Some tools for studying uncertainties in geotechnical engineering

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Some Tools for Studying Uncertainties in Geotechnical Engineering

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Abstract:

Uncertainties in geotechnical engineering are generally approached using probabilistic tools. A tentative is made to mention number of other approaches for dealing with lack of data and scarce information in geotechnical problems. Fuzzy sets and interval-based studies are of interest. We show through number of examples the applicability of such approaches and how they can offer valuable tools for uncertainty analysis in geotechnical engineering. A general view will be given on the use of intervals, fuzzy sets and random sets in optimization of geotechnical investigations, diffusion of stresses in granular media and in consolidation.

Keywords: Uncertainties, Geotechnical Engineering, Probabilities, Fuzzy Sets, Random Sets, Intervals.

1. Introduction

Uncertainty analysis in geotechnical engineering became an important subject of study during the last twenty years. It relates to many aspects of geotechnics such as foundations, slope stability, soil investigation, soil modelling and so on. Most of the codes of practice in geotechnical engineering rely on deterministic approaches even though uncertainty is inherent to most of the systems. The difficulty in analysing uncertainties made it a secondary concern for geotechnical engineers.

Magnan (2000) introduced by number of examples the usual sources of uncertainty in geotechnics such as; being unlucky in soundings positions, human errors in manipulating or interpreting data, remoulding of samples for lab testing. In this context characterizing uncertainties defined as the gap or deviation between the engineer's model and reality becomes complex and cannot be reduced to the use of statistics or simple probabilistic methods.

Boissier (2000) showed that in the geotechnics field data are very imprecise and uncertain, and proposed to introduce the use of fuzzy sets and belief functions theories, many examples were shown in the study which confirm the difficulty of dealing with uncertainty by using only probabilities. Incompleteness being a key parameter in many problems of geotechnics, the pape presents a concise view with practical examples.

Xiao et al. (2017) proposed a simplified reliability analysis method for efficient full probabilistic design of soil slopes in spatially variable soils which in the views of the authors improves the computational efficiency of soil slope design in comparison with simulation-based full probabilistic design.

Phoon (2017) summarises the role of reliability calculations in geotechnical design; approaches that enable taking into account different types of uncertainty in geotechnical design. The author points the importance of reliability calculations. Uncertainties amenable to probabilistic treatment typically fall under the category of "known unknowns" where some measured data and/or past experience exist for limited site-specific data to be supplemented by both objective regional data and subjective judgement derived from comparable sites elsewhere. Within this category, reliability is very useful in handling complex real-world information (multivariate correlated data) and information imperfections (scarcity of information or incomplete information).

Engineer's judgement is of vital importance in geotechnical reliability studies because of the uncertainties that arise. According to Duncan & Sleep (2017) a core principle of geotechnical engineering practice is the need to exercise judgement in evaluating soil and site conditions and in performing analyses. Geotechnical reliability studies inevitably involve significant uncertainties, and judgement is needed to perform the analyses and to evaluate the results. From Two case histories, the authors showed that judgement and experience are essential prerequisites for meaningful assessment of geotechnical reliability.

In his article "Is Probability the Only Coherent Approach to Uncertainty?" Colyvan (2008) asks a fundamental question if probability is the unique way to deal with uncertainty. In the view of the author the claim that probability theory is the only coherent means of dealing with uncertainty is implausible. It is ill-suited to the uncertainty arising from vagueness.

For historical and conceptual reasons most of the studies on uncertainty use probability as a primary tool. Historically the first studies on uncertainty have automatically used probability approaches because they were developed for the situation. When uncertainty arises as a result of vagueness and ambiguity other methods are required.

The difficulty in using probabilistic approaches locates essentially in the need for large data to achieve accurate analyses. According to Baecher & Christian (2015) geotechnical engineer is facing problems with few data available, he or she faces extremely limited numbers of observations, measurements of differing types and quality, a blend of qualitative and quantitative information, and a need to make sequential decisions as data arrive.

The article is intended to present other methods than probability approaches used for analysing uncertainty in geotechnical engineering. A presentation of types of uncertainty and how to deal with them depending on the context is given, followed by number of examples.

