Analysis Von Bertalanffy Equation With Variation Coefficient

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Abstract. Growth is the increase of size, both its length and weight at a specific time period. In studying the behavior of the growth of fish used a growth model that is Von Bertalanffy models. However Von Bertalanffy models showed that the growth rate is a constant function. Meanwhile these assumptions can only describe the dynamics of growth of marine life in an environment of constant and fail to describe the dynamics of the growth of marine organisms whose growth varies seasonally or by time. Von Bertalanffy models thus developed with a coefficient of variation which gives additional biological realism of Von Bertalanffy models into a population that enables the growth rate by changing the size of the body with seasonal variations. This study aims to find solutions of to the equations Von Bertalanffy Model with a coefficient of variation and actualize the model on Wader pari fish and skipjack, with the incorporation coefficient time varied significantly will increase the the ability of Von Bertalanffy's model to describe the changes of fish's body size up to the long asymtot by considering the growth factors such as temperature, water temperature and food availability that exist within the environment.

Keywords: Growth, Von Bertalanffy, Varying Coefficient, actualization

INTRODUCTION

The mathematical model is an attempt to explain some sections that deal with the real world in the form of mathematical equations. One real-life example that can be applied in the mathematical model is a fisheries science [1]. Evidenced by the biologist and mathematician, Ludwig Von Bertalannfy (1957) known as the Von Bertalanffy growth model. Von Bertalanffy growth model is a model that represents the process of growth which is related to the length, weight or size of two opposing processes, namely catabolism and anabolism [2].

according to the previous research, which deals with the model of Von Bertalanffy model by Warda Susaniati in 2014 of the Study of Biology Skipjack (Katsuwonus pelamis) in the waters of the Flores Sea South Sulawesi. This research aimed to analyze the structure of the size and age groups conform to the area and fishing season, analyzed the relationship between fish's age and length in the area and fishing season, and estimated growth arrest by region and season. In the research, the value of the growth rate b <0.00138 for each day, and in theory that fish can achievied maximum length growth(L_{max}) by 106 cm. In these research mentioned before the Von Bertalanffy models are used to distinguish between the growth of the population or sub-populations of fish by counting Von Bertalanffy model parameters, namely asimptot length (L_{max}), growth coefficient (b), and the theoretical age (t (0)), but in the research assumed that the growth coefficient (b) is a constant [3]. So to study in detail we need for research on models of Von Bertalanffy with a coefficient of variation, where the coefficient of variation is the growth coefficent of fish according to a function of time is b(t) [4].

This case provides additional biological realism into populations with seasonal variations or trends conform to time conditions. This research is important in order to know in detail the growth dynamics of fish in a population using the Von Bertalanffy growth model with a coefficient of variation that has not been developed at this time. This research is expected to be one of the information subtance about growing conditions and can be used for the management of fishery resources, particularly in the conservation, domestication and this development until that the presence of fish in nature can be preserved.

THEORITICAL FRAMEWORK

First orde Linear Ordinary Differential Equation

First orde linear ordinary differential equations is defined as below

$$\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{P(x)y} = \mathrm{Q(x)} \tag{2.1}$$

Then the general solution of First order linear ordinary differential equations, namely

$$y\mathrm{e}^{\int \mathrm{P(x)dx}} = \int \mathrm{Q(x)e}^{\int \mathrm{P(x)dx}} dx + C$$

C as constanta[5].

Bernoulli Equation

Bernoulli equation is defined as below

$$\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{P(x)y} = \mathrm{Q(x)y^n} \tag{2.2}$$

such that the acquired form (2.1) with transform

$$y^{-n+1} = v, y^{-n}.\frac{dy}{dx} = \frac{1}{1-n}.\frac{dv}{dx}$$

Combined Trapezoid Method

Trapezoidal method is a method of numerical integral approach with first order polynomial equation, then the rules of integration are obtained composite trapezoidal rule as follows[6]

$$\begin{aligned}
\int_a^b \mathrm{f(x)dx} \quad &= \int_{x_0}^{x_1} \mathrm{f(x)\ dx} + \int_{x_1}^{x_2} \mathrm{f(x)\ dx} + \ldots + \int_{x_{n-1}}^{x_n} \mathrm{f(x)\ dx} \\
&= \frac{h}{2}[f(x_0) + f(x_1)] + \frac{h}{2}[f(x_1) + f(x_2)] + \cdots + \frac{h}{2}[f(x_{n-1}) + f(x_n)] \\
&= \frac{h}{2}([f(x_0) + f(x_1)] + [f(x_1) + f(x_2)] + \cdots + [f(x_{n-1}) + f(x_n)]) \\
&= \frac{h}{2}\left(f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right)
\end{aligned}$$

