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High School Preparation for College Calculus: Is the Story the Same for Males and Females?

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Using data from the first national study on high school preparation for college calculus, the Factors Including College Success in Mathematics (FICSMath) project, this paper connects males' (n=3,648) and females' (n=2,033) instructional experiences from their high school precalculus or calculus course to their college calculus performance. A hierarchical linear model identifies several significant instructional experiences that predict college calculus performance. Our findings show that high school instructional practices affect college calculus performance similarly for males and females.

Keywords: Gender, High School, Instructional Practices, College Preparation

Gender and the School to College Rite of Passage

The high school to college transition in mathematics is complex, and research about the transition for males and females is sparse. While it is common for this transition to be referred to as the secondary-tertiary transition, this paper will use the term *school to college transition*. Potential gender differences in mathematics preparation and performance across the school to college transition are important to investigate because approximately one million more STEM graduates, both males and females, will be needed over the next decade to meet the demands of the U.S. workplace (Ellis, Fosdick, Rasmussen, 2016; Olson & Riordan, 2012). While there is no straightforward answer as to when this transition begins and ends, educational research that covers the last two years in high school and the first two years in college or university most likely present school to college transition issues (Gueudet, 2008).

Clark and Lovric (2008) suggested applying the rite of passage theory to better understand the school to college transition in mathematics. The change from one well-defined situation (high school) to another equally well-defined situation (college or university) can be considered as a rite of passage for both males and females. There are three transition phases across the rite of passage. First, the separation phase takes place while students are still in high school and includes anticipation of forthcoming university life. In this paper, the separation phase is limited to students who took precalculus or calculus during their senior year in high school. Second, the liminal phase expands across the end of high school to the first year at university and includes typical transition celebrations, such as high school graduation and college orientation. Lastly, the incorporation phase includes roughly the first year at university (Clark & Lovric, 2009). This paper considers students in single variable college calculus to be in the incorporation phase.

The need for more STEM graduates is one reason educational research continues to address the under-representation of females in STEM careers. While females reportedly take the same mathematics courses in high school as males, and, in 2000, high school females took calculus at the same rate as males, females in colleges and universities account for less than half of the students in college calculus (46-47%) even though they make up the majority of undergraduates (Bressoud, 2014). In the end, most females (53%) in calculus tend to pursue a degree in the biological sciences or teaching, while only 20% major in the physical sciences, engineering, or computer science. This is reversed for males, where 53% intend to major in the physical sciences, engineering, or computer science and only 23% go into the biological sciences or teaching (Bressoud, 2014). Females were almost twice as likely as males to not continue to Calculus II, even though it was a requirement for their intended major. When comparing females and males with above-average mathematical abilities and preparedness, females start and end college calculus with significantly lower mathematical confidence than males, so the lack of persistence to Calculus II for females seems to align with lack of mathematical confidence instead of lack of mathematical ability (Ellis, Fosdick & Rasmussen, 2016).

Yet, this does not eliminate males from the discussion of how to better prepare high school students for college calculus and STEM majors. Ellis, Fosdick, and Rasmussen (2016) reported that from the 70% of males interested in science their senior year in high school, only 49% of males actually complete a STEM degree in a college. In the end, 30% of males (and only 19% of females) who completed a STEM degree actually enter into the STEM workforce. So the concern about STEM preparation and gender extends to both males and females.

In light of these concerns, this research investigated *high school instructional experiences* from precalculus and calculus courses that prepare males and females for college calculus. Using the first national study of high school preparation for college calculus success, the Factors Influencing College Success in Mathematics (FICSMath) study, students in college calculus reported instructional experiences from their most recent high school mathematics course. These experiences were then linked to each respondent's actual performance in college calculus, thus investigating the passage across the school to college transition.

