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Predicting parity progression ratios for young women by the end of their childbearing life

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ABSTRACT

Parity progression ratios (PPR's) have been extensively described in literature on demography and have played an important role in fertility, unlike the idea of calculating projected parity progression ratios proposed by Brass (1985). However, we decided to use this method in our paper to analyse future fertility trends, firstly by assessing age-specific parity progression ratios for women in childbearing ages, and then by comparing these ratios with ratios at the end of women's reproductive life, as well as by comparing the latter with the completed PPR's. More specifically, the aim of this study is to adopt a modified Brass method to calculate the projected parity progression ratios using the age-period fertility data sourced from the Human Fertility Database (HFD). We progress to use the observed and predicted age-specific PPR's to examine parity progressions in Poland as a case study.

Key words: fertility rates, parity, projected parity progression ratios.

1. Introduction

The long-term decline in cohort fertility rates across developed countries has been widely studied and documented (Frejka and Calot 2001, Kohler et al. 2002, Billiari and Kohler 2004, Frejka 2008, Myrskyla et al. 2013, Sobotka 2013).

Empirical findings and evidence from the EU countries over last decades fit to the dominant demographic theories such as demographic transition and second demographic transition postulating that as societies progress, fertility tends to decrease. The period total fertility rate TFR declined considerably between 1980 and 2003 in most of the EU countries reaching the level below 1.30 between 2000 and 2003. According to Kohler et al. (2002) such low levels of TFR are termed "lowest-low" fertility. During the 1990s there were several lowest-low fertility countries in Southern, Central, South-Eastern and Eastern Europe, e.g. in Bulgaria, Czechia, Greece, Spain, Italy, Latvia, Slovenia, Slovakia. In several European countries fertility started to increase gradually around 2005, and decrease again with the financial crisis in 2008. More recently, according to the annual Eurostat reports, the period EU-wide total fertility rate attained 1.59 live births per woman in 2017, ranging from 1.26 in Malta to 1.90 in France. Moreover, almost half of children born in the EU in 2017 were first-born children.

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Several social and economic factors can serve as a response to observed fertility patterns and spatial differences, e.g. economic uncertainty, recession, increased incentives to invest in higher education and labour market experience, new lifestyle opportunities, reproductive behaviour, contraceptive use, abortion availability, or even late home-leaving by young adults, which is strongly correlated with high costs of formation of separate households.

Studying parity progression ratios can deliver more interesting details for understanding fertility changes and differences in parity distributions (Henry 1980, Paradysz 1995, Preston et al. 2001). Parity at a given point in time is defined as the number of children ever born by a women and the Parity Progression Ratio (PPR) of an i-th order reflects the proportion of women with i children who continue to an $i+1$-th live birth during their reproductive life. Thus, parity progression ratios allow to assess how frequently women are moving from the lower to higher parity.

Changes in particular PPR's may provide insight into processes of fertility with respect to the propensity of women to have children. Frejka (2008) found that decreasing PPR's to first and second births played a key role in fertility declines among European women born after 1955. In the Central and Eastern Europe fertility decline was driven primarily by falling PPR's to second births. Kohler et al. (2002) and Billiari and Kohler (2004) suggested that a pattern of the lowest-low fertility in Europe during the 1990s is characterized by a delay of childbearing especially for first births as well as by low progression probability after the first child but not by low probability of the first childbearing.

Usually, parity projection ratios are calculated for cohorts of women who have finished their reproductive life. A particular PPR of an i-th order is then defined as the ratio of the number of women at parity $i+1$ or more to the number of women at parity i or more and is treated as a fixed and completed cohort measure. PPR's for younger women are also calculated but are considered as uncompleted age-specific parity measures since women in reproductive ages move to higher parities and the distribution of their parities is changing. Brass (1985) proposed a methods which enables one to use parity data on younger women to calculate the so-called Projected Parity Progression Ratios (PPPR's) considered as completed progression ratios expected to be achieved by younger women by the end of their reproductive life. The method is based on the assumption that the current age pattern of specific fertility remains constant at the level observed at a given point in time.

In the paper a modified formula of the projected parity progression ratio is applied to investigate changes in the parity distribution in Poland as a case study. Some *ex post* comparisons are also conducted, i.e. between the observed and predicted PPR's for women in various age groups and between the latter and the completed PPR's observed for women attaining age 49 in a particular calendar year. Findings formulated from the comparisons allow one to assess the prediction accuracy of the modified Brass method as well as to make a contribution to explaining the future change in parity distribution over ten-year time horizon. The procedure is illustrated in details on the age-period fertility data of Poland sourced from the Human Fertility Database.

2. Notation and assumptions

In our analysis we use the following notation adopted to the population of women and to live births in a particular year:

N – total exposure-to-risk,

B – the total number of births,

N_x – exposure-to-risk in age interval $[x, x+1)$ for women attaining age x,

B_x – the number of births delivered by women aged x (in completed years),

$N_x(i)$ – exposure-to-risk in age interval $[x, x+1)$ for women attaining age x and of parity i,

$B_x(i)$ – the number of births delivered by women aged x and of parity i,

$N(i)$ – total exposure-to-risk for women of an i-th parity,

$B(i)$ – the total number of births to women of an i-th parity.