2. Sources of uncertainties

According to the scientific literature in engineering there are mainly two types of uncertainties; Epistemic and Aleatory uncertainty. Epistemic uncertainty is associated with the knowledge of a system, it includes uncertainty due to limitations of measurement, insufficient data, extrapolations and interpolations (Colyvan, 2008). This type of uncertainty is dealt with by various statistical methods and probabilistic approaches. Aleatory uncertainty is related to stochasticity; in geotechnics it is related to natural randomness.
Vagueness is another source of uncertainty which arises out of vagueness in the language, from vague predicates. One of the approaches used to deal with this type of uncertainty is fuzzy sets theory and fuzzy reasoning in general. The term fuzziness is sometimes used for vagueness when working with fuzzy sets theory.

3. Uncertainty Analysis Methods

Random Sets and Point Estimate Methods

Nasekhian (2011) investigated two selected non-probabilistic and probabilistic methods with the potential of being combined with numerical methods –without requiring any modification to the core of the numerical code, to demonstrate what useful information can be provided for more rational engineering judgments and decision making in tunnelling. Among non-probabilistic methods, Random Set Finite Element Method (RS-FEM) has demonstrated its attractive results in practical geotechnical problems. The results obtained from different random set analyses are combined using evidence aggregation methods. The author made comparison between the Point Estimate Method categorised as one of the probabilistic approaches and RS-FEM, using the same case study. It is shown that under certain assumptions, a similar conclusion regarding the range of most probable behaviour of a tunnel problem obtained from both PEM and RS-FEM approaches can be drawn.

Fuzzy-based methods

Within a reliability analysis using scarce information Beer et al. (2013) investigated the problem in a geotechnical engineering context. In this article common probabilistic methods are compared with interval analysis as an alternative non-probabilistic approach. The potential of imprecise probabilities is discussed as an option for combining probabilistic and non-probabilistic information. The authors paid attention to low-probability-but-high consequence events, which are often essential for risk assessment. A retaining wall structure was adopted for the study, and the failure mechanisms are considered as known in a deterministic form. It was found that interval modeling and fuzzy modeling provide certain advantages if the available information is very limited.
Wenping et al. (2014) presented an interesting paper on a fuzzy set–based Robust Geotechnical Design (RGD) approach for the design of earth slopes in which uncertain soil parameters are represented as fuzzy sets. The Factor of safety obtained after analysis is a fuzzy set. The failure probability of the slope can readily be determined based on the obtained fuzzy factor of safety; The purpose of the RGD is to derive a design that is robust against the uncertainty in the input parameters, with an explicit consideration on safety and cost. A multi-objective optimization is performed and a three dimensional or two-dimensional Pareto front showing a trade-off between objectives can be obtained, which allows for an informed design decision. The proposed fuzzy set based RGD approach is an effective tool in this type of problems, in which the authors showed an illustrative example. The authors used Pareto front to show the effectiveness of such an approach. The proposed fuzzy set based RGD approach requires only a range estimate of uncertain soil parameters and can be implemented and processed in a deterministic manner. The proposed fuzzy set based RGD approach has potential as a practical design tool.
Water transport modelling in a layered soil profile was studied by Karsten and Huwe (1999), they used fuzzy set theory to account for uncertainty. The methodology is based on fuzzy set theory to express imprecision of input data in a non-probabilistic sense. Imprecision may originate from indirect measurements, estimation routines, subjective interpretation, and expert judgement of available information. A numerical finite difference solution scheme was chosen to solve one-dimensional steady-state water flow in the unsaturated zone of a layered soil profile. The authors concluded that resulting imprecision not only depends on the degree of imprecision and the number of uncertain parameters but also very much on the system context (e.g. boundary conditions and spatial distribution). In using fuzzy set theory to express imprecision in the parameters, the membership function of the dependent variables (here soil water pressures) represents the resulting imprecision of the outcomes. To evaluate the obtained membership functions, a certain a-level must be chosen

(corresponding to an interval of real numbers) on which a decision will be linked. This corresponds to choosing a 1*s*, 2*s*, or 3*s* confidence interval in a stochastic framework.

