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Scheduling Dynamic Positioned Tankers with Variable Travel Time for Offshore Offloading Operations

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Leonardo Salsano de Assis

Scheduling Dynamic Positioned Tankers with Variable Travel Time for Offshore Offloading Operations

Dissertation presented to the Graduate Program in Automation and Systems Engineering in partial fulfillment of the requirements for the degree of Master in Automation and Systems Engineering.
Advisor: Prof. Eduardo Camponogara, Dr.
Co-Advisor: Prof. Agustinho Plucenio, Dr.

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Leonardo Salsano de Assis

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This dissertation is recommended in partial fulfillment of the requirements for the degree of "Master in Automation and Systems Engineering", which has been approved in its present form by the Graduate Program in Automation and Systems Engineering.

Florianópolis, July 2nd 2015.

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To my family Bruno, Rossana and Larissa, and to Jessica.

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The power of rational thinking is the primary source of freedom in the world.

Neil deGrasse Tyson

Resumo Extendido

A operação de campos de petróleo em alto mar implica na transferência de óleo que acumula em Unidades Flutuantes de Produção, Armazenamento e Descarregamento (FPSOs em inglês) para terminais em terra. Uma frota de Petroleiros Dinamicamente Posicionados (DPTs em inglês), ou navios aliviadores, é utilizada para a transferência de óleo das plataformas flutuantes até os terminais terrestres, onde depois o petróleo é transportado em grandes petroleiros ou por dutos até as refinarias. O escalonamento de uma frota de navios aliviadores, que minimiza os custos operacionais e que atenda às restrições do sistema, consiste em um problema complexo.

Este trabalho propõe uma formulação em Programação Linear Inteira Mista (MILP em inglês) que avança em relação à trabalhos anteriores pela contabilização de tempos de viagem variáveis entre plataformas e terminais terrestres. As viagens dos navios aliviadores são modeladas como caminhos em um grafo direcionado tendo o terminal terrestre, plataformas flutuantes e pontos de controle como nós, enquanto os arcos representam os possíveis movimentos e operações de carregamento/descarregamento dos navios aliviadores.

Do ponto de vista econômico, a frota de navios aliviadores deve ser escalonada para maximizar a produção de petróleo nas plataformas flutuantes enquanto minimiza os custos gerados pelas viagens. A combinação da formulação MILP com um *solver* constitui uma ferramenta para auxiliar os engenheiros na tomada de decisões. Este problema pode ser resolvido diariamente utilizando a estratégia de *rolling-horizon* para responder a eventos inesperados.

Palavras-chave: Logística para Transporte de Petróleo, FPSO, Navios Aliviadores, *Rolling-Horizon*, *Relax-and-Fix*.

No capítulo 1 apresentamos o problema do Planejamento do Suprimento de Petróleo que envolve desde a produção de óleo e gás nas plataformas até o atendimento da demanda do mercado com os subprodutos beneficiados nas refinarias. O escalonamento de navios aliviadores constitui um subproblema da cadeia produtiva de petróleo e possui uma importância tática dentro do planejamento das operações.

No capítulo 2 discutimos alguns conceitos importantes no campo da otimização. Primeiro definimos o que é otimização e o que é um modelo de otimização utilizando como exemplo o Problema da Mochila. Em seguida, apresentamos o conceito de Programação Inteira (IP) e Programação Linear Inteira Mista (MILP) e fazemos uma breve discussão sobre algoritmos para a solução de IPs e MILPs. Também introduzimos o conceito de relaxação e sua relevância na solução de problemas de otimização. Uma seção é dedicada para a introdução de conceitos relacionados com problemas de escalonamento e roteamento e por último apresentamos alguns métodos de otimização dinâmica como o *rolling-horizon* e *relax-and-fix*.

No capítulo 3, apresentamos primeiro uma formulação prévia do problema que considera os tempos de viagem entre as plataformas e os terminais terrestres como fixos e em seguida, propomos uma nova formulação que considera os tempos de viagem variáveis. Uma revisão da literatura é feita com o intuito de relacionar o modelo proposto com outros trabalhos. O capítulo termina com resultados teóricos obtidos a partir do novo modelo.

No capítulo 4 desenvolvemos a relaxação Lagrangeana do problema que origina o problema Dual Lagrangeano. Em seguida, apresentamos os algoritmos utilizados para a solução do problema dual e finalizamos o capítulo desenvolvendo a decomposição da função Dual Lagrangeana que quebra a função dual em diversas funções, uma para cada navio aliviador e plataforma, permitindo assim computação paralela.

No Capítulo 5 definimos uma instância exemplo com campos de petróleo, terminais terrestres e navios aliviadores, no qual as análises computacionais são feitas. A primeira análise consiste em comparar os limites obtidos pela relaxação Lagrangeana e a relaxação linear do problema. Os métodos do subgradiente e de geração de restrições sob demanda foram utilizados para a solução do problema Dual Lagrangeano. Em seguida, comparamos a solução estática, onde o problema é resolvido para todo o horizonte de planejamento, com a solução dinâmica obtida utilizado as heurísticas de *rolling-horizon* e *relax-and-fix*. Estas em geral fornecem soluções sub-ótimas mas respondem de forma satisfatória às grandes instâncias e incertezas do problema. Fechando o capítulo, a estratégia de *rolling-horizon* é avaliada em um simulador que gera perturbações para variáveis do modelo.

No capítulo 6 concluímos a dissertação com uma análise geral e contribuições do trabalho e propomos algumas direções para pesquisas futuras.

Abstract

The logistics of operating oil fields off the coast entails transferring oil that accumulates in Floating Production Storage and Offloading Units (FPSOs) to onshore terminals. A fleet of Dynamically Positioned Tankers (DPTs), or shuttle tankers, is deployed for transferring oil from the floating platforms to onshore terminals, where the oil is transported in large tanker ships or by pipelines to refineries. The scheduling of a fleet of shuttle tankers that minimizes the operating costs while satisfying the system constraints consists of a complex problem. To this end, this work proposes a formulation in Mixed-Integer Linear Programming (MILP) that advances previous works by accounting for variable time travel between floating platforms and the onshore terminal. The trips of the shuttle tankers are modeled as paths in a directed graph having the onshore terminal, floating platforms, and control points as nodes and arcs representing possible moves and offloading operations for the shuttle tankers. As a business case, the fleet of shuttle tankers should be scheduled to maximize oil production from the floating platforms while factoring in the transportation costs. The combination of the MILP formulation with an optimization solver constitutes a tool to aid operations engineers in making advised decisions. This formulation can be systematically solved daily in a rolling-horizon framework to respond to unanticipated events.

Keywords: Oil Transportation Logistics, FPSO, MILP, Shuttle Tankers, Rolling-Horizon, Relax-and-Fix.

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Chapter 1

Introduction

One of the many challenges in the Oil & Gas Industry is the Petroleum Supply Planning Problem (PSPP). In this problem the crude oil is produced in offshore platforms which is then transferred to onshore terminals by sub-sea pipelines or Dynamically Positioned Tankers (DPTs), here simply called shuttle tankers. After arriving to the onshore terminals, the crude oil is shipped to refineries to supply domestic demands with its sub-products which include: gaseous fuels, liquid fuels (gasoline, kerosene and others), lubricants, paraffin wax, sulfur, petroleum coke, asphalt, petrochemicals, among others.

When deep water offshore oil exploration is considered, in most of the cases there are no pipelines available and the oil produced by a group of wells must be stored in Floating Production Storage and Offloading Units (FPSOs), or platforms for short. Although these platforms have a large storage capacity, often they receive shuttle tankers to perform offloading operations. For the large number of platforms, a fleet of shuttle tankers is needed due to high volume of oil that must be transferred from the platforms to onshore terminals. This need gives rise to the necessity of scheduling trips of shuttle tankers among platforms and onshore terminals over a planning horizon. This problem is known as the Shuttle Tanker Scheduling Problem (STSP) which is illustrated in Figure 1.1[1]. This scenario is composed by three platforms (named FPSO 1, FPSO 2 and FPSO 3), two shuttle tankers (named

[1]Image Credit: Tracey Saxby, IAN Image Library (ian.umces.edu/imagelibrary/).

ST1 and ST2) and onshore terminals.

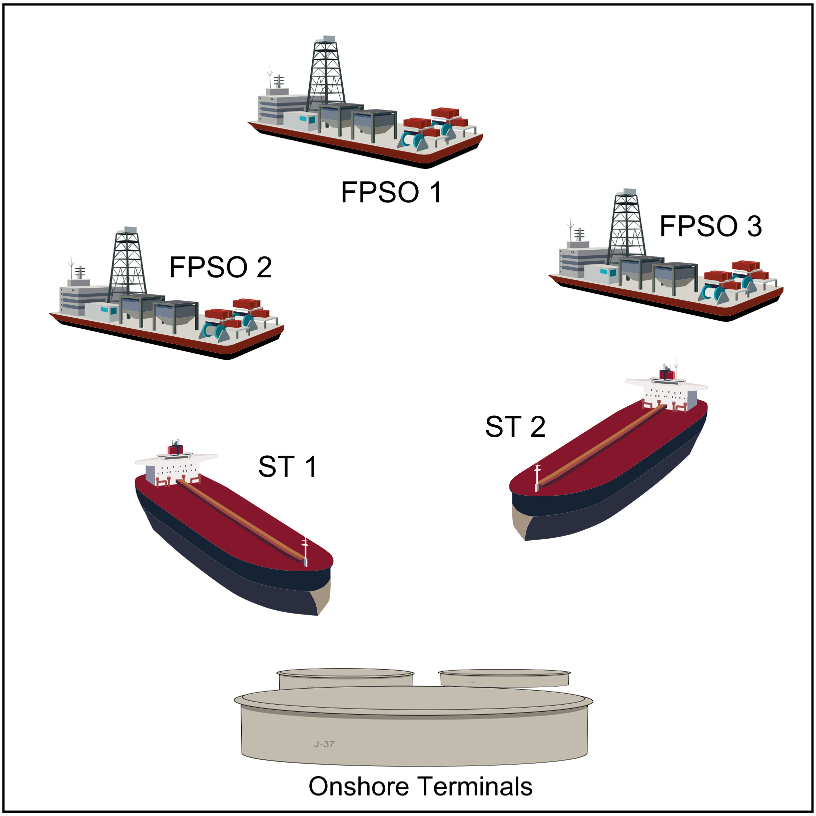

Figure 1.1 – Shuttle Tanker Scheduling Problem (STSP).

The shuttle tanker scheduling problem can be viewed as a business case. First, it is not acceptable to halt production at a platform due to the lack of storage capacity. Besides being a complex operation, the shutting off wells is a risk procedure that can compromise future production, since wells can take a considerable time to be brought back to full operation and production downtime represents a major loss of revenue. Second, the amount of oil left in the platforms incurs a loss of revenue. In other words, the oil owner would have to pay interest to access the money equivalent left in the platforms in the form of oil. On the other hand, there is a cost to bring the oil to the onshore terminals. This cost is related to the rental and operation of shuttle tankers. However, much more important than the costs involved in the operation of shuttle tankers, is the need to keep the platforms with sufficient storage capacity to receive the daily production of its wells. Therefore, the optimization algorithm must guarantee a proper offloading of the platforms.

The scheduling of shuttle tankers is a logistics problem. Logistics decisions consist in deciding when and how materials should be acquired, moved and stored and they can be divided in three hierarchical levels: strategic, tactical and operational. Strategic is in highest level and the decisions are planned considering the long-term (usually over years). They deal with fleet sizing, facility location and layout and capacity sizing. Tactical decisions consider the medium-term (months or quarterly) and deal with production and distribution planning, transportation mode selection, storage allocations and order picking strategies. Finally, in the lowest level, operational decisions deal with warehouse order picking, shipment and vehicle dispatching and are made on a daily basis using very detailed data (GHIANI; LAPORTE; MUSMANNO, 2004). This dissertation deals with the tactical level, since we assume that strategic decisions such as fleet sizing, platforms and onshore terminal location, layout and capacity sizing, are already known.

For the platforms important constraints and parameters like production rate, storage capacity, minimum volume of oil that the platforms should have stored to guarantee stability, the number of shuttle tankers that can offload platforms simultaneously and inventory balance equations should be considered. For the shuttle tankers, constraints and parameters like storage capacity, volume of oil to be offloaded from platforms and the inventory balance play an important role.

1.1 Objectives

The overall objective of this dissertation is the development, validation and analysis of a Mixed-Integer Linear Programming Model for scheduling shuttle tankers for offshore oil offloading operations. The specific goals are the following:

- Extension of a previous formulation to account variable travel time for shuttle tankers and other operating constraints.

- Theoretical assessment of the computational hardness of the variable travel-time formulation and its relation with preceding formulation.

- Proposition and analysis of Lagrangean relaxation as strategy to generate dual bounds.

- Computational analysis of the variable travel-time formulation for scheduling shuttle tankers in representative offshore oil fields.

- Computational analysis of dynamic optimization strategies to handle large problem instances, namely the rolling-horizon and relax-and-fix strategies.

1.2 Dissertation Organization

This dissertation is divided in six chapters. They are the following:

- In chapter 2, some optimization background is presented. We define what is optimization and what is an optimization model; the concept of Integer Programming (IP) and Mixed-Integer Linear Programming (MILP) and algorithms to solve it; the definition of a relaxation and its relevance to solving optimization problems; scheduling and routing problems; and last but not least we discuss about dynamic optimization methods used to solve the Shuttle Tanker Scheduling problem.

- In chapter 3, we first present a prior formulation for the STSP, with fixed travel times, and then propose a revised model with variable travel times. A review of literature was carried out to relate this dissertation to other technical works. The chapter ends with theoretical results regarding some properties of the revised model.

- In chapter 4, we present the Lagrangean relaxation of the STSP, algorithms to solve the Lagrangean dual problem and the Lagrangean dual function decomposition.

- In chapter 5, we propose an offshore oil field scenario for the computational analysis and the computational set up. A bound analysis regarding the Lagrangean relaxation and the linear relaxation of the problem is presented. The rolling-horizon and relax-and-fix heuristics are compared with respect to their performance in the scenario. Further, the rolling-horizon strategy is evaluated in a prototype simulator which generates random perturbations to the variables of the model.
- In chapter 6, we conclude the dissertation with a general view and contributions of the work and propose some directions for further research.

Chapter 2

Fundamentals

In this chapter, we present some optimization concepts considering Integer Programming (IP) and Mixed-Integer Linear Programming (MILP) to provide the mathematical foundation for the work developed in this dissertation.

2.1 Introduction to Mathematical Optimization

A classical optimization problem is the Knapsack Problem (NEMHAUSER; WOLSEY, 1988). In this problem we have a set of items (each one with a mass and a price) and a knapsack. We would like to know which is the most valuable collection of items that we can carry, given a fixed knapsack with a limited capacity. The selection of the best items from a set, regarding some constraints (in this case, the size of the knapsack), in order to maximize or minimize a quantity can be a simple and direct way to illustrate optimization. This seems trivial, however, optimization problems can become complex depending on their nature and size.

An example of a Knapsack Problem taken from (FISHER, 1985) can be seen in Problem 2.1. KS is formed by Equations (2.1a)-(2.1f) and represents the model of the Knapsack Problem. Equation (2.1a) is called objective function or cost function and it is the quantity that we want to maximize. Equations (2.1b)-(2.1d) are the constraints of the problem. The domain is established by Equation (2.1e) and Equation (2.1f) defines the nature of the variables $x_j,\ j = 1, \ldots, 4$.

Problem 2.1 belongs to a set of problems classified as Integer Programming, since its variables $x_j,\ j = 1, \ldots, 4$ are integers. Besides Integer Programming problems, the field of optimization is composed by other sets of problems. Figure 2.1 shows the main fields of optimization.

$$
\begin{aligned}
KS: \ f = \max_{\boldsymbol{x}} \quad & 16x_1 + 10x_2 + 4x_4 && \text{(2.1a)} \\
\text{s.t.}: \quad & && \\
& 8x_1 + 2x_2 + x_3 + 4x_4 \leq 10 && \text{(2.1b)} \\
& x_1 + x_2 \leq 1 && \text{(2.1c)} \\
& x_3 + x_4 \leq 1 && \text{(2.1d)} \\
& 0 \leq x_j \leq 1,\ j = 1 \ldots 4 && \text{(2.1e)} \\
& x_j \in \mathcal{Z},\ j = 1 \ldots 4 && \text{(2.1f)}
\end{aligned}
$$

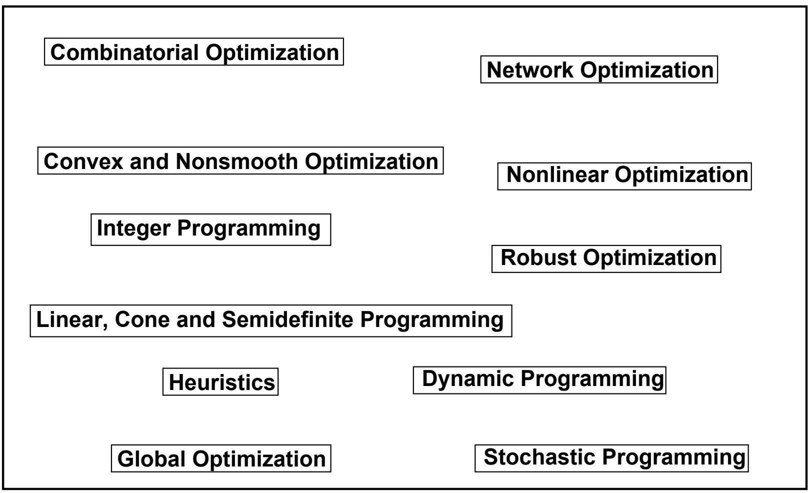

Figure 2.1 – Classical Optimization Fields.

2.1.1 Integer Programming

A wide variety of practical problems can be formulated and solved using integer programming: train scheduling, airline crew scheduling, production planning, electricity generation planning and cutting problems, among others (WOLSEY, 1998). An integer program has its objective function and constraints linear and its variables are discrete and continuous. A generic integer programming problem with only integer variables is defined as follows:

$$IP: \; f = \max_{\boldsymbol{x}} \; \boldsymbol{c}^T\boldsymbol{x} \tag{2.2a}$$

$$\text{s.t.}:$$

$$A\boldsymbol{x} \leq \boldsymbol{b} \tag{2.2b}$$

$$\boldsymbol{x} \in \mathbb{Z}^n_+ \tag{2.2c}$$

A is a $m \times n$ matrix, $\boldsymbol{x}$ and $\boldsymbol{c}$ are n-dimensional column vectors and $\boldsymbol{b}$ is a m-dimensional column vector. Some variations of Problem 2.2 are: mixed-integer linear programming (MILP), binary integer programming (BIP) and combinatorial programming (CP).

