carrillo, E., Tiffe, A., & Knobloch, M. (2009a). *Study on equity release schemes in EU. Part I: General Report.* Institut für Finanzdienstleistungen e.V. Retrieved May 20, 2018, from http://ec.europa.eu/internal_market/finservices-retail/docs/credit/equity_release_part1_en.pdf

Reifner, U., Clerc-Renaud, S., Pérez Carrillo, E., Tiffe, A., & Knobloch, M. (2009b). *Study on equity release schemes in EU. Part II: Country Reports.* Institut für Finanzdienstleistungen e.V. Retrieved May 20, 2018, from http://www.eurofinas.org/uploads/documents/policies/ equity_release_part2_en.pdf

Reifner, U., Clerc-Renaud, S., Pérez Carrillo, E., Tiffe, A., & Knobloch, M. (2009c). *Study on equity release schemes in EU. Part III: Annexes.* Institut für Finanzdienstleistungen e.V. Retrieved May 20, 2018, from https://online.ercep.com/media/attachments/annexes-en-55094.pdf

Richman, R. & Wüthrich, M. V. (2018). A neural network extension of the Lee Carter model to multiple populations. *Annals of Actuarial Science*, 1–â21. doi:10.1017/S1748499519000071

Ronald, R., Lennartz, C., & Kadi, J. (2017). What ever happened to asset-based welfare? Shifting approaches to housing wealth and welfare security. *Policy & Politics*, *45*(2), 173–193. doi:doi:10.1332/030557316X14786045239560

Rosenblatt, F. (1958). The perceptron: A probabilistic model for information storage and organization in the brain. *Psychological Review*, *6*(65), 386–408.

Rumelhart, D. E., Hinton, G. E., & Williams, R. J. (1988). Neurocomputing: Foundations of research. In J. A. Anderson & E. Rosenfeld (Eds.), (Chap. Learning Representations by Back-propagating Errors, pp. 696–699). Cambridge, MA, USA: MIT Press. Retrieved from http://dl.acm.org/citation.cfm?id=65669.104451

Rumelhart, D. E., McClelland, J. L., & PDP Research Group, C. (Eds.). (1986). *Parallel distributed processing: Explorations in the microstructure of cognition, vol. 1: Foundations.* Cambridge, MA, USA: MIT Press.

Sánchez, A. J. Q. (2009). La hipoteca inversa: Una opciòn realmente atractiva? *Revista del Ministerio de Trabajo e Inmigraciòn*, (81).

Sass, S. A. (2017). *Is home equity an underutilized retirement asset?* (No. 17-6). Center for Retirement Research at Boston College. Retrieved May 26, 2018, from https://dlib.bc.edu/islandora/object/bc-ir:107323

Schmäl, W. (2007). Dismantling an Earnings-Related Social Pension Scheme: Germany's New Pension Policy. *Journal of Social Policy*, *36*(2), 319–340. doi:10.1017/S0047279406000626

Scholz, J. K., Seshadri, A., & Khitatrakun, S. (2006). Are Americans Saving "Optimally" for Retirement? *Journal of Political Economy*, *114*(4), 607–643. doi:10.1086/506335

Shahroudi, A. (2019). *Probabilistic Forecasting with Monte-Carlo Dropout in Neural Networks* (Master's thesis, University of Tartu).

Shan, H. (2011). Reversing the trend: The recent expansion of the reverse mortgage market. *Real Estate Economics*, *39*(4), 743–768. Retrieved May 5, 2018, from https://EconPapers.repec.org/RePEc:bla:reesec:v:39:y:2011:i:4:p:743-768

Shao, A. W., Sherris, M., & Hanewald, K. (2012). *Equity release products allowing for individual house price risk*. 11th Emerging Researchers in Ageing Conference. doi:10.2139/ssrn.2233688

Sherris, M. & Sun, D. (2010). Risk based capital and pricing for reverse mortgages revisited. *UNSW Australian School of Business Research Paper No.2010ACTL04*.

Sulaiman, N., Ishaq, M., & Ghani, Z. (2018). Factors Influencing Reverse Mortgage (RM) Product Market. *Advanced Science Letters*, *24*(6), 4623–4625.

Sun, H. (2015). *Longevity risk securitization of housing reverse mortgages in future China* (Master's thesis).

Sun, W., Rachev, S., Chen, Y., & Fabozzi, F. J. (2008). Measuring intra-daily market risk: A neural network approach. Retrieved October 20, 2018, from https://www.efmaefm.org/0EFMSYMPOSIUM/2009-Nantes/paper/Fabozzi.pdf

Szymanoski, E., Enriquez, J. C., & DiVenti, T. R. (2007). Home equity conversion mortgage terminations: Information to enhance the developing secondary market. *Cityscape: A Journal of Policy Development and Research*, *9*(1), 5–46.

