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A design of PID controllers using FRIT-PSO

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Abstract—**This paper proposes the Fictitious Reference Iterative Tuning-Particle Swarm Optimization (PSO-FRIT) method to design PID controllers for control systems. The proposed method is an offline PID parameter tuning method and it is not necessary to derive any mathematical models of objected control systems. The proposed method is demonstrated by comparing with the FRIT method in numerical examples.**

Keywords-PID control; particle swarm optimization; fictitious reference iterative tuning; systems and control

I. INTRODUCTION

Recently it is strongly needed to improve productivity and cost-saving in some process industries, which are chemical process, oil process and steel process and so on. In these process control systems, PID controllers are embedded to achieve stability and some performances which are good responses for reference signals. The PID controller is described as the following form and only three parameters K_P, K_I, K_D are designed.

$$C(s) = K_p + \frac{K_I}{s} + sK_d. \qquad (1)$$

Because the performance of the control system is directly dependent to K_P, K_I, K_D, the PID parameter tuning is very important.

Traditionally PID controllers are designed based on dynamical models of the considered systems. In the paper [1-4], the models are described as simple transfer functions such as first-order systems with a time-delay. In the book [6] and the references, general transfer functions are considered and PID controllers can be designed based on the Bode plots in viewpoint from the loop shaping method. However usual process control systems are difficult to derive exact transfer functions of the considered systems. Thus data-driven PID tuning techniques have been focused on [7-8]. In the data-driven PID tuning method, mathematical models of the considered control systems are not necessary at all.

The data-driven PID tuning is an optimization problem. By using input and output data for process control systems, a given performance index which is dependent on PID parameters is optimized. One of data-driven PID tuning techniques is FRIT (Fictitious Reference Iterative Tuning) [8]. The advantage of FRIT is that PID tuning is possible based on a set of one-shot data and offline. However FRIT has a disadvantage such that local solution is easy to obtain after PID tuning because the nonlinear and non-convex optimization problem is considered.

In this article, FRIT and the optimization technique, that is particle swarm optimization (PSO) [9], is applied to PID tuning and the FRIT-PSO method is proposed. This paper shows that the disadvantage of FRIT is solved by using the proposed FRIT-PSO and FRIT-PSO achieves better performances than FRIT. This is because the proposed FRIT-PSO method can avoid to obtain local solution for nonlinear optimization problems.

II. OPTIMAZATION PROBLEM IN FRIT

A. A set of input and output data

The control system is shown in the Fig. 1. Here assume that the control object is described as $P(s)$ in Fig. 1 but the mathematical model is not known in advance or is not necessary. Since PID gains are tuning parameters, the PID controller $C(s)$ in the equation (1) is described as the following form.

$$C(\rho, s) = \rho_1 + \frac{\rho_2}{s} + s\rho_3,$$

$$\rho = [\rho_1 \quad \rho_2 \quad \rho_3]^T = [K_p \quad K_I \quad K_D]^T.$$

In Fig. 1, the reference signal is r, the control input is u and the output is y. A reference model is given as $T_d(s)$ and the error signal is e between y and $T_d(s)r$. It is assumed that a one-shot data set $\{u_0, y_0\}$ is given in advance by using an initial PID parameter ρ^0. The data u_0 is time series of the control input and y_0 is time series of the output by using ρ^0.

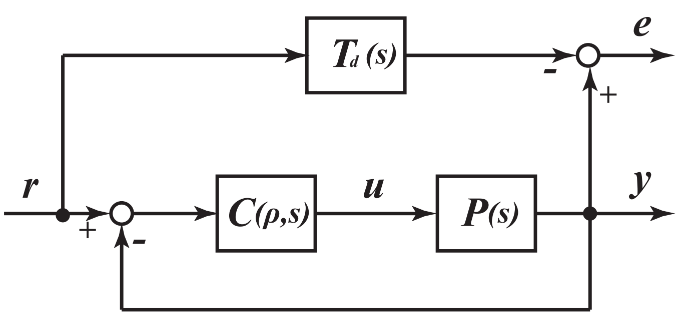


Figure 1. Feedback control systems and reference models

B. *Optimization problem in FRIT*

Since the reference model is given as $T_d(s)$ in Fig. 1, the error signal can be described as

$$e(\rho,s) = y - T_d(s)r,$$
$$= (T(\rho,s) - T_d(s))r,$$

where the transfer function $T(\rho,s)$ from r to y is given as the following equation.