In relation to vertical capacity of piles, Elton et al. (2000) submitted a study on using fuzzy sets for predicting the ultimate capacity of single piles. They used fuzzy set theory to account for uncertainty in the soil parameters and the prediction methods. The validity of the program results is evaluated using a FHWA pile load test database. The authors reported good comparison between predicted and measured ultimate capacities of piles in sand, clay and mixed soils. Use of fuzzy approach was dictated by the lack of understanding of the phenomena of soil-pile interaction, and the limited quantity and inexact quality of subsurface soil information. This approach helps handling the uncertainty in the soil parameters and the prediction methods. The authors performed number of simulations which have shown the capabilities of the analysis.

A variety of applications of fuzzy models in civil engineering was presented by Fetz et al. (2005), they addressed uncertainties in geotechnical engineering, fuzzy finite-element computation of a foundation raft, fuzzy dynamic systems, and processing uncertainty in project scheduling and cost planning. In this article different applications of fuzzy models show the accuracy of the approaches in dealing with different types of problems involving uncertainties.

Majumdar (2012) The parameters in the differential equation of heat conduction were considered with uncertainties using fuzzy finite element method. Those parameters are found in general by some measurements or experiments. The material properties are uncertain and may be considered to vary in an interval or as fuzzy. Representation of interval/fuzzy numbers may give the clear picture of uncertainty and interval/fuzzy arithmetic is applied in the finite element method to solve a steady state heat conduction problem. The author also proposed new methods to handle such type of fuzzy system of linear equations. The approach used to solve heat conduction problems can be used also in similar problems of soil mechanics. The interest of the proposed approach resides in the use of interval analysis as a base for constructing fuzzy sets.

An interesting analysis was carried out by Valliappan and Pham (1995) which deals with elastoplasticity in soils based on fuzzy mathematics and finite element method. The Drucker-Prager yield criterion and the elasto-plastic matrix are fuzzified for the non-linear analyses which adopt the initial stress method for the solution. The authors presented a numerical example to illustrate the possible variations in the displacements and the extent of plastic zones at the discrete membership functions. A reliability index of a membership grade based on the concept of non-probabilistic entropy is also proposed. The interest of the study is in its ability to treat reasonably the soil properties as fuzzy values in a combination with finite element method. The results of the numerical example indicate the usefulness of such an analysis. A reliability index was also defined and calculated within a fuzzy environment, it was applied to measure the certainty of a membership grade, depending on its fuzzy entropy and the global behaviour of the fuzzy set.

Entropy-based methods

Other studies can be cited (Singh, 2009) on the use of entropy theory for movement of moisture in soils. According to the author the entropy theory seems capable of simulating soil moisture profiles for wet, dry and mixed cases reasonably well. Entropy theory was also used by Dalvi et al. (2013) for identifying significant parameters for seismic soil liquefaction. The authors argued that the entropy method is preferable as it considers varied range of parameters involved in soil liquefaction. Entropy in geotechnics is gaining interest and could offer valuable alternatives for analysing uncertainty, it was used by Imre et al. (2012) for grading of soils. Benchmark examples are given to illustrate its application to the characterisation of piping, softening and dispersive soils, and to filtering problems in the context of a leachate collection system for a landfill site.

4. Examples from author's work

Based on a probabilistic approach of diffusion of stresses in a granular media, Boumezerane (2018) used an interval-based approach for characterizing parameters that influence the diffusion phenomena, and showed that uncertainty can be handled at first glance by intervals as most of parameters values are given in ranges of values in general. The purpose was to show the ability of interval valued method to handle uncertainty due to lack of knowledge and data. The obtained results are quite interesting, comparisons with results from laboratory experiments can be explored for more understanding of the influence of parameter uncertainties in problems of mechanics of particulate media.