Literature Review

There have been several recent meta-analyses and research articles that address gender differences in mathematics performance (Gallagher et al., 2000; Haciomeroglu, Chicken & Dixon, 2013; Linberg, Hyde, Peterson, & Linn, 2010; Pahlke, Hyde, & Allison, 2014; Spelke, 2005) and a large-scale national study that investigated gender and persistence in college calculus (Bressoud, 2014; Ellis, Fosdick, & Rasmussen, 2016). While these studies considered variance in mathematics performance across gender, and presented various hypotheses for performance differences, there have been no large-scale studies that focused on high school preparation for college calculus with respect to gender. Considering gender and high school mathematics preparation is important for STEM studies because high school mathematics has been found to be the only high school predictor of performance across college freshman-level biology, chemistry, and physics (Sadler & Tai, 2007).

At the forefront of educational concerns is student preparation for STEM degrees and careers (Sadler, Sonnert, & Tai, 2012). While middle school students' interest in STEM may be high, the academic challenges presented in learning high school science and mathematics, as well as challenges faced in college science and mathematics, have led to the "leaky pipeline" description of the movement into STEM majors (Lewis, Menzies, Najera, & Page, 2009; Sadler, Sonnert, & Tai, 2012, p. 412). This is particularly true among females (Sadler, Sonnert, & Tai, 2012). For example, fourth-grade girls and boys reported similar interest in STEM, but, by the twelfth grade, 34% of females and 48% of males report such an interest (Cunningham, Hoyer, & Sparks, 2015; Ellis, Fosdick, & Rasmussen, 2016). Then, upon college entrance, 22% of females intend to major in a STEM field, compared with 34% of males. So, females are often considered to be a significant part of the STEM leaky pipeline.

More recently, researchers have been using the metaphor "pathways to STEM" because students, males and females, do not just move out of STEM degrees but also move into them (Lewis et al., 2009; Sadler, Sonnert, & Tai, 2012)., This flow into STEM degrees has for a long time been considered a path mostly taken by males. Mau (2003) found that females were significantly less likely to persist in science and engineering career preparation by their sophomore year in college; and, again, this opt-out has been connected to the struggles experienced or performance issues in college calculus (Ellis, Fosdick, Rasmussen, 2016). Thus, the STEM career choice pattern that persists in the U.S. scientific and technical workforce has been, in most disciplines, predominately male (Sadler, Sonnert, & Tai, 2012). At the forefront of concern for STEM degree and career preparation, for both males and females, is high school mathematics. The extent and quality of high school preparation, including pedagogical practices and teacher-student relationships experienced during the separation phase of the rite of passage, have been shown to contribute to students' decision to persist in or move out of STEM once they are in college (Ellis, Fosdick, Rasmussen, 2016).

SAT-M Performance & Gender

While current reports show little difference across gender at the high school level in mathematics course taking and performance, Benbow and Stanley (1980) found that males outperformed females in both mean score and highest score on the mathematics section of the SAT (SAT-M). In fact, SAT-M scores have not changed much since the 1980s. SAT data between 1987 and 2006 indicate that males' scores have consistently been 30-40 points higher on the SAT-M than females' scores (Hannon, 2012). Ryan and Ryan (2005) attributed gender performance differences on the SAT-M to social-psychological barriers, such as stereotype threat and implicit association (Fennema &

Sherman, 1977; Linberg et al., 2010; Nosek et al., 2009). The stereotype threat theory assumes that females' mathematical performance is disrupted under threat, not because of insufficient skill but because females feel threatened by the possibility that their performance will confirm the negative stereotype associated with their gender (Tomasetto, Alparone, & Cadinu, 2011). In light of this theory, one might surmise that specific instructional tactics in mathematics would lessen the threat for females (while others might exacerbate it) and thus lead to different outcomes by gender.

