

# A New Method for Interference Reduction in the Smoothed Pseudo Wigner-Ville Distribution

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**Abstract**— This article presents a new method facilitating interference reduction in the Wigner-Ville distribution, which is used for nonstationary signal analysis, for example in machine condition monitoring. The algorithm is based on multiple Smoothed Pseudo Wigner-Ville distributions: differently smoothed time-frequency planes are compared and, for every point, a cross-term free value is calculated on the basis of optimal smooth estimation. The proposed approach is compared with the Gabor-Wigner transform, the Zhao-Atlas-Marks distribution, and the Choi-Williams distribution. Five time-frequency Gaussian atoms and a bat echolocation chirp are used as the testing signals. The Rényi entropy, the ratio of norms, the Stanković measure, and the mean squared error are used as quantitative measures to demonstrate the promising results of the proposed method.

**Keywords**- *Wigner-Ville Distribution; Smoothed Pseudo Wigner-Ville Distribution; Reduced Interference; Time-Frequency Distribution; Quantitative measure*

## I. INTRODUCTION

As real life signals change over time (in, for example, the analysis of heart sounds, machine condition monitoring, and engine or gear fault diagnosis), transitional or nonstationary components are an inevitable part of these signals. The components can be hardly analyzed based on only the time or the frequency, and therefore time-frequency tools were introduced. Nevertheless, the widely used methods (such as the Short Time Fourier Transform or the Wavelet Transform) do not offer sufficient crisp resolution to facilitate detailed examination of the above-mentioned components. Consequently, a variety of Time-Frequency Distributions (TFD) with different resolution levels and properties were developed, see Boashash et al. [1].

The Wigner-Ville Distribution (WVD) is one of the basic TFDs with optimal resolution, and it appears to be ideal for the linear frequency modulated (FM) signal. In addition, the WVD exhibits many beneficial mathematical properties, including energy conservation, time and frequency shift invariants, or compatibility with filters; the details are described by Cohen [2].

Good Time-Frequency (TF) resolution and other properties are balanced by the presence of interferences. These constitute an indivisible portion of the result if multi-component or nonlinear FM signals are included in the analyzed signal. More

components in the signal result in outer interferences, whereas a nonlinear FM signal produces inner interferences. Details concerning the interferences and their formation are well described by Hlawatsch and Flandrin [3].

### A. Interference reduction

The presence of cross terms diminishes the readability of the result; thus, many methods for their reduction were proposed, see Shafi et al. [4]. All such techniques attempt to significantly reduce or completely remove the interferences, preserve autoterm clarity (good resolution), and maintain the maximum of the distribution properties. One of the basic methods is the Pseudo WVD. Here, a filtering window is introduced, whereby the result is filtered in the frequency plane, and the interferences formed by distantly placed components in the time plane are suppressed. However, cross terms created by distant autoterms in the frequency plane cannot be removed by frequency filtering.

For the removal of the cross terms formed in the frequency plane, windowing in time is needed. In consequence of this fact, smoothing in time and frequency was introduced. The derived Smoothed Pseudo Wigner-Ville Distribution (SPWVD) enables the reduction of all types of interferences at the cost of resolution reduction, see Flandrin [5] or Fig. 1.

Smoothing by wider time filtering (window width  $L_g$ ) and frequency filtering (window width  $L_h$ ) leads to interference reduction and slow degradation of the resolution, as is visible in Fig. 1 (a)-(c) demonstrating the case of three Gaussian TF atoms.

There is a number of other well-developed methods based on the WVD; however, their details reach beyond the scope of this article. The majority of effective algorithms are briefly described and referenced in a review article by Shafi et al. [5]. In general, algorithms with good interference reduction are iterative, signal-dependent, or parametric. The reduced Interference Distributions (RIDs) derived from the general TFDs constitute another class of solutions to the interference issue, see Williams and Jeong [6].

