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QUASI-CONFORMAL CURVATURE TENSOR OF GENERALIZED SASAKIAN-SPACE-FORMS

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Abstract. The present paper deals with the study of generalized Sasakian-space-forms with the conditions $C^q(\xi, X).S = 0$, $C^q(\xi, X).R = 0$ and $C^q(\xi, X).C^q = 0$, where R , S and C^q denote Riemannian curvature tensor, Ricci tensor and quasi-conformal curvature tensor of the space-form, respectively. In the end of the paper, we have given some examples to support our results.

Keywords: Quasi-conformal curvature tensor; generalized Sasakian-space-forms; Einstein manifold; Pseudosymmetric manifold.

1. Introduction

An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be a generalized Sasakian-space-form if the curvature tensor of the manifold has the following form

$$\begin{aligned}
 R(X, Y)Z &= f_1 \{g(Y, Z)X - g(X, Z)Y\} \\
 &+ f_2 \{(g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z)\} \\
 &+ f_3 \{(\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\
 &+ g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi)\},
 \end{aligned}
 \tag{1.1}$$

for any vector fields X, Y, Z on M^{2n+1} . By taking $f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$, where c denotes constant ϕ -sectional curvature tensor, we get different kind of generalized Sasakian-space-forms. This idea was introduced by P. Alegre, D. Blair and A. Carriazo [13] in 2004. P. Alegre and Carriazo [15], A. Sarkar, S. K. Hui, etc. [19, 21, 22] studied generalized Sasakian-space-forms by considering the cosymplectic space of Kenmotsu space form as particular types of generalized Sasakian-space-forms. In 2006, U. Kim [22] studied conformally flat generalized Sasakian-space-form and locally symmetric generalized Sasakian-space-form. He proved some geometric properties of generalized Sasakian-space-forms which depends on the nature

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of the functions f_1, f_2 and f_3 . Also, he proved that if a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ is locally symmetric then $(f_1 - f_3)$ is constant. In [21] De and Sarkar studied the projective curvature tensor of generalized Sasakian-space-forms and proved that generalized Sasakian-space-forms is projectively flat if and only if $f_3 = \frac{3f_2}{1-2n}$. D.G. Prakasha and H. G. Nagaraja [8] studied quasiconformally semi-symmetric generalized Sasakian-space-forms. They proved that a generalized Sasakian-space-forms is quasiconformally semi-symmetric if and only if either space form is quasiconformally flat or $f_1 = f_2$. Recently, Hui and Prakasha [17] have studied C -Bochner curvature tensor of generalized Sasakian-space-forms. S. K. Hui and D. G. Prakasha [17] studied the C -Bochner pseudosymmetric generalized Sasakian-space-forms. The generalized Sasakian-space-forms have also been studied in ([9], [18], [10], [11], [23]) and many others. Throughout their study, C -Bochner curvature tensor B satisfied the conditions $B(\xi, X).S = 0$, $B(\xi, X).R = 0$ and $B(\xi, X).B = 0$, where R and S denoted the Riemannian curvature tensor and Ricci curvature tensor of the space form respectively. After investigations of the above mentioned developments, we plan to study the quasi-conformal curvature tensor of generalized Sasakian-space-forms.

2. Preliminaries

A Riemannian manifold (M^{2n+1}, g) of dimension $(2n + 1)$ is said to be an almost contact metric manifold [7] if there exists a tensor field ϕ of type $(1, 1)$, a vector field ξ (called the structure vector field) and a 1-form η on M such that

$$(2.1) \quad \phi^2(X) = -X + \eta(X)\xi,$$

$$(2.2) \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

and

$$(2.3) \quad \eta(\xi) = 1,$$

for any vector fields X, Y on M . In an almost contact metric manifold, we have $\phi\xi = 0$ and $\eta\phi = 0$. Then such type of manifold is called a contact metric manifold if $d\eta = \Phi$, where $\Phi(X, Y) = g(X, \phi Y)$ is called the fundamental 2-form of $M^{(2n+1)}$. A contact metric manifold is said to be K -contact manifold if and only if the co-varient derivative of ξ satisfies

$$(2.4) \quad \nabla_X \xi = -\phi X,$$

for any vector field X on M .

The almost contact metric structure of M is said to be normal if

$$(2.5) \quad [\phi, \phi](X, Y) = -2d\eta(X, Y)\phi,$$

for any vector fields X and Y, where $[\phi, \phi]$ denotes the Nijenhuis torsion of ϕ . A normal contact metric manifold is called Sasakian manifold. An almost contact metric manifold is Sasakian if and only if

$$(2.6) \quad (\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X,$$

for any vector fields X, Y.