Pedagogy, Cognition, and Gender

The continuing stereotype that females lack mathematical ability calls for up-to-date and generalizable information about gender and various types of mathematics performance. One such meta-analysis concluded that females performed relatively better on tests with a higher proportion of algebra items whereas, on tests with a higher proportion of measurement items, males performed relatively better (Lindberg, Hyde, Peterson & Linn, 2010). Showing similarity in mathematical literacy and problem solving skills, the most recent Program for International Student Assessment (PISA) revealed that scores from 15-year-old males in the U.S. were not measurably different from those of females (Organization for Economic Cooperation and Development, Program for International Student Assessment, 2014, Figures M2a & PS2a). There is evidence that, as the disparity in mathematics course taking decreased between males and females, performance differences decreased, too. For example, Hyde et al. (2008) found that, in 2000, high school females were taking calculus at the same rate as males, but the disparity of calculus course taking still exists for females at the college calculus level (Bressoud, 2014).

There is little agreement in the literature regarding gender and mathematical cognition, which is often discussed in gender studies as visual or analytic processing (Haciomeroglu, Chicken & Dixon, 2013). Typically, visual solution strategies are image-based, and analytic processing requires analyzing algebraic equations or understanding function characteristics. Battista (1990) presented a study from high school geometry where visualization was at the root of performance differences between males and females. Although there were no significant gender differences in the use of geometric problem-solving strategies, males scored significantly higher than females on spatial visualization and geometric problem solving. In contrast, Galindo and Morales (1994) reported a study where college calculus students were classified based on their preference for visual or analytic processing; they found no significant gender-related performance differences for either group. Haciomeroglu, Chicken, and Dixon (2013) researched the relationship between cognition, gender, and calculus performance, studying 150 high school students, 82 males and 68 females, in AP Calculus. Using six different tests to measure spatial ability and verbal-logical ability, they found that the preference for visual or analytic processing was independent of gender. Specific to AP Calculus performance, Bremigan (2005) found that males used fewer diagrams than did females on free response problems. This suggested that females might be more dependent on the use of drawing strategies. Ferrini-Mundy (1987) found significant differences in college calculus achievement, favoring females, and in spatial visualization ability, favoring males.

In sum, the picture from extant research is far from clear. But if there are gender differences in the approach to mathematics, or if there is at least the possibility that they exist, it behooves us to explore whether various pedagogies used in high school mathematics might affect college calculus performance differently by gender.

Research Questions
Using the FICSMath dataset, we addressed the following research questions: What precalculus and calculus high school instructional experiences, recalled by students in college calculus, predicted their performance in college calculus? And, were there any differences by gender?

Methods

The FICSMath Study
Funded by the National Science Foundation, the FICSMath study was carried out in the Science Education Department of the Harvard-Smithsonian Center for Astrophysics at Harvard University. The survey was administered to students in college calculus courses across the U.S. near the beginning of the fall semester of 2009. Students were asked to complete the FICSMath survey and report what their high school teachers did to prepare them for college calculus success. After the students completed the survey, the professors held the surveys until the end of the semester so students' final grades earned in the course could be recorded before returning the surveys back to Harvard University. In this way, the FICSMath survey connects students' perceptions of their high school teachers' instructional practices that prepare them for college calculus to students' actual performance in college calculus.

The items on the FICSMath survey were created from three sources. The first was a broad literature review of the school to college transition. The second source was an online survey sent to precalculus and calculus teachers and mathematics professors from across the nation asking their views of how to best prepare students for college calculus (For a review of the findings see Wade, Sonnert, Sadler, Hazari, & Watson, 2016). Lastly, a panel was created including mathematics educators, mathematicians, and researchers from various colleges and universities across the U.S. that met at WWW on two separate occasions to discuss the development of the FICSMath survey items.