## II. NEW METHOD

As mentioned above, the different time and frequency window widths in the SPWVD affect the resulting presence of the interferences and TF resolution. If we focus on the growing window impact at different points in the TF plane, we can

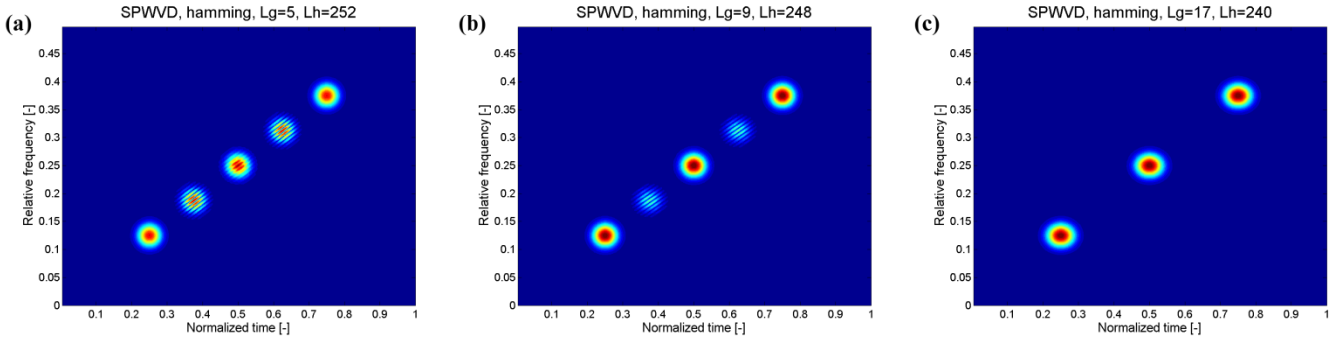


Figure 1. Illustration of time and frequency smoothing on three TF Gaussian atoms and their three interferences: more intensive filtering reduces the interferences and degrades the resolution. (a) The SPWVD of three TF Gaussian atoms,  $L_g=5$  and  $L_h=252$ ; (b) The SPWVD of the same signal, where the interferences are reduced,  $L_g=9$  and  $L_h=248$ ; (c) The SPWVD of the same signal: interference-free and with a worse resolution,  $L_g=17$  and  $L_h=240$ .

distinguish three general cases. Firstly, the autoterm is affected only a little in the beginning; however, with the growing window width, its amplitude decreases, and the resolution degrades; this process is shown in Fig. 2 (a). Secondly, the interference amplitude rapidly decreases to zero – the interferences are removed, see Fig. 2 (b). Lastly, if the interference covers the autoterm, smoothing will rapidly remove the interference, and the consecutive evolution matches that of the autoterm, see Fig. 2 (c).

The proposed method needs to calculate a number of SPWVDs with different filtrations. For a single TF point, our algorithm finds the minimal difference between two consecutive SPWVDs. Two SPWVDs with such minimal difference then constitute the ideal smoothing for a concrete TF point. The actual estimated value is calculated as the mean of the two values provided by the two SPWVDs for the concrete TF point. The described method is repeated for all TF points within the whole TF plane.

For the TF point corresponding to the autoterm, the first difference (almost nonsmoothed) should be found as the best estimation. While zero value should be chosen in the case of a cross term, the first value with sufficiently reduced interference is to be selected as the estimated one for the TF point containing the interference and autoterm. The disadvantage of the algorithm is the need of multiple calculations of the SPWVD; however, this part of the algorithm can be parallelized.

### III. VERIFICATION OF METHOD PERFORMANCE

#### A. Testing signals

For the TFD comparison, a set of simulated and real-life signals is often used, see Shafi et al. [5]. The simulated signal can be a number of TF Gaussian atoms (the number of atoms ranges between one and five), a set of linear chirps, parabolic or sinusoidal FM signals, or a combination of the above-mentioned components. The applied real-life signals are either a bat echolocation chirp or a whale echo, see Boashash et al. [1] or Shafi et al. [5].

The first chosen testing signal is the five TF Gaussian atoms. The signal forms 10 outer interferences in the WVD. The entire variety of outer cross terms is present, namely the time, frequency, and TF interferences. In addition, one atom in the center of the TF plane is masked by double interference, and the majority of existing algorithms have problems with autoterms overlaid by cross terms.