Terry, R., Rachel; Gibson. (2007). Overcoming obstacles to equity release. *Housing Care and Support*, *10*(1). doi:10.1108/14608790200700004

Toulson, S. (1996). Use of neural network ensembles for portfolio selection and risk management. Retrieved September 20, 2018, from http://citeseerx.ist.psu.edu/viewdoc/download?doi= 10.1.1.199.4370&rep=rep1&type=pdf

Towers Watson. (2013). Equity release: Accessing housing wealth in retirement. Retrieved May 22, 2018, from https://www.towerswatson.com/en/Insights/IC-Types/Ad-hoc-Point-of-View/2013/06/Equity-release-Accessing-housing-wealth-in-retirement

Verstyuk, S. (2018). Modeling multivariate time series in economics: From auto-regressions to recurrent neural networks. Retrieved from https://ssrn.com/abstract=3297736%20or% 20http://dx.doi.org/10.2139/ssrn.3297736

Visit, L., Christopher, G., & Lee, M. (2004). House price prediction: Hedonic price model vs. artificial neural network. *American Journal of Applied Sciences*, *1*(3). doi:10.3844/ajassp. 2004.193.201

Wang, C. W., Huang, H. C., & Lee, Y. T. (2014). On the valuation of reverse mortgage insurance. *Scandinavian Actuarial Journal*. doi:10.1080/03461238.2014.925967

Wang, J. L., Hsieh, M.-H., & Chiu, Y.-f. (2011). Using reverse mortgages to hedge longevity and financial risks for life insurers: A generalised immunisation approach. *Geneva Papers on Risk and Insurance - Issues and Practice*, *36*(4). doi:10.1057/gpp.2011.22

Wang, L., Valdez, E., & Piggott, J. (2008). Securitization of longevity risk in reverse mortgages. *12*, 345–371. doi:10.1080/10920277.2008.10597529

Warshawsky, M. J. (2018). Retire on the house: The possible use of reverse mortgages to enhance retirement security. *The Journal of Retirement*, *5*(3), 10–31. doi:10.3905/jor.2018.5.3.010

Werbos, P. J. (1974). *Beyond regression: New tools for prediction and analysis in the behavioral sciences* (Doctoral dissertation, Harvard University).

Widrow, B. & Hoff, M. E. (1960). Adaptive switching circuits. In *1960 IRE WESCON convention record, part 4* (pp. 96–104). New York: IRE.

Widrow, B. & Lehr, M. A. (1990). 30 years of adaptive neural networks: Perceptron, madaline, and backpropagation. *Proceedings of the IEEE*, *78*(9), 1415–1442. doi:10.1109/5.58323

Wiener, N. (1948). *Cybernetics, or, control and communication in the animal and the machine / norbert wiener*. Technology Press [Cambridge, Mass.]

Wiśniewski, R. (2017). Modeling of residential property prices index using committees of artificial neural networks for PIGS, the European-G82, and Poland. *Argumenta Oeconomica*, *38*(1), 145–194.

Yang, S. S. (2011). Securitisation and tranching longevity and house price risk for reverse mortgage products. *The Geneva Papers on Risk and Insurance - Issues and Practice*, *36*(4), 648–674. doi:10.1057/gpp.2011.26

Yilmazer, T. & Scharff, R. L. (2014). Precautionary savings against health risks: Evidence from the health and retirement study. *Research on Aging*, *36*(2), 180–206. doi:10.1177/ 0164027512473487

Zaremba, W., Sutskever, I., & Vinyals, O. (2014). Recurrent neural network regularization. Retrieved from http://arxiv.org/abs/1409.2329

Zheng, H. & Xikun, S. (2016). The interest rate risk of reverse mortgages. *Management Science and Engineering, 10*(2), 24–26. doi:10.3968/8261

Zhou, Z.-H., Wu, J., & Tang, W. (2002). Ensembling neural networks: Many could be better than all. *Artificial Intelligence, 137*(1), 239–263.

LONG SHORT TERM MEMORY NEURAL NETWORKS PARAMETERS AND OUTPUTS

A.1 Termination Risk Model

Layer	Number of Neurons	Activation Function
LSTM	60	ReLu
Dense	30	sigmoid
Dense	30	sigmoid
Dropout Layer	30	none
Output Layer	1	linear