Von Bertalanffy Model

The process of how an individual change according to the time rooted in physiological processes that occur in these individuals. Individuals receiving energy from food, and that energy can be converted into growth, reproductive development, or activity. According to Von Bertalannfy (1957), growth (which is related to the length, weight, or size) is the result of two opposing processes, catabolism and anabolism. An anabolic process of protein synthesis, while recast protein is a catabolic process. Von Bertalanffy growth model shown in equation below [2]

$$\mathrm{L(t)} = \mathrm{L_{max}} - (\mathrm{L_{max}} - \mathrm{L_{min}})\mathrm{e}^{-\mathrm{b}(\mathrm{t}-\mathrm{t_0})}$$

DISCUSSION

Equation Analysis of Von Bertalanffy Model With Coefficient of Variation

Ludwig Von Bertalanffy (1957) explains that anabolime growth exceeded katabolime growth. If the growth of the organism against time (t) is proportional to the speed of anabolism (η) is the multiplication of the growth of organisms through time t (Y (t)) with the degree k is reduced by the speed of catabolism (λ) is the product of the growth of organisms through time t (Y (t)), then formulated by Von Bertalanffy growth model in the form of mathematical models as follows

$$\frac{dY(t)}{dt} = \eta Y(t)^k - \lambda Y(t)$$

can be written as below

$$\frac{dY(t)}{dt} + \lambda Y(t) = \eta Y(t)^k \tag{3.1}$$

which is a bernoulli differential equation where k represents a real number except 0 or 1 and λ, η respectively are constants or functions in (t). So if both sides of the equation (3.1) multiplied by $Y(t)^{-k}$ then

$$Y(t)^{-k}\frac{dY(t)}{dt} + \lambda Y(t)^{-(k-1)} = \eta \tag{3.2}$$

given $x(t) = Y(t)^{1-k}$, the derivative of the function x (t) versus t is

$$\frac{dx(t)}{dt} = (1-k)Y(t)^{-k}\frac{dY(t)}{dt}$$

or

$$\frac{1}{1-k}Y(t)^k\frac{dx(t)}{dt} = \frac{dY(t)}{dt} \tag{3.3}$$

by substituting the equation (3.3) into the equation (3.1), then obtained

$$\frac{1}{1-k}Y(t)^k\frac{dx(t)}{dt} + \lambda Y(t) = \eta Y(t)^k \tag{3.4}$$

devide both sides of the equation (3.4) and $Y(t)^k$ to obtain equation

$$\frac{1}{1-k}\frac{dx(t)}{dt} + \lambda x(t) = \eta$$

or

$$\frac{dx(t)}{\eta - \lambda x(t)} = (1-k) \tag{3.5}$$

than if both sides of the equation (3.5) are integrated obtained

$$\int \frac{dx(t)}{\eta - \lambda x(t)} = \int (1-k)dt$$

$$-\lambda^{-1}\ln(\eta - \lambda x(t)) = (1-k)t + \ln C$$

$$\ln(\eta - \lambda x(t)) = -\lambda(1-k)t - \lambda \ln C$$

$$\eta - \lambda x(t) = \alpha e^{-\lambda(1-k)t}, (\alpha = C^{-\lambda})$$

$$x(t) = \frac{\eta}{\lambda} - \frac{\alpha}{\lambda}e^{-\lambda(1-k)t} \tag{3.6}$$

by substituting $Y(t)^{1-k}$ to x (t) in equation (3.6) and to degree both sides by $\frac{1}{(1-k)}$ then the equation becomes

$$Y(t) = \left[\frac{\eta}{\lambda} - \frac{\alpha}{\lambda}e^{-\lambda(1-k)t}\right]^{1/(1-k)} \tag{3.7}$$