The second chosen testing signal is a brown bat echolocation chirp. The signal was selected because of its real life origin, wide use for performance comparison, and availability.

#### B. Compared algorithms

For comparison with other methods, we first selected the Gabor transform (GT) as the initial tool. The GT is a short-time Fourier transform with a Gaussian window; although the procedure provides only poor resolution, it completely eliminates interferences. The next choice, characterized

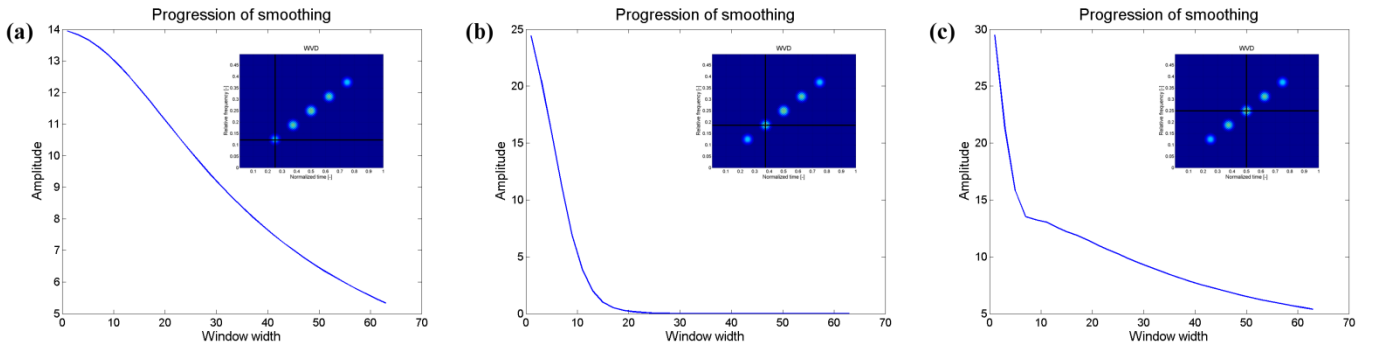


Figure 2. Evolution of TF points with respect to growing filtering in time and frequency. (a) Evolution of the TF point corresponding to the autoterm; (b) Evolution of the TF point corresponding to the interference; (c) Evolution of the TF point corresponding to the autoterm masked by the interference.