Uncertainties in stress diffusion parameter

The equation of diffusion of vertical stresses $\tilde{S}$ in a granular medium can be written in terms of intervals as :

$$\frac{\partial \tilde{S}}{\partial z} = \widetilde{D}\,\frac{\partial^2 \tilde{S}}{\partial x^2} \quad (1)$$

The diffusion parameter $\widetilde{D}$ is subjected to epistemic uncertainties which have influence on the diffusion process in the granular medium. The main sources of uncertainty of $\widetilde{D}$ come from evaluation of lateral pressure coefficient $\tilde{k}$ as:

$$\widetilde{D} = \tilde{k}.z \quad (2)$$

In situ and laboratory measures are used to evaluate lateral pressure coefficient in soils via empirical relations ($k = 1 - sin\varphi$ Jaky's formula for example). Empirical correlations combining Jaky's formula with OCR from laboratory are also used to determine the lateral pressure coefficient at rest. When experimental results are given in terms of intervals the average values are generally used.
Uncertainties are inherent to the testing method, and interval values can be suitable to handle the perturbations. In different types of tests uncertainties propagate during measuring procedures and from the use of different devices. In using Jaky's formula for example uncertainty lies essentially in measuring the material angle of friction φ . Whatever the techniques used for measuring such a parameter, there are uncertainties to consider.

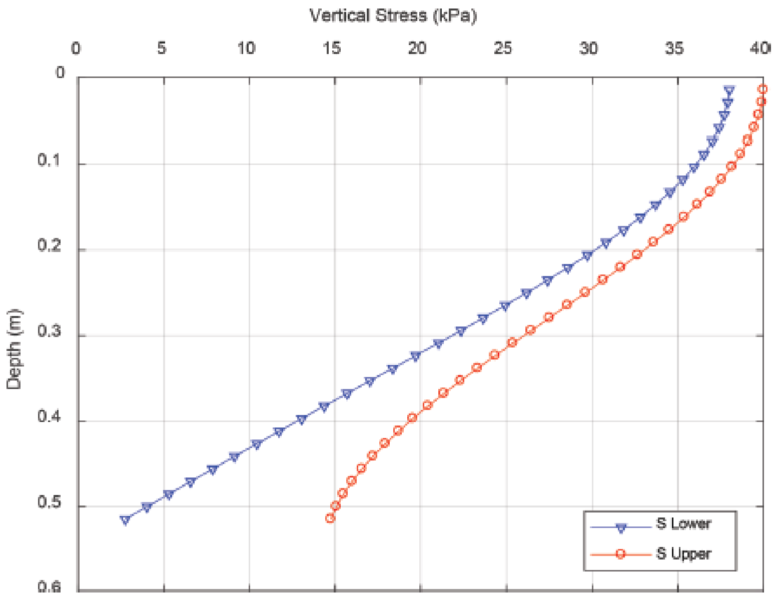

Figure 1. Vertical stress distribution under axis for interval surficial pressure $\tilde{q}$ and interval coefficient $\tilde{k}$

Figure 1 shows distribution of vertical stress $\tilde{S}$ with depth, based on uncertain input parameter $\tilde{k}$ taken as interval based. It was shown that uncertainty in the diffusion parameter affects significantly the distribution of vertical stress in the granular medium. And, when combined with the uncertainty from surficial pressure the stress distribution shows more dispersion with depth. Interval valued parameters are suitable when dealing with lack of data and uncertainty.

Soundings density estimation for geotechnical investigation

In another aspect of geotechnical problems in relation with geotechnical investigations Boumezerane (2015) used a Random Sets-Based System for estimation of soundings density during an investigation. The required number of soundings for geotechnical site investigation depends on different factors such as geology of the site, soil variability and the type of project to build. Indications on the number of soundings are given in codes of practice (Eurocode 7) but they focus mainly on a minimal number which depends only on the area of the project.
Geotechnical investigation is a process conducted, in general, in two steps. The first step consists in collecting available information and executing a limited number of soundings on site, while the second step of investigation is based on the first and requires more soundings for soil testing. The preliminary information is expressed by using random sets of the influencing factors (soil variability, geology…). Intervals of values are proposed by experts (engineers) concerning the influence of each factor based on preliminary information from site (soil variability, geology, type of project). The expert proposes an interval of values based on his degree of belief.
A *random set*, sometimes also referred to as a *Dempster-Shafer structure* is given by finitely any subsets Ai, $i = 1, . . . , n$ of a given set X, called the *focal sets*, each of which comes with a *probability weight* $mi = m(Ai)$, $\sum m(Ai) = 1$. An example of a random set is shown in Fig.2