The generalized Sasakian-space-forms $M^{2n+1}(f_1, f_2, f_3)$ satisfies the following relations [13]

$$(2.7) \quad R(X, Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\},$$

$$(2.8) \quad R(\xi, X)Y = (f_1 - f_3)\{g(X, Y)\xi - \eta(Y)X\},$$

$$(2.9) S(X, Y) = (2nf_1 + 3f_2 - f_3)g(X, Y) - \{3f_2 + (2n - 1)f_3\}\eta(X)\eta(Y).$$

Replacing Y by ξ in the equation (2.9), we get

$$(2.10) \quad S(X, \xi) = 2n(f_1 - f_3)\eta(X).$$

Replacing X and Y by ξ in the equation (2.9), we get

$$(2.11) \quad S(\xi, \xi) = 2n(f_1 - f_3),$$

from the equation (2.9), we have

$$(2.12) \quad r = 2n(2n + 1)f_1 + 6nf_2 - 4nf_3.$$

Again from (2.9), we have

$$(2.13) \quad QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n - 1)f_3)\eta(X)\xi.$$

Replacing X by ξ in the above equation, we get

$$(2.14) \quad Q\xi = 2n(f_1 - f_3)\xi.$$

In a Riemannian manifold of dimension $(2n + 1)$ the quasi-conformal curvature tensor is defined by [12]

$$(2.15) \quad \begin{aligned} C^q(X, Y)Z &= aR(X, Y)Z + b(S(Y, Z)X - S(X, Z)Y \\ &+ g(Y, Z)QX - g(X, Z)QY) \\ &- \frac{r}{2n + 1} \left[\frac{a}{2n} + 2b \right] \{g(Y, Z)X - g(X, Z)Y\}, \end{aligned}$$

where a and b are constants such that $a, b \neq 0$, Q is the Ricci operator, i.e., $g(QX, Y) = S(X, Y)$, for all X and Y and r is scalar curvature of the manifold.

Using the equations (2.7)-(2.15), we have

$$(2.16) \quad C^q(X, Y)\xi = \left[\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} \right] \{ \eta(X)Y - \eta(Y)X \},$$

$$(2.17) \quad C^q(\xi, Y)Z = \left[\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} \right] \{ \eta(Z)Y - g(Y, Z)\xi \},$$

and

$$(2.18) \quad \begin{aligned} \eta(C^q(X, Y)Z) &= \left[\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} \right] (g(Z, X)\eta(Y) \\ &- g(Y, Z)\eta(X)). \end{aligned}$$

This is required quasi-conformal curvature tensor in generalized Sasakian-space-forms.

3. Quasi-conformal Pseudosymmetric generalized Sasakian-Space-Forms

Let (M, g) be a Riemannian manifold and let ∇ be the Levi-Civita connection of (M, g) . A Riemannian manifold is called locally symmetric if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M, g) . The locally symmetric manifolds have been studied by different differential geometry through various approaches and they extended semisymmetric manifolds by [2, 3, 4, 5, 6, 24], recurrent manifolds by Walker [1], conformally recurrent manifold by Adati and Miyazawa [20]. According to Z. I. Szabó [24], if the manifold M satisfies the condition

$$(3.1) \quad (R(X, Y).R)(U, V)W = 0, \quad X, Y, U, V, W \in \chi(M)$$

for all vector fields X and Y , then the manifold is called semi-symmetric manifold. For a $(0, k)$ - tensor field T on M , $k \geq 1$ and a symmetric $(0, 2)$ -tensor field A on M the $(0, k+2)$ -tensor fields $R.T$ and $Q(A, T)$ are defined by

$$(3.2) \quad \begin{aligned} (R.T)(X_1, \dots, X_k; X, Y) &= -T(R(X, Y)X_1, X_2, \dots, X_k) \\ &- \dots - T(X_1, \dots, X_{k-1}, R(X, Y)X_k), \end{aligned}$$

and

$$(3.3) \quad \begin{aligned} Q(A, T)(X_1, \dots, X_k; X, Y) &= -T((X \wedge_A Y)X_1, X_2, \dots, X_k) \\ &- \dots - T(X_1, \dots, X_{k-1}, (X \wedge_A Y)X_k), \end{aligned}$$