if the equation (3.7) is given t = 0, then

$$Y(0) = \left[\frac{\eta}{\lambda} - \frac{\alpha}{\lambda}\right]^{1/(1-k)}$$

obtained

$$\frac{\alpha}{\lambda} = \frac{\eta}{\lambda} - Y(0)^{1-k} \qquad (3.8)$$

if substituted for the equation (3.8) to equation (3.7) then

$$Y(t) = \left[\frac{\eta}{\lambda} - \left(\frac{\eta}{\lambda} - Y(0)^{1-k}\right) e^{-\lambda(1-k)t}\right]^{1/(1-k)}$$

for t → ∞ or during the long time it can be said that

$$\lim_{t\to\infty} Y(t) = Y(\infty) = \left(\frac{\eta}{\lambda}\right)^{1/(1-k)}$$

from the statement of equation (3.9) can be written as

$$Y(t) = \left[Y(\infty)^{1-k} - \left(Y(\infty)^{1-k} - Y(0)^{1-k}\right)e^{-\lambda(1-k)t}\right]^{1/(1-k)} \qquad (3.9)$$

Equation (3.10) is a common form Von Bertalanffy growth model.
With:
Y (t) = Growth of the organism against time t
Y (∞) = maximum growth of the organism
Y (0)= The minimum growth of organisms
λ = Parameter growth

From the general Von Bertalanffy models above, then redeveloped by James E. Cloern and Frederic H. Nichols to form the Von Bertalanffy growth model based on the concept of physiological, assuming that the characteristics common changes in body size of organisms is given that as follows

- at the recruitment of time $(t = t_0)$, the size of the body is in a state of constant minimum(L_{min}).
- The maximum growth rate at the time $\frac{dL(t)}{dt} = L'_{max}$ when the minimum body size.
- There is a constant upper limit to the level of an organism's body size.
- speed reduction in growth of the organism is decreases of linier function from the body size as below.

$$\frac{dL(t)}{dt} = L'_{max} - b(L(t) - L_{min}) \qquad (3.10)$$

with solution

$$x(t) = L_{max} - (L_{max} - L_{min})e^{-b(t-t_0)} \qquad (3.11)$$

The solution of equation (3.11) shows that the growth rate is a constant function. While these assumptions can only describe the dynamics of growth of marine life in an environment of constant and fail to describe the dynamics of the growth of marine organisms whose growth varies seasonally or by time. Von Bertalanffy models with a coefficient of variation provides additional biological realism of Von Bertalanffy models into a population that enables the growth rate by changing the size of the body with seasonal variations. If seasonal variations in the growth rate used as the seasons change in the coefficient b, then each function that describes time variation in b can be substituted into the equation (3.10).

Variation coefficient showed that the growth of shellfish Macoma Balthica coefficient changes during the annual cycle in San Francisco. with the substitution function b (t) be a real function of time, with

$$b(t) = a1 + a2.exp\left(a3.sin\left(\frac{\pi}{180}(t + \theta)\right)\right)$$

if the function $b(t)$substituted into the equation (3.10) then

$$\frac{dL(t)}{dt} = L'_{max} - b(t)(L(t) - L_{min}) \qquad (3.18)$$

then that the equation (3.18) becomes

$$L(t) = L_{max} - (L_{max} - L_{min}).\, e^{-a1(t-t_0) - \int_{t_0}^{t} a2.exp\left(a3.sin\left(\frac{\pi}{180}(t+\theta)\right)\right)dt} \qquad (3.19)$$

integral solutions to the equation (3.19) were completed using an integral approach trapezoid method, with the assistance of Matlab program.

Actualization Von Bertalanffy Model With a Coefficient of Variation

Skipjack (Katsuwonus pelamis) is often called Skipjack tuna. Morphological characteristics of Cakalang are torpedo-shaped body (fusiform), elongated and roundish, has gill raker 53-62 pieces. There are two separate dorsal fins, the first dorsal fin are On 14-16 of fingers, the fingers are weak in the second dorsal fin followed by 7-8 finlet. Pectoral fins short, there are two flops between the pelvic fins. Pelvic fins (anal) followed with 7-8 finlet. Body scales except on the diaper body (corcelets) and the lateral line are tiny dots. Another hallmark Cakalang on the back of blue-black (dark) side down and belly silvery, with 4-6 pieces of black stripes extending on the side of the body (Figure 3.1).