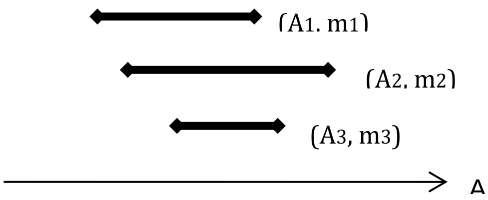

Figure 2. A random set.

In the Dempster-Shafer approach (Alvarez, 2008), the random set allows to define a degree of belief $\gamma(S)$ and a degree of plausibility $\eta(S)$, respectively, that the realizations of the parameter A lie in S by;

$$\gamma(S) = \sum_{Ai \subset S} m(Ai) \quad (3)$$

$$\eta(S) = \sum_{Ai \cap S \neq \emptyset} m(Ai) \quad (4)$$

The belief function $\gamma(S)$ or *Bel*, of a subset S is a set-valued function obtained through summation of basic probability assignments of subsets Ai included in S and the plausibility function $\eta(S)$, or *Pl*, of subset S is a set-valued function obtained through summation of basic probability assignments of subsets Ai having a non-zero intersection with S. They are envelopes of all possible cumulative distribution functions compatible with the data.

If one considers for instance a Dempster Shafer (D.S) structure which is formed by gathering the information provided by four different sources (e.g. books, experts, previous analysis, etc.) on the friction angle of some soil; each of those opinions will form one element Ai of the focal set A. Suppose that A = {A1= [20°,22°], A2 = [21°,23°], A3 = [18°,20°], A4 =[20°,25°]}. The basic mass assignment given to each of those focal elements will represent the importance of each of those opinions in our assessments. Suppose for example that ($m(A_1) = 0.4$, $m(A_2) = 0.2$, $m(A_3) = 0.1$, $m(A_4) = 0.3$; this means that we are giving to our first source of information the largest relevance (Alvarez, 2008).

Using Eurocode7 recommendations for site soundings, an "objective function" f(X) is constructed to rely "soil variability" to the number of soundings. It permits constructing the random set and obtain the number of soundings by unit area for each expert (engineer). The same reasoning is applied with other parameters such as geology and an inference system is obtained. Information is aggregated from the available parameters (Soil variability, Geology…) and the random sets computed from which upper and lower probabilities (probability boxes) are built. They permit optimizing the number of soundings to be carried out on site.

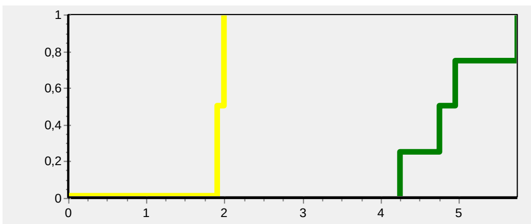

Figure 3. Construction of Upper and Lower probabilities vs Number of soundings

The random set-based approach introduced here can be considered as a valuable tool to estimate the number of soundings for geotechnical investigations. Information gathered from an expert and containing uncertainties can be expressed in form of intervals and degree of belief. The construction of random sets is another step. After aggregation Upper and Lower probabilities are built, which gives the limits of the decision.

5. Conclusion

The study of uncertainties in geotechnical problems can be conducted using other methods and approaches than probability-based ones only. In many problems those methods proved their ability to deal with different types of uncertainties like vagueness for example. Scarcity of information and lack of knowledge encountered generally in geotechnics makes it often difficult to study the underlying uncertainties using only probabilistic methods which need a lot of data to be validated. Fuzzy sets, random sets and interval-based approaches constitute valuable tools for dealing with epistemic uncertainties. Several examples are shown in the articles with applications in different aspects of geotechnical engineering.

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