Formulating an integer programming problem consists in defining the parameters, variables, the set of constraints and the objective function of the problem. According to Wolsey (1998) the concept of *polyhedron* and *formulation* are defined as follows:

Definition 1. *A subset of $\mathcal{R}^n$ described by a finite set of linear constraints $P = \{\boldsymbol{x} \in \mathcal{R}^n : A\boldsymbol{x} \leq \boldsymbol{b}\}$ is defined as a polyhedron.*

Definition 2. *A polyhedron $P \subseteq \mathcal{R}^{n+p}$ is a formulation for a set $X \subseteq (\mathcal{Z}^n \times \mathcal{R}^p)$ if and only if $X = P \cap (\mathcal{Z}^n \times \mathcal{R}^p)$, X being the set of all feasible solutions.*

A problem can have an infinite number of formulations. Considering that, it is possible to think that there should be a best one. When the perfect formulation is obtained it does not matter if the integer programming or its linear relaxation is solved, since they will both give the same result. Wolsey (1998) defines the ideal formulation as follows:

Definition 3. *Given a set $X \subseteq \mathcal{R}^n$, the convex hull of X, denoted by conv(X), is the convex combination of all points in X and it is defined* *as: conv(X) =* $\{x : x = \sum_{i=1}^{t} \lambda_i x^i, \sum_{i=1}^{t} \lambda_i = 1, \lambda_i \geq 0$ *for* $i = 1, \ldots, t,$ *over all finite subsets* $\{x^1, \ldots, x^t\}$ *of* $X\}$.

Considering Definition 3 it is possible to conclude that conv(X) is a polyhedron and its extreme points all lie in X.

2.1.2 MILPs

The Shuttle Tanker Scheduling problem, the focus of this dissertation, belongs to a specific set of optimization problems classified as Mixed-Integer Linear Programming (MILP). MILPs are a subset of integer programming problems whose objective function and constraints are linear and variables are discrete and continuous. A generic MILP problem is formulated as follows:

$$MILP: \; f = \max_{\boldsymbol{x},\boldsymbol{y}} \; \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{g}^T\boldsymbol{y} \tag{2.3a}$$

$$\text{s.t.}:$$

$$A\boldsymbol{x} \leq \boldsymbol{b} \tag{2.3b}$$

$$C\boldsymbol{y} \leq \boldsymbol{d} \tag{2.3c}$$

$$\boldsymbol{x} \in \mathcal{Z}_+^n \tag{2.3d}$$

$$\boldsymbol{y} \in \mathcal{R}_+^m \tag{2.3e}$$

where $\boldsymbol{c}$, $\boldsymbol{g}$, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{b}$ and $\boldsymbol{d}$ are vectors and A and C are matrices (all vectors and matrices are with the right dimensions).

2.2 Solving Strategies

Considering the complexity of integer programming, some strategies are necessary to solve this type of problems. In this section we discuss in a basic way the *Branch and Bound*, *Cutting Plane*, *Branch and Cut* and *Heuristics.* Other strategies like the Dantzig-Wolf (based on column generation) and Benders' (based on row generation) decompositions can also be employed, but will not be discussed.

2.2.1 Branch and Bound

The branch and bound strategy can be seen as a decomposition algorithm based on a divide and conquer approach, in which a problem is broken into smaller and easier subproblems and its lower and upper bounds are used to solve the original problem. This algorithm has the structure of a tree, with nodes representing the subproblems and the edges the constraints (or bounds) to be added to the new subproblems.

2.2.2 Cutting Plane

From Definition 3 we can conclude that it is possible to reformulate an integer program as a linear program $\max\{\boldsymbol{c}^T\boldsymbol{x} : \widetilde{A}\boldsymbol{x} \leq \widetilde{\boldsymbol{b}}, \boldsymbol{x} \geq \boldsymbol{0}\}$. In other words, an optimal extreme point solution of a linear program is an optimal solution of an integer program provided that $\widetilde{P} = \{\boldsymbol{x} : \widetilde{A}\boldsymbol{x} \leq \widetilde{\boldsymbol{b}}, \boldsymbol{x} \geq \boldsymbol{0}\} = conv(X)$ (WOLSEY, 1998). The same holds for MILPs.

For $\mathcal{NP}$-Hard problems like the Shuttle Tanker Scheduling problem it is very difficult to describe $conv(X)$. The aim of cutting plane algorithms is to try to approximate $conv(X)$ for a given instance. Said that, an important concept is the definition of a valid inequality:

Definition 4. *An inequality $\boldsymbol{\pi}^T\boldsymbol{x} \leq \boldsymbol{\pi_0}$ is a valid inequality for $X \subseteq \mathcal{R}^n$ if $\boldsymbol{\pi}^T\boldsymbol{x} \leq \boldsymbol{\pi_0}$ for all $\boldsymbol{x} \in X$.*

Valid inequalities can be seen simply as constraints that are satisfied by all $\boldsymbol{x} \in X$.

2.2.2.1 Chavátal-Gomory Procedure

Assuming that we do not have a family of valid inequalities, we can use the Chavátal-Gomory procedure to obtain them (Theorem 1).

Theorem 1. *Every valid inequality for the set $X = P \cap \mathcal{Z}^n$, where $P = \{\boldsymbol{x} \in \mathcal{R}^n_+ : A\boldsymbol{x} \leq \boldsymbol{b}\}$, A is a $m \times n$ matrix with columns $\{a_1, a_2, \ldots, a_n\}$ and $\boldsymbol{u} \in \mathcal{R}^m_+$, can be obtained by applying the Chavátal-Gomory procedure a finite number of times.*

The procedure is defined as follows:

1. The inequality $\sum_{j=1}^{n} ua_j x_j \leq ub$ is valid for P, since $u \geq 0$ and $\sum_{j=1}^{n} a_j x_j \leq b$ is valid for P,
2. The inequality $\sum_{j=1}^{n} \lfloor ua_j \rfloor x_j \leq ub$ is valid for P as $x \geq 0$,
3. The right hand side must also be integer. Therefore, the inequality $\sum_{j=1}^{n} \lfloor ua_j \rfloor x_j \leq \lfloor ub \rfloor$ is valid for X as x is integer.

Consequently, the procedure generated the valid inequality $\sum_{j=1}^{n} \lfloor ua_j \rfloor x_j \leq \lfloor ub \rfloor$.

This procedure is sufficient to generate all valid inequalities for an integer program. The challenge consists in defining the non-negative weights vector u and verify how useful these valid inequalities are for the problem at hand.

2.2.2.2 Cutting Plane Algorithm

Consider that $X = P \cap \mathcal{Z}^{\mathrm{n}}$ and that we have a family $\mathcal{F}$ of valid inequalities $\boldsymbol{\pi}^T \boldsymbol{x} \leq \boldsymbol{\pi_0}, (\boldsymbol{\pi}, \boldsymbol{\pi_0}) \in \mathcal{F}$ for X. The set $\mathcal{F}$ can contain an enormous quantity of inequalities to be added a priori and some of them may not be useful for the problem instance. An alternative is to add valid inequalities from $\mathcal{F}$ interactively, so as to cut off infeasible solutions (solutions that violated some inequality of the family $\mathcal{F}$)

First we solve the linear relaxation of the problem. The optimal value is tested for being an integer solution. If not, we can search for a linear inequality that separates this fractional solution from the feasible set. The problem of finding such an inequality is called the separation problem and the inequality is a cut. When the cut is added to the linear program, the current non-integer solution is no longer feasible to the relaxation. This process is repeated until an optimal integer solution is reached or no cutting planes can be found.

Some other strategies to generate valid inequalities like the Gomory's Fractional Cutting Plane Algorithm, the Gomory Mixed Integer Cut and Disjunctive Inequalities can also be used.

Ideally, we would find *Strong Valid Inequalities* that are expected to be more effective and to lead to a stronger formulation. The interested reader may refer to Wolsey (1998) for a detailed discussion on strong valid inequalities.

The possibility of introducing cutting planes on demand has given rise to the *Branch and Cut Method*, a strategy based on branch and bound which adds cutting planes at the nodes of the enumeration tree. It is the branch and bound with cutting planes been generated along its tree. Rather than reoptimizing fast at each node, like the branch and bound strategy, the new philosophy attempts to tighten the dual bound by introducing cutting planes and consequently reduce the number of nodes. There is a trade-off in the sense that if many cuts are added at each node, reoptimizing can take much longer and keeping all information in the tree can take much more memory.

2.2.3 Heuristics

Many practical problems that we wish to solve are $\mathcal{NP}$-Hard and approximation algorithms or heuristics play an important role in that matter. Heuristics do not guarantee optimality, instead they attempt to quickly achieve a good feasible solution (WOLSEY, 1998). Greedy, local search, and genetic algorithms are examples of heuristics for integer programming problems.

2.3 Lagrangean Duality

According to Geoffrion (1974), the relaxation of a optimization problem (a maximization one) is defined as follows.

Definition 5. *The problem* $RP_{\max} : \max\{g(\boldsymbol{x})|\boldsymbol{x} \in \mathcal{W}\}$ *is a relaxation of the problem* $P_{\max} : \max\{f(\boldsymbol{x})|\boldsymbol{x} \in \mathcal{V}\}$*, with the same decision variables* $\boldsymbol{x}$*, if and only if:*

1. *The feasible set of* $RP_{\max}$ *contains the feasible set of* $P_{\max}$*, i.e.,* $\mathcal{W} \supseteq \mathcal{V}$.

2. *Over the feasible set of* $P_{\max}$*, the objective function of* $RP_{\max}$ *is always equal or greater than the objective function of* $P_{\max}$*, i.e.,* $\forall \boldsymbol{x} \in \mathcal{V}, g(\boldsymbol{x}) \geq f(\boldsymbol{x})$.

Considering $v(P_{\max})$ the optimal value of problem $P_{\max}$, a consequence of Definition 5 is that $v(RP_{\max}) \geq v(P_{\max})$. In other words, $RP_{\max}$ can be considered as an optimistic version of $P_{\max}$.

Two types of relaxations can be mentioned: the *Linear Relaxation* which ignores the integrality constraint on the decision variables and the *Lagrangean Relaxation* which will be discussed below.

The objectives of relaxing a problem are two: obtain bounds on the optimal value of complex problems; use their solutions, that most of the cases are infeasible for the original problem, as initial points for specialized heuristics to obtain a primal feasible solution.

2.3.1 Lagrangean Relaxation

The Lagrangean relaxation is a technique which can be used to simplify an optimization problem and obtain dual bounds for the objective function. This technique is presented and discussed in the works of Guignard (2003), Geoffrion (1974), Fisher (1985) and Wolsey (1998), among others.

A generic integer programming problem is given in Problem 2.4, where $\boldsymbol{c}$, $\boldsymbol{x}$, $\boldsymbol{b}$ and $\boldsymbol{d}$ are vectors and A and C are matrices (all vectors and matrices are with the right dimensions). Suppose that the set of constraints (2.4b) is complicating, in the sense that if we remove it from the problem, it becomes easier to solve.

$$Z: \; f = \max_{\boldsymbol{x}} \; \boldsymbol{c}^T \boldsymbol{x} \tag{2.4a}$$

$$\text{s.t.}: \; A\boldsymbol{x} \leq \boldsymbol{b} \tag{2.4b}$$

$$C\boldsymbol{x} \leq \boldsymbol{d} \tag{2.4c}$$

$$\boldsymbol{x} \in X \tag{2.4d}$$

where X defines the integrality constraints. The linear relaxation LP of Z is obtained by dropping the integratility constraints from problem (2.4), namely the problem given by Equations (2.4a) through (2.4c).

The Lagrangean relaxation of this problem is defined as follows:

$$
\begin{aligned}
Z_{LGR}:\ l(\boldsymbol{\lambda}) = \max_{\boldsymbol{x}}\ \ & \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) && (2.5a)\\
\text{s.t.}:\ \ & C\boldsymbol{x} \leq \boldsymbol{d} && (2.5b)\\
& \boldsymbol{x} \in X && (2.5c)
\end{aligned}
$$

where $l(\boldsymbol{\lambda})$ is the Lagrangean dual function. Note that Problem 2.5 is a relaxation of Problem 2.4 for the following reasons:

1. The objective function $c^T\boldsymbol{x} + \boldsymbol{\lambda}^T(b - A\boldsymbol{x}) \geq c^T\boldsymbol{x}$, since $\boldsymbol{\lambda} \geq 0$ and $b - A\boldsymbol{x} \geq 0$ for any feasible solution of Problem 2.4. For an infeasible solution of Problem 2.4, the added term will have negative sign.

2. The feasible region of Problem 2.4 is contained in the feasible region of Problem 2.5.

It is important to note that the bound provided by the Lagrangean dual function depends on which constraints were dualized. The influence of the dualized constraints on the effectiveness of the dual bound will be described latter in this chapter.

2.3.2 Lagrangean Dual Problem

Given the Lagrangean relaxation problem Z_{LGR}, which $\boldsymbol{\lambda}$ gives the tightest bound for the integer problem Z? The goal is to minimize the increase that the dualized constraints provide in the objective function, meaning the minimization of $l(\boldsymbol{\lambda})$. From this arises the Lagrangean dual problem:

$$
\begin{aligned}
Z_{LD}:\ \min_{\boldsymbol{\lambda}}\ \ & l(\boldsymbol{\lambda}) && (2.6a)\\
\text{s.t.}:\ \ & \boldsymbol{\lambda} \geq \boldsymbol{0} && (2.6b)
\end{aligned}
$$

Let $\mathcal{X} = \{\boldsymbol{x} \in X : C\boldsymbol{x} \leq \boldsymbol{d}\}$ be the set of feasible points of the Lagrangean relaxation problem. Notice that $\mathcal{X} = \{\boldsymbol{x}^1, \ldots, \boldsymbol{x}^m\}$ is a countable set assuming that the polyhedron $\{\boldsymbol{x} \in \mathbb{R}^n : C\boldsymbol{x} \leq \boldsymbol{d}\}$ is bounded.

Now, the Lagrangean dual function $l(\boldsymbol{\lambda})$ can be seen as the upper envelope of a set of affine functions of $\boldsymbol{\lambda}$ which lead to the following reformulation of Z_{LD}:

$$
\begin{aligned}
Z_{LD} : \min_{\boldsymbol{\lambda} \geq \mathbf{0}} l(\boldsymbol{\lambda}) &= \min_{\boldsymbol{\lambda} \geq \mathbf{0}} \max_{\boldsymbol{x} \in \mathcal{X}} f(\boldsymbol{x}) + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) \\
&= \min \ \eta \\
&\quad \text{s.t.} : \eta \geq f(\boldsymbol{x}) + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}), \forall \boldsymbol{x} \in \mathcal{X} \\
&\quad \boldsymbol{\lambda} \geq \mathbf{0}
\end{aligned}
$$

The Lagrangean dual problem Z_{LD} becomes a convex non-differentiable piecewise-linear minimization problem of $\boldsymbol{\lambda}$ (Figure 2.2). Considering the characteristics of problem Z_{LD}, special algorithms like the *Subgradient* method and the *Constraint Generation Method* can be applied to find the lowest upper bound for the optimal f.

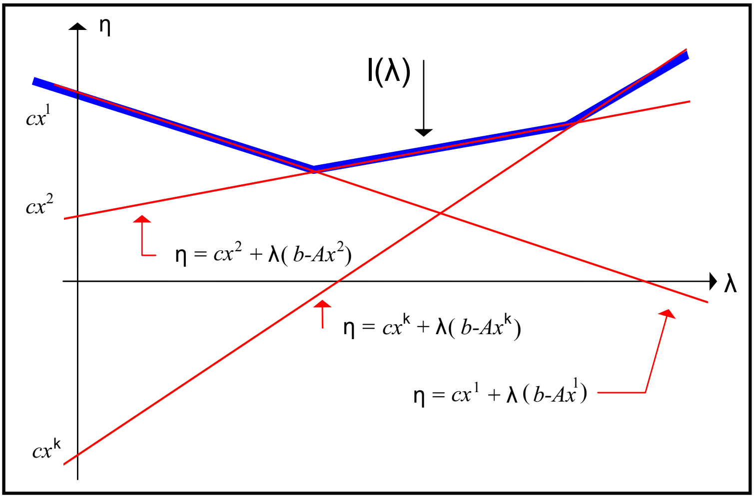

Figure 2.2 – Lagrangean Dual Function $l(\boldsymbol{\lambda})$ (GUIGNARD, 2003).

2.3.3 Subgradient Method

The Lagrangean dual function $l(\boldsymbol{\lambda})$ is known to be convex and therefore can be minimized using descent methods (BAZARAA; SHERALI; SHETTY, 2006; FISHER, 1985). Because $l(\boldsymbol{\lambda})$ is non-differentiable, the gradient descent method is not applicable and therefore the subgradient algorithm is an alternative.

Let $\boldsymbol{x}(\boldsymbol{\lambda})$ denote a solution for the Lagrangean dual function $l(\boldsymbol{\lambda})$ associated to $\boldsymbol{\lambda}$. The first step is computing the subgradient $\partial l(\boldsymbol{\lambda})$ for l at $\boldsymbol{\lambda}$, which is easily obtained from $\boldsymbol{x}(\boldsymbol{\lambda})$ as follows:

$$\partial l(\boldsymbol{\lambda}) = \boldsymbol{b} - A\boldsymbol{x}(\boldsymbol{\lambda}) \tag{2.7}$$

At each iteration k the subgradient method solves $l(\boldsymbol{\lambda}_k)$ to obtain the dual solution $\boldsymbol{x_k}(\boldsymbol{\lambda}_k)$. Then, the subgradient $\partial l(\boldsymbol{\lambda}_k)$ of l at $\boldsymbol{\lambda}_k$ is computed according with Equation (2.7) and the updating rule is used:

$$\boldsymbol{\lambda}_{k+1} = \max\{\boldsymbol{\lambda}_k - \mu_k \partial l(\boldsymbol{\lambda}_k), \mathbf{0}\}$$

to obtain the next Lagrange multiplier $\boldsymbol{\lambda}_{k+1}$, where $\mu_k > 0$ is a scalar. The subgradient method is described by Algorithm 1. The input data are the initial guess $\boldsymbol{\lambda_0}$ for the Lagrange multiplier vector, the initial step μ, the decrement of the step dec_μ and the maximum number of iterations $it_{\max}$. While the maximum number of iterations is not achieved, we solve the Lagrangean relaxation problem Z_{LGR} for the current $\boldsymbol{\lambda_k}$ and update $\boldsymbol{x_k}(\boldsymbol{\lambda_k})$ and the Lagrangean dual bound $actualLGR_{BOUND}$. Next, the subgradient and the Lagrange multiplier $\boldsymbol{\lambda_{k+1}}$ for the next iteration are computed. The policy for updating the step of the method μ is done by checking if the Lagrangean dual bound is decreasing in each iteration; if not, μ is multiplied by dec_μ.

Let $\{\mu_k\}_{k=1}^{\infty}$ be a sequence of positive steps satisfying a convergence condition such as the decreasing and non-summing condition, namely $\sum_{k=1}^{\infty} \mu_k = \infty$ and $\lim_{k\to\infty} \mu_k = 0$. For such a step sequence, the subgradient method yields a sequence of dual (upper) bounds $\{l(\boldsymbol{\lambda}_k)\}_{k=1}^{\infty}$ converging to $l(\boldsymbol{\lambda}^\star)$ (WOLSEY, 1998) which is the lowest higher bound induced by the optimal dual solution $\boldsymbol{\lambda}^\star$ of the Lagrangean dual problem Z_{LD}. Although this method is not expensive computationally, it does not have a stopping criteria and guarantees convergence only in theory.