The FICSMath Survey
Along with many demographic items, the FICSMath survey contained five sections with 70 items about students' course and instructional experiences in their most recent mathematics course. The survey asked questions about: the organization and structure of the mathematics course (16 items); textbooks, homework, and in-class assignments (13 items); tests and quizzes given in the course (13 items); teacher characteristics (six items); and class time and methods used during instruction (22 items). The format of the items varied. Some were dichotomous questions, others were Likert scale, such as one (very little) to six (a lot), others required marking all that applied, and when appropriate, scales were linearized. The gender item was placed under the *Concerning Your Family's and Your Own Background* section and read "Are you male or female?" Out of the 10,492 responses to the survey there were 9,896 responses to the gender question (6% missing data). For the specific research that we report, there were 5,985 students who moved from high school precalculus or calculus into college calculus and 5,681 of them reported their gender (5% missing data). If the gender question had not been asked in a dichotomous fashion, but had included the various gender expression options, there may have been fewer missing data.

Validity and Reliability
The literature review, the responses from teachers and professors to the online survey, and the discussions of the FICSMath items by the panel of mathematics educators, mathematicians, and researchers were measures to assure content validity of the survey. To gauge reliability, we conducted a separate study in which 174 students from

three different colleges took the survey twice, two weeks apart. Our analysis found that for groups of 100, less than a 0.04% chance of reversal between the 50th and 75th percentiles existed (Thorndike, 1997, p. 117).

The Sample
The National Center for Education Statistics (NCES) kindly transmitted an Integrated Postsecondary Education Data System (IPEDS) table with enrollment numbers for two and four year degree-granting institutions. From large, medium, and small colleges and universities across the nation, participants were recruited by contacting the mathematics department heads at the institutions. The department heads were requested to allow students in their college calculus courses to take the 20-minute FICSMath survey. Of the 276 institutions contacted, 182 (65.9 %) agreed to participate. From the 134 institutions that returned the surveys, a stratified national random sample of 10,437 students was obtained, with 6,113 males and 3,448 females. From this sample, 5,985 students had taken either: precalculus (1416 males; 788 females); non-AP Calculus (920 males; 526 females); AP Calculus AB (1078 males; 609 females), or AP Calculus BC (234 males; 110 females) during their senior year in high school. Of the 3,648 males included in this study, only 158 (4.3%) reported attending an all male high school, and from the 2,033 females, 118 (5.8%) reported attended an all female high school. Because these numbers are so small, the results generally reflect the situation in co-gendered high schools.

Analysis
To answer the research questions, we used Hierarchical Linear Modeling (HLM) to account for the nested structure of the data. Students who responded to the XXX survey were prepared for college calculus in various high schools across the US. After high school graduation these students were in various college calculus courses within two and four year small, medium, or large colleges and universities across the US. The HLM method allowed us to analyze the data using the nested data structure (students within calculus courses within institutions) of the FICSMath survey (Osborne, 2000). Interactions between gender and instructional items that were perceived as preparing students for college calculus were also investigated.

Findings
We first report findings across gender about various high school performance measures and mean performance in college calculus. Second, a hierarchical linear model is presented where gender and instructional experiences were entered as main effects. Then, we discuss a model in which gender was interacted with instructional experiences and other predictors.

Prior research revealed gender differences in performance on the SAT-mathematics (SAT-M), with males scoring significantly higher than females. Figure 1 shows this holds true for this sample when using both the SAT-M score by itself and the SAT-M–American College Test (ACT) mathematics concordance score. The concordance score combines the SAT-M and the ACT mathematics (ACT-M) scores by mapping the ACT-M scores onto the SAT-M scale (Schneider & Dorans, 1999). The benefit of using the ACT-SAT mathematic concordance score is that it captures more data from the sample.

Despite performing significantly lower on the SAT-M, Figure 2 shows that females in high school precalculus or calculus courses scored significantly higher than males, except in AP Calculus BC. The difference between the three high school calculus courses is typically one of scope and not difficulty. AP Calculus BC moves faster than AP Calculus AB, with more material covered, specifically parametric, polar, and vector functions as well as sequences and series (The College Board, 2015). The non-AP calculus course most often covers the same material as AP Calculus AB, but more time can be spent on learning content because no test preparation time is required for the AP Exam. The mean high school grades ranged from 3.25 (B) to 3.61 (B+) on a scale of 4.3 (A+) to 0 (F).