The following relations hold

$$N_x = \sum_{i=0}^{\pi} N_x(i), \quad N(i) = \sum_{x=\alpha}^{\beta} N_x(i), \quad N = \sum_{i=1}^{\pi} N(i) = \sum_{x=\alpha}^{\beta} N_x, \tag{1}$$

and

$$B_x = \sum_{i=0}^{\pi} B_x(i), \quad B(i) = \sum_{x=\alpha}^{\beta} B_x(i), \quad B = \sum_{i=1}^{\pi} B(i) = \sum_{x=\alpha}^{\beta} B_x, \tag{2}$$

where π is the highest parity in the data set, and α, β define the limits of the reproductive age range $[\alpha, \beta+1)$. Further, we will assume $\alpha = 15$ and $\beta = 49$.

We will also assume that in the given calendar year women had at most one birth, i.e. there are neither multiple deliveries nor multiple confinements, and that age-specific fertility rates for the reference period will continue to characterize future fertility patterns.

Note, that numbers of women P_x, P_{x+1} attaining respective ages x and $x+1$ during the reference year are closely related to exposure-to-risk N_x. Let us assume that the birthdays of females are distributed uniformly within the calendar year. Then each of the P_x females contributes on average $\frac{1}{2}$ of person-years to exposure N_x. Similarly, each of the P_{x+1} females contributes on average $\frac{1}{2}$ of person-years to N_x. On the other hand, assuming uniform distribution of deaths within the year, each of the D_x^L deaths (i.e. deaths in the lower triangle according to the Lexis diagram) among P_x females reduces exposure-to-risk by $\frac{1}{3}$ of person-years, on average, while each of the D_x^U deaths (i.e. deaths in the upper Lexis triangle) contributes an average $\frac{1}{3}$ of person-years to exposure N_x. Thus, N_x can be written as

$$N_x \approx \frac{1}{2}(P_x + P_{x+1}) + \frac{1}{3}\left(D_x^U - D_x^L\right). \tag{3}$$

Assuming $D_x^U \approx D_x^L$, expression (3) comes down to

$$N_x \approx \frac{1}{2}(P_x + P_{x+1}). \tag{4}$$

Analogous approximate equality refers to $N_x(i)$, i.e.

$$N_x(i) \approx \frac{1}{2}(P_x(i) + P_{x+1}(i)). \tag{5}$$

Further, exposure-to-risk N_x will be treated as an approximate average number of women aged x (in completed years) and similarly $N_x(i)$ – as an approximate average number of women aged x and of parity i.

3. Period Specific Fertility Rates and Average Parity

The Age-Specific Fertility Rate (ASFR) and the Age-Specific i-th Order Fertility Rate (ASOFR) for women aged x are defined as follows

$$ASFR_x = \frac{B_x}{N_x} \qquad \text{and} \qquad ASOFR_x(i) = \frac{B_x(i)}{N_x}. \tag{6}$$

Note that $ASOFR_x(i)$ cannot be termed "order-specific rate" as the denominator N_x is unidentified by parity. Observe also that

$$ASFR_x = \sum_{i=1}^{\pi} ASOFR_x(i). \tag{7}$$

Average parity P in a population is calculated by dividing the total number of children ever born by the number of women N, i.e.

$$P = \frac{1}{N} \cdot \sum_{j=0}^{\pi} j \cdot N(j) = \frac{1 \cdot N(1)}{N} + \frac{2 \cdot N(2)}{N} + \frac{3 \cdot N(3)}{N} + \ldots + \frac{\pi \cdot N(\pi)}{N}. \tag{8}$$

4. Cumulated Age-Specific *i*-th Order Fertility Rates

In this section we will employ age-specific and age-specific *i*-th order fertility rates given in (6) for one-year age bands $[x, x+1)$ to define total and total order fertility rates as well as cumulated age-order fertility rates.

Using the α, β as the limits for the summation, the Total Fertility Rate (TFR) and the Total *i*-th Order Fertility Rate (TOFR) are defined as

$$TFR = \sum_{x=\alpha}^{\beta} ASFR_x \qquad \text{and} \qquad TOFR(i) = \sum_{x=\alpha}^{\beta} ASOFR_x(i). \tag{9}$$

Then the Cumulated Age-Specific *i*-th Order Fertility Rate is determined by summing age-specific *i*-th order specific fertility rates up to the desired age x. Thus, we have

$$TOFR_y(i) = \sum_{x=\alpha}^{y} ASOFR_x(i). \tag{10}$$

It follows from (9) and (10) that $TOFR_y(i) = TOFR(i)$ for $y = \beta$.

5. Conventional and Projected Parity Progression Ratios

The concept of a parity progression ratio was introduced by Henry in 1953 as a useful measure of fertility. Later, many researchers have proposed methods to evaluate parity progression ratios (PPR's) or the projected parity progression ratios (PPPR's) (Srinivasan 1968, Feeney 1983, Yadava and Bhattacharya 1985, Brass 1985, Feeney and Jingyuan 1987, Yadava et al. 1992, Islam and Yadava 1997, Bhardwaj et al. 2010, Yadava and Kumar 2011).

The conventional PPR of an *i*-th order is the proportion of women who progress from *i*-th to $i+1$-th parity. In other words, it is the chance that a female after giving birth to *i*-th child will ever deliver another child. Projected parity progression ratios indicate the possible future evolution of parity progression for younger women, taking into account both current fertility and the women's childbearing history.