Table A.1: Network Layers - Female

Epochs	53
Loss function	Mean Squared Error
Optimizer	Adaptive Moment Estimation

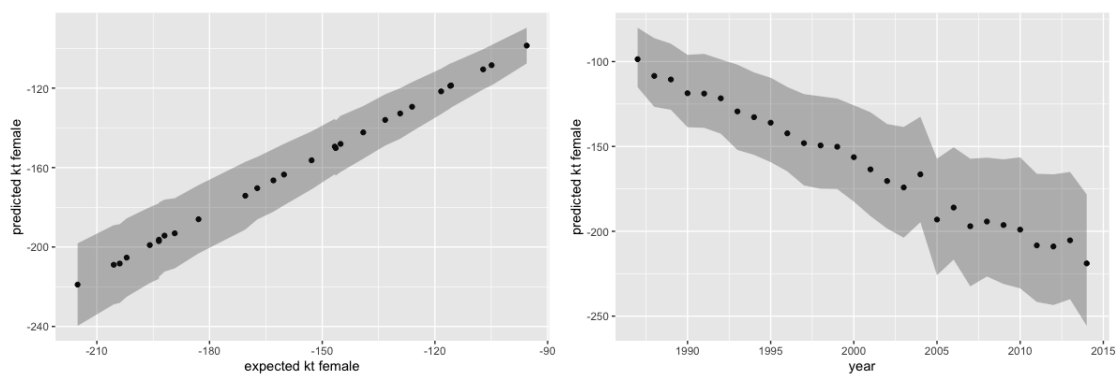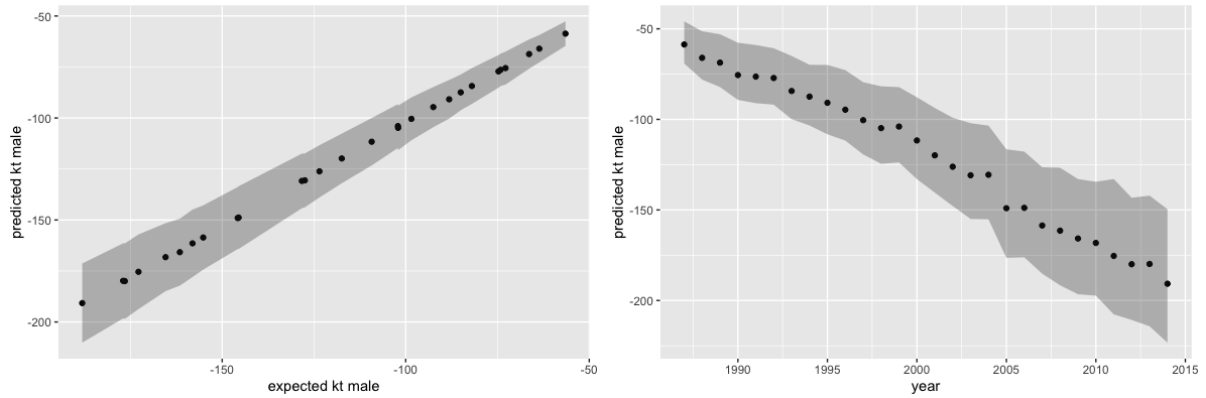Table A.2: Other Specifications - Female

Figure A.1: k_t expected vs. predicted with epistemic and aleatory uncertainty (left) and predictions with confidence intervals at 97.5% (right) - Female.

Layer	Number of Neurons	Activation Function
LSTM	50	ReLu
Dense	25	sigmoid
Dense	25	sigmoid
Dropout Layer	25	none
Output Layer	1	linear

Table A.3: Network Layers - Male

Epochs	51
Loss function	Mean Squared Error
Optimizer	Adaptive Moment Estimation

Table A.4: Other Specifications - Male

Figure A.2: k_t expected vs. predicted with epistemic and aleatory uncertainty (left) and predictions with confidence intervals at 97.5% (right) - Male.

A.2 Financial Risk Model

Layer	Number of Neurons	Activation Function
LSTM	50	ReLu
LSTM stacked	50	sigmoid
LSTM stacked	25	sigmoid
Dropout Layer	25	none
Output Layer	5	linear

Table A.5: Network Layers

Epochs	165
Loss function	Mean Absolute Error
Optimizer	Root Mean Square Propagation

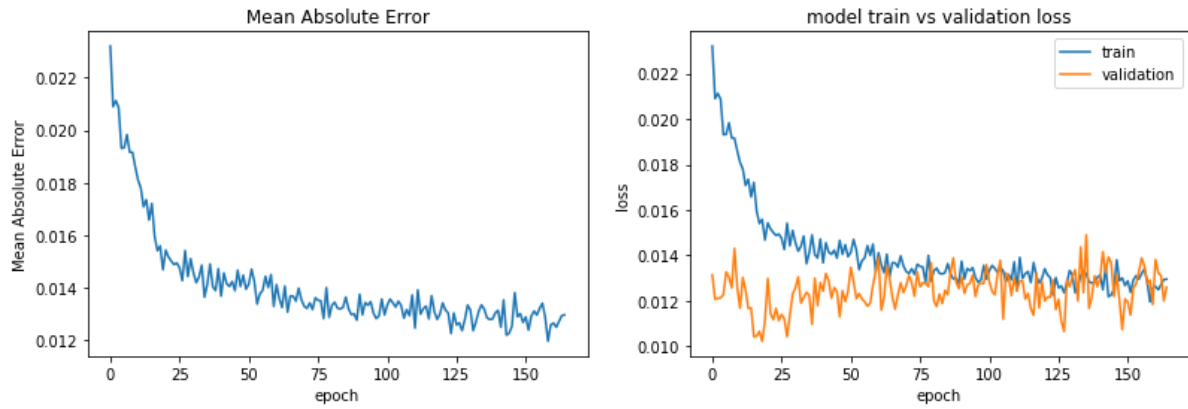Table A.6: Other Specifications

Figure A.3: Mean Absolute Error (left) and loss function behaviour for both training and validation set (right).