Figure 3.1 Skipjack (Katsuwonus pelamis)

In a study entitled "Study of Biology Skipjack (Katsuwonus pelamis) in South Sulawesi Sea waters flores" gives Von Bertalanffy model parameters are$L_{max} = 106$ cm and$b = 0{,}000972$ per day and the value of $t(0)$obtained by -0,00114 for each day So when parameters are substituted into the solution of equation (2.17) then becomes

$$L(t) = 106(1 - e^{-0{,}000972(t+0{,}00114)}) \quad (3.20)$$

From the research, the function $b(t)$ closest to Von Bertalanffy growth curve with constant coefficients of the equation (3.11) is

$$b(t) = 2{,}9x10^{-4} + 0{,}7x10^{-4}.e^{\left(3{,}7.sin\left(\frac{\pi}{180}(t+14{,}4)\right)\right)}$$

with $b(t)$shown in Figure (3.2)

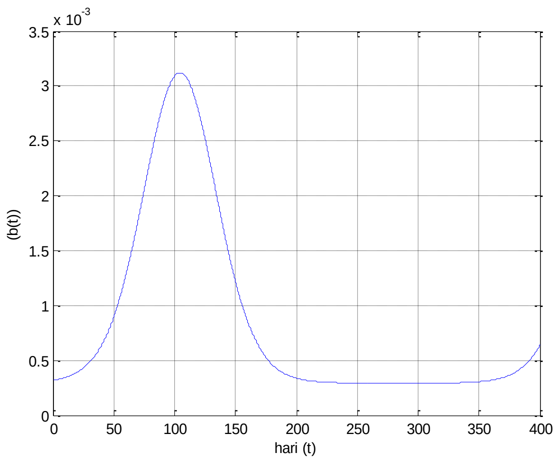

Figure 3.2 $b(t) = 2{,}9x10^{-4} + 0{,}7x10^{-4}.e^{\left(3{,}7.sin\left(\frac{\pi}{180}(t+14{,}4)\right)\right)}$

the picture 3.3 shows the function of b (t) is used as the controller determines the amplitude and length of time on the increase and decrease speed Von Bertalanffy growth models with a coefficient of variation. So if the function b (t) is substituted into the equation (3.10) has a solution

$$L(t) = L_{max} - (L_{max} - L_{min}) \; e^{-2{,}9x10^{-4}(t-t_0) - 0{,}7x10^{-4}\int_{-0{,}000888}^{1200} e^{\left(3{,}7.\sin\left(\frac{\pi}{180}(t+14{,}4)\right)\right)} dt} \quad (3.21)$$

integral solutions to the equation (3.21) were completed using an integral approach trapezoid method, with the assistance of Matlab program. So the solution of equation (3.21) is shown in Figure 3.3.

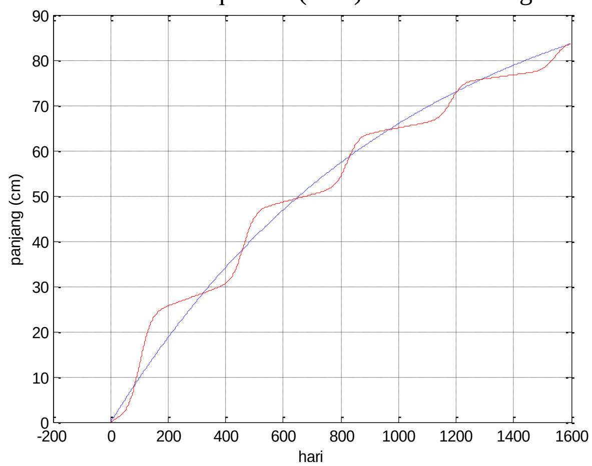

Figure 3.3 Comparison between the model and the Von Bertalanffy constant coefficient of variation Skipjack fish

based on the picture 3.4 can be seen clearly comparisons between Von Bertalanffy growth curve with constant coefficients and variation. Where the Von Bertalanffy growth curve with a coefficient of variation followed the Von Bertalanffy growth curve with constant coefficients, which in detail describes the growth fluctuated. It can be seen that the growth curve of Von Bertalanffy with a coefficient of variation at the age of 100 days the length up to 12 cm, at the age of 240 days reached a length of 26 cm, at the age of 720 days (2 years) reaches a length of 50 cm, at the age of 1600 days reaches length 83 cm up towards the long asiymptot at the age of 3600 days (10 years) with a length of 106 cm.