Parity progression ratios can be calculated on a cohort or period basis depending on the data available. For cohorts, they are usually calculated for women who have completed their childbearing, e.g. for women aged 49. Cohort ratios are often calculated from the census data whereas period ratios use probabilities of giving birth in a defined reference period. In our analysis we use the age-period fertility data sourced from the Human Fertility Database to calculate PPR's and generalized PPPR's for the female population in Poland (see Section 6).

5.1. Parity Progression Ratio

Let us consider the number $W(i)$ of women in a population having attained parity i or higher. Note that

$$W(i) = \sum_{j=i}^{\pi} N(j) = N(i) + N(i+1) + \cdots + N(\pi). \tag{11}$$

It is clear that $W(0)$ is equal to the total number of women in the population

$$W(0) = \sum_{j=0}^{\pi} N(j) = N. \tag{12}$$

The proportion $M(i)$ of women ever-attaining parity i, i.e. the share of women who have at least i children, can be expressed as

$$M(i) = \frac{W(i)}{N} = \frac{1}{N} \cdot \sum_{j=i}^{\pi} N(j). \tag{13}$$

The corresponding proportion at parity zero or higher is $M(0) = N/N = 1$.

By analogy, the number $W_x(i)$ and the proportion $M_x(i)$ of women aged x and having attained parity i or higher are as follows

$$W_x(i) = \sum_{j=i}^{\pi} N_x(j), \tag{14}$$

$$M_x(i) = \frac{W_x(i)}{N_x} = \frac{1}{N_x} \cdot \sum_{j=i}^{\pi} N_x(j). \tag{15}$$

Then, the associated parity progression ratios $PPR(i)$ and $PPR_x(i)$ can be expressed as

$$PPR(i) = \frac{W(i+1)}{W(i)} = \frac{W(i+1)/N}{W(i)/N} = \frac{M(i+1)}{M(i)}, \tag{16}$$

$$PPR_x(i) = \frac{W_x(i+1)}{W_x(i)} = \frac{W_x(i+1)/N_x}{W_x(i)/N_x} = \frac{M_x(i+1)}{M_x(i)}. \tag{17}$$

It is worth noting that ratios $PPR_x(i)$ calculated for younger women should be treated as uncompleted age-specific PPR's. In such cases it is also reasonable to calculate projected parity progression ratios $PPPR_x(i)$ in order to estimate completed parity progressions for younger women by the end of their reproductive life.

5.2. Projected Parity Progression Ratio and its generalization

According to the Brass concept (see, e.g. Moultrie et al. 2013, p. 74) the difference between the Total Order Fertility Rate and the Cumulated Age-Specific i-th Order Fertility

Rate,

$$TOFR(i) - TOFR_x(i), \quad i = 1, 2, \ldots, \pi, \tag{18}$$

can be treated as an estimate of an additional proportion of women aged x expected to achieve parity i by the end of their childbearing years. This interpretation is admissible under the assumption that the current fertility pattern will remain constant until the end of women's reproductive life and that in the given year every women had at most one birth.

Let us consider a modified version of formula (18) by substituting $TOFR_y(i)$ for $TOFR(i)$ in (18), where $y > x$. Under assumptions as stated above, the difference of the form

$$TOFR_y(i) - TOFR_x(i), \quad i = 1, 2, \ldots, \pi, \quad y > x, \tag{19}$$

can be treated as an estimate of an additional proportion of women aged x expected to achieve parity i at age y. Note that formula (19) reduces to (18) when $y = \beta$.

Then, the proportions of women aged x projected to achieve at least parity i at age $y > x$ can be defined as

$$M^*_{x,y}(i) = M_x(i) + TOFR_y(i) - TOFR_x(i), \qquad M^*_{x,y}(0) = M_x(0) = 1. \tag{20}$$

For $y = \beta$ we will write $M^*_x(i)$ instead of $M^*_{x,y}(i)$. Thus, in this case we have

$$M^*_x(i) = M_x(i) + TOFR(i) - TOFR_x(i), \tag{21}$$

Generalized projected parity progression ratios for women aged x will be considered as progression ratios expected to be reached after $y - x$ years. They will be expressed as

$$PPPR_{x,y}(i) = \frac{M^*_{x,y}(i+1)}{M^*_{x,y}(i)}. \tag{22}$$

6. Parity distribution in Poland – analysis based on projected parity progression ratios

To examine the measures of fertility presented in previous sections we applied the most recent fertility data on the distribution of the female population in Poland tabulated by one-year age groups and by parity as well as the distribution of births attained to this population and tabulated by birth order and mothers' age (in completed years).

6.1. The input data

The input data contained in Tables 1 and 2 were sourced from the Human Fertility Database. The body of Table 1 shows age-parity exposure of Polish female population in the last available year 2016, whereas Table 2 displays counts of live births in Poland in the same year tabulated by birth order and mothers' age.