where $X \wedge_A Y$ is the endomorphism given by

$$(3.4) \quad (X \wedge_A Y)Z = A(Y, Z)X - A(X, Z)Y.$$

According to R. Deszcz [16], a Riemannian manifold is said to be pseudosymmetric if

$$(3.5) \quad R.R = L_R Q(g, R),$$

holds on $U_r = \left\{ x \in M \mid R - \frac{r}{n(n-1)}G \neq 0 \text{ at } x \right\}$, where G is (0, 4)-tensor defined by $G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)$ and L_R is some smooth function on U_R . A Riemannian manifold M is said to be quasi-conformal pseudosymmetric if

$$(3.6) \quad R.C^q = L_{C^q} Q(g, C^q),$$

holds on the set $U_{C^q} = \{x \in M : C^q \neq 0 \text{ at } x\}$, where L_{C^q} is some function on U_{C^q} and C^q is the quasi-conformal curvature tensor.

Let $M^{2n+1}(f_1, f_2, f_3)$ be quasi-conformal pseudosymmetric generalized Sasakian-space-form then from the equation(3.6), we have

$$(3.7) \quad (R(X, \xi).C^q)(U, V)W = L_{C^q}[(X \wedge_g \xi).C^q](U, V)W].$$

Using the equations (3.2) and (3.3) in the equation (3.7), we get

$$(3.8) \quad \begin{aligned} R(X, \xi)C^q(U, V)W &= C^q(R(X, \xi)U, V)W - C^q(U, R(X, \xi)V)W \\ &= C^q(U, V)R(X, \xi)W \\ &= L_{C^q}((X \wedge_g \xi)C^q(U, V)W \\ &= C^q((X \wedge_g \xi)U, V)W \\ &= C^q(U, (X \wedge_g \xi)V)W - C^q(U, V)(X \wedge_g \xi)W). \end{aligned}$$

Again, using the equations (2.7) and (3.4) in (3.8), we conclude the following

$$(3.9) \quad \begin{aligned} (f_1 - f_3)(g(\xi, C^q(U, V)W)X &= g(X, C^q(U, V)W)\xi \\ &= \eta(U)C^q(X, V)W \\ &+ g(X, U)C^q(\xi, V)W - \eta(V)C^q(U, X)W \\ &+ g(X, V)C^q(U, \xi)W - \eta(W)C^q(U, V)X \\ &+ g(X, W)C^q(U, V)\xi \\ &= L_{C^q}(g(\xi, C^q(U, V)W)X - g(X, C^q(U, V)W)\xi \\ &= \eta(U)C^q(X, V)W \\ &+ g(X, U)C^q(\xi, V)W - \eta(V)C^q(U, X)W \\ &+ g(X, V)C^q(U, \xi)W - \eta(W)C^q(U, V)X \\ &+ g(X, W)C^q(U, V)\xi). \end{aligned}$$

The above expression can be written as

$$(3.10) \quad \begin{aligned} (f_1 - f_3 - L_{C^q})(g(\xi, C^q(U, V)W)X &= g(X, C^q(U, V)W)\xi \\ &= \eta(U)C^q(X, V)W + g(X, U)C^q(\xi, V)W \\ &= \eta(V)C^q(U, X)W + g(X, V)C^q(U, \xi)W \\ &= \eta(W)C^q(U, V)X + g(X, W)C^q(U, V)\xi = 0, \end{aligned}$$

which implies either $L_{C^q} = f_1 - f_3$ or

$$\begin{aligned}
 (g(\xi, C^q(U, V)W)X & - g(X, C^q(U, V)W)\xi - \eta(U)C^q(X, V)W \\
 & + g(X, U)C^q(\xi, V)W - \eta(V)C^q(U, X)W \\
 & + g(X, V)C^q(U, \xi)W - \eta(W)C^q(U, V)X \\
 & + g(X, W)C^q(U, V)\xi) = 0.
 \end{aligned}
 \tag{3.11}$$

Putting $W = \xi$ in the equation (3.11) and using the equations (2.17) and (2.18), we have

$$\begin{aligned}
 C^q(U, V)X & = \left[\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} \right] (g(X, U)V \\
 & - g(X, V)U),
 \end{aligned}
 \tag{3.12}$$

contracting V in the above equation, we have

$$\left[\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} \right] 2ng(U, X) = 0,
 \tag{3.13}$$

this implies that

$$\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} = 0,
 \tag{3.14}$$

from the above equation two conditions arise, either

$$a = -(2n - 1)b
 \tag{3.15}$$

or

$$f_3 = \frac{3f_2}{(1 - 2n)}.
 \tag{3.16}$$

Using the equations (3.15) or (3.16) in (2.16) and (2.17), we get

$$C^q(\xi, Y)Z = 0,
 \tag{3.17}$$

and

$$C^q(X, Y)\xi = 0,
 \tag{3.18}$$

this means $M^{2n+1}(f_1, f_2, f_3)$ is quasi-conformally flat.