Algorithm 1: Subgradient Method.

Data: $\boldsymbol{\lambda_0}, \mu, dec_\mu, it_{\max}$.
Result: $\boldsymbol{\lambda_k}$ that minimizes $l(\boldsymbol{\lambda})$.

```
prevLGR_BOUND = 10e10;
actualLGR_BOUND = 10e10;
while (it ≤ it_max) do
    prevLGR_BOUND ← actualLGR_BOUND;
    x_k, actualLGR_BOUND ← solve Z_LGR for λ_k;
    subgrad_k ← (b − A ∗ x_k λ_k);
    λ_{k+1} ← max{0, λ_k − μ ∗ subgrad_k};
    if (actualLGR_BOUND < prevLGR_BOUND) then
        μ ← μ;
    else
        μ ← μ ∗ dec_μ;
    end
    it = it + 1;
end
```

2.3.4 Constraint Generation Method

This method relies on the fact that the Lagrangean dual function $l(\boldsymbol{\lambda})$ is the upper envelope of a family of linear functions.

$$
\begin{aligned}
Z_{LD}: \; &= \min \; \eta \\
&\text{s.t.}: \eta \geq f(\boldsymbol{x}) + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}), \; \forall \boldsymbol{x} \in \mathcal{X} \\
&\boldsymbol{\lambda} \geq \mathbf{0}
\end{aligned}
$$

where $\mathcal{X}$ is the set of all feasible solutions of the Lagrangean relaxation Z_{LGR}. At each iteration, the method generates constraints with the following form:

$$\eta \geq f(\boldsymbol{x}) + \boldsymbol{\lambda}^T(b - A\boldsymbol{x})$$

by solving the Lagrangean relaxation $l(\boldsymbol{\lambda}_\text{k})$ for the current Lagrange multiplier $\boldsymbol{\lambda}_k$ to obtain the solution $\boldsymbol{x}_\text{k}$. These constraints are added to those generated previously to form the linear programming master problem MP:

$$MP: \ \eta_k = \min \quad \eta \tag{2.8a}$$
$$\text{s.t.}: \ \eta \geq f(\boldsymbol{x_l}) + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x_l}), \, l = 1, \ldots, k \tag{2.8b}$$
$$\boldsymbol{\lambda} \geq \mathbf{0} \tag{2.8c}$$

whose solution yields the next iterate $\boldsymbol{\lambda}_{k+1}$ for the Lagrangean relaxation. The process terminates when $\eta_k = l(\boldsymbol{\lambda}_{k+1})$, which is the optimal value of the Lagrangean dual problem Z_{LD}. The constraint generation method is described in Algorithm 2. The entry data is the initial guess $\boldsymbol{\lambda_0}$ for the Lagrange multiplier. The algorithm runs until the optimal value of the master problem MP (η_k) is equal to the value of the Lagrangean dual function $l(\boldsymbol{\lambda}_{k+1})$. In each iteration we solve the Z_{LGR} for the current $\boldsymbol{\lambda_k}$ and and update the solution $\boldsymbol{x_k}$. Then, we calculate the angular and coefficients for the new cut to be added to the master problem MP. The solution of the master problem generates the new Lagrange multiplier $\boldsymbol{\lambda_{MP}}$ to be used in the next iteration.

Algorithm 2: The Constraint Generation Method.

Data: $\boldsymbol{\lambda_0}$.
Result: $\boldsymbol{\lambda_k}$ that minimizes $l(\boldsymbol{\lambda}_\text{k})$.
while $(\eta_k \neq l(\boldsymbol{\lambda}_{k+1}))$ **do**
- $\boldsymbol{x}_\mathbf{k} \leftarrow solveZ_{LGR} \ for \ \boldsymbol{\lambda}_\mathbf{k}$;
- $linearCOEF \leftarrow \mathbf{c}^T\boldsymbol{x_k}$;
- $angularCOEF \leftarrow (\boldsymbol{b} - A * \boldsymbol{x_k})$;
- $AddCut(\eta \geq linearCOEF + \boldsymbol{\lambda_{MP}} * angularCOEF)$;
- $\boldsymbol{\lambda_{MP}} \leftarrow solveMP$;
- $\boldsymbol{\lambda_{k+1}} \leftarrow \boldsymbol{\lambda_{MP}}$;
- $k = k + 1$;

end

2.3.5 Bound Analysis

According to Geoffrion (1974) it is possible to prove that the Lagrangean dual problem Z_{LD} is equivalent to the following *Primal*

Relaxation:

$$PR:\ f = \max_{\boldsymbol{x}}\ \boldsymbol{c}^T\boldsymbol{x} \tag{2.9}$$

s.t. :

$$A\boldsymbol{x} \leq \boldsymbol{b} \tag{2.10}$$

$$conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\} \tag{2.11}$$

Proof.

$$v(Z_{LD}) = \min_{\boldsymbol{\lambda} \geq \boldsymbol{0}} v(Z_{LGR}) \tag{2.12}$$

$$= \min_{\boldsymbol{\lambda} \geq \boldsymbol{0}} \max_{\boldsymbol{x}} \{\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) | C\boldsymbol{x} \leq \boldsymbol{d}, \boldsymbol{x} \in X\} \tag{2.13}$$

$$= \min_{\boldsymbol{\lambda} \geq \boldsymbol{0}} \max_{\boldsymbol{x}} \{\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) | \boldsymbol{x} \in conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\}\} \tag{2.14}$$

$$= \max_{\boldsymbol{x}} \{\boldsymbol{c}^T\boldsymbol{x} | A\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\}\} \tag{2.15}$$

$$= v(PR) \tag{2.16}$$

where $v(Z_{LD})$, $v(Z_{LGR})$ and $v(PR)$ are respectively the optimal values of the Lagrangean dual, Lagrangean relaxation and the Primal relaxation.

Equation (2.14) is true because the maximum of a linear function over a bounded, discrete set of points is equal to the maximum of that linear function over the convex hull of this set of points. Equation (2.15) is true by linear programming duality because $conv\{\boldsymbol{x} \in \mathbb{R}^n | C\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in X\}$ is a bounded polyhedron and the problem in Equation (2.15) is a feasible bounded linear program. This proof is detailed in Appendix A. □

Considering the demonstration above and Figure 2.3, it is possible to make the following observations:

1. If $conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\} = \{\boldsymbol{x} | C\boldsymbol{x} \leq \boldsymbol{d}\}$, then $v(Z) \leq v(PR) = v(Z_{LD}) = v(LP)$, where $v(Z)$, $v(PR)$, $v(Z_{LD})$ and $v(LP)$ stand for, respectively, to the optimal values of the original problem, the primal relaxation, the Lagrangean dual problem and the linear relaxation of the original problem. In this case, the Lagrangean dual problem has the Integrality Property. In other words, if we

solve the Lagrangean dual problem relaxing the integrality of its integer variables, they would still get integer values. In this case, the Lagrangean relaxation bound is equal to the linear relaxation bound.

2. If $conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\} \subset \{\boldsymbol{x} | C\boldsymbol{x} \leq \boldsymbol{d}\}$, then $v(Z) \leq v(PR) = v(Z_{LD}) \leq v(LP)$ and the Lagrangean relaxation bound is better than the linear relaxation bound.

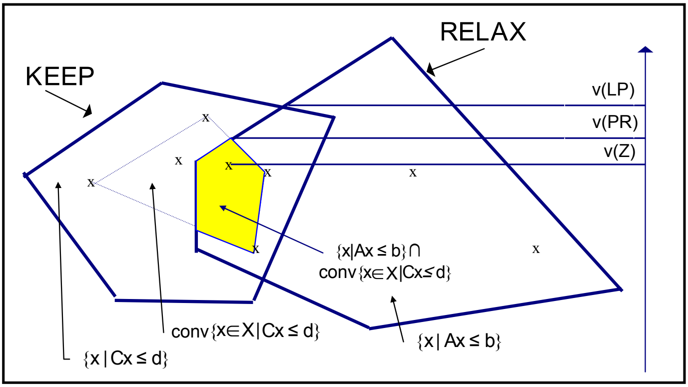

Figure 2.3 – Bound Analysis (GUIGNARD, 2003).

2.4 Dynamic Optimization

Here we present the rolling-horizon and the relax-and-fix strategies that were designed and implemented to solve the shuttle tanker scheduling problem. A synthesis of these strategies is given below.

2.4.1 Rolling-Horizon Strategy

Consider that we would like to plan the trips of the shuttle tankers for a planning horizon of length T periods. The rolling-horizon strategy (RHS) (MOHAMMADI et al., 2010; LI; IERAPETRITOU, 2010; TANG; JIANG; LIU, 2010; BERALDI et al., 2008), as illustrated in Figure 2.4, consists in defining a time window, also called prediction horizon (PH), which is smaller than the planning horizon

T. The optimization algorithm runs over this time window and only the actions of the first period are implemented (highlighted in green). Then, the time window slides one period towards the future, the state of the system (state variables) is updated and the process is repeated until reaching the end of the planning horizon T. In general this strategy gives sub-optimal solutions for the problem but is useful to deal with large problems instances.

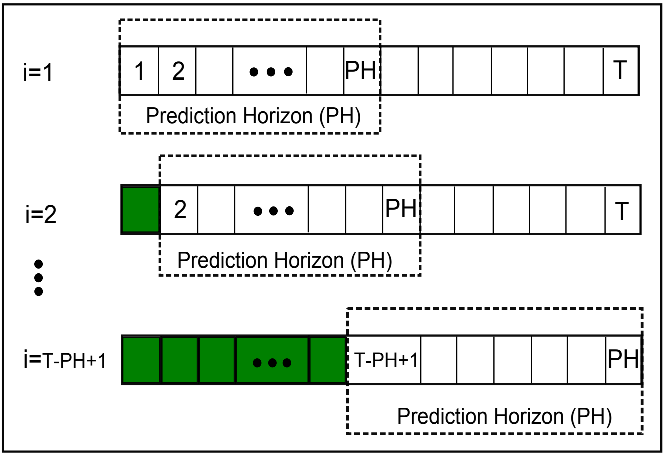

Figure 2.4 – Rolling-Horizon Strategy.

2.4.2 Relax-and-Fix Strategy

The relax-and-fix strategy is a branch of the rolling-horizon strategy (DILLENBERGER et al., 1994; MOHAMMADI et al., 2010; BERALDI et al., 2008). Like the RHS, we define a prediction horizon smaller than the planning horizon T. The difference is that the optimization algorithm also takes into account the periods beyond the prediction horizon (highlighted in red). However, the variables beyond are relaxed and the variables inside preserve their integral nature. Only the actions of the first period are implemented (highlighted in green). Then, the time window slides one period towards the future, the state of the system (state variables) is updated and the process is repeated until reaching the end of the planning horizon T. Figure 2.5 illustrates the process. Like the rolling-horizon, this strategy gives sub-optimal solutions for the problem but is useful to deal with large problems instances.

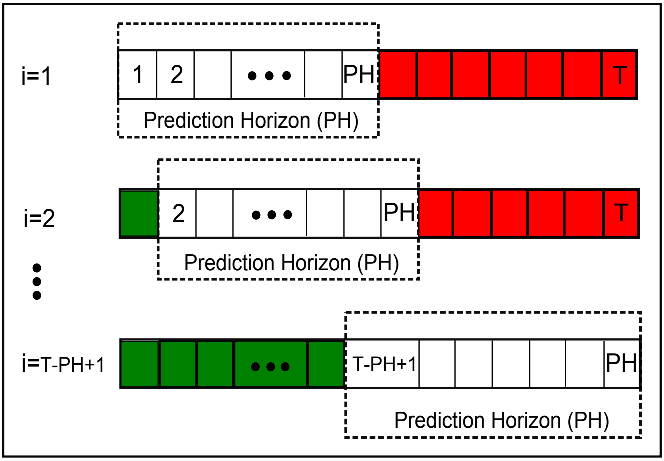

Figure 2.5 – Relax-and-Fix Strategy.

2.5 Summary

In this chapter we introduced some optimization concepts such as what is optimization and what is an optimization model using as example the Knapsack problem. Following, we presented the concept of Integer Programming (IP) and Mixed-Integer Linear Programming (MILP) and made a brief discussion on strategies to solve IPs and MILPs. We also introduced the concept of relaxation and its relevance for solving optimization problems. Finally, a section was dedicated to discuss rolling-horizon and relax-and-fix heuristics.

Chapter 3

Problem Formulation

In this chapter, we first present a former formulation for the shuttle tanker scheduling problem with fixed travel times. Then, we propose a revised new model which accounts with variable travel times. Next, a review of literature is carried out to relate the new proposed model to other technical works. The chapter ends with results regarding some theoretical properties of the revised model.

3.1 Fixed Travel Time Formulation (FTTF)

In this section we present and discuss the formulation proposed in (CAMPONOGARA; PLUCENIO, 2014) because this dissertation will extend this previous formulation to account for variable travel times for shuttle tankers.

The following parameters appear in the formulation:

- $\mathcal{F} = \{1, \ldots, F\}$ is the set of platforms.
- $\mathcal{S} = \{1, \ldots, S\}$ is the set of shuttle tankers to be scheduled.
- $\mathcal{T} = \{1, \ldots, T\}$ is the set of periods, which defines a time horizon of length T, over which the shuttle tanker moves and offloading/uploading operations are to be carried out. The periods of the horizon correspond to days.

- $\mathcal{U} = \{0\} \cup \mathcal{F}$ is the set of platforms augmented with the onshore terminal, represented by 0.
- $\mathcal{G} = (\mathcal{U}, \mathcal{E})$ is a graph whose nodes $\mathcal{U}$ represent the positions where shuttle tankers can be stationed, with $\mathcal{E} = \mathcal{U} \times \mathcal{U}$ being the set of arcs representing their possible moves.
- $US_s^0 \in \mathcal{U}$ is the initial position and VD_s^0 defines the initial volume of oil stored in a shuttle tanker s, in m^3, which establish the initial conditions for the shuttle tankers.
- $VD_s^{\max}$ is the storage capacity of tanker s in m^3.
- VF_u^0 is the initial oil stock, $VF_u^{\max}$ is the storage capacity and $VF_u^{\min}$ is the minimum volume that should be stored, in m^3, in a platform u.
- $VF_{u,s}^{\text{ofl}}$ is the pre-defined volume of oil, in m^3, that must be offloaded from platform u by shuttle tanker s each time a shuttle tanker visits the platform.
- QF_u^t is the daily production rate of oil to be produced by platform u during period t, given in m^3/day.
- $C_u^{\text{h},t}$ is the inventory holding cost tied up to the oil left in platforms, in \$.
- $C_{0,u}^t$ is cost of a shuttle tanker trip between the onshore terminal and a platform. This cost might be different from the average cost of tanker trips between platforms.
- $C_{u,v}^t$ is the cost of a trip between a given platform to another, which is considered the same for all pairs of platforms since the differences between these distances are negligible. In an offshore oil field, the distances between platforms are relatively small with respect to the distance to the onshore terminal.

The variables of the formulation are the following:

- $x_{u,s}^t \in \mathbb{B} = \{0, 1\}$ takes on value 1 if shuttle tanker s is at node u at the end of period t, otherwise it assumes value 0. This variable defines the scheduling of the shuttle tankers.
- vd_s^t is the volume of oil stored in shuttle tanker s at the end of period t, given in m^3.

- vf_u^t is the volume of oil stored by platform u at the end of period t, in m^3.

- $vd_s^{\text{aux},t}$ is an auxiliary variable used to implement the shuttle tanker unloading at the onshore terminal, given in m^3.

- $y_{0,s}^t \in \mathbb{B}$ takes on value 1 if tanker s arrives at or departs from the onshore terminal during period t and 0 otherwise.

- $y_{u,s}^t \in \mathbb{B}$ takes on value 1 if tanker s arrives at platform u during period t coming from another platform.

3.1.1 Formulation

After introducing all sets, parameters and variables, the STSP can be cast as the following MILP:

$$\begin{aligned} P: \ \min \quad f = & \sum_{u \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_u^{\text{h},t} \left(vf_u^t - VF_u^{\min} \right) \\ & + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} C_{0,u}^t y_{0,s}^t \\ & + \sum_{u \in \mathcal{F}} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} C_{u,v}^t y_{u,s}^t \end{aligned} \tag{3.1a}$$

subject to:

$$\sum_{u \in \mathcal{U}} x_{u,s}^t = 1, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T} \cup \{0\} \tag{3.2a}$$

$$\left\{ \begin{array}{l} x_{u,s}^0 = 1, \ u = US_s^0 \\ x_{u,s}^0 = 0, \ \forall u \in \mathcal{U} \setminus \{US_s^0\} \\ vd_s^0 = VD_s^0 \end{array} \right. \qquad \forall s \in \mathcal{S} \tag{3.2b}$$

$$vf_u^0 = VF_u^0, \ \forall u \in \mathcal{F} \tag{3.2c}$$

$$\sum_{s \in \mathcal{S}} x_{u,s}^t \leq 1, \ \forall u \in \mathcal{F}, \ \forall t \in \mathcal{T} \tag{3.2d}$$

$$\begin{cases} vf_u^t = vf_u^{t-1} + QF_u^t - \sum\limits_{s \in \mathcal{S}} VF_u^{\text{ofl}} x_{u,s}^t \\ vf_u^t \leq VF_u^{\max} \\ vf_u^t \geq VF_u^{\min} \end{cases} \quad \forall u \in \mathcal{F}, \forall t \in \mathcal{T} \tag{3.2e}$$

$$\begin{cases} vd_s^{\text{aux},t} = vd_s^{t-1} + \sum\limits_{u \in \mathcal{F}} VF_u^{\text{ofl}} x_{u,s}^t \\ vd_s^t \leq VD_s^{\max}(1 - x_{0,s}^t) \\ vd_s^t \geq vd_s^{\text{aux},t} - VD_s^{\max} x_{0,s}^t \\ vd_s^t \leq vd_s^{\text{aux},t} \end{cases} \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \tag{3.2f}$$

$$-y_{0,s}^t \leq x_{0,s}^t - x_{0,s}^{t-1} \leq y_{0,s}^t, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T} \tag{3.2g}$$

$$x_{u,s}^t - x_{u,s}^{t-1} - x_{0,s}^{t-1} \leq y_{u,s}^t, \ \forall u \in \mathcal{F}, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T} \tag{3.2h}$$

$$vd_s^t \geq 0, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T} \cup \{0\} \tag{3.2i}$$

$$vd_s^{\text{aux},t} \geq 0, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T} \tag{3.2j}$$

$$vf_u^t \geq 0, \ \forall u \in \mathcal{F}, \ \forall t \in \mathcal{T} \cup \{0\} \tag{3.2k}$$

$$x_{u,s}^t \in \{0, 1\}, \ \forall u \in \mathcal{U}, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T} \cup \{0\} \tag{3.2l}$$

$$y_{u,s}^t \in \{0, 1\}, \ \forall u \in \mathcal{U}, \ \forall s \in \mathcal{S}, \ \forall t \in \mathcal{T}. \tag{3.2m}$$

The objective function, defined by Eq. (3.1a), minimizes the inventory holding in the platforms and the transportation costs of trips among platforms and between the platforms and the onshore terminal.