Table 1: Age-parity female exposure $N_x(i)$ of the 2016 female population in Poland (average number of females by age and parity)

Age x	$N_x(i)$ $i=0$	$i=1$	$i=2$	$i=3$	$i \geq 4$	total
-15	699085.42	165.06	5.45	0	0	699255.93
16	184257.97	625.43	15.58	0	0	184898.98
17	185483.01	1798.32	58.44	1.98	0	187341.75
18	189041.11	4135.90	222.94	5.45	0.49	193405.89
19	194699.88	8154.46	656.40	37.69	1.99	203550.42
20	194719.18	13174.49	1460.10	110.39	6.94	209471.10
21	196404.47	19087.15	2868.78	285.58	27.31	218673.29
22	201459.53	26170.88	4972.21	591.05	73.55	233267.22
23	198931.98	32937.34	7624.92	1034.25	169.31	240697.80
24	197317.30	40905.71	11273.04	1640.15	301.11	251437.31
25	194496.67	50425.01	16150.27	2433.19	503.30	264008.44
26	181595.98	59855.72	22111.05	3501.29	801.03	267865.07
27	167695.53	69161.72	29309.41	4701.49	1159.01	272027.16
28	157336.79	78246.95	38409.45	6152.04	1573.96	281719.19
29	144603.24	84933.58	47992.62	7766.47	1965.86	287261.77
30	137485.75	92972.07	60900.12	10248.90	2636.65	304243.49
31	131856.06	98881.18	75666.38	13357.25	3552.52	323313.39
32	122603.24	97951.45	86973.82	16151.36	4282.29	327962.16
33	113050.33	94418.04	96023.29	18921.63	5066.54	327479.83
34	101354.89	87457.72	99899.98	20814.14	5747.44	315274.17
35	92601.81	83963.21	105242.26	23310.58	6632.15	311750.01
36	85900.89	82755.66	110598.42	26078.02	7683.17	313016.16
37	77920.47	80183.70	112355.60	28034.34	8720.38	307214.49
38	70508.67	78048.62	113088.62	30099.28	9948.63	301693.82
39	65645.77	77014.28	114896.35	32326.78	11178.48	301061.66
40	61221.21	74909.35	114266.77	33816.61	12380.23	296594.17
41	56286.57	70929.06	110469.02	34585.13	13553.56	285823.34
42	52374.76	66343.63	106479.98	35118.83	14795.63	275112.83
43	49362.05	61801.96	102184.14	35642.20	15832.16	264822.51
44	45290.59	57896.05	98669.16	36265.96	16929.75	255051.51
45	41136.72	54218.25	95804.85	36888.67	18122.17	246170.66
46	36632.17	51109.92	93088.02	37827.05	19325.51	237982.67
47	33993.21	48309.58	90251.72	39001.04	20315.62	231871.17
48	32899.15	45636.39	88909.58	39772.78	21526.43	228744.33
49	31780.03	43773.84	89020.08	40891.76	23095.80	228561.51

Source: HUMAN FERTILITY DATABASE. Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria), available at www.humanfertility.org (data downloaded on [06/06/2019]).

Table 2: Number of births $B_x(i)$ by mothers' age x and birth order i, Poland, 2016

	$B_x(i)$				
Age x	$i=1$	$i=2$	$i=3$	$i\geq 4$	total
-15	250	5	0	0	255
16	752	14	0	0	766
17	1591	91	3	0	1685
18	3007	263	5	0	3275
19	4642	611	60	5	5318
20	5770	1148	108	10	7036
21	6718	1836	246	32	8832
22	7556	2637	378	65	10636
23	8879	3554	572	123	13128
24	10543	4519	876	205	16143
25	12817	6025	1058	272	20172
26	14392	7331	1313	406	23442
27	14981	8903	1662	511	26058
28	14894	10711	2037	606	28248
29	13735	12074	2384	692	28885
30	12589	13594	2844	879	29906
31	10394	13742	3220	965	28322
32	8476	13249	3568	1008	26301
33	6558	11758	3662	1203	23181
34	4862	9443	3273	1126	18704
35	3823	7630	3114	1236	15804
36	2877	5968	2938	1261	13045
37	2121	4397	2505	1153	10177
38	1547	2981	2043	1085	7655
39	1138	2130	1525	998	5791
40	744	1369	1046	812	3971
41	515	839	714	615	2683
42	275	439	494	439	1647
43	127	235	239	300	901
44	68	98	109	184	460
45	28	45	49	85	207
46	10	15	20	30	75
47	8	12	7	20	47
48	5	2	0	3	10
49	1	4	1	2	8

Source: As in table 1.

6.2. Results

A. As a first step of the analysis, the number $W_x(i)$ and the proportion $M_x(i)$ of women aged x and having attained parity i or higher are calculated. For this purpose, we use formulas (14) and (15). Next, parity progression ratios $PPR_x(i)$ from (17) are computed. Table 3 reveals results concerning $M_x(i)$ and $PPR_x(i)$.