Thus, we conclude:

Theorem 3.1. *Let $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional generalized Sasakian-space-form. If $M^{2n+1}(f_1, f_2, f_3)$ is quasi-conformal pseudosymmetric then $M^{2n+1}(f_1, f_2, f_3)$ is quasiconformally flat if at least one of the following conditions holds:*

$$(i) f_3 = \frac{3f_2}{(1 - 2n)} \quad (ii) a = -(2n - 1)b, \quad (iii) L_{C^q} = f_1 - f_3.$$

Now we propose:

Theorem 3.2. *Let $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional generalized Sasakian-space-form. Then $M^{2n+1}(f_1, f_2, f_3)$ satisfies $C^q(\xi, X).S = 0$ if and only if at least one of the following conditions holds:*

$$(i) f_3 = \frac{3f_2}{(1-2n)}, \quad (ii) a = -(2n-1)b, \quad (iii) S(X, U) = 2n(f_1 - f_3)g(X, U).$$

Proof. If generalized Sasakian-space-form satisfies $C^q(\xi, X).S = 0$. Then from the equation (3.2), we have

$$(3.19) \quad S(C^q(\xi, X)U, \xi) + S(U, C^q(\xi, X)\xi) = 0,$$

From the equation (2.10), we have

$$(3.20) \quad S(C^q(\xi, X)U, \xi) = 2n(f_1 - f_3)\eta(C^q(\xi, X)U).$$

Now with the help of equations (2.17) and (3.20), we can write

$$(3.21) \quad \begin{aligned} S(C^q(\xi, X)U, \xi) &= 2n(f_1 - f_3) \left[\frac{(a + (2n-1)b)((2n-1)f_3 + 3f_2)}{2n+1} \right] (\eta(X)\eta(U) \\ &- g(X, U)). \end{aligned}$$

Again in view of the equation (2.17), we have

$$(3.22) \quad \begin{aligned} S(C^q(\xi, X)\xi, U) &= \left[\frac{(a + (2n-1)b)((2n-1)f_3 + 3f_2)}{2n+1} \right] (S(X, U) \\ &- 2n(f_1 - f_3)\eta(X)\eta(U)). \end{aligned}$$

By using the expressions (3.21) and (3.22) in (3.19), we infer

$$(3.23) \quad \begin{aligned} &\left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right] (S(X, U) \\ &- 2n(f_1 - f_3)g(X, U)) = 0, \end{aligned}$$

which implies that if $C^q(\xi, X).S = 0$ then either $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or $S(X, U) = 2n(f_1 - f_3)g(X, U)$.

Conversely, it is clear that if $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or $S(X, U) = 2n(f_1 - f_3)g(X, U)$ then from (2.17), we have \square

$$(3.24) \quad C^q(\xi, X).S = 0.$$

Now we take $C^q(\xi, U).R = 0$.

Then from the equation (3.2), we have

$$(3.25) \quad \begin{aligned} &C^q(\xi, U)R(X, Y)Z - R(C^q(\xi, U)X, Y)Z \\ &- R(X, C^q(\xi, U)Y)Z - R(X, Y)C^q(\xi, U)Z = 0, \end{aligned}$$

which in view of the equation (2.17), we have

$$(3.26) \quad \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right] (\eta(R(X, Y)Z)U - g(U, R(X, Y)Z)\xi - \eta(X)R(U, Y)Z + g(U, X)R(\xi, Y)Z - \eta(Y)R(X, U)Z + g(U, Y)R(X, \xi)Z - \eta(Z)R(X, Y)U + g(U, Z)R(X, Y)\xi) = 0,$$

using $Z = \xi$ and (2.2) in the above equation, we infer

$$(3.27) \quad \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right] ((f_1 - f_3)(g(U, Y)X - g(U, X)Y) - R(X, Y)U) = 0,$$

which implies that if $C^q(\xi, X).R = 0$ then either $a = -(2n - 1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or

$$R(X, Y)U = (f_1 - f_3)(g(U, Y)X - g(U, X)Y).$$

Thus, we conclude:

Theorem 3.3. *Let $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional generalized Sasakian-space-form. If $M^{2n+1}(f_1, f_2, f_3)$ satisfying $C^q(\xi, U).R = 0$ then at least one of the following necessarily holds:*

$$(i) \quad f_3 = \frac{3f_2}{(1-2n)}, (ii) \quad a = -(2n - 1)b, \\ (iii) \quad R(X, Y)U = (f_1 - f_3)(g(U, X)Y - g(U, Y)X).$$

Now we propose:

Theorem 3.4. *Let $M^{2n+1}(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional generalized Sasakian-space-form. Then $M^{2n+1}(f_1, f_2, f_3)$ satisfies $C^q(\xi, X).C^q = 0$ if and only if either $f_3 = \frac{3f_2}{(1-2n)}$ or $a = -(2n - 1)b$.*

Proof. If generalized Sasakian-space-form satisfies $C^q(\xi, X).C^q = 0$. Then, from the equation (3.2) we have

$$(3.28) \quad C^q(\xi, X)C^q(U, V)W - C^q(C^q(\xi, X)U, V)W - C^q(U, C^q(\xi, X)V)W - C^q(U, V)C^q(\xi, X)W = 0,$$

by which in view of the equation (2.16) we get

$$(3.29) \quad \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right] (\eta(C^q(U, V)W)X - g(X, C^q(U, V)W)\xi - \eta(U)C^q(X, V)W + g(X, U)C^q(V, \xi)W - \eta(V)C^q(U, X)W + g(X, V)C^q(U, \xi)W + g(W, X)C^q(U, V)\xi - \eta(W)C^q(U, V)X) = 0.$$

By using $V = \xi$ in the above equation, we infer

$$(3.30) \quad \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right] ((C^q(U, X)W + \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right] (g(X, W)U) - g(U, W)X) = 0,$$

which implies that either $a = -(2n - 1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or

$$(3.31) \quad C^q(U, X)W = \left(\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} \right) (g(U, W)X - g(X, W)U),$$

contracting U in the above equation, we have

$$(3.32) \quad \left[\frac{(a + (2n - 1)b)((2n - 1)f_3 + 3f_2)}{2n + 1} \right] 2ng(X, V) = 0,$$

this implies that either $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$. Conversely, if $M^{2n+1}(f_1, f_2, f_3)$ satisfies $a = -(2n - 1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$, then in view of (2.17) we have $C^q(\xi, X).C^q = 0$. \square

4. Examples

Example 4.1. [13] Let $N(\lambda_1, \lambda_2)$ be generalized Sasakian-space-forms of dimension 4, then by the warped product $M \times N$ endowed with the almost contact metric structure (ϕ, ξ, η, g_f) , Sasakian space form $M(f_1, f_2, f_3)$ is generalized with

$$(4.1) \quad f_1 = \frac{\lambda_1 - (f')^2}{f^2}, \quad f_2 = \frac{\lambda_2}{f^2}, \quad f_3 = \frac{\lambda_1 - (f')^2}{f^2} + \frac{f''}{f},$$

where λ_1 and λ_2 are constants, $f = f(t)$, $t \in R$ and f' denotes the derivative of f with respect to t.

If we take $\lambda_1 = -\frac{3\lambda_2}{7}$ and $f(t) = e^{Kt}$, K is constant, then $f_1 = -\frac{1}{e^{2Kt}}[\frac{3\lambda_2}{7} + K^2 e^{2Kt}]$, $f_2 = \frac{\lambda_2}{e^{2Kt}}$ and $f_3 = -\frac{1}{e^{2Kt}}[\frac{3\lambda_2}{7}]$. Hence $f_3 = \frac{3f_2}{(1-2n)}$, if $n = 4$.

Example 4.2. [14] Let $N(c)$ be a complex space form, and by the warped product $M = (-\frac{\pi}{2}, \frac{\pi}{2}) \times_f N$ endowed with the almost contact metric structure (ϕ, ξ, η, g_f) , Sasakian space form $M(f_1, f_2, f_3)$ is generalized with functions

$$(4.2) \quad f_1 = \frac{c - 4(f')^2}{4f^2}, \quad f_2 = \frac{c}{4f^2}, \quad f_3 = \frac{c - 4(f')^2}{4f^2} + \frac{f''}{f},$$

where $f = f(t)$, $t \in R$ and f' denotes the derivative of f with respect to t.

If we take $c = 0$ and $f(t) = e^{Kt}$, K is constant,

then $f_1 = -K^2$, $f_2 = f_3 = 0$. Hence $f_3 = \frac{3f_2}{(1-2n)}$.

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