Although Skipjack at certain times have increased and decreased in growth, but the curve shows that the growth of Skipjack in general that the initial phase of life Skipjack experiencing rapid growth, followed by slow growth along with increased age or when it reaches old age. Increases and decreases in the growth of Skipjack occur because difference food availability or abundance of food in the habitat that provides quite energy for the growth of fish body length,then length growth also become relatively larger. Skipjack eat a variety of foods consisting of Teri, Peperek, Layang, squid, shrimp, and worms as an energy source.

Von Bertalanffy models with a coefficient of variation can also be used to predict the growth of tuna fish in the Flores Sea South Sulawesi for every recruitment (t_0). In this thesis are given on recruitment(t_0)= 0, 120 and 240 with the parameter value L_{max}= 106 cm, L_{min}= 0cm and the value of $b(t) = 2{,}9\text{x}10^{-4} + 0{,}7\text{x}10^{-4}.\, e^{3{,}7.\sin\left(\frac{\pi}{180}(t+14{,}4)\right)}$which the growth curve is shown in Figure 3.4.

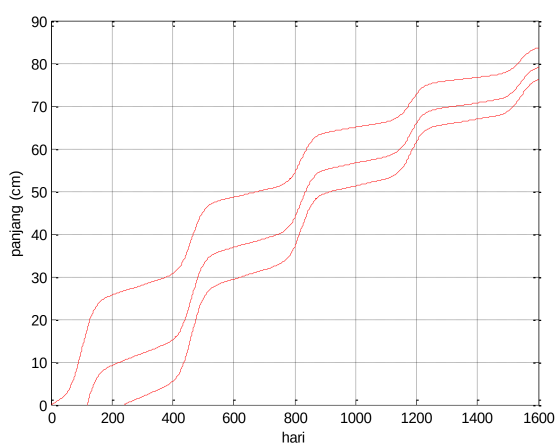

figure 3.4 Growth prediction Skipjack with t_0Different Parameters (0, 120, 240)

based on the figure growth curve Von Bertalanffy with a variation coefficient of the different parameter t_0, it explains that the skipjack within one year are hatched on the day to 120 faster growth compared with individuals who hatch in two other time, and the initial growth very slower experienced by individuals hatched on day 240. Although individuals hatch on 240 days to slow the growth in the beginning, but people at the end of the year (day 240) will grow faster to catch older fish length in the next season. It explains the phenomenon cathing-up where the smallest sized cohort at the end of the season long chase older cohort in the following season.

Differences in growth Skipjack different every time because in the transitional seasons west to east is a transition season that led to the salinity and water temperature increases so Skipjack grow faster and reproduce. It is because at a suitable temperature Skipjack will have a good appetite. In accordance with the nature of Skipjack is strongly influenced by the water conditions in warm climates and high salinity.

CONCLUSION

Based on the research conducted, it can be given the following conclusion:

1. $b(t)$ the closest Von Bertalanffy growth curve with constant coefficients on Skipjack with parameters
$$L(t) = 106(1 - e^{-0{,}000972(t+0{,}00114)})$$
is
$$b(t) = 2{,}9x10^{-4} + 0{,}7x10^{-4}.e^{\left(3{,}7.\sin\left(\frac{\pi}{180}(t+14{,}4)\right)\right)}$$
2. Von Bertalanffy models with a coefficient of variation shows in detail the increase in body size of marine life which at one particular season the fish will grow fast and in other seasons the fish will grow more slowly because it is influenced by water temperature, factors abundance and quality of the food.
3. Von Bertalanffy models with a coefficient of variation with time of recruitment (t_0) different clearly illustrates the phenomenon of catching-up where the smallest sized cohort at the end of the season chasing pamnajang older cohort in the following season.

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