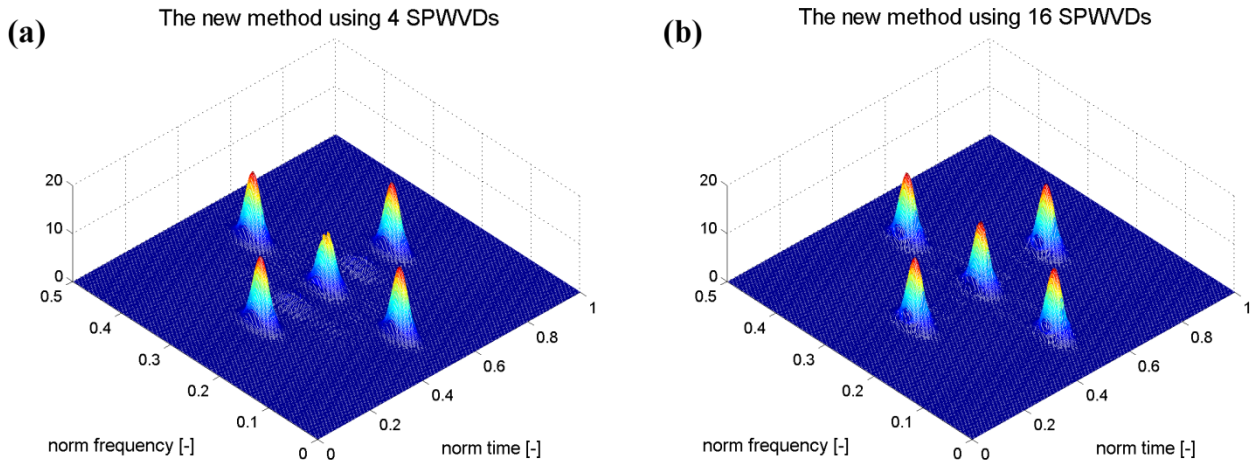


Figure 3. Comparison of results: (a) The proposed method based on 4 SPWVDs; the image shows degradation of the middle term, while the rest of the interferences are visible: (b) The proposed method based on 16 SPWVDs: no degradation of the center term, and complete removal of the cross terms.

by simplicity and good properties, was a combination of Gabor and the WVD: the Gabor-Wigner Transform (GWT), see Pei and Ding [7]. Even though the GWT has four different realizations, merely one of them is capable of reasonable processing of the signal masked by interference (as in the case of our first testing signal, namely the five TF atoms). The corresponding GWT equation is

$$GWT = \min\{|GT(t, \omega)|^2, |WVD(t, \omega)|\}. \quad (1)$$

Finally, two representatives of the RIDs were chosen: the cone-kernel representation, also named the Zhao-Atlas-Marks distribution (ZAMD) by Zhao et al. [8], and the Choi-Williams distribution (CWD) by Choi and Williams [9]. These two RIDs are often compared with the SPWVD, see Hlawatsch et al. [10]. Excepting the GWT, the reference algorithms were realized using the GNU “The Time-Frequency Toolbox”.

### C. Quantitative measures

For quantitative comparison, more methods can be used. Information theory offers the Shannon entropy; however, Flandrin et al. [11] recommends the use of the Rényi entropy. For our purpose, we applied the normalized Rényi entropy presented by Sang and Williams [12]. Subsequently, the ratio of norms (RN) by Jones and Parks [13] and the measure introduced by Stanković [14] were considered. All these techniques are commonly used to compare TFD algorithms. Better distribution minimizes the Rényi entropy and the Stanković measure. By contrast, the maximal RN value corresponds to favorable distribution. For the simulated case of the five TF atoms, we can calculate the ideal distribution result as the sum of the five resulting WVDs of the mono-component Gaussian atoms. It is therefore possible to use the mean squared error (MSE) as the measure to compare the result quality with the ideal WVD.

## IV. RESULTS

Even a small number of distributions is sufficient for significant reduction of the discussed interferences in the proposed method. In Fig. 3 (a), the result created on the basis of the four SPWVDs is affected by remains of the cross terms,

and the middle component is slightly degraded; with the growing number of used distributions, the estimation quality nevertheless increases, see Fig. 3 (b). The results of the quantitative measures applied to the two tested signals are shown in Tabs. 1 and 2.

Out of all the tested techniques, the proposed method (based on the four SPWVDs) provides the best performance for the bat echolocation chirp. For the five Gaussian atoms, the GWT produces slightly better results; however, as the masked autoterm in the GWT incorporates oscillations, the MSE in the proposed method performs better than the MSE applied in the GWT.

TABLE I. COMPARISON OF THE EXAMINED METHODS USING THE BAT ECHOLOCATION; THE PROPOSED METHOD IS BASED ON 4 SPWVDs.

Used TFD	Bat echolocation signal		
	Rényi	RN	Stanković
CWD	13.46	2.89	7.34
ZAMD	13.47	2.94	6.37
GT	14.90	0.73	8.99
GWT	12.98	3.09	3.73
Proposed	12.01	6.08	2.27

TABLE II. COMPARISON OF THE EXAMINED METHODS USING THE FIVE TF GAUSSIAN ATOMS; THE PROPOSED METHOD IS BASED ON 4 SPWVDs.

Used TFD	Five time-frequency Gaussian atoms			
	Rényi	RN	Stanković	MSE
CWD	12.97	3.31	3.54	0.29
ZAMD	12.96	3.93	3.92	0.42
GT	13.84	1.18	3.28	0.45
GWT	11.84	4.68	0.92	0.04
Proposed	11.89	4.56	1.05	0.02

In its basic form, the designed algorithm is parametric. The optimal choice of the window widths is generally a signal-dependent parameter.