To ensure that each shuttle tanker is exactly at only one node (platform or onshore terminal) at the end of each period, constraint (3.2a) is required. The initial conditions for the shuttle tankers are determined by the set of constraints (3.2b). Constraint (3.2c) establishes the initial volume of oil in the platforms and constraint (3.2d) ensures that at most one shuttle tanker can perform offloading operations in a platform during a time period.

In the constraint set (3.2e), the first one defines the inventory balance at the platforms, along with their volume capacity bounds. The inventory balance equation establishes that the current volume stored in a platform consists of the volume in the previous period added to the production and discounted the volume offloaded by a shuttle tanker, if stationed at the platform.

The constraint set (3.2f) establishes the inventory balance for the shuttle tankers. The variable $vd_s^{\mathrm{aux},t}$ defines the volume of oil in the shuttle tanker at the end of a period, which corresponds to its previous volume added by the amount taken from a platform (when $x_{u,s}^t = 1$). Its stock will be driven to zero at the end of the period if the shuttle tanker is positioned at the onshore facility (when $x_{0,s}^t = 1$). In other words, every time a shuttle tanker visits the onshore terminal, it will upload all of its volume of oil into the terminal.

The constraints (3.2g) and (3.2h) express the behavior of the variables $y_{u,s}^t$. It is possible to see that according with Eq. (3.2g), $y_{0,s}^t \geq |x_{0,s}^t - x_{0,s}^{t-1}|$. Since $|x_{0,s}^t - x_{0,s}^{t-1}|$ can be either 0 or 1: $y_{0,s}^t = 1$ when the value of the expression is 1 (meaning that a shuttle tanker arrived or departed from the onshore terminal); otherwise, $y_{0,s}^t = 0$ (meaning that the shuttle tanker stayed at the onshore terminal). This leads to the conclusion that $y_{0,s}^t \in \{0, 1\}$.

In Eq. (3.2h), the left-hand side can assume one of the values in the set $\{-1, 0, 1\}$. If the value of the left-hand side is 1, $y_{u,s}^t$ clearly assumes value 1, meaning that a shuttle tanker arrived in a platform. Otherwise, $y_{u,s}^t \geq 0$ because it cannot be negative.

The remaining constraints (3.2i) through (3.2m) define the nature of the decision variables.

For this formulation it is possible to define the vector of decision variables $\boldsymbol{\phi} = (\boldsymbol{vd}, \boldsymbol{vd}^{\mathbf{aux}}, \boldsymbol{vf}, \boldsymbol{x}, \boldsymbol{y})$ for any solution to P where $\boldsymbol{vd} = (vd_s^t : s \in \mathcal{S},\, t \in \mathcal{T} \cup \{0\})$ and $\boldsymbol{vd}^{\mathbf{aux}}$, $\boldsymbol{vf}$, $\boldsymbol{x}$, and $\boldsymbol{y}$ are defined similarly.

The Fixed Travel Time Formulation (FTTF) for the STSP can be represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ such as the simple scenario illustrated in Figure 3.1. The brown nodes (labeled "F_u") represent the platforms and the black node (labeled "0") represents the onshore terminal. Based on this representation and the formulation, it is possible to conclude the following:

- Every displacement between nodes is performed in exactly 1 period of time no matter the distance among platforms or between the onshore terminal and the platforms. Consider for instance that 1 period of time is equivalent 1 day. This means that if in a real operation the travel time for a shuttle tanker to go from platform $F2$ to the onshore terminal is 1 day and to go from platform $F2$ to platform $F3$ is half of a day (12 hours), so in terms of the model they all will be performed in 1 period of time, which takes 1 day for this instance.

- When an uploading or offloading operation is scheduled, it takes place in the same period of time used by the shuttle tanker to travel to the node where the operation will happen.

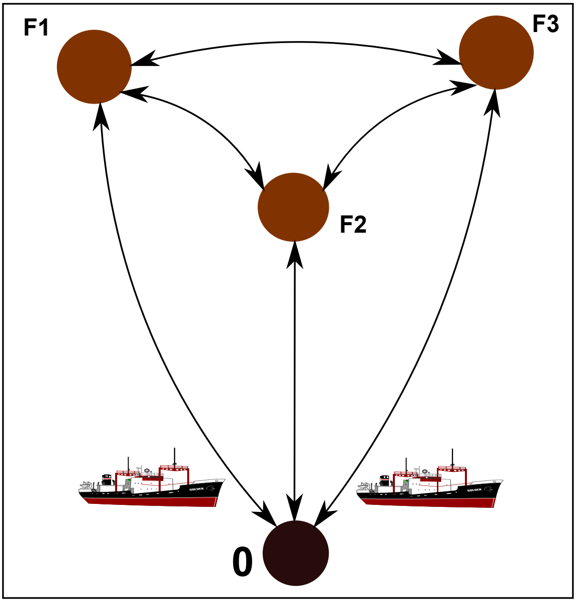

Figure 3.1 – Fixed Travel Time Representation.

3.2 Variable Travel Time Formulation (VTTF)

This section extends the formulation of (CAMPONOGARA; PLUCENIO, 2014) to consider variable travel time among the platforms and between the onshore terminal and the platforms.

3.2.1 Problem Representation

For the extended formulation, the STSP can be represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ such as in Figure 3.2. The brown nodes (labeled "F_u") represent the platforms; the black node (labeled "0") represents the onshore terminal; the blue nodes (labeled with positive numbers) are control points, placed in the model to introduce variable travel times. The control points can be used by the shuttle tankers as waiting points, since they cannot perform offloading operations at a platform if there is already another shuttle tanker stationed at the platform.

The possible moves for the shuttle tankers are represented by the arcs in Figure 3.2. Each arc represents an one-period trip (one period corresponds to one day). Note that the arcs do not represent physical paths but regions where the shuttle tankers can travel. Each shuttle tanker can perform two types of moves:

- go from node u to v, during a period t, along the arc $(u, v) \in \mathcal{E}$;
- stay at node u, during a period t, represented by the self-loop arcs $(u, u) \in \mathcal{E}$. If the shuttle tanker performs this move when visiting a platform, then an amount of oil can be offloaded from the platform; when visiting a control point (waiting point), it will stay there without performing any action; and when visiting the onshore terminal, the total volume of oil stored in its tanks will be offloaded.

The graph can be detailed as needed, enabling not only to represent days with the arcs, but also hours or minutes.

3.2.2 Problem Formulation

The parameters required for problem statement are as follows:

- $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a graph representing the moves that shuttle tankers can perform in one time period, where $\mathcal{V} = \{0, \ldots, V\}$ is the set of nodes and $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ is the set of arcs.
- The set of nodes is $\mathcal{V} = \mathcal{F} \cup \mathcal{I} \cup \{0\}$ with $\mathcal{F}$ being the set of platforms, $\mathcal{I}$ the set of intermediate nodes, and 0 the onshore terminal. It is assumed that $\mathcal{E}$ contains self-loops for the nodes, that is $\{(u, u) : u \in \mathcal{V}\} \subset \mathcal{E}$. A self-loop at a platform represents an offloading operation, whereas a self-loop at the onshore terminal

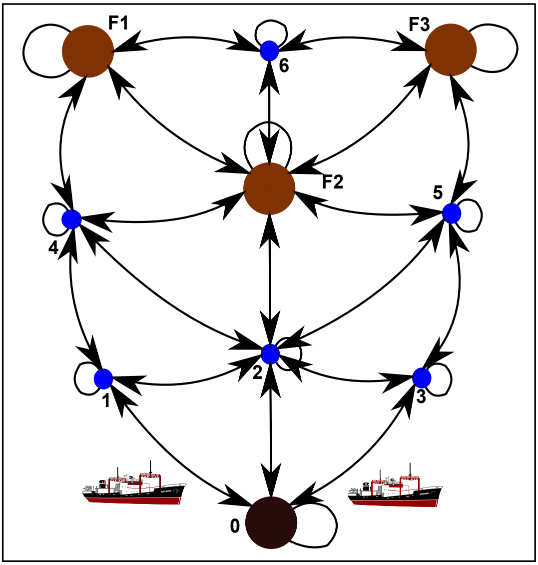

Figure 3.2 – Variable Travel Time Representation.

represents an uploading operation by a shuttle tanker. Self-loops at the intermediate nodes model periods during which a shuttle tanker is stationed.

- $\mathcal{T} = \{1, \ldots, T\}$ is the set of periods that defines a planning horizon of length T, over which the shuttle tanker moves and offloading/uploading operations are to be carried out. The periods of the horizon correspond to days.
- $\mathcal{S} = \{1, \ldots, S\}$ is the set of shuttle tankers.
- VF_u^0 is the initial oil stock, $VF_u^{\max}$ is the storage capacity, and $VF_u^{\min}$ is the minimum volume that should be stored in a platform $u \in \mathcal{F}$.
- $QF_u^{\min,t}$ and $QF_u^{\max,t}$ are the minimum and maximum volume of oil, respectively, that can be produced by platform $u \in \mathcal{F}$ in the period $[T_{t-1}, T_t)$, $t \in \mathcal{T}$.
- VD_s^0 is the initial volume of oil stored in shuttle tanker s, $VD_s^{\max}$ is its capacity, and US_s^0 is the initial position of the shuttle tanker, which can be a platform, the onshore terminal or a control point.

- $VF_{u,s}^{\text{ofl,min,t}}$ and $VF_{u,s}^{\text{ofl,max,t}}$ are the minimum and maximum volume of oil that can be offloaded from platform u by shuttle tanker s during period t.

- NS_u is the maximum number of shuttle tankers that can offload from a platform u or upload at the onshore facility (when $u = 0$).

The following variables are required:

- $x_{u,v}^{s,t} \in \mathbb{B}$ is 1 if shuttle tanker s moves from node u to v during period t, which stretches from time T_{t-1} to T_t, where $\mathbb{B} = \{0, 1\}$.

- vf_u^t is the volume of oil stored in platform $u \in \mathcal{F}$ at end of period t, while Δvf_u^t is the volume of oil produced during this period.

- vd_s^t is the volume of oil stored in shuttle tanker s at the end of period t.

- $\Delta vd_{u,s}^{\text{ofl},t}$ is the volume of oil offloaded from platform u or the amount uploaded at the onshore facility by shuttle tanker s, during period t.

A feasible route for shuttle tanker $s \in \mathcal{S}$ is established by the following constraints:

$$\sum_{(v,u)\in\mathcal{E}} x_{v,u}^{s,t} = \sum_{(u,v)\in\mathcal{E}} x_{u,v}^{s,t+1},\ u \in \mathcal{V},\ t \in (\mathcal{T}\backslash\{T\} \cup \{0\}), \tag{3.3a}$$

$$\sum_{(u,v)\in\mathcal{E}} x_{u,v}^{s,1} = 1,\ u = US_s^0, \tag{3.3b}$$

$$x_{u,v}^{s,1} = 0,\ u \in \mathcal{V}\backslash\{US_s^0)\}, (u,v) \in \mathcal{E}. \tag{3.3c}$$

The initial stocks at the shuttle tankers and the platforms are established by the following constraints:

$$vf_u^0 = VF_u^0,\ u \in \mathcal{F} \tag{3.4a}$$

$$vd_s^0 = VD_s^0,\ s \in \mathcal{S} \tag{3.4b}$$

The number of shuttle tankers that can perform offloading operations at the platforms and uploading operations at the onshore terminal, simultaneously, is limited:

$$\sum_{s \in \mathcal{S}} x_{u,u}^{s,t} \leq NS_u, \; u \in \mathcal{F} \cup \{0\}, \; t \in \mathcal{T}. \tag{3.5}$$

The inventory balance at the platforms are regulated by the following oil conservation equations for all $u \in \mathcal{F}, t \in \mathcal{T}$:

$$vf_u^t = vf_u^{t-1} + \Delta vf_u^t - \sum_{s \in \mathcal{S}} \Delta vd_{u,s}^{\text{ofl},t}, \tag{3.6a}$$

$$VF_u^{\min} \leq vf_u^t \leq VF_u^{\max}, \tag{3.6b}$$

$$QF_u^{\text{min,t}} \leq \Delta vf_u^t \leq QF_u^{\text{max,t}}, \tag{3.6c}$$

$$VF_{u,s}^{\text{ofl,min,t}} x_{u,u}^{s,t} \leq \Delta vd_{u,s}^{\text{ofl},t} \leq VF_{u,s}^{\text{ofl,max,t}} x_{u,u}^{s,t}, \; s \in \mathcal{S}. \tag{3.6d}$$

The inventory balance at the shuttle tanker is governed the following equations for all $s \in \mathcal{S}$ and $t \in \mathcal{T}$:

$$vd_s^t = vd_s^{t-1} - \Delta vd_{0,s}^{\text{ofl},t} + \sum_{u \in \mathcal{F}} \Delta vd_{u,s}^{\text{ofl},t}, \tag{3.7a}$$

$$vd_s^t \leq VD_s^{\max}. \tag{3.7b}$$

Oil transfer equations are necessary at the onshore facility for all $s \in \mathcal{S}$ and $t \in \mathcal{F}$:

$$\Delta vd_{0,s}^{\text{ofl},t} \geq vd_s^{t-1} - VD_s^{\max}(1 - x_{0,0}^{s,t}), \tag{3.8a}$$

$$\Delta vd_{0,s}^{\text{ofl},t} \leq vd_s^{t-1}, \tag{3.8b}$$

$$\Delta vd_{0,s}^{\text{ofl},t} \leq VD_s^{\max} x_{0,0}^{s,t}, \tag{3.8c}$$

Having introduced the notation and constraints, the problem of scheduling shuttle tankers to transport oil from platforms to an onshore facility is cast as:

$$
\begin{aligned}
P: \ \min \quad f = & \sum_{u \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_u^{\mathrm{h},t}(vf_u^t - VF_u^{\min}) \\
& + \sum_{u \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_u^{\mathrm{up},t}(QF_u^{\max,\mathrm{t}} - \Delta vf_u^t) \\
& + \sum_{(u,v) \in \mathcal{E}} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} C_{u,v}^{s,t} x_{u,v}^{s,t} \qquad (3.9\mathrm{a})
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.}: \ & (3.3)\text{–}(3.8) && (3.9\mathrm{b}) \\
& vf_u^t \geq 0, \ u \in \mathcal{F}, \ t \in \mathcal{T} \cup \{0\} && (3.9\mathrm{c}) \\
& \Delta vf_u^t \geq 0, \ u \in \mathcal{F}, \ t \in \mathcal{T} && (3.9\mathrm{d}) \\
& x_{u,v}^{s,t} \in \{0,1\}, (u,v) \in \mathcal{E}, \ s \in \mathcal{S}, \ t \in \mathcal{T} && (3.9\mathrm{e}) \\
& vd_s^t \geq 0, \ s \in \mathcal{S}, \ t \in \mathcal{T} \cup \{0\} && (3.9\mathrm{f}) \\
& \Delta vd_{u,s}^{\mathrm{ofl},t} \geq 0, \ u \in \mathcal{F} \cup \{0\}, \ s \in \mathcal{S}, \ t \in \mathcal{T} && (3.9\mathrm{g})
\end{aligned}
$$

whose objective accounts for inventory holding cost, production below capacity, and transportation costs where:

- $C_u^{\mathrm{h},t}$ is the inventory holding cost tied up to the oil left in platforms, which represents the tax the oil company has to pay in order to anticipate access to the money left in the platforms in the form of oil.

- $C_u^{\mathrm{up},t}$ being the cost for under production at platform $u \in \mathcal{F}$.

- $C_{u,v}^{s,t}$ being the cost for shuttle tanker s to travel along arc (u,v) during period t.

An economic study should be carried out to define values for the following parameters: inventory holding cost ($C_u^{\mathrm{h},t}$), underproduction cost($C_u^{\mathrm{up},t}$) and tanker-travel cost ($C_{u,v}^{s,t}$).

It is possible to define $\boldsymbol{\phi} = (\mathbf{vf}, \boldsymbol{\Delta}\mathbf{vf}, \mathbf{x}, \mathbf{vd}, \boldsymbol{\Delta}\mathbf{vd}^{\mathrm{ofl}})$ as the vector of decision variables for P where $\mathbf{vf} = (vf_u^t : u \in \mathcal{F}, \ t \in \mathcal{T} \cup \{0\})$ and $\boldsymbol{\Delta}\mathbf{vf}$, $\mathbf{x}$, $\mathbf{vd}$, and $\boldsymbol{\Delta}\mathbf{vd}^{\mathrm{ofl}}$ are defined similarly. With these vector definitions, the STSP can be recast in the following compact form:

$$\begin{aligned} P: \quad \min \quad & f(\phi) && (3.10a)\\ \text{s.t.}: \quad & \phi \text{ satisfies Eqs. } (3.3)-(3.8) && (3.10b)\\ & \phi \in \text{dom}(f) && (3.10c) \end{aligned}$$

where dom(f) is the domain of f established by Eqs. (3.9c)-(3.9g).

3.3 Literature Review

Several models for optimizing oil transportation are found in literature considering different characteristics. In this section we relate the new proposed model to other technical works. We divided this literature in two subsections. In the first one the works address the same problem considered in this dissertation. The second one deals with ship scheduling and oil transportation problems considering other contexts.

3.3.1 Similar Works

In (ROCHA; GROSSMANN; ARAGãO, 2013), the authors addressed the problem of transporting crude oil from offshore platforms to onshore terminals. This is a sub-problem of the Petroleum Supply Chain Problem, and the objective is to obtain the optimal schedule for the shuttle tankers, that satisfies economic and operational constraints (refinery demands, inventory capacity at the platforms and inventory capacity at the terminals). In this problem, a company has several platforms producing oil that is transported to terminals to supply refineries. The platforms produce only crude oil and they can only receive shuttle tankers of a specific class for offloading operations. The trips are only made between platforms and terminals, in others words, there are no trips between platforms. It is assumed that the daily production of each platform, the demands of the refineries, the travel time and routes between platforms and terminals are known in advance for the entire planning horizon. Also, when a shuttle tanker offloads a platform, it must be loaded to full capacity. The number of shuttle tankers is considered unlimited for each class of tanker. An initial formulation is presented, which considers inventory balance equations and storage capacity for the platforms and for the terminals. The cost function minimizes the transportation cost over the planning horizon. The authors propose a reformulation of the inventory balance constraints, named Cascading Knapsack inequalities, which has a special structure that can be exploited by solvers. The new formulation accomplishes to write those constraints as Knapsack Inequalities, in which inventory variables are eliminated, bringing forth to a 0-1 integer programming problem. The new formulation is as tight as the initial one, however, its structure can be exploited by MILP solvers, as well as providing a basis for the design of stronger formulations.