In the same vein, females (n=2,033) scored significantly higher than males (n=3,648) in college calculus (scores reported by respondents' professors). The mean score for females was 82 compared to the mean score of 79 for males ($p<0.001$). These scores were reported on a 100 grading scale with mean scores between 79.4 (C+) and 81.9 (B-) on a scale where: 94.5=A; 84.5=B; and 74.5=C. This aligns with Bressoud's (2014) findings.

The Hierarchical Linear Model

This section presents a hierarchical linear model used to investigate gender and instructional experiences that predicted performance in college calculus. At the student level, high school seniors were in either precalculus or calculus courses and experienced various instructional practices with their teachers. At the college course level, these students were in STEM or non-STEM major calculus courses that were nested within different colleges and universities across the nation. Eighty-nine percent of the variability in the dependent variable (college calculus grade [100-point scale]) came from the student level, 7% from the course level, and 4% from the college or university level, so a hierarchical linear model (HLM) was used to account for the nested data structure. To deal with missing data, multiple imputation (Horton & Kipsitz, 2001) was used.

The controls in the model at the student level were: grades from students' last high school mathematics course, either precalculus or calculus; a dummy variable used to differentiate between which course was their most recent, either precalculus (coded as 0) or calculus (coded as 1); the SAT/ACT concordance score; high school final grades from algebra-2 and geometry. At the college calculus course level, performance was compared between STEM major calculus courses (coded as 1) and non-STEM major courses (coded as 0). At the school level, 2-year small, medium, and large colleges (coded as 0) were compared to 4-year small, medium, and large colleges and universities (coded as 1); however, this variable was not a significant predictor of college calculus performance, so it was removed from the model.

After multiple imputation, the HLM model (Table 1) captured 5,765 responses from the 5,985 respondents in the sample and 20% of the variability in the dependent variable. Gender (males=1; females=0) and instructional experiences from senior level precalculus or calculus courses were entered as main effects. The high school performance measure that had the strongest impact on college calculus performance was grade earned in precalculus or calculus course. The second strongest performance

Figure 1: Comparison of Males and Females average scores on the SAT-M and the SAT-ACT Mathematics Concordance Score.

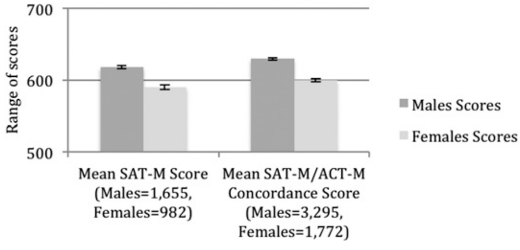
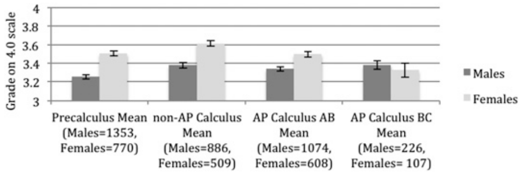


Figure 2: Mean performance grades in students' last high school mathematics class (precalculus or any level of calculus) with the number of males and females who self reported their grades on the FICSMath survey.



measure was the calculus dummy variable, which differentiated between students most recent high school mathematics course being either precalculus or calculus. The 5.7 parameter estimate indicated that students who had calculus their senior year (any level) scored almost six points higher in collge calculus than did students whose last high school course was precalculus. The -1.7 parameter estimate for gender indicates that, on average, females scored almost 2 points higher than males in college calculus. The -3.6 parameter estimate for STEM or non-STEM major college calculus course indicates that students in non-STEM major courses scored almost 4 points higher than students in STEM major courses. This may indicate that STEM major college calculus courses are more rigorous.