Table 3: Proportions $M_x(i)$ of women aged x attaining parity i and parity progression ratios $PPR_x(i)$

	$M_x(i)$					$PPR_x(i)$			
Age x	$i=0$	$i=1$	$i=2$	$i=3$	$i\geq 4$	$i=0$	$i=1$	$i=2$	$i=3$
-15	1	0.0002	0.0000	0.0000	0.0000	0.0002	0.0000		
16	1	0.0035	0.0001	0.0000	0.0000	0.0035	0.0286	0.0000	
17	1	0.0099	0.0003	0.0000	0.0000	0.0099	0.0303	0.0000	
18	1	0.0226	0.0012	0.0000	0.0000	0.0226	0.0531	0.0000	
19	1	0.0435	0.0034	0.0002	0.0000	0.0435	0.0782	0.0588	0.0000
20	1	0.0704	0.0075	0.0006	0.0000	0.0704	0.1065	0.0800	0.0000
21	1	0.1018	0.0145	0.0014	0.0001	0.1018	0.1424	0.0966	0.0714
22	1	0.1364	0.0242	0.0028	0.0003	0.1364	0.1774	0.1157	0.1071
23	1	0.1735	0.0367	0.0050	0.0007	0.1735	0.2115	0.1362	0.1400
24	1	0.2152	0.0526	0.0077	0.0012	0.2152	0.2444	0.1464	0.1558
25	1	0.2633	0.0723	0.0111	0.0019	0.2633	0.2746	0.1535	0.1712
26	1	0.3221	0.0986	0.0161	0.0030	0.3221	0.3061	0.1633	0.1863
27	1	0.3835	0.1293	0.0215	0.0043	0.3835	0.3372	0.1663	0.2000
28	1	0.4415	0.1638	0.0274	0.0056	0.4415	0.3710	0.1673	0.2044
29	1	0.4966	0.2009	0.0339	0.0068	0.4966	0.4046	0.1687	0.2006
30	1	0.5481	0.2425	0.0424	0.0087	0.5481	0.4424	0.1748	0.2052
31	1	0.5922	0.2863	0.0523	0.0110	0.5922	0.4835	0.1827	0.2103
32	1	0.6262	0.3275	0.0623	0.0131	0.6262	0.5230	0.1902	0.2103
33	1	0.6548	0.3665	0.0733	0.0155	0.6548	0.5597	0.2000	0.2115
34	1	0.6785	0.4011	0.0842	0.0182	0.6785	0.5912	0.2099	0.2162
35	1	0.7030	0.4336	0.0960	0.0213	0.7030	0.6168	0.2214	0.2219
36	1	0.7256	0.4612	0.1079	0.0245	0.7256	0.6356	0.2340	0.2271
37	1	0.7464	0.4854	0.1196	0.0284	0.7464	0.6503	0.2464	0.2375
38	1	0.7663	0.5076	0.1327	0.0330	0.7663	0.6624	0.2614	0.2487
39	1	0.7820	0.5261	0.1445	0.0371	0.7820	0.6728	0.2747	0.2567
40	1	0.7936	0.5410	0.1558	0.0417	0.7936	0.6817	0.2880	0.2677
41	1	0.8031	0.5549	0.1684	0.0474	0.8031	0.6909	0.3035	0.2815
42	1	0.8096	0.5685	0.1814	0.0538	0.8096	0.7022	0.3191	0.2966
43	1	0.8136	0.5802	0.1944	0.0598	0.8136	0.7131	0.3351	0.3076
44	1	0.8224	0.5954	0.2086	0.0664	0.8224	0.7240	0.3504	0.3183
45	1	0.8329	0.6126	0.2235	0.0736	0.8329	0.7355	0.3648	0.3293
46	1	0.8461	0.6313	0.2402	0.0812	0.8461	0.7461	0.3805	0.3381
47	1	0.8534	0.6450	0.2558	0.0876	0.8534	0.7558	0.3966	0.3425
48	1	0.8562	0.6567	0.2680	0.0941	0.8562	0.7670	0.4081	0.3511
49	1	0.8705	0.6945	0.3057	0.1142	0.8705	0.7978	0.4402	0.3735

Source: Authors' own calculations.

Interpretation of the data in Table 1 is rather straightforward. For instance, while 26.33% women aged 25 have had at least one birth ($M_{25}(1) = 0.2633$), only 7.23% have had two or more births ($M_{25}(2) = 0.0723$). On the other hand, the parity progression ratios suggest that 15.35% women aged 25 who had two children went on to have a third ($PPR_{25}(2) = 0.1535$).

B. In the second step the Cumulated Age-Specific i-th Order Fertility Rates and the Total Order Fertility Rates, i.e. $TOFR_x(i)$, $TOFR(i)$ from (10) and (9), respectively, are derived. Based on differences $TOFR(i) - TOFR_x(i)$, additional proportions of women aged x expected to attain parity i by the end of their childbearing years are found. Results are given in Table 4.

Table 4: Cumulated Age-Specific i-th Order Fertility Rates $TOFR_x(i)$ and differences $TOFR(i) - TOFR_x(i)$