The work presented in (AIZEMBERG et al., 2014) and (ROCHA; GROSSMANN; ARAGãO, 2013) deals with the same problem . This work has two objectives. The first one is to compare mathematical formulations found in the literature for the oil transportation problem. From that, the authors proposed a new formulation that outperformed all previous ones when given to a solver. The second one is to propose a column generation-based heuristics to solve difficult instances of the problem.

The formulation developed in this dissertation differs in some aspects from the one presented in (ROCHA; GROSSMANN; ARAGãO, 2013). They are presented as follows:

1. **Overall Objective**. The work presented in (ROCHA; GROSSMANN; ARAGãO, 2013) aims to determine the optimal shipment schedule in order to satisfy refinery demands, avoid shutting down the platforms and minimize the total cost of trips.

   The formulation proposed in this dissertation has the goal to determine the optimal schedule for the shuttle tankers, in order to avoid shutting down platforms for lack of storage capacity, while respecting operational constraints and minimizing a combination of inventory holding, underproduction and total cost of trips (Net Present Value - NPV).

2. **Assumptions**. In (ROCHA; GROSSMANN; ARAGãO, 2013), the authors assume that offloading operations must load the shuttle tankers to full capacity; the number of shipments per period of time between each platform-terminal pair is limited to at most one; the number of shutter tankers is unlimited; offloading and uploading operations and the displacement between a platform-terminal pair are done in the same period of time.

For the formulation presented in this dissertation we considered that the onshore terminal has unlimited capacity and the number of shuttle tankers is limited.

3. **Constraints**. The work proposed in (ROCHA; GROSSMANN; ARAGãO, 2013) considers inventory balance for platforms and onshore terminals; refinery demands; limited capacity for platforms and onshore terminals; restrictions on the types of platforms that can supply each onshore terminal and types of vessels that can perform offloading operations in each platform.

   The work presented in this dissertation considered inventory balance for platforms and shuttle tankers; limited capacity for platforms and shuttle tankers; network constraints; bounds for the oil production rate and bounds for the volume of oil offloaded from a platform.

4. **Output Decisions**. As output, the model proposed in (ROCHA; GROSSMANN; ARAGãO, 2013) provides for each period of time if a shuttle tanker is assigned to offload a specific platform and delivery oil at a specific onshore terminal.

   The formulation proposed in this dissertation provides as output, for each period of time, the schedule of the shuttle tankers, the oil production rate and the amount of oil to be offloaded at each platform.

3.3.2 Related Works

The work of Rocha (2010) aims to solve the Petroleum Supply Chain Problem (PSCP) at Petrobras. First, the author describes the logistic process of petroleum supply to refineries in a strategic, tactical and operational hierarchical structure. In a general way, this problem consists in planning the shipments of crude oil from platforms to refineries on a daily basis. The crude oil can be nationally produced or imported. The locally produced oil, mostly offshore, can be transported to onshore terminals by shuttle tankers or pipelines. The imported oil is transported to the onshore terminals by shuttle tankers. At the onshore terminals, the stored oil can be exported or sent to refineries to supply its demands. The proposed mathematical model is based in a network flow structure and a discrete time representation. At the platforms, it is considered inventory balance constraints; limits in the storage capacity; limits on the number of shuttle tankers that leave a platform in each period; and limits on the number of shuttle tankers that arrive at the onshore terminal in each period of time. Regarding the onshore terminals, the source of crude oil can be from the platforms or from an other terminal and a shuttle tanker can upload all its volume or only a partition of it at a specific terminal. Constraints such as maximum capacity and inventory balance at the terminals are considered. For the refineries, there is an optimal range of inventory that should be respected. Volumes above (can generate logistical problems for the refinery) and below (refinery may need to shut down production units) that range are penalized. Also, an inventory balance constraint and a strategic plan for the refineries supply (always considering the next two months) are considered. For the shuttle tankers, it is defined a constraint on the maximum number of additional tankers needed during the planning horizon. The objective function minimizes shipping cost, penalty for out of range inventory at the refineries, penalty for deviation from the strategic planning and cost for additional tankers.

Al-Yakoob (1997) deals with the Oil Tanker Scheduling Problem for the Kuwait Petroleum Corporation (KPC). The problem consists in ship crude oil and refined oil-related products from ports in Kuwait to customers (ports) located in Europe, North America and Japan. Deliveries are undertaken by KPC or by other shipping companies and two routes are available for the tankers. Each vessel is a full shipload, and is characterizes by its type (oil, refined products, etc), loading port, loading date, discharging port, and discharging date.

In (CHOI; TCHA, 2007), the authors propose a tight integer programming model for the heterogeneous fleet vehicle routing problem and its linear programming relaxation is solved using Column Generation technique. The problems consists in defining a set of routs, each starting and ending at a depot, for a heterogeneous fleet of vehicles which services customers with known demands. Each customer is visit exactly one time and total demand of the route does not exceed the capacity of the vehicle type assigned to it. The routing cost of a vehicle is the sum of its fixed cost and a variable cost related to the travel distance. The objective is to minimize the total of routing costs and the number of vehicles of each type is assumed to be unlimited. The problem is represented in a direct graph and based in a discrete time representation. The nodes represent the central depot and customers and the arcs the available routes.

The works of (CHU et al., 2012) and (NISHI; YIN; IZUNO, 2011) address the problem of crude oil transportation among ports. Chu et al. (2012) demonstrate how the crude oil transportation problem, by shuttle tankers or trucks, can be transformed in a single item lot sizing problem with limited production and inventory capacity. In this problem the crude oil is shipped from a supplier port to n client ports to satisfy demands over a planning horizon of length T using a fleet of identical shuttle tankers with limited capacity. Constraints regarding inventory capacity of customers are considered and vary in time. The objective function consists in minimizing the total cost over the planning horizon. The assumption that the number of tankers are unlimited and demands are independent allows independent shipments to customers, which can be seen as n independent problems. These problems can be transformed into n single item lot sizing problems with limited production and inventory capacity, where the tanker capacity corresponds to production capacity in classical lot sizing models.

In (JETLUND; KARIMI, 2004), the authors consider the problem of scheduling of a fleet of multi-parcel tankers engaged in shipping bulk liquid chemicals. The work presents a MILP model and proposes a decomposition strategy to transform the fleet scheduling model in several one-ship model, which solves in reasonable time.

3.4 Theoretical Results

3.4.1 NP-Hardness

Proposition 1. *The problem of scheduling shuttle tankers is NP-Hard.*

Proof. The computational hardness of P can be shown by a reduction from the Hamiltonian path. Let C denote the Hamiltonian path problem for which one wants to find a simple path visiting each node of an undirected graph $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{E}_C)$, assuming w.l.o.g. that $\mathcal{V}_C = \{1, \ldots, V\}$.

The reduction of C to P is as follows:

1. Generate graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \mathcal{V}_C \cup \{0\}$ and $\mathcal{E} = \{(u, v), (v, u) : \{u, v\} \in \mathcal{E}_C\} \cup \{(u, u), (0, u), (u, 0) : u \in \mathcal{V}\}$ with platforms corresponding to the nodes of the original graph.

2. $VF_u^0 = 1$, $VF_u^{\max} = 1$, and $VF_u^{\min} = 0$ for each $u \in \mathcal{F}$, where $\mathcal{F} = \mathcal{V}_C$ and so that $\mathcal{I} = \emptyset$.

3. the planning horizon is $\mathcal{T} = \{1, 2, \ldots, 2V\}$, $QF_u^{\min,t} = QF_u^{\max,t} = 0$ for $t = 1, 2, \ldots, 2V-1$ but $QF_u^{\min,2V} = QF_u^{\max,2V} = 1$ for all $u \in \mathcal{F}$.

4. $S = \{1\}$ with $US_s^0 = 0$, $VD_s^{\max} = V$, and $VD_s^0 = 0$.

5. $VF_{u,1}^{\text{ofl},\min,t} = 0$, $VF_{u,1}^{\text{ofl},\max,t} = 1$ and $NS_u = 1$ for all $u \in \mathcal{F}$ and $t \in \mathcal{T}$.

6. $C_u^{\text{h},t} = 0$ and $C_u^{\text{up},t} = 0$ for all $u \in \mathcal{F}$ and $t \in \mathcal{T}$.

7. $C_{u,v}^{1,t} = 0$ for all $(u, v) \in \mathcal{E}$ and $t \in \mathcal{T}$.

Clearly C contains a Hamiltonian path if, and only if, P is feasible.

If C has a Hamiltonian path $H = \langle u_1, \ldots, u_V \rangle$, then the shuttle tanker can visit the platforms in the order H, offloading 1 unit from each platform following the route $(0, u_1), (u_1, u_1)$, (u_1, u_2), $(u_2, u_2), \ldots, (u_{V-1}, u_V)$, (u_V, u_V) which is clearly feasible in $T = 2V$ periods, since H is a Hamiltonian path and each platform will be empty at time $2V$, when one unit is required to be produced.

On the other hand, if C does not have a Hamiltonian path, then the shuttle tanker cannot visit and offload the volume stored in each platform during a planning horizon of $2V$ periods, which is required for all platforms to receive one unit of production at time $2V$, thereby implying that P is infeasible. □

3.4.2 Generalization

Proposition 2. *The problem of scheduling shuttle tankers with variable travel time generalizes the problem with unique travel time that was presented in (CAMPONOGARA; PLUCENIO, 2014).*

Proof. This generalization is demonstrated by means of a problem reduction. Let $\mathcal{H}$ be an instance of the scheduling problem with unique travel time.

An instance $\mathcal{H}_{\text{vtt}}$ of the scheduling problem with variable travel time corresponding to $\mathcal{H}$ is obtained as follows.

- Let the number of nodes be equal to the number of platforms $V = F$, $\mathcal{F} = \{1, \ldots, V\}$, $\mathcal{V} = \mathcal{F} \cup \{0\}$ with $\mathcal{I} = \emptyset$, and $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{E} = \{(u, v) : u \in \mathcal{V}, v \in \mathcal{V}\}$. Notice that $\mathcal{G}$ will have two arcs between each pair of nodes, as well as self-loops for all nodes.
- Let $\mathcal{T} = \{1, \ldots, 2T\}$, which will have twice the number of periods in $\mathcal{H}$ because in this version a shuttle tanker travels and performs an offloading operation during a single period, whereas in the variable travel-time version these operations are distinct.
- Let $VF_u^0 = VF_u^{\text{zero}}$, and $VF_u^{\max}$ and $VF_u^{\min}$ be as in $\mathcal{H}$, and $NS_u = 1$ for all $u \in \mathcal{F}$.
- Let $QF_u^{\max,t}$ be as given in $\mathcal{H}$ and $QF_u^{\min,t} = QF_u^{\max,t}$ when t is even, whereas and $QF_u^{\min,t} = QF_u^{\max,t} = 0$ when t is odd, for all $u \in \mathcal{F}$ and $t \in \mathcal{T}$.
- Let $US_s^0 = u_s^{\text{zero}}$, $VD_s^{\max}$ be as in $\mathcal{H}$, and $VD_s^0 = VD_s^{\text{zero}}$ for all $s \in \mathcal{S}$.
- Let $VF_{u,s}^{\text{ofl},\min,t} = VF_{u,s}^{\text{ofl},\max,t} = VF_u^{\text{ofl}}$ when t is even, whereas $VF_{u,s}^{\text{ofl},\min,t} = VF_{u,s}^{\text{ofl},\max,t} = 0$ when t is odd for all $u \in \mathcal{F}$, $s \in \mathcal{S}$, and $t \in \mathcal{T}$.
- Let $C_u^{\text{h},t} = c_1$ for t even and $C_u^{\text{h},t} = 0$ for t odd, and $C_u^{\text{up},t} = 0$ for all $u \in \mathcal{F}$, $t \in \mathcal{T}$.
- Let $C_{0,u}^{s,t} = C_{u,0}^{s,t} = c_2$ when t is odd, whereas $C_{0,u}^{s,t} = C_{u,0}^{s,t} = \infty$ when t is even, for all $u \in \mathcal{F}$, $t \in \mathcal{T}$, and $s \in \mathcal{S}$.
- Let $C_{u,v}^{s,t} = C_{v,u}^{s,t} = c_3$ when t is odd, whereas $C_{u,v}^{s,t} = C_{v,u}^{s,t} = \infty$ when t is even, for all $u \in \mathcal{F}$, $v \in \mathcal{F} \backslash \{u\}$, $t \in \mathcal{T}$, and $s \in \mathcal{S}$.
- Let $C_{u,u}^{s,t} = 0$ for all $u \in \mathcal{V}$, $s \in \mathcal{S}$, and $t \in \mathcal{T}$.

Let $X(\mathcal{H})$ be a solution to the instance $\mathcal{H}$. Then, a corresponding solution $X(\mathcal{H}_{\text{vtt}})$ to $\mathcal{H}_{\text{vtt}}$ is obtained as follows. For all $t \in \mathcal{T}$, $u, v \in \mathcal{V}$, and $s \in \mathcal{S}$, define $x_{u,v}^{s,2t-1} = 1$ and $x_{v,v}^{s,2t} = 1$ in $X(\mathcal{H}_{\text{vtt}})$ if only if $x_{u,s}^{t-1} = 1$ and $x_{v,s}^{t} = 1$ in $X(\mathcal{H})$, whereas the remaining variables $x_{u,v}^{s,t}$ are set to zero.

From the structure of $\mathcal{G}$, the definitions of the costs, production planning, and offloading operations, one can verify that at the end of period $2t$ in $X(\mathcal{H}_{\mathrm{vtt}})$, the state of the platforms, shuttle tankers (position and volumes), and accumulated transportation and holding costs will be the same at the end of period t in $X(\mathcal{H})$.

Similar reasoning can show that a solution $X(\mathcal{H}_{\mathrm{vtt}})$ to $\mathcal{H}_{\mathrm{vtt}}$ corresponds to an equivalent solution $X(\mathcal{H})$ to $\mathcal{H}$. The shuttle tankers in $\mathcal{H}_{\mathrm{vtt}}$ can effectively move from one node to another only in odd-number periods, or else stay stationed at any node in such periods without incurring cost, while offloading and uploading operations occur only in even-numbered periods by moving along self-loops. $\square$

3.4.3 A Family of Valid Inequalities

Let $\mathbf{x} = (x_{u,v}^{s,t} : (u,v) \in \mathcal{E},\, s \in \mathcal{S},\, t \in \mathcal{T})$ be a vector for the $x_{u,v}^{s,t}$ variables, and define vectors $\mathbf{vf}$, $\Delta\mathbf{vf}$, $\mathbf{vd}$, and $\Delta\mathbf{vd}$ in a similar manner. Further define $\boldsymbol{\theta} = (\mathbf{x}, \mathbf{vf}, \Delta\mathbf{vf}, \mathbf{vd}, \Delta\mathbf{vd})$ as a vector encompassing all of the decision variables. Then, the polyhedron $\mathcal{P} = \{\boldsymbol{\theta} : A_{\mathrm{i}}\boldsymbol{\theta} \leq \boldsymbol{b}_{\mathrm{i}},\, A_{\mathrm{ii}}\boldsymbol{\theta} = \boldsymbol{b}_{\mathrm{ii}},\, \boldsymbol{\theta} \geq \mathbf{0},\, \mathbf{x} \leq \mathbf{1}\}$ is a formulation for the problem of concern in this work if the rows of the systems $(A_{\mathrm{i}}, \boldsymbol{b}_{\mathrm{i}})$ and $(A_{\mathrm{ii}}, \boldsymbol{b}_{\mathrm{ii}})$ correspond to the constraints (3.3)-(3.8). This means that $\mathcal{X} = \mathcal{P} \cap (\mathbb{Z}^{\mathrm{size}(\mathbf{x})} \times \mathbb{R}^{\mathrm{size}(\boldsymbol{\theta})-\mathrm{size}(\mathbf{x})})$ with $\mathcal{X}$ being the feasible set. According to convex analysis, the convex hull $\mathrm{conv}(\mathcal{X})$ of $\mathcal{X}$ is a polyhedron.

A valid inequality can be imposed on the number of offloading operations performed at a subset $\mathcal{U} \subseteq \mathcal{F}$ of the platforms until time t.

Let

$$AVF_{\mathcal{U}}^{t} = \sum_{u \in \mathcal{U}} VF_{u}^{0} + \sum_{l=1}^{t} \sum_{u \in \mathcal{U}} QF_{u}^{\min,l} \tag{3.11}$$

be the minimum accumulated volume by the platforms $\mathcal{U}$ at end of period t. Then,

$$NOF_{\mathcal{U}}^{t} = \left\lceil \frac{AVF_{\mathcal{U}}^{t} - \sum_{u \in \mathcal{U}} VF_{u}^{\max}}{\max\{VF_{u,s}^{\mathrm{ofl},\max,l} : 1 \leq l \leq t,\, s \in \mathcal{S},\, u \in \mathcal{U}\}} \right\rceil \tag{3.12}$$

is a valid lower bound on the number of offloading operations that must be performed at the platforms $\mathcal{U}$ from period 1 through t in order to avoid capacity violation. This leads to the following valid inequality.

Proposition 3. *Given a subset $\mathcal{U} \subseteq \mathcal{F}$ of the platforms and period t, the inequality*

$$\sum_{l=1}^{t} \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} x_{u,u}^{s,l} \geq NOF_{\mathcal{U}}^{t} \tag{3.13}$$

is valid for conv($\mathcal{X}$).

Table 3.1 – Instance Parameters.

Platforms	Storage Capacity $VF_u^{\max}$ $[10^3 bbl]$	Initial Volume VF_u^0 $[10^3 bbl]$	Minimum Production Rate $QF_u^{\min,l}$ $[10^3 bbl/day]$	Maximum Offloading Volume $VF_{u,s}^{\text{ofl},\max,l}$ $[10^3 bbl]$
1	1400	1100	180	500
3	1450	600	230	400

Consider the sample instance parameters shown in Table 3.1 and the platform set $\mathcal{U} = \{1, 3\}$. Then,

$$AVF_{\mathcal{U}}^{10} = (600 + 1100) + 10(180 + 230) = 5800$$
$$\max\{VF_{u,s}^{\text{ofl},\max,l} : 1 \leq l \leq 10,\ s \in \mathcal{S},\ u \in \mathcal{U}\} = 500$$
$$\sum_{u \in \mathcal{U}} VF_u^{\max} = 1400 + 1450 = 2850.$$

Consequently, $NOF_{\mathcal{U}}^{10} = \lceil (5800 - 2850)/500 \rceil = \lceil 5.9 \rceil = 6$. Similar calculations lead to conclude that $NOF_{\mathcal{U}}^{19} = \lceil 13.28 \rceil = 14$.