The instructional items that were positive predictors of performance in college calculus, in order of their standardized coefficients (indicating the strength of the prediction) were: required new insight and creativity (on test and quizzes) (dichotomous variable: 1=yes; 0=no); the extent of conceptual understanding (Likert scale: 0=very little, 5=a lot); emphasis on functions (Likert scale: 0=not emphasized at all, 5=emphasized heavily); and emphasis on vocabulary (Likert scale: 0=not emphasized at all, 5=emphasized heavily). There were five significant negative instructional predictors of college calculus performance, with frequency of problems with proofs being the strongest (linearized item for frequency per week: 0=none, 45=high). The remaining negative predictors were: teacher highlighted more than one way of solving a problem (Likert scale: 0=low, 5=high); emphasis on hands-on activities or labs (Likert scale: 0=not emphasized at all, 5=emphasized heavily); whole class discussions were held (linearized item for frequency per week: low=0.10, high=5); and connected math to real life applications (linearized item for frequency per week: low=0.10, high=5).

To answer the research question about gender differences in the effects of instructional experiences, we tested the significance of gender interactions with high school instructional experiences (as well as with high school performance measures). However, no interaction variables were significant, indicating similarity between the males' and females' instructional experiences that predicted college calculus performance. Therefore the discussion that follows is mainly about high school instructional experiences that equally support (or hinder) males' and females' performance in college calculus.

Limitations and Future Work

While there are many factors that influence the preparation for college calculus across gender, this research focused on how males' and females' instructional experiences in their senior level precalculus or calculus course predicted their college calculus performance. One limitation of this study, which it shares with all correlational studies of this type, is that it cannot prove causality. Another may be that it relies on students' self-reports. How students recalled their instructional experiences might not align with their teachers' views of what they actually did during class (even though the teachers' views may, in some cases, also be less than accurate). Second, students' experiences in

Table 1: *Hierarchical Linear Model with Gender (Males=1; Females=0) and Instructional Variables (Entered as Main Effects) that were Significant Predictors of Performance in College Calculus (n=5,765, r^2=0.199, beta naught (B_0) or y-intercept value is 42)*

Variable and Description Name	Parameter Estimate	Standard Error	Standardized Coefficients	Min	Max	Mean
Precalculus or Calculus Grade	3.784***	0.248	0.212	0.000	4.333	3.391
Calculus Dummy	5.708***	0.371	0.197	0.000	1.000	———
SAT/ACT Math	0.017***	0.002	0.111	200.000	800.000	631.860
Grade in Algebra 2	1.449***	0.333	0.063	0.000	4.333	3.648
Grade in Geometry	1.344***	0.315	0.060	0.000	4.333	3.651
Gender	-1.697***	0.365	-0.057	0.000	1.000	———
Required new insight and creativity	1.923***	0.402	0.059	0.000	1.000	———
Extent of Conceptual Understanding	0.619***	0.160	0.050	0.000	5.000	3.680
Emphasis on Functions	0.565**	0.174	0.043	0.000	5.000	3.930
Emphasis on vocabulary	0.403**	0.140	0.039	0.000	5.000	2.340
Frequency of problems with proofs solved in class	-0.096***	0.019	-0.060	0.000	7.000	4.942
Teacher highlighted more than one way of solving a problem	-0.484**	0.142	-0.046	0.000	5.000	3.660
Emphasis on hands-on activities/labs	-0.427**	0.125	-0.045	0.000	5.000	1.740
Whole class discussions were held	-0.288***	0.090	-0.040	0.100	5.000	2.295
Connected math to real life applications	-0.323**	0.121	-0.034	0.100	5.000	1.554
STEM or non-STEM major college calculus course	-3.612***	0.684	-0.063	0.000	1.000	———

*p<0.05; **p<0.01; ***p<0.001

high school may not align with how college professors teach in college calculus courses. Having both the teachers' and professors' perspectives would be valuable complements. More research is needed to understand the experiences that best prepare male and female students for college calculus success.