We obtained good results in selecting the window widths depending on the number of the calculated SPWVDs ( $R$ ) and data length ( $N$ ). The actual choice is expressed as follows:

$$\forall i \in \{R, \dots, 1\} : F_{smooth} = \left\lfloor \frac{iN}{2R} \right\rfloor, \quad (2)$$

$$\forall i \in \{1, \dots, R\} : T_{smooth} = \left\lfloor \frac{iN}{8R} \right\rfloor. \quad (3)$$

In this form, the algorithm is nonparametric and exhibits satisfactory results. The performance on the real-life bat echolocation chirp is presented in Fig. 4 (a) and (b), and the result of the basic WVD is shown in Fig. 4 (b) for comparison.

## V. CONCLUSION

Even though a low number of SPWVDs is used, the proposed algorithm shows robust results with a potential for further enhancement based on a higher number of distributions. In contrast to iterative algorithms, the algorithm applied in this paper facilitates parallelization of its computationally most intensive section (SPWVD calculations). The proposed method showed the best result for both tested signals (the bat echolocation chirp and five time-frequency atoms) when compared to the GWT, ZAMD, and CWD. In particular, the result of the new method is notable for the case of the five time-frequency atoms, since many existing interference reduction methods have problems analyzing this type of signal. To conclude, the proposed method is the one closest to the ideal WVD from all the tested algorithms.

In the future, we plan to compare the above-outlined technique with iterative algorithms, mainly because the method offers the perspective of much faster computation and, simultaneously, a resolution crisp close to the result expected from the iterative algorithms. As a practical test, we plan to carry out noise measurement of ventilator starts and stops in a noised environment.

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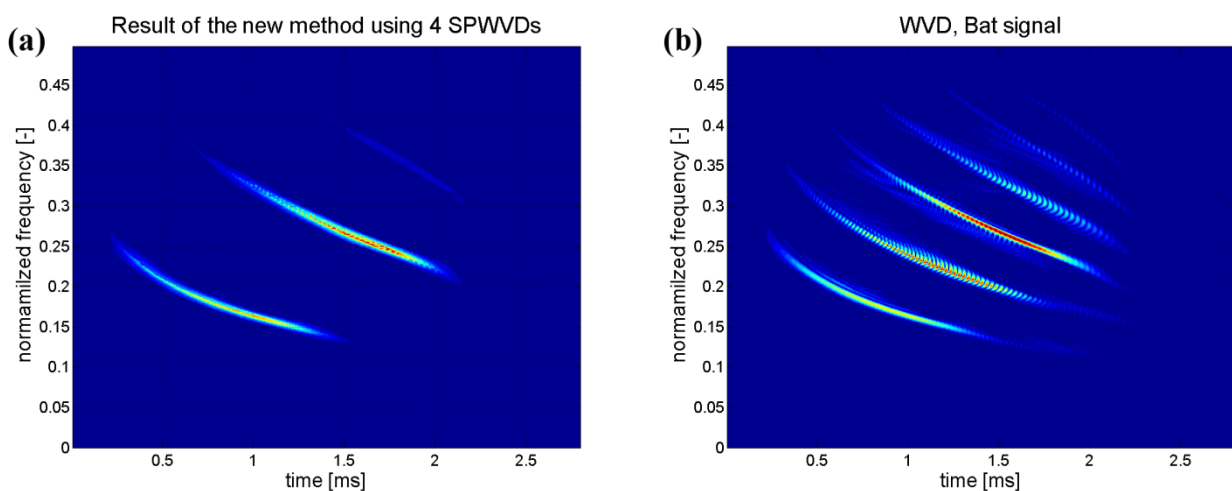


Figure 4. Comparison of the results: (a) Result of the proposed method based on Bat echo: cross term free and with maximally preserved resolution; (b) The Wigner-Ville distribution of the bat echo-location chirp (the result incorporates severe interferences).

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