Let $p_{u,v}^{t_1,t_2} = \langle u_1, u_2, \ldots, u_{t_2-t_1+1} \rangle$ be a path in graph $\mathcal{G}$ from $u_1 = u$ to $u_{t_2-t_1+1} = v$, such that $u_j \neq 0$ for all $j \in J_{\text{ofl}}(p_{u,v}^{t_1,t_2}) = \{1, 2, \ldots, t_2 - t_1 + 1\}$. Let $J_{\text{ofl}}(p_{u,v}^{t_1,t_2}) \subseteq J(p_{u,v}^{t_1,t_2})$ be the subset of maximum cardinality such that $j \in J_{\text{ofl}}(p_{u,v}^{t_1,t_2})$ if, and only, if $j < |J|$, $u_j = u_{j+1}$ and $u_j \in \mathcal{F}$.

Proposition 4. *For $t_1, t_2 \in \mathcal{T}$ with $t_2 > t_1$ and $u, v \in \mathcal{V} \backslash \{0\}$, let $p_{u,v}^{t_1,t_2}$*

be a given path in $\mathcal{G}$. *If, for a given shuttle tanker* $s \in \mathcal{S}$,

$$\sum_{j \in J_{\text{ofl}}(p_{u,v}^{t_1,t_2})} VF_{u_j,s}^{\text{ofl,min}} > VD_s^{\max} \tag{3.14}$$

then the inequality

$$\sum_{j \in J_{\text{ofl}}(p_{u,v}^{t_1,t_2})} x_{u_j,u_j}^{s,t_1+j-1} \leq |J_{\text{ofl}}(p_{u,v}^{t_1,t_2})| - 1 + \sum_{j=1}^{t_2-t_1} x_{0,0}^{s,t_1+j-1} \tag{3.15}$$

is valid for $\text{conv}(\mathcal{X})$.

3.5 Summary

In this chapter we presented a former formulation for the shuttle tanker scheduling problem with fixed travel times. Then, we proposed a revised new model, which accounts with variable travel times, and described its parameters, variables, constraints and objective function. Next, a review of literature was carried out to relate the new proposed model to other technical works.

The chapter ended with the presentation of theoretical results establishing the computational hardness of the problem and the generalization of the previous model which assumes the travel times of the shuttle tankers as fixed. Also, a family of cutting planes derived from the classic knapsack cover inequalities was presented.

Chapter 4

Lagrangean Duality

In this chapter we present the Lagrangean relaxation of the shuttle tanker scheduling problem, algorithms to solve the Lagrangean dual problem and the Lagrangean dual function decomposition.

4.1 Lagrangean Relaxation

The use of the Lagrangean relaxation technique for this problem is justified by two factors:

1. The distance between the primal optimal solution and the lower bound provided by the linear relaxation is significantly high. The technique can be used to try to obtain tighter bounds for the primal problem.

2. The technique provided significantly better bounds than the linear relaxation in the work presented in (CAMPONOGARA; PLUCENIO, 2014).

We can use the Lagrangean relaxation to obtain lower bounds for the Shuttle Tanker Scheduling Problem with Variable Travel Time by dualizing the coupling constraints (3.5) and (3.6a). To this end, let $\boldsymbol{\lambda} = (\boldsymbol{\lambda}^{(3.5)}, \boldsymbol{\lambda}^{(3.6a)})$ where $\boldsymbol{\lambda}^{(3.5)} = (\lambda_u^{(3.5),t} \geq 0 : u \in \mathcal{F} \cup \{0\}, t \in \mathcal{T})$

and $\boldsymbol{\lambda}^{(3.6a)} = (\lambda_u^{(3.6a),t} \in \mathcal{R} : u \in \mathcal{F},\, t \in \mathcal{T})$ are vectors of Lagrange multipliers associated with constraints (3.5) and (3.6a), respectively.

The Lagrangean dual function $l(\boldsymbol{\lambda})$, with respect to $\boldsymbol{\lambda}$, consists in solving the following Lagranagean subproblem:

$$LGR(\boldsymbol{\lambda}) :\ l(\boldsymbol{\lambda}) = \min_{\boldsymbol{\phi}} f(\boldsymbol{\phi}) + \sum_{u \in \mathcal{F}\cup\{0\}} \sum_{t \in \mathcal{T}} \lambda_u^{(3.5),t} \left(\sum_{s \in \mathcal{S}} x_{u,u}^{s,t} - NS_u \right)$$

$$+ \sum_{u \in \mathcal{F}} \sum_{t \in \mathcal{T}} \lambda_u^{(3.6a),t} \left(vf_u^t - vf_u^{t-1} - \Delta vf_u^t + \sum_{s \in \mathcal{S}} \Delta vd_{u,s}^{\text{ofl},t} \right) \quad (4.1a)$$

$$\text{s.t.} :\ \boldsymbol{\phi} \text{ satisfies Eqs. } (3.3)-(3.4),\ (3.6b)-(3.6d),\ (3.7)-(3.8) \quad (4.1b)$$

$$\boldsymbol{\phi} \in \text{dom}(f) \quad (4.1c)$$

where dom(f) is the domain of f established by Equations (3.9c)-(3.9g). It is important to note that the bound provided by the Lagrangean dual function depends on which constraints were dualized.

A Lagrange vector $\boldsymbol{\lambda} = (\boldsymbol{\lambda}^{(3.5)}, \boldsymbol{\lambda}^{(3.6a)})$ is dual feasible if $\boldsymbol{\lambda}^{(3.5)} \geq \mathbf{0}$, while $\boldsymbol{\phi}$ is primal feasible if $\boldsymbol{\phi} \in \text{dom}(f)$ and further satisfies Equations (3.3)-(3.8). From duality theory, $l(\boldsymbol{\lambda}) \leq f(\boldsymbol{\phi})$ for any dual feasible $\boldsymbol{\lambda}$ and primal feasible $\boldsymbol{\phi}$ (BAZARAA; SHERALI; SHETTY, 2006).

Having introduced the Lagrangean dual function, we want to find $\boldsymbol{\lambda}$ that gives the tightest lower bound for problem P. By maximizing the Lagranagean dual function $l(\boldsymbol{\lambda})$ we obtain the Lagrangean dual problem:

$$LD :\ \max_{\boldsymbol{\lambda}}\ l(\boldsymbol{\lambda}) \quad (4.2a)$$

$$\text{s.t.} :\ \boldsymbol{\lambda}^{(3.5)} \geq \mathbf{0} \quad (4.2b)$$

$$\boldsymbol{\lambda}^{(3.6a)} \in \mathbf{R} \quad (4.2c)$$

$$\boldsymbol{\lambda} = (\boldsymbol{\lambda}^{(3.5)}, \boldsymbol{\lambda}^{(3.6a)}) \quad (4.2d)$$

This is a concave, non differentiable and piecewise linear maximization problem on $\boldsymbol{\lambda}$. Two methods were implemented to solve the Lagrangean dual problem: the subgradient method and the constraint generation method. The numerical results will be described in Section 5.2.

4.2 Lagrangean Dual Function Decomposition

It is possible to decompose the computation of the Lagranagean dual function $l(\boldsymbol{\lambda})$ in $S{+}F$ subproblems, S being the number of shuttle tankers and F the number of platforms.

From the structure of the primal problem P, the dualization of the coupling constraints in the objective function renders the Lagrangean dual function separable, with independent terms for the shuttle tankers and platforms as follows:

$$l(\boldsymbol{\lambda}) = \sum_{s\in\mathcal{S}} l_s(\boldsymbol{\lambda}) + \sum_{u\in\mathcal{F}} l_u(\boldsymbol{\lambda}) - \sum_{u\in\mathcal{F}\cup\{0\}} \sum_{t\in\mathcal{T}} \lambda_u^{(3.5),t} NS_u + \sum_{u\in\mathcal{F}} \sum_{t\in\mathcal{T}} \left(C_u^{\mathrm{up},t} QF_u^{\max,t} - C_u^{\mathrm{h},t} VF_u^{\min}\right) \tag{4.3}$$

where l_s and l_u are computed by solving subproblems concurrently or in parallel as defined below.

Given the Lagrange vector $\boldsymbol{\lambda}$, $l_u(\boldsymbol{\lambda})$ is calculated for platform $u \in \mathcal{F}$ by solving:

$$\begin{aligned}
&L_u(\boldsymbol{\lambda}) : l_u(\boldsymbol{\lambda}) = \\
&= \min \sum_{t\in\mathcal{T}} \left(C_u^{\mathrm{h},t} vf_u^t - C_u^{\mathrm{up},t} \Delta vf_u^t\right) + \sum_{t\in\mathcal{T}} \lambda_u^{(3.6\mathrm{a}),t} \left(vf_u^t - vf_u^{t-1} - \Delta vf_u^t\right) \\
&= \min \sum_{t\in\mathcal{T}} \left(C_u^{\mathrm{h},t} + \lambda_u^{(3.6\mathrm{a}),t}\right) vf_u^t - \sum_{t\in\mathcal{T}} \left(\lambda_u^{(3.6\mathrm{a}),t}\right) vf_u^{t-1} \\
&- \sum_{t\in\mathcal{T}} \left(C_u^{\mathrm{up},t} + \lambda_u^{(3.6\mathrm{a}),t}\right) \Delta vf_u^t \\
&= \min \; - \lambda_u^{(3.6\mathrm{a}),1} vf_u^0 + \sum_{t=1}^{T-1} \left(C_u^{\mathrm{h},t} + \lambda_u^{(3.6\mathrm{a}),t} - \lambda_u^{(3.6\mathrm{a}),t+1}\right) vf_u^t \\
&+ \left(C_u^{\mathrm{h},T} + \lambda_u^{(3.6\mathrm{a}),T}\right) vf_u^T - \sum_{t\in\mathcal{T}} \left(C_u^{\mathrm{up},t} + \lambda_u^{(3.6\mathrm{a}),t}\right) \Delta vf_u^t
\end{aligned} \tag{4.4a}$$

$$\text{s.t.} : vf_u^0 = VF_u^0 \tag{4.4b}$$

$$\left\{\begin{array}{l} VF_u^{\min} \leq vf_u^t \leq VF_u^{\max} \\ QF_u^{\min,t} \leq \Delta vf_u^t \leq QF_u^{\max,t} \end{array}\right. \quad t \in \mathcal{T} \tag{4.4c}$$

$$vf_u^t \geq 0,\ t \in \mathcal{T} \cup \{0\} \tag{4.4d}$$

$$\Delta vf_u^t \geq 0,\ t \in \mathcal{T} \tag{4.4e}$$

which is a linear programming problem. Actually, $L_u(\boldsymbol{\lambda})$ is solved analytically as follows:

$$vf_u^0 = VF_u^0 \tag{4.5a}$$

$$vf_u^t = \left\{\begin{array}{l} VF_u^{\min} \text{ if } (C_u^{\mathrm{h},t} + \lambda_u^{(3.6\mathrm{a}),t} - \lambda_u^{(3.6\mathrm{a}),t+1}) \geq 0 \\ VF_u^{\max} \text{ otherwise} \end{array}\right. \quad \text{for } t \in (\mathcal{T} \backslash \{T\}) \tag{4.5b}$$

$$vf_u^T = \left\{\begin{array}{l} VF_u^{\min} \text{ if } (C_u^{\mathrm{h},T} + \lambda_u^{(3.6\mathrm{a}),T}) \geq 0 \\ VF_u^{\max} \text{ otherwise} \end{array}\right. \tag{4.5c}$$

$$\Delta vf_u^t = \left\{\begin{array}{l} QF_u^{\max,t} \text{ if } (C_u^{\mathrm{up},t} + \lambda_u^{(3.6\mathrm{a}),t}) \geq 0 \\ QF_u^{\min,t} \text{ otherwise} \end{array}\right. \quad \text{for } t \in \mathcal{T} \tag{4.5d}$$

Similarly, $l_s(\boldsymbol{\lambda})$ is calculated for a shuttle tanker $s \in \mathcal{S}$ by solving the MILP program:

$$\begin{aligned} L_s(\boldsymbol{\lambda}):\ l_s(\boldsymbol{\lambda}) = \min \ & \sum_{(u,v)\in\mathcal{E}} \sum_{t\in\mathcal{T}} C_{u,v}^{s,t} x_{u,v}^{s,t} \\ & + \sum_{u\in\mathcal{F}\cup\{0\}} \sum_{t\in\mathcal{T}} \lambda_u^{(3.5),t} x_{u,u}^{s,t} \\ & + \sum_{u\in\mathcal{F}} \sum_{t\in\mathcal{T}} \lambda_u^{(3.6\mathrm{a}),t} \Delta vd_{u,s}^{\mathrm{ofl},t} \end{aligned} \tag{4.6a}$$

$$\text{s.t.}:\ \sum_{(v,u)\in\mathcal{E}} x_{v,u}^{s,t} = \sum_{(u,v)\in\mathcal{E}} x_{u,v}^{s,t+1},\ u \in \mathcal{V},\ t \in (\mathcal{T}\backslash\{T\} \cup \{0\}) \tag{4.6b}$$

$$\sum_{(u,v)\in\mathcal{E}} x_{u,v}^{s,1} = 1,\ u = US_s^0 \tag{4.6c}$$

$$x_{u,v}^{s,1} = 0,\ u \in \mathcal{V}\backslash\{US_s^0)\}, (u,v) \in \mathcal{E} \tag{4.6d}$$

$$vd_s^0 = VD_s^0 \tag{4.6e}$$

$$VF_{u,s}^{\text{ofl,min},t} x_{u,u}^{s,t} \leq \Delta vd_{u,s}^{\text{ofl},t} \leq VF_{u,s}^{\text{ofl,max},t} x_{u,u}^{s,t},\ u \in \mathcal{F},\ t \in \mathcal{T} \tag{4.6f}$$

$$\left\{\begin{array}{l} vd_s^t = vd_s^{t-1} - \Delta vd_{0,s}^{\text{ofl},t} + \sum_{u\in\mathcal{F}} \Delta vd_{u,s}^{\text{ofl},t} \\ vd_s^t \leq VD_s^{\max} \\ \Delta vd_{0,s}^{\text{ofl},t} \geq vd_s^{t-1} - VD_s^{\max}(1 - x_{0,0}^{s,t}) \qquad t \in \mathcal{T} \\ \Delta vd_{0,s}^{\text{ofl},t} \leq vd_s^{t-1} \\ \Delta vd_{0,s}^{\text{ofl},t} \leq VD_s^{\max} x_{0,0}^{s,t} \end{array}\right. \tag{4.6g}$$

$$x_{u,v}^{s,t} \in \{0,1\}, (u,v) \in \mathcal{E},\ t \in \mathcal{T} \tag{4.6h}$$

$$vd_s^t \geq 0,\ t \in \mathcal{T} \cup \{0\} \tag{4.6i}$$

$$\Delta vd_{u,s}^{\text{ofl},t} \geq 0,\ u \in \mathcal{F} \cup \{0\},\ t \in \mathcal{T} \tag{4.6j}$$

4.3 Summary

In this chapter we presented the Lagrangean relaxation of the shuttle tanker scheduling problem. From the Lagrangean relaxation we derived the Lagrangean dual problem which can be solved using the subgradient and the constraint generation methods. At the end of the chapter we presented the Lagrangean dual function decomposition that generated $S+F$ subproblems, S being the number of shuttle tankers and F the number of platforms, which can be solved concurrently or in parallel to reduce computational time.

Chapter 5

Computational Analysis

In this chapter we present a basic offshore oil field scenario and the computational set up in which the analysis are carried out. The first analysis compares the Lagrangean dual bound with the linear relaxation bound. The second analysis compares the static solution, when the problem is solved for the entire planning horizon, with the dynamic solution obtained using the rolling-horizon and relax-and-fix heuristics. Finally, we evaluate the rolling-horizon strategy in a prototype simulator which generates random perturbations to the variables of the model.

5.1 Offshore Oilfield Scenario

The offshore oil field scenario is formed by 3 platforms, 2 shuttle tankers and 1 onshore terminal. Table 5.1 presents the following parameters for the platforms:

- Storage capacity $VF_u^{\max}$, in $10^3 bbl$;
- Minimum volume that must remain on the platforms $VF_u^{\min}$, in $10^3 bbl$;
- Initial volume $VF_u(0)$, in $10^3 bbl$;
- Production rate interval $[QF_u^{\min,t}, QF_u^{\max,t}]$, in $10^3 bbl/day$;

- Maximum and minimum offloading volume where $VF^{\text{ofl,min},t}_{u,s} = VF^{\text{ofl,max},t}_{u,s}$, in $10^3 bbl$;

where *bbl* and *bbl/day* stands for barrels and barrels per day respectively.

Table 5.1 – Parameters for the Platforms.

FPSO	Storage Capacity [$10^3 bbl$]	Min. Vol. [$10^3 bbl$]	Initial Volume [$10^3 bbl$]	Prod. Rate [$10^3 bbl/day$]	Offload. Vol. [$10^3 bbl$]
1	1400	500	650	[75, 80]	450
2	1350	500	750	[95, 100]	500
3	1450	500	600	[125, 130]	400

Table 5.2 gives the required parameters of the shuttle tankers, namely storage capacity ($VD^{\max}_s$) and initial volume (VD^0_s).

Table 5.2 – Parameters for the Shuttle Tankers.

Shuttle Tanker	Capacity [$10^3 bbl$]	Initial Volume [$10^3 bbl$]
1	2000	0
2	1800	0

For this scenario three categories of costs are defined and they can be seen in Table 5.3. Parameter $C^{s,t}_{u,v}$ is the cost to travel from u to v in dollars [$]; $C^{\text{up},t}_u$ is the cost for under production in dollars per barrel per day [$\$ * barrel^{-1} * day^{-1}$]; and $C^{\text{h},t}_u$ represents the cost for inventory holding in dollars per barrel per day [$\$ * barrel^{-1} * day^{-1}$]. It is assumed that only one shuttle tanker can perform offloading operations in a platform at a time, however up to two shuttle tankers can perform uploading operations at the onshore terminal simultaneously. Also, all shuttle tankers are initially stationed at the onshore terminal.

The computational set up consisted in expressing the MILP formulation of the scenario in AMPL (FOURER; GAY; KERNIGHAN, 2003), which was optimized with IBM ILOG CPLEX 12.2.0. The computational experiments were performed in a workstation with an Intel CPU i7 @ 2.00 GHz, Windows operating system and 8 GB of RAM.

Table 5.3 – Operating Costs.

Operating Costs	$C_u^{\mathrm{h},t}$ $[\frac{\$}{bbl*day}]$	$C_u^{\mathrm{up},t}$ $[\frac{\$}{bbl*day}]$	$C_{u,v}^{s,t}$ [$]
Low Cost	17	17	15 000
Baseline Cost	17	17	45 000
High Cost	17	17	80 000

5.2 Lagrangean Dual Bound

Table 5.4 presents the solution of the original problem (Primal Solution) and its respective linear relaxation bound (LP Solution) for four lengths of planning horizon ($T = 10$, $T = 15$, $T = 20$ and $T = 25$ days). The scenario is presented in Section 5.1 and the baseline cost is shown in Table 5.3. The GAP[1] and the CPU Time, in seconds, for solving the original problem are also presented. From Table 5.4, one can observe two points as the planning horizon varies:

- The GAP is significantly high and increases with the length of the planning horizon.
- The CPU Time grows exponentially with the length of the planning horizon.