Discussion and Conclusion

This research connects gender and high school precalculus and calculus instructional experiences to actual performance in college calculus. The FICSMath study, the first national study on secondary preparation for college calculus success, provided an opportunity to investigate gender and the rite of passage transition to college calculus, using hierarchical linear models that accounted for controls at the student level (high school performance measures) and the college calculus level (differentiation between STEM and non-STEM college calculus courses).

We found that a number of teaching methods in high school mathematics during the separation phase of the rite of passage made a difference in the students' long-term mathematics success, and that the effects of these methods were similar for males and females. In light of Ferrini-Mundy (1987 finding that, during problem solving, females were more dependent on drawing strategies and males had more visualization ability, it is notable that the FICSMath items concerning the use of graphing or sketching drawings, be it by hand or by calculator, did not affect college calculus performance for males nor females.

The instructional item from high school precalculus or calculus courses that was the strongest predictor of performance in college calculus was 'required new insight and creativity' (on tests and quizzes). The parameter estimate for this variable shows that overall, college calculus performance increased about 2 points if this instructional practice was experienced. Spelke (2005) distinguished conventional and unconventional problems, the latter of which were defined as requiring students to "grapple with new and unfamiliar tasks when the relevant solution methods are not known" (p. 56). Solving problems that require insight and creativity may align with solving unconventional problem. If so, our findings indicate that including such problems in junior and senior high school level assessments is beneficial for both males and females in the school to college transition.

Requiring conceptual understanding in respondents' senior level precalculus or calculus course was the next strongest positive predictor of performance in college calculus. The parameter estimate indicates that, per unit increase in this instructional experience, performance increased almost one point. Skemp (2006) distinguished two different types of understanding in school mathematics, relational and instrumental understanding. Relational understanding implies that students know what to do and why, while instrumental understanding indicates that students know rules without reason. Relational understanding may align with conceptual understanding and our study suggests that it may be beneficial for the students in the long run if the teacher emphasizes relational or conceptual understanding. However, what specific instruction supports students in high school precalculus and calculus courses to conceptually understand mathematics? More work needs to be done to better identify instructional practices that support relational understanding for students in the school to college transition.

The next item that was a significant and positive predictor of performance in college calculus was emphasis on functions. The parameter estimate in the model shows that college calculus performance increased almost one point per unit increase in this type of instruction. Wade, Sonnert, Sadler, Hazari, and Watson (2016) reported precalculus and calculus teachers' and professors' perspectives of preparing students for a successful college transition into calculus. Consistent with the findings in the present study, one of the views expressed by professors was that “students need a deeper understanding of trigonometric and circular functions beyond right triangle trigonometry” (p. 11). Trigonometric functions are part of the transcendental functions studied in the school to college transition.

Lastly, emphasis on vocabulary was the smallest positive predictor of performance in college calculus. For this instructional experience, the parameter estimate in the model shows a per unit increase of almost one-half of a point in college calculus. Mathematics is a language that includes definitions, vocabulary, numerals, symbols, and syntax that are at times interrelated and interdependent, and at other times disjointed and autonomous (Adams, 2003). Our result aligns with Tall’s (1993) study, which discussed students’ difficulties in college calculus and pinpointed one of them as mathematical language, which includes vocabulary.

What this communicates about the separation phase of the rite of passage is the importance of high school precalculus and calculus teachers placing emphasis on requiring new insight and creativity (on tests), conceptual understanding, functions, and vocabulary. These are the elements that foreshadow a successful incorporation phase. By multiplying the parameter estimate by the maximum value of each of these items, our model shows that a student’s final grade in college calculus could be raised by 10 points through a high focus on these instructional experiences in during the separation phase.