	$TOFR_x(i)$				$TOFR(i) - TOFR_x(i)$			
Age x	$i=1$	$i=2$	$i=3$	$i \geq 4$	$i=1$	$i=2$	$i=3$	$i \geq 4$
-15	0.0004	0.0000	0.0000	0.0000	0.6568	0.5048	0.1406	0.0350
16	0.0044	0.0001	0.0000	0.0000	0.6527	0.5047	0.1406	0.0350
17	0.0129	0.0006	0.0000	0.0000	0.6442	0.5042	0.1406	0.0350
18	0.0285	0.0019	0.0000	0.0000	0.6287	0.5029	0.1405	0.0350
19	0.0513	0.0049	0.0003	0.0000	0.6059	0.4999	0.1403	0.0350
20	0.0788	0.0104	0.0009	0.0001	0.5783	0.4944	0.1397	0.0349
21	0.1095	0.0188	0.0020	0.0002	0.5476	0.486	0.1386	0.0348
22	0.1419	0.0301	0.0036	0.0005	0.5152	0.4747	0.1370	0.0345
23	0.1788	0.0449	0.0060	0.0009	0.4783	0.4599	0.1346	0.0341
24	0.2207	0.0628	0.0095	0.0015	0.4364	0.4420	0.1311	0.0335
25	0.2693	0.0857	0.0135	0.0023	0.3878	0.4191	0.1271	0.0327
26	0.3230	0.1130	0.0184	0.0034	0.3341	0.3918	0.1222	0.0316
27	0.3781	0.1458	0.0245	0.0047	0.2790	0.3590	0.1161	0.0303
28	0.4310	0.1838	0.0317	0.0062	0.2262	0.3210	0.1089	0.0288
29	0.4788	0.2258	0.0400	0.0079	0.1784	0.2790	0.1006	0.0271
30	0.5202	0.2705	0.0494	0.0098	0.1370	0.2343	0.0912	0.0252
31	0.5523	0.3130	0.0593	0.0118	0.1048	0.1918	0.0813	0.0232
32	0.5781	0.3534	0.0702	0.0139	0.0790	0.1514	0.0704	0.0211
33	0.5982	0.3893	0.0814	0.0163	0.0590	0.1155	0.0592	0.0187
34	0.6136	0.4193	0.0918	0.0186	0.0435	0.0856	0.0488	0.0164
35	0.6259	0.4437	0.1017	0.0212	0.0313	0.0611	0.0388	0.0138
36	0.6351	0.4628	0.1111	0.0238	0.0221	0.0420	0.0295	0.0112
37	0.6420	0.4771	0.1193	0.0262	0.0152	0.0277	0.0213	0.0088
38	0.6471	0.4870	0.1261	0.0283	0.0101	0.0178	0.0145	0.0067
39	0.6509	0.4941	0.1311	0.0303	0.0063	0.0107	0.0095	0.0047
40	0.6534	0.4987	0.1346	0.0318	0.0038	0.0061	0.0059	0.0032
41	0.6552	0.5016	0.1371	0.0329	0.0020	0.0032	0.0034	0.0021
42	0.6562	0.5032	0.1389	0.0337	0.0010	0.0016	0.0016	0.0012
43	0.6567	0.5041	0.1398	0.0343	0.0005	0.0007	0.0007	0.0006
44	0.6569	0.5045	0.1403	0.0347	0.0002	0.0003	0.0003	0.0003
45	0.6570	0.5047	0.1405	0.0349	0.0001	0.0001	0.0001	0.0001
46	0.6571	0.5047	0.1406	0.0349	0.0001	0.0001	0.0000	0.0000
47	0.6571	0.5048	0.1406	0.0350	0.0000	0.0000	0.0000	0.0000
48	0.6571	0.5048	0.1406	0.0350	0.0000	0.0000	0.0000	0.0000
49	0.6571	0.5048	0.1406	0.0350	0.0000	0.0000	0.0000	0.0000

Source: Authors' own calculations.

For example, the cumulated age-order fertility rate up to the age of 25 for parity $i = 2$ would be

$$TOFR_{25}(2) = 0.0857.$$

The Total Order Fertility Rate for the same parity, $TOFR(i)$, is equal to the cumulated age-order fertility rate up to the end of women's reproductive life, i.e. up to the age of 49. Thus, we have

$$TOFR(2) = TOFR_{49}(2) = 0.5048.$$

It implies that that difference

$$TOFR(2) - TOFR_{25}(2) = 0.4191$$

estimates the additional proportion of women aged 25 expected to achieve parity 2 by the end of their childbearing years. In other words, it is the anticipated future increment of proportion of women at parity 2.

This interpretation is valid under the assumptions that women had at most one birth in the given year and that current fertility will remain constant until the end of women's reproductive life.

C. Next, we derive projected proportions of women aged x who will attain at least parity i by the end of their childbearing years using formula (21). Thus, projected proportions $M_x^*(i)$ are calculated by adding the future order increments (18) to $M_x(i)$. Finally, the Projected Parity Progression Ratios between for parity i are computed using formula (22), i.e. as ratios of proportions $M_x^*(i)$ of women expected to attain each successive parity at any given age (Table 5).

For instance, the proportions of women aged 25 projected to achieve at least parity 2 by the end of their childbearing life equals

$$M_{25}^*(2) = M_{25}(2) + TOFR(2) - TOFR_{25}(2) =$$

$$= 0.0723 + 0.4191 = 0.4914.$$

It follows that the proportion of women aged 25 with one child who are projected to have at least two children is 75.47% ($PPPR_{25}(1) = 0.7547$), whereas the proportion of women with two births who are projected to have at least three children is 28.12% ($PPPR_{25}(2) = 0.2812$).