These observations motivate the search for tighter lower bounds. To this end, the Lagrangean relaxation technique was applied.

Table 5.4 – Primal Solution and LP Solution GAP.

Planning Horizon [days]	Primal Solution [10^3\$]	LP Solution [10^3\$]	GAP	CPU Time [s]
10	132 650	63 507.2	52.12%	0.515
15	229 150	63 957.2	71.76%	73.25
20	336 700	64 407.2	80.87%	1 282.09
25	462 100	64 857.2	85.96%	75 445.03

[1] GAP = $\frac{PrimalSolution - LPSolution}{PrimalSolution} * 100$.

To find optimal Lagrange multipliers, two methods were implemented to solve the Lagrangean dual problem: subgradient and constraint generation. In this analysis, we consider the scenario presented in Section 5.1, a planning horizon of $T = 10$ days and the baseline cost shown in Table 5.3.

5.2.1 Subgradient Solution

The subgradient $\partial l(\boldsymbol{\lambda}) = (\partial l(\boldsymbol{\lambda}^{(3.5)}), \partial l(\boldsymbol{\lambda}^{(3.6a)}))$ for the Lagrangean dual function $l(\boldsymbol{\lambda})$ at $\boldsymbol{\lambda}$ is defined as follows:

$$\partial l(\lambda_u^{(3.5),t}) = \sum_{s \in \mathcal{S}} x_{u,u}^{s,t} - NS_u,\ u \in \mathcal{F} \cup \{0\},\ t \in \mathcal{T} \tag{5.1a}$$

$$\partial l(\lambda_u^{(3.6a),t}) = vf_u^t - vf_u^{t-1} - \Delta vf_u^t + \sum_{s \in \mathcal{S}} \Delta vd_{u,s}^{\text{ofl},t},\ u \in \mathcal{F},\ t \in \mathcal{T} \tag{5.1b}$$

Figure 5.1 shows the convergence of the subgradient method. One can observe in green the primal solution $P = 132\,650\ 10^3\$$, in red the LP bound $LP = 63\,507.2\ 10^3\$$ and in blue the bounds provided by Lagrangean dual function $l(\boldsymbol{\lambda}_k)$ for all $\boldsymbol{\lambda}_k$. Table 5.5 presents the chosen parameters of the subgradient method.

Table 5.5 – Subgradient Parameters.

Parameters	Value
Initial step size α_k	2.0
Step decrement α_{dec}	0.7
Number of iterations	3 000

Geoffrion (1974) shows that the Lagrangean relaxation must provide a bound at least as good as the LP bound, which is not accomplished for this instance using the subgradient method (Figure 5.1). This method has proved to be not practical for this instance since it does not have a stopping criteria and guarantees convergence only in theory.

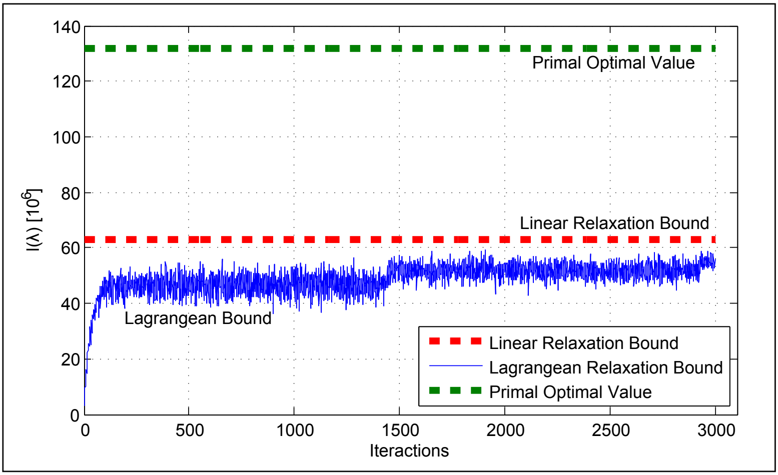

Figure 5.1 – Subgradient Convergence.

5.2.2 Constraint Generation Solution

Figure 5.2 shows the convergence of the constraint generation method. It is possible to observe in green the primal solution $P = 132\,650\ 10^3\$$, in red the LP bound $LP = 63\,507.2\ 10^3\$$ and in blue the bounds provided by Lagrangean dual function $l(\boldsymbol{\lambda}_k)$ for all $\boldsymbol{\lambda}_k$.

The method converged after 141 iterations, providing a bound $LGR = 63\,498\ 10^3\$$ which is approximately the same as the LP bound. If the Lagrangean relaxation had provide a better bound than the LP solution, it would be interesting to use it as the relaxation strategy in the branch-and-bound strategy. However, for this instance the Lagrangean dual bound do not offer advantages since it is almost the same as the LP bound, although with a higher computation time.

5.3 Static and Dynamic Analysis

In this section we compare the results obtained from the static and dynamic solutions. The static solution consists in solving the problem for the entire planning horizon T and the dynamic solution in solving the problem using rolling-horizon and relax-and-fix strategies to respond to unanticipated events and large instances of the problem.

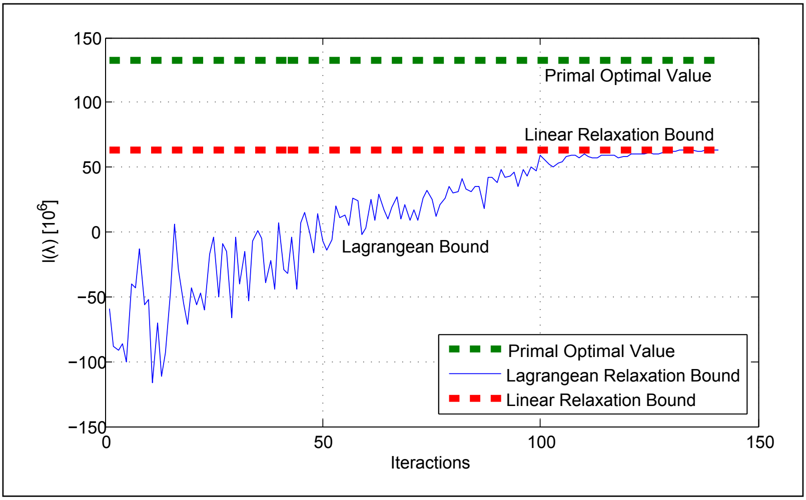

Figure 5.2 – Constraint Generation Convergence.

5.3.1 Static Solution

Here, we assume that there are no uncertainties in the operations. In other words, there will not be shutdowns or failures at the platforms, shuttle tankers or terminals; the production volume, the offloading and uploading volumes, among other parameters and variables, are known in advance for the entire planning horizon and will be as predicted.

The computational results, for the three types of transportation cost, are shown in Table 5.6, for a planning horizon of $T = 20$. CPU time (in seconds) is the time taken by CPLEX to find the optimal schedule.

Table 5.6 – Solution for Planning Horizon $T = 20$.

Transportation Cost	Optimal Value [10^3\$]	CPU Time [s]
Low Cost	335 500	855.51
Baseline Cost	336 700	1282.09
High Cost	338 100	4215.17

5.3.2 Dynamic Solution (Rolling-Horizon and Relax-and-Fix)

If uncertainties in the operations are not considered and reliable predictions of the parameters are known for the entire planning horizon T, then it is possible to solve the problem and obtain an optimal solution. However, two difficulties can arise when we run the optimizer for the entire planning horizon:

- Problem size. Depending on the size of the problem, determined by the number of platforms, shuttle tankers and the length of the planning horizon T, it can become hard to find a feasible solution, let alone an optimal one.
- Uncertainties. It is not reliable to schedule the shuttle tankers over the planning horizon based only on the available predictions at the current time. At the end of each period, the state variables (the position of each shuttle tanker; the volume of oil of each floating platform; and the volume of oil of the shuttle tankers) may differ from what was planned due to uncertainties and thereby should be revised.

To effectively respond to unanticipated events and handle large instances of the problem, two strategies are considered to solve the scheduling problem: rolling-horizon and relax-and-fix.

5.3.2.1 Rolling-Horizon Strategy (RHS)

Table 5.7 presents the computational results obtained by applying the RHS to the experimental scenario for a varying length of the prediction horizon and considering the size of the planning horizon as $T = 20$ days. The table gives the best value obtained with RHS, the CPU time, and the solution GAP[2], which is the relative distance between the objective value induced by the RHS and the optimal solution, namely the one obtained by solving the problem for the entire planning horizon (see Table 5.6).

[2] GAP $= \frac{Rolling-HorizonBestValue-OptimalValue}{OptimalValue} * 100$. The same formula is applied to the relax-and-fix GAP.

By comparing the results depicted in Tables 5.6 and 5.7, for the three types of transportation cost, it can be noticed that the RHS achieves the optimal solution with a relatively short prediction horizon and a substantially smaller CPU time. The RHS reaches the optimum with only 11 periods for the PH, taking less than 50 seconds, whereas the standard solution can take more than 4 000 seconds.

Table 5.7 – Rolling-Horizon Solution.

Transp. Cost	Prediction Horizon [days]	Best Value [$10^3$$]	GAP	CPU Time [s]
	7	367 800	9.62%	5.15
	8	367 800	9.62%	6.85
	9	367 800	9.62%	11.76
Low Cost	10	367 800	9.62%	24.65
	11	335 500	0%	40.01
	12	335 500	0%	78.03
	13	335 500	0%	145.60
	14	335 500	0%	952.31
	7	369 000	9.59%	5.23
	8	369 000	9.59%	10.07
	9	369 000	9.59%	11.32
Baseline Cost	10	369 000	9.59%	22.68
	11	336 700	0%	39.62
	12	336 700	0%	95.00
	13	336 700	0%	237.54
	14	336 700	0%	625.76
	7	370 400	9.55%	5.21
	8	370 400	9.55%	8.03
	9	370 400	9.55%	11.18
High Cost	10	370 400	9.55%	22.48
	11	338 100	0%	47.28
	12	338 100	0%	93.78
	13	338 100	0%	171.53
	14	338 100	0%	453.79

5.3.2.2 Relax-and-Fix Strategy (RFS)

Table 5.8 gives the computational results for the experimental scenario for a varying length of the prediction horizon and transportation costs. The size of the planning horizon is $T = 20$ days.

Table 5.8 – Relax-and-Fix Solution.

Transp. Cost	Prediction Horizon [days]	Best Value [10^3\$]	GAP	CPU Time [s]
Low Cost	7	335 500	0%	39.93
	8	378 000	12.66%	100.73
	9	335 500	0%	131.48
	10	364 400	8.61%	258.73
	11	343 150	2.28%	447.89
	12	343 150	2.28%	803.89
	13	335 500	0%	1181.78
	14	335 500	0%	1306.73
Baseline Cost	7	336 700	0%	42.12
	8	379 200	12.62%	110.92
	9	336 700	0%	148.15
	10	365 600	8.58%	287.76
	11	344 350	2.27%	484.62
	12	344 350	2.27%	907.37
	13	336 700	0%	1079.23
	14	336 700	0%	1415.57
High Cost	7	338 100	0%	45.79
	8	380 600	12.57%	86.59
	9	338 100	0%	95.90
	10	367 000	8.54%	210.64
	11	345 750	2.26%	357.79
	12	345 750	2.26%	791.23
	13	338 100	0%	1267.92
	14	338 100	0%	949.81

When Table 5.6 and 5.8 are compared, for all types of transportation cost, it is possible to observe that the RFS presents a smaller CPU Time. However, for some choices of prediction horizon, the CPU time is higher and the relax-and-fix does not reach the optimal value. A comparison between relax-and-fix and rolling-horizon suggests that relax-and-fix can reach the optimum with a shorter prediction horizon, however the behavior is somewhat erratic since solution quality can deteriorate with the increase of prediction horizon length.

5.4 Uncertainty Solution

The goal of this analysis is to model uncertainties for the shuttle tanker scheduling operation and use the rolling-horizon strategy to respond to these unanticipated events. A simulator is used to generate the uncertainties. An analysis regarding the best value of the objective function and different lengths of prediction horizon is presented.

5.4.1 Prototype Simulator

At the end of each period of the planning horizon $\mathcal{T}$, the system can be characterized by three state variables:

- The volume of oil vf_u^t at each platform $u \in \mathcal{F}$;
- The volume of oil vd_s^t in each shuttle tanker $s \in \mathcal{S}$;
- The position v of each shuttle tanker s, considering that $x_{u,v}^{s,t} = 1$.

When the operation obeys the predictions, in other words, there are no uncertainties or measurement errors, the values of the state variables will be as planned for the entire planning horizon T. However, in a real operational environment, variations in the predictions may occur and they should be taken into account. Some examples are described as follows:

- A platform can fully or partially halt the production due to technical problems. This may cause the oil production rate (Δvf_u^t) to be out of the acceptable range $[QF_u^{\min,t}, QF_u^{\max,t}]$, violating constraint (3.6c).
- The volume (vf_u^t) in a platform can be lower or higher than the prediction due to a variation in the oil production rate (Δvf_u^t) or in the offloaded volume ($\Delta vd_{u,s}^{\text{ofl},t}$). This may cause an infeasibility problem, violating constraint (3.6b), in the minimum or maximum volume of oil that a platform should have in storage.

- The volume in a shuttle tanker (vd_s^t) can vary depending on the offloaded volume from a platform ($\Delta vd_{u,s}^{\text{ofl},t}$), violating constraint (3.7b).

- A shuttle tanker can stop operating due to technical problems or maintenance reasons. In this case, the fleet may have its capacity diminished or a new shuttle tanker, with or without the same capacities, can replace it. Also, a shuttle tanker may have to follow another path due to the operator's decision or meteorological conditions. In all cases, this change should be taken into account in the optimization process.

One way to account for uncertainties and unanticipated events consists in relying on feedback and reoptimizing at the end of each period. The goal of designing a prototype simulator is to generate uncertainties to simulate real operations. These uncertainties consist of deviations from the predicted problem data. For the moment, we will only consider variations on the oil production rate at the platforms (Δvf_u^t). However, to represent in a more faithful way the uncertainties that arise in operations, a much more complete simulator would be necessary, taking into account the following situations:

- Variations in the oil production rate.

- Partial production reduction or complete shutdown of a floating platform.

- Failure in a shuttle tanker.

- Interruption of operations at the onshore terminal.

- Deviation from the planned route of shuttle tankers.

- Measurement errors.

5.4.1.1 Modeling Uncertainty for the Oil Production Rate

To avoid economic loss, a platform must produce at its maximum capacity but uncertainties or unanticipated events can lead to under production. For all $u \in \mathcal{F}$ and $t \in \mathcal{T}$ the simulator will generate the uncertainty parameter $\widehat{\Delta vf_{u,t}}$ for the oil production rate Δvf_u^t.

The uncertainty is modeled as follows:

$$\widehat{\Delta vf_{u,t}} = \max\left\{\min\left\{N(0,\sigma^2),0\right\},-\sigma\right\} \qquad (5.2)$$

where $\widehat{\Delta vf_{u,t}}$ is in barrels of oil per day and according to the model will be limited to the range $[-\sigma, 0]$. $N(0,\sigma^2)$ is a normal distribution with mean 0 and standard deviation σ. If the parameter $\widehat{\Delta vf_{u,t}} = 0$, there are no uncertainties and if $\widehat{\Delta vf_{u,t}} = -\sigma$ the platform will have its oil production rate diminished by σ.

5.4.2 Analysis

Table 5.9 shows the best values obtained using the rolling-horizon strategy for several types of prediction horizon and three types of standard deviation: $\sigma = 0$ (no uncertainties), $\sigma = 500$ and $\sigma = 1000$ barrels per day. We considered the scenario presented in Section 5.1, a planning horizon of $T = 25$ days and the baseline cost parameters shown in Table 5.3.

Table 5.9 – Solution Considering Uncertainties.

Prediction Horizon [days]	Best Value [10^3\$] (for $\sigma = 0$ barrels/day)	Best Value [10^3\$] (for $\sigma = 500$ barrels/day)	Best Value [10^3\$] (for $\sigma = 1\,000$ barrels/day)
7	407 105	441 846	439 205
8	398 605	441 889	439 343
9	398 605	441 892	439 355
10	406 510	459 777	457 346
11	405 150	459 848	449 037
12	395 375	451 364	449 098
13	403 620	451 364	449 098
14	402 175	448 049	445 783
15	382 030	427 060	425 006

The objective function of the problem accounts for three types of costs: inventory holding, underproduction and traveling cost. Before solving the problem using the rolling-horizon heuristic, intuition led us to think that with the increase of the standard deviation the value of the objective function would also increase since the oil production rate would get lower. However, from Table 5.9, we observe, for all prediction horizons, that the values using the standard deviation $\sigma = 500$ barrels per day are the largest, followed up by the values using $\sigma = 1000$ barrels per day and then by the values considering no uncertainties.

1. **Traveling Cost**[3]. The costs related to the trips of the shuttle tankers were all the same for the three types of standard deviation. However, they presented a different schedule and performed different offloading and uploading operations.

2. **Underproduction Cost**[4]. A higher value of the uncertainty generates a smaller value for the oil production rate, which produces a higher value of underproduction. In that sense, the solution using $\sigma = 1000$ barrels per day had the higher cost related to the underproduction, followed up by the solution using $\sigma = 500$ barrels per day and then by the solution with no uncertainties. Is worth mentioning that the differences between the underproduction costs, for the three types of standard deviation, were small.

3. **Inventory Holding Cost**[5]. The solution using $\sigma = 500$ barrels per day removed the smallest amount of oil from the platforms, generating the largest inventory holding cost, followed up by the solution using $\sigma = 1000$ barrels per day and then by the solution with no uncertainties.

For this instance, the solution using $\sigma = 500$ barrels per day achieved the largest values for its objective function, which was caused by the impact of the inventory holding cost. However, since we are dealing with heuristics it is not possible to generalize this analysis.

5.5 Summary

In this chapter we presented a basic offshore oil field scenario and the computational set up in which the analysis were carried out. The first analysis consisted in comparing the Lagrangean dual

[3]Traveling Cost $= \sum_{(u,v)\in\mathcal{E}} \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} C_{u,v}^{s,t} x_{u,v}^{s,t}$.
[4]Underproduction Cost $= \sum_{u\in\mathcal{F}} \sum_{t\in\mathcal{T}} C_u^{\mathrm{up},t}(QF_u^{\max,t} - \Delta vf_u^t)$.
[5]Inventory Holding Cost $= \sum_{u\in\mathcal{F}} \sum_{t\in\mathcal{T}} C_u^{\mathrm{h},t}(vf_u^t - VF_u^{\min})$.

bound, obtained via the Lagrangean relaxation, with the linear relaxation bound. The subgradient and the constraint generation methods were used to solve the Lagrangean dual problem.

Next we compared the static solution, when the problem is solved for the entire planning horizon, with the dynamic solution obtained using the rolling-horizon and relax-and-fix heuristics which in general give sub-optimal solutions but perform well in large problems instances and with unanticipated events such as uncertainties.

Closing the chapter, the rolling-horizon strategy was evaluated in a prototype simulator which generated random perturbations to the variables of the model.