The model also revealed five instructional experiences from high school that had a significant negative effect on future learning in college calculus. Such practices may support students’ learning in high school, which is important, yet do not predict future learning and performance in college calculus. The first significant negative predictor was frequency of problems with proofs in class. There is documented difficulty that novice students experience with proofs (Gueudet, 2008; Moore, 1994). Yet, professors expect proofs to be understood in college calculus because they are used to demonstrating the correctness of a result or the truth of a mathematical statement (Peressini et al., 2004). Interestingly, heightened exposure to proofs in high school did not prepare students for the college calculus experience. Peressini et al. (2004) stated that high school mathematics students have traditionally conceived of proofs as formal and often meaningless exercises that teachers required them to do. Thus, an emphasis on proofs in high school mathematics classes might have the unintended effect of being confusing or off-putting. The second item was teachers ‘highlighted more than one way to solve a problem.’ As Tall (2003) noted, there are multiple ways to solve mathematical problems, including graphic, numeric, and symbolic solutions. While instruction highlighting these multiple ways may support learning at the high school level, it may be that professors do not implement such instruction. Bressoud (2010) stated that many colleges now teach a more theoretical differential calculus. Moreover, highlighting multiple ways of solving problems, while *prima facie* useful, may hold a potential for confusion.

The next two negative predictors of performance in college calculus were ‘emphasis on hands-on activities/lab’ and ‘whole class discussions were held.’ These items represent progressive ways of learning, doing, and interacting with mathematics during instruction, yet these may not be common instructional practices in college calculus. Likewise, the pedagogical method of connecting mathematics to real life applications has been found, in high school mathematics, to align more with an increased motivation to learn mathematics than with better student performance (Beswick, 2010). With college calculus being more theoretical (Bressoud, 2010), the negative impact of these instructional practices may reveal a disconnect in the school to college transition.

These negative predictors may align with what the rite of passage research refers to as a synthetic model. A synthetic model is a network of misconceptions of mathematics learned in high school that cannot assimilate well into college calculus. While much is yet to be researched, it is known that many students moving from high school mathematics into college calculus find developing an appropriate new framework for higher levels of learning very challenging (Clark & Lovric, 2009). In sum, the negative predictors of performance in college calculus imply that, if teachers in the separation phase frequently solved problems with proofs in class, highlighted more than one way of solving a problem, emphasized hands on activities, whole class discussions, or connected math to real life applications, students would be less prepared for college calculus. By multiplying the parameter estimate of these items by the maximum value (representing a high focus on these student perceived instructional experiences) our model shows that a student's final predicted grade in college calculus would drop by eight points. These findings may seem to misalign with the findings from Peressini et al (2004), Tall (2003), and Beswick (2010), yet their work was not focused on transition research across the rite of passage. Importantly, when studying the passage from the separation phase to the incorporation phase, one must take into account expectations and teaching practices of professors in college calculus.

Facilitating students' progress on their trajectory of mathematics learning merits the attention of the mathematics education community. What we know about the school to college rite of passage is that the mathematical under-preparedness of students entering into college mathematics is an issue that may compromise their success in university mathematics (Hourigan & O'Donoghue, 2007; Selden, 2005; Hong et al., 2009). We have found homogeneity between the genders in the effects of high school instructional experiences on college calculus success. In this respect (long-term mathematics success), it appears unnecessary that high school precalculus and calculus teachers in co-gendered classrooms attempt to differentiate instruction based on gender. This result aligns with, and seems to confirm, the findings of a lack of performance differences across gender reported in the Pedagogy, Cognition, and Gender section above. Thus high school mathematics instruction modified for the genders in a co-educational environment, or even through single-sex classes and single-sex schools, may be beneficial in addressing social issues, motivation, or even performance at the high school level, but not for *future* learning. Here, when it comes to the question of which high school instructional experiences are helpful, and which are not, the story is the same for males and females.

We hope these findings are informative for mathematics education researchers, secondary mathematics teachers preparation programs, and high school precalculus and calculus teachers who seek to prepare all students for the school to college transition.

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