Table 5: Projected proportions $M_x^*(i)$ of women expected to attain at least parity i and projected parity progression ratios $PPPR_x(i)$

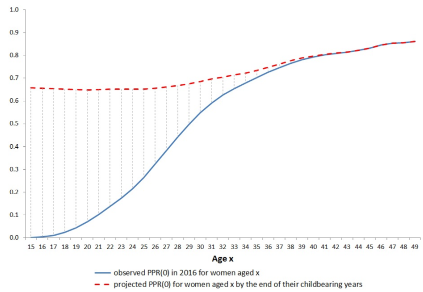
	$M_x^*(i)$				$PPPR_x(i)$			
Age x	$i=1$	$i=2$	$i=3$	$i=4$	$i=0$	$i=1$	$i=2$	$i=3$
-15	0.6570	0.5048	0.1406	0.0350	0.6570	0.7683	0.2785	0.2489
16	0.6562	0.5048	0.1406	0.0350	0.6562	0.7693	0.2785	0.2489
17	0.6541	0.5045	0.1406	0.0350	0.6541	0.7713	0.2787	0.2489
18	0.6513	0.5041	0.1405	0.0350	0.6513	0.7740	0.2787	0.2491
19	0.6494	0.5033	0.1405	0.0350	0.6494	0.7750	0.2792	0.2491
20	0.6487	0.5019	0.1403	0.0349	0.6487	0.7737	0.2795	0.2488
21	0.6494	0.5005	0.1400	0.0349	0.6494	0.7707	0.2797	0.2493
22	0.6516	0.4989	0.1398	0.0348	0.6516	0.7657	0.2802	0.2489
23	0.6518	0.4966	0.1396	0.0348	0.6518	0.7619	0.2811	0.2493
24	0.6516	0.4946	0.1388	0.0347	0.6516	0.7591	0.2806	0.2500
25	0.6511	0.4914	0.1382	0.0346	0.6511	0.7547	0.2812	0.2504
26	0.6562	0.4904	0.1383	0.0346	0.6562	0.7473	0.2820	0.2502
27	0.6625	0.4883	0.1376	0.0346	0.6625	0.7371	0.2818	0.2515
28	0.6677	0.4848	0.1363	0.0344	0.6677	0.7261	0.2811	0.2524
29	0.6750	0.4799	0.1345	0.0339	0.6750	0.7110	0.2803	0.2520
30	0.6851	0.4768	0.1336	0.0339	0.6851	0.6960	0.2802	0.2537
31	0.6970	0.4781	0.1336	0.0342	0.6970	0.6859	0.2794	0.2560
32	0.7052	0.4789	0.1327	0.0342	0.7052	0.6791	0.2771	0.2577
33	0.7138	0.4820	0.1325	0.0342	0.7138	0.6753	0.2749	0.2581
34	0.7220	0.4867	0.1330	0.0346	0.7220	0.6741	0.2733	0.2602
35	0.7343	0.4947	0.1348	0.0351	0.7343	0.6737	0.2725	0.2604
36	0.7477	0.5032	0.1374	0.0357	0.7477	0.6730	0.2731	0.2598
37	0.7616	0.5131	0.1409	0.0372	0.7616	0.6737	0.2746	0.2640
38	0.7764	0.5254	0.1472	0.0397	0.7764	0.6767	0.2802	0.2697
39	0.7883	0.5368	0.1540	0.0418	0.7883	0.6810	0.2869	0.2714
40	0.7974	0.5471	0.1617	0.0449	0.7974	0.6861	0.2956	0.2777
41	0.8051	0.5581	0.1718	0.0495	0.8051	0.6932	0.3078	0.2881
42	0.8106	0.5701	0.1830	0.0550	0.8106	0.7033	0.3210	0.3005
43	0.8141	0.5809	0.1951	0.0604	0.8141	0.7135	0.3359	0.3096
44	0.8226	0.5957	0.2089	0.0667	0.8226	0.7242	0.3507	0.3193
45	0.8330	0.6127	0.2236	0.0737	0.8330	0.7355	0.3649	0.3296
46	0.8462	0.6314	0.2402	0.0812	0.8462	0.7462	0.3804	0.3381
47	0.8534	0.6450	0.2558	0.0876	0.8534	0.7558	0.3966	0.3425
48	0.8562	0.6567	0.2680	0.0941	0.8562	0.7670	0.4081	0.3511
49	0.8610	0.6694	0.2800	0.1010	0.8610	0.7775	0.4183	0.3607

Source: Authors' own calculations.

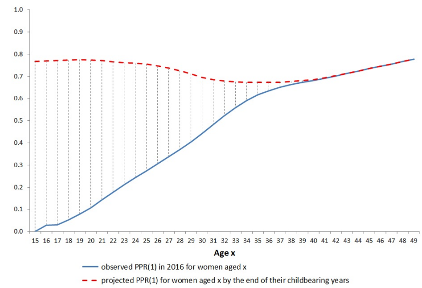
6.3. Graphical illustration

Figures 1 and 2 allow for more detailed comparison between the projected and observed age-specific PPR's by parity summarized in Tables 3, 5.

As expected, there are substantial differences between both types of PPR's, especially for young women, although differences vanish as age is getting older.



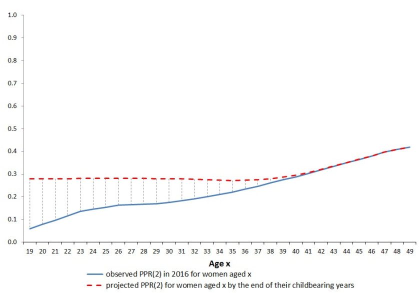
(a) Parity $i = 0$



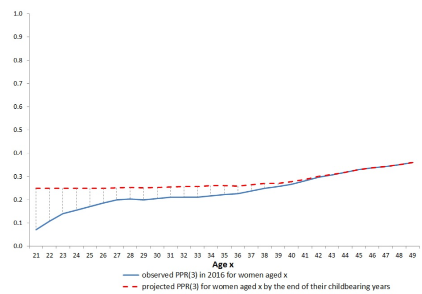
(b) Parity $i = 1$

Figure 1: Projected and observed parity progression ratios for parities $i = 0,1$

Source: Developed by the authors.