Chapter 6

Conclusion

In this dissertation we have modeled and solved the Shuttle Tanker Scheduling problem, an important sub-problem of the Petroleum Supply Planning. The proposed model advances the previous work of Camponogara e Plucenio (2014) by proposing a mathematical formulation for scheduling shuttle tankers to account for variable travel times. From the formulation, we were able to establish theoretical properties such as the computational hardness, the generalization of the previous model and a family of cutting planes derived from classic knapsack cover inequalities. In addition, we have presented Lagrangean duality, Lagrangean decomposition and dual bounds obtained via subgradient and constraint generation methods. Also, we developed rolling-horizon and relax-and-fix heuristics for solving large instances of the problem and to respond to uncertainties in the operations. Finally, these strategies were evaluated computationally in a prototype simulator in order to assess their effectiveness in response to uncertainties in operations.

6.1 Contributions

This dissertation achieved the following contributions:

- An MILP formulation for scheduling shuttle tankers for offloading operations in large offshore oil fields with variable travel times for the vessels.

- Theoretical properties establishing the computational hardness of the problem and the generalization of the previous model which assumes the travel times of the shuttle tankers as fixed.
- A family of cutting planes derived from the classic knapsack cover inequalities.
- Rolling-horizon and relax-and-fix heuristics for solving large problem instances.
- A prototype simulator to assess the effectiveness of the model and the heuristics in response to uncertainties in operations.
- Lagrangean duality, Lagrangean decomposition and dual bounds obtained by subgradient and constraint generation methods.

6.2 Future Works

There are some topics that were not investigated during the dissertation and that we believe could generate significant contributions:

- Implementation of the family of cutting planes and analysis of the improvement on computational results in a typical offshore oilfield scenario.
- Extension of the prototype simulator to include other uncertainties such as partial production reduction or complete shutdown of a floating platform; failure in a shuttle tanker; interruption of operations at the onshore terminal; deviation from the planned route of shuttle tankers; and measurement errors.
- Identify and treat degeneracies and symmetries, that are usually present in scheduling problems, in order to reduce computation time.
- Extend the proposed formulation to consider other constraints such as inventory holding, inventory balance and limits on the storage capacity at the onshore terminals; demands at the refineries; platforms producing different types of oil; restrictions on which platforms can supply each onshore terminal; restrictions

on which tankers can offload each platform and upload each onshore terminal; oil batches at the onshore terminals reserved for the exporting or originated from the importing or from pipelines.

- The necessity to take into account uncertainties has been recognized as an important issue in logistics problems. Two problems can arise. The first one is how to model the uncertainties involved in the problem. The second problem is that, usually, mathematical models that incorporate uncertainties have a large number of variables and constraints, and are computationally hard to solve. From that, some research directions can be pursued such as: uncertainty modeling, development of stochastic formulations to deal with uncertainties and the design of optimization strategies to solve large-sized instances.

Bibliography

AHUJA, R.; MAGNANTI, T.; ORLIN, J. *Network flows: theory, algorithms, and applications*. [S.l.]: Prentice Hall, 1993.

AIZEMBERG, L. et al. Formulations for a problem of petroleum transportation. *European Journal of Operational Research*, v. 237, n. 1, p. 82–90, 2014.

AIZEMBERG, L. et al. Optimization model for the problem of oil transportation with local search by MIP and simulation. In: *Proceedings of the XLIII Brazilian Symposium on Operations Research (SBPO)*. [S.l.: s.n.], 2011.

AL-YAKOOB, S. *Mixed-Integer Mathematical Programming Optimization Models and Algorithms for an Oil Tanker Routing and Scheduling Problem*. Tese (Doutorado) — Virginia Polytechnic Institute and State University, 1997.

ASSIS, L. S. de; CAMPONOGARA, E.; PLUCENIO, A. A MILP formulation for scheduling oil tankers for offloading operations with variable travel time. In: *Proceedings of the International Conference on Engineering Optimization*. [S.l.: s.n.], 2014. p. 873–878.

BALAS, E. Facets of the knapsack polytope. *Mathematical Programming*, v. 8, p. 146–164, 1975.

BALAS, E.; ZEMEL, E. Facets of the knapsack polytope from minimal covers. *SIAM Journal on Applied Mathematics*, v. 34, n. 1, p. 119–148, January 1978.

BANASZEWSKI, R. et al. Planning transport of crude oil derivatives with simultaneous auctions. In: *Proceedings of the IEEE International*

Conference on Systems Man and Cybernetics (SMC). [S.l.: s.n.], 2010. p. 2820–2827.

BAZARAA, M. S.; SHERALI, H. D.; SHETTY, C. M. *Nonlinear Programming: Theory and Algorithms*. 3rd edition. ed. [S.l.]: John Wiley & Sons, 2006.

BERALDI, P. et al. Rolling-horizon and fix-and-relax heuristics for the parallel machine lot-sizing and scheduling problem with sequence-dependent set-up costs. *Computers & Operations Research*, v. 35, n. 11, p. 3644 – 3656, 2008.

BURDEN, R.; FAIRES, J. *Numerical Analysis*. [S.l.]: Cengage Learning, 2004.

CAMPONOGARA, E.; PLUCENIO, A. Scheduling dynamically positioned tankers for offshore oil offloading. *International Journal of Production Research*, v. 52, n. 24, p. 7251–7261, 2014.

CAMPONOGARA, E.; PLUCENIO, A.; PEREIRA, F. A model for optimizing trips of dynamically positioned shuttle tankers for offshore oil transfer. In: *Proceedings of the International Conference on Engineering Optimization*. [S.l.: s.n.], 2012.

CHOI, E.; TCHA, D.-W. A column generation approach to the heterogeneous fleet vehicle routing problem. *Computers & Operations Research*, v. 34, n. 7, p. 2080–2095, 2007.

CHU, C. et al. A polynomial dynamic programming algorithm for crude oil transportation planning. *IEEE Transactions on Automation Science and Engineering*, v. 9, n. 1, p. 42–55, 2012.

CROWDER, H.; JOHNSON, E. L.; PADBERG, M. Solving large-scale zero-one linear programming problems. *Operations Research*, v. 31, n. 5, p. 803–834, 1983.

DILLENBERGER, C. et al. On practical resource allocation for production planning and scheduling with period overlapping setups. *European Journal of Operational Research*, v. 75, n. 2, p. 275–286, 1994.

FISHER, M. L. An applications oriented guide to lagrangian relaxation. *Interfaces*, v. 15, p. 10–21, 1985.

FOURER, R.; GAY, D.; KERNIGHAN, B. *AMPL: a modeling language for mathematical programming*. [S.l.]: Thomson/Brooks/Cole, 2003.

GEOFFRION, A. M. Lagrangian relaxation for integer programming. *Mathematical Programming Study 2*, p. 82–114, 1974.

GHIANI, G.; LAPORTE, G.; MUSMANNO, R. *Introduction to Logistics Systems Planning and Control*. [S.l.]: Wiley, 2004. (Wiley Interscience Series in Systems and Optimization).

GHIANI, G.; LAPORTE, G.; MUSMANNO, R. *Introduction to Logistics Systems Management*. [S.l.]: Wiley, 2013. (Wiley Essentials in Operations Research and Management Science).

GUIGNARD, M. Lagrangean relaxation. *TOP: An Official Journal of the Spanish Society of Statistics and Operations Research*, Springer-Verlag, p. 151–200, 2003.

HAKSEVER, C.; RENDER, B. *Service Management: An Integrated Approach to Supply Chain Management and Operations*. [S.l.]: Pearson Education, 2013.

JETLUND, A. S.; KARIMI, I. Improving the logistics of multi-compartment chemical tankers. *Computers & Chemical Engineering*, v. 28, n. 8, p. 1267 – 1283, 2004.

LI, Z.; IERAPETRITOU, M. G. Rolling horizon based planning and scheduling integration with production capacity consideration. *Chemical Engineering Science*, v. 65, n. 22, p. 5887–5900, 2010.

MOHAMMADI, M. et al. Rolling-horizon and fix-and-relax heuristics for the multi-product multi-level capacitated lotsizing problem with sequence-dependent setups. *Journal of Intelligent Manufacturing*, v. 21, n. 4, p. 501–510, 2010.

NEMHAUSER, G. L.; WOLSEY, L. A. *Integer and Combinatorial Optimization*. [S.l.]: Wiley-Interscience, 1988.

NISHI, T.; YIN, S.; IZUNO, T. Column generation approach to ship scheduling problems for international crude oil transportation. In: *Proceedings of the IEEE Conference on Automation Science and Engineering (CASE)*. [S.l.: s.n.], 2011. p. 468–473.

ROCHA, R. *Petroleum supply planning: models, reformulations and algorithms*. Tese (Doutorado) — Departamento de Informática, PUC-Rio, 2010.

ROCHA, R.; GROSSMANN, I. E.; ARAGãO, M. V. Poggi de. Petroleum allocation at petrobras: Mathematical model and a solution algorithm. *Computers & Chemical Engineering*, v. 33, n. 12, p. 2123–2133, 2009.

ROCHA, R.; GROSSMANN, I. E.; ARAGãO, M. V. Poggi de. Cascading knapsack inequalities: reformulation of a crude oil distribution problem. *Annals of Operations Research*, v. 203, n. 1, p. 217–234, 2013.

SAWIK, T. *Scheduling in Supply Chains Using Mixed Integer Programming*. [S.l.]: Wiley, 2011.

SILVA, T. L.; CAMPONOGARA, E. A computational analysis of multidimensional piecewise-linear models with applications to oil production optimization. *European Journal of Operational Research*, v. 232, n. 3, p. 630–642, 2014.

TAHA, H. *Operations Research: An Introduction, 9/E*. [S.l.]: Pearson, 2011.

TANG, L.; JIANG, S.; LIU, J. Rolling horizon approach for dynamic parallel machine scheduling problem with release times. *Industrial & Engineering Chemistry Research*, v. 49, n. 1, p. 381–389, 2010.

WOLSEY, L. *Integer Programming*. [S.l.]: John Wiley & Sons, 1998.

Appendix A

Mathematical Proofs

A.1 Lagrangean Dual Problem Equivalence

In this section we intend to show in details the mathematical proof of the following proposition:

Proposition 5. *According to Geoffrion (1974) it is possible to prove that the Lagrangean dual problem Z_{LD} is equivalent to the following Primal Relaxation:*

$$PR: \; f = \max_{\boldsymbol{x}} \; \boldsymbol{c}^T \boldsymbol{x} \tag{A.1}$$

$$\text{s.t.}: \; A\boldsymbol{x} \leq \boldsymbol{b} \tag{A.2}$$

$$conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\} \tag{A.3}$$

Proof:

Consider the following generic integer programming problem:

$$Z: \; f = \max_{\boldsymbol{x}} \; \boldsymbol{c}^T \boldsymbol{x} \tag{A.4a}$$

$$\text{s.t.}: \; A\boldsymbol{x} \leq \boldsymbol{b} \tag{A.4b}$$

$$C\boldsymbol{x} \leq \boldsymbol{d} \tag{A.4c}$$

$$\boldsymbol{x} \in X \tag{A.4d}$$

where $\boldsymbol{c}$, $\boldsymbol{x}$, $\boldsymbol{b}$ and $\boldsymbol{d}$ are vectors, A and C are matrices (all vectors and matrices are with the right dimensions); and X defines the integrality constraints.

Suppose that the set of constraints (A.4b) is complicating, in the sense that if we remove it from the problem, it becomes easier to solve. The Lagrangean relaxation of this problem is defined as follows:

$$\begin{aligned} Z_{LGR}: \ l(\boldsymbol{\lambda}) = \max_{\boldsymbol{x}} \ & \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) \\ \text{s.t.}: \ & C\boldsymbol{x} \leq \boldsymbol{d} \\ & \boldsymbol{x} \in X \end{aligned}$$

where $l(\boldsymbol{\lambda})$ is the Lagrangean dual function.

Given the Lagrangean relaxation problem Z_{LGR}, which $\boldsymbol{\lambda}$ gives the tightest bound for the integer problem Z? The goal is to minimize the increase that the dualized constraints provide in the objective function, meaning the minimization of $l(\boldsymbol{\lambda})$. From this arises the Lagrangean dual problem:

$$Z_{LD}: \min_{\boldsymbol{\lambda}} \ l(\boldsymbol{\lambda}) \tag{A.6a}$$

$$\text{s.t.}: \ \boldsymbol{\lambda} \geq \mathbf{0} \tag{A.6b}$$

Having introduced PR, Z_{LGR} and Z_{LD}, it is possible to write the following:

$$v(Z_{LD}) = \min_{\boldsymbol{\lambda}\geq\mathbf{0}} v(Z_{LGR}) \tag{A.7}$$

$$= \min_{\boldsymbol{\lambda}\geq\mathbf{0}} \max_{\boldsymbol{x}} \{\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) | C\boldsymbol{x} \leq \boldsymbol{d}, \boldsymbol{x} \in X\} \tag{A.8}$$

$$= \min_{\boldsymbol{\lambda}\geq\mathbf{0}} \max_{\boldsymbol{x}} \{\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) | \boldsymbol{x} \in conv\{\boldsymbol{x} \in X | C\boldsymbol{x} \leq \boldsymbol{d}\}\} \tag{A.9}$$

where $v(Z_{LD})$ and $v(Z_{LGR})$ are respectively the optimal values of the Lagrangean dual and the Lagrangean relaxation. Assuming that $\{\boldsymbol{x} \in \mathbb{R}^n : C\boldsymbol{x} \leq \boldsymbol{d}\}$ is a bounded polyhedron, the step of going from Equation (A.8) to Equation (A.9) is true because the maximum of a linear function over a bounded, discrete set of points is equal to the maximum of that linear function over the convex hull of this set of points.

Let $\mathcal{X} = \{\boldsymbol{x} \in X : C\boldsymbol{x} \leq \boldsymbol{d}\}$ be the set of feasible points of the Lagrangean relaxation problem. Notice that $\mathcal{X} = \{\boldsymbol{x}^1, \ldots, \boldsymbol{x}^m\}$ is a countable set assuming that the polyhedron $\{\boldsymbol{x} \in \mathbb{R}^n : C\boldsymbol{x} \leq \boldsymbol{d}\}$ is bounded.

Now, the Lagrangean dual function $l(\boldsymbol{\lambda})$ can be seen as the upper envelope of a set of affine functions of $\boldsymbol{\lambda}$ which lead to the following reformulation of Z_{LD}:

$$
\begin{aligned}
v(Z_{LD}) = \min_{\boldsymbol{\lambda} \geq \mathbf{0}} l(\boldsymbol{\lambda}) &= \min_{\boldsymbol{\lambda} \geq \mathbf{0}} \max_{\boldsymbol{x} \in \mathcal{X}} \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) \\
&= \min \ \eta \\
&\quad \text{s.t.} : \eta \geq \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}), \ \forall \boldsymbol{x} \in \mathcal{X} \\
&\quad \quad \boldsymbol{\lambda} \geq \mathbf{0} \\
&= \min \ \eta \\
&\quad \text{s.t.} : \eta - \boldsymbol{\lambda}^T(\boldsymbol{b} - A\boldsymbol{x}) \geq \boldsymbol{c}^T\boldsymbol{x}, \ \forall \boldsymbol{x} \in \mathcal{X} \\
&\quad \quad \boldsymbol{\lambda} \geq \mathbf{0}
\end{aligned}
$$

which is a linear programming problem. This problem can be placed in a matrix form as follows:

$$v(Z_{LD}) = \min_{\boldsymbol{\lambda} \geq \mathbf{0}} \eta \tag{A.10a}$$

s.t. :

$$
\begin{bmatrix} 1 & -(\boldsymbol{b} - A\boldsymbol{x}^1)^T \\ 1 & -(\boldsymbol{b} - A\boldsymbol{x}^2)^T \\ \vdots & \vdots \\ 1 & -(\boldsymbol{b} - A\boldsymbol{x}^m)^T \end{bmatrix} \times \begin{bmatrix} \eta \\ \boldsymbol{\lambda} \end{bmatrix} \geq \begin{bmatrix} \boldsymbol{c}^T\boldsymbol{x}^1 \\ \boldsymbol{c}^T\boldsymbol{x}^2 \\ \vdots \\ \boldsymbol{c}^T\boldsymbol{x}^m \end{bmatrix} \tag{A.10b}
$$

$$\boldsymbol{\lambda} \geq \mathbf{0} \tag{A.10c}$$

In a linear programming problem, the optimal values of the objective function of the primal and dual problem are equal (strong duality). The dual of the primal Problem (A.10), with variables $\mu^1, \ldots, \mu^m$ being the dual variables associated with each constraint, can be defined as:

$$v(Z_{LD}) = \max \quad \boldsymbol{c}^T\boldsymbol{x}^1\mu^1 + \boldsymbol{c}^T\boldsymbol{x}^2\mu^2 + \ldots + \boldsymbol{c}^T\boldsymbol{x}^m\mu^m \tag{A.11a}$$

s.t. :

$$\mu^1 + \mu^2 + \ldots + \mu^m = 1 \tag{A.11b}$$

$$\mu^j \geq 0,\ j = 1, \ldots, m. \tag{A.11c}$$

$$\begin{bmatrix} -(\boldsymbol{b} - A\boldsymbol{x}^1) & \ldots & -(\boldsymbol{b} - A\boldsymbol{x}^m) \end{bmatrix} \times \begin{bmatrix} \mu^1 \\ \vdots \\ \mu^m \end{bmatrix} \leq \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{A.11d}$$

which can be rewritten as:

$$v(Z_{LD}) = \max_{\boldsymbol{x}} \quad \boldsymbol{c}^T\boldsymbol{x} \tag{A.12a}$$

$$\text{s.t. :} \quad A\boldsymbol{x} \leq \boldsymbol{b} \tag{A.12b}$$

$$\mu^1 + \mu^2 + \ldots + \mu^m = 1 \tag{A.12c}$$

$$\boldsymbol{x} = \boldsymbol{x}^1\mu^1 + \boldsymbol{x}^2\mu^2 + \ldots + \boldsymbol{x}^m\mu^m \tag{A.12d}$$

$$\mu^j \geq 0,\ j = 1, \ldots, m. \tag{A.12e}$$

Notice that Equations (A.12c), (A.12d) and (A.12e) define the convex hull of the set $\mathcal{X} = \{\boldsymbol{x}^1, \ldots, \boldsymbol{x}^m\}$. Hence,

$$v(Z_{LD}) = \max_{\boldsymbol{x}} \quad \boldsymbol{c}^T\boldsymbol{x} \tag{A.13}$$

$$\text{s.t. :} \quad A\boldsymbol{x} \leq \boldsymbol{b} \tag{A.14}$$

$$\boldsymbol{x} \in conv\{\boldsymbol{x} \in X : C\boldsymbol{x} \leq \boldsymbol{d}\} \tag{A.15}$$

which is the definition of the Primal Relaxation. Therefore, $v(Z_{LD}) = v(PR)$, that is, the Lagragean dual problem is equivalent to the Primal Relaxation.