(a) Parity $i = 2$



(b) Parity $i = 3$

Figure 2: Projected and observed parity progression ratios for parities $i = 2,3$

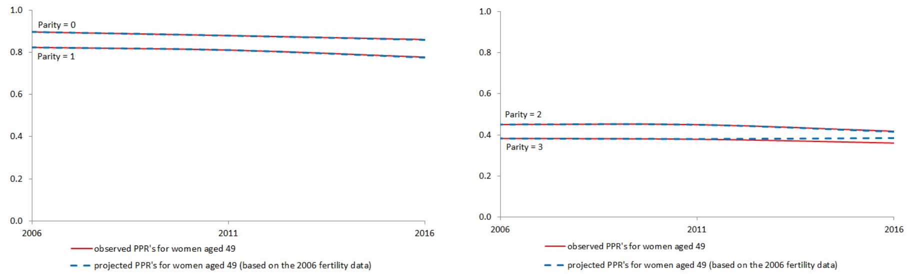
Source: Developed by the authors.

We can observe typical shapes of observed age-specific PPR's with curves increasing with age whereas projected PPR's tend to level. Both observed and projected PPR's decrease as parity increases.

6.4. Prediction

The major thrust of this section is to see if the projected parity progression ratios derived for younger women give a good prediction of their completed parity progression ratios. To achieve this, the projected ratios for parities $i = 0,1,2,3$ (predictions based on the 2006 fertility data) are compared with completed ratios for women aged 49 in the years 2006, 2011 and 2016. Results are illustrated on Figure 3.

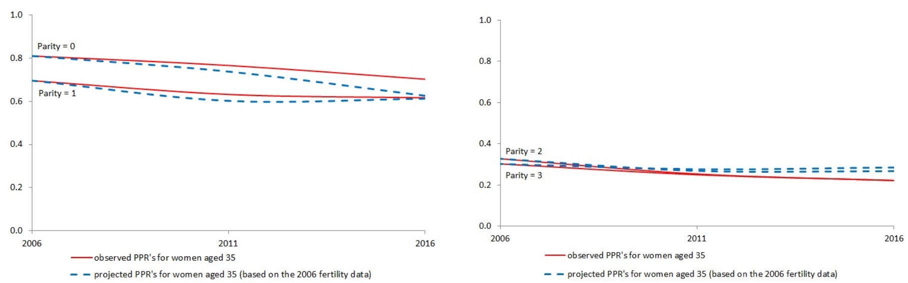
Analogous comparisons are made between the projected parity progression ratios (based on the 2006 fertility data) and the parity progression ratios for women aged 35 observed in the years 2006, 2011 and 2016. Figure 4 shows these two comparisons.



(a) (b)

Figure 3: Projected and observed (completed) PPR’s for women aged 49

Source: Developed by the authors.



(a) (b)

Figure 4: Projected and observed (uncompleted) PPR’s for women aged 35

Source: Developed by the authors.

Figure 3 shows that the projected and observed age-specific PPR’s are very similar. Given that women aged 44 or 39 in 2006 are close to the end of their childbearing life and that there is little time for a significant fertility change, the projected ratios are almost the same as the completed PPR’s observed for women aged 49 in 2011 or in 2016, respectively.

Based on the comparisons of the projected and observed PPR’s for females attaining age 35 in 2011 or in 2016, we can conclude (see Figure 4) that the projections fit much better for the five-year forecast horizon compared to the ten-year horizon. Moreover, they underestimate the completed PPR’s for parity $i = 0$ and overestimate the completed PPR’s for parities $i = 2,3$. This effect results from the fact that in this case the main assumption about time-invariant fertility rates is not satisfied.

In general, the projected parity progression ratios seem to provide a satisfactory prediction in the ten-year or shorter time horizon.

7. Conclusion

Parity progression ratios are important indicators explaining the pattern of fertility. They provide an alternative to conventional age-based studies of fertility trends. Traditional age-specific fertility rates and their sum, i.e. the total fertility rate, use age as a main structural feature of the female population that may influence the number of births in a given period. However, another important structural feature is parity.

The parity analysis facilitates the interpretation of trends in the number of births and the age of women who decide to have a child. What is more, parity measures can be related more directly to behavioural factors, because a woman makes her decision about having a child not only based on how old she is but also how many children she already has.

In the paper age-specific parity progression ratios and projected parity progression ratios for the Polish female population were investigated in greater details. Based on the numerical results presented in Section 6 the following principal findings can be formulated: the decline in fertility in Poland in the near future will be caused by the gradual decrease in the propensity of women to have more than two children. There is still no problem with the desire to have one child. About 86% of young childless Polish women decide to have a child. Most of them have also a second child. The situation is much worse in the case of higher order births. The results obtained indicate that PPR's for higher parities drop down rapidly by more then half compared to the above mentioned rate.

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