$$T(\rho,s) = \frac{P(s)C(\rho,s)}{1+P(s)C(\rho,s)}.$$

Here note that $T(\rho,s)$ is not known because $P(s)$ is not known in advance. The main optimization problem is to find the following optimal parameter based on the data set $\{u_0, y_0\}$ only.

$$\rho^* = \arg\min_{\rho} J(\rho),$$
$$J(\rho) = \int_0^{\infty} e(\rho,t)^2 dt, \qquad (2)$$
$$e(\rho,t) = L^{-1}[e(\rho,s)].$$

On the FRIT, the following fictitious reference signal is introduced to solve the optimization problem in the equation (2).

$$\tilde{r}(\rho,s) = C(\rho,s)^{-1}u_0 + y_0.$$

Moreover a new error signal and a new performance index are defined as the following forms.

$$\tilde{e}(\rho,s) = y_0 - T_d(s)\tilde{r}(\rho,s),$$
$$\tilde{J}(\rho) = \int_0^{\infty} \tilde{e}(\rho,t)^2 dt, \qquad (3)$$
$$\tilde{e}(\rho,t) = L^{-1}[\tilde{e}(\rho,s)].$$

Then the main optimization problem is reduced to find

$$\rho^* = \arg\min_{\rho} \tilde{J}(\rho), \qquad (4)$$

instead of the problem in the equation (2).

C. *Gradient method in FRIT*

To compute the optimal solution ρ^* in the equation (4), the iterative method is given as the equation (5). This is same as the steepest descent method [10].

$$\rho^{i+1} = \rho^i - \alpha^i R(\rho^i)^{-1} \left.\frac{\partial \tilde{J}(\rho)}{\partial \rho}\right|_{\rho=\rho^i} \qquad (5)$$

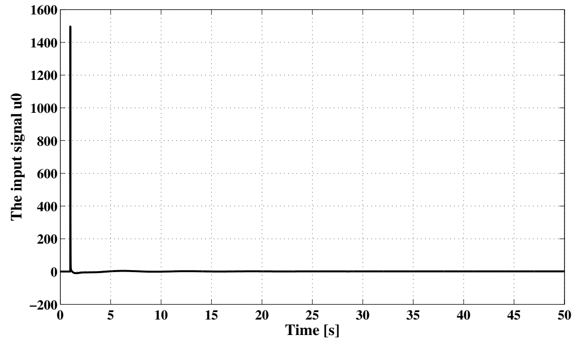
where $\alpha^i = diag(\gamma_1, \gamma_2, \gamma_3)$ for $\gamma_1 > 0, \gamma_2 > 0, \gamma_3 > 0$ and

$$R(\rho^i) = \left(\left.\frac{\partial \tilde{J}(\rho)}{\partial \rho}\right|_{\rho=\rho^i}\right)^T \left(\left.\frac{\partial \tilde{J}(\rho)}{\partial \rho}\right|_{\rho=\rho^i}\right), \qquad (6)$$

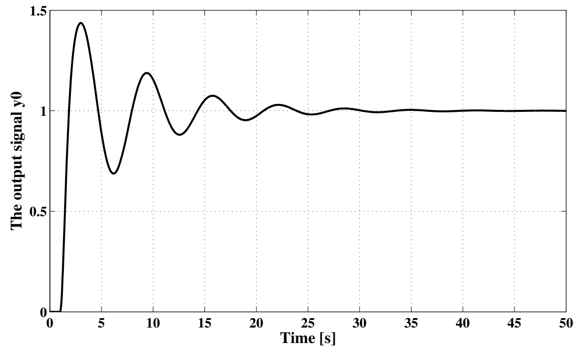
$$\left.\frac{\partial \tilde{J}(\rho)}{\partial \rho}\right|_{\rho=\rho^i} = 2\int_0^{\infty} \left.\frac{\partial \tilde{e}(\rho,t)}{\partial \rho}\right|_{\rho=\rho^i} \tilde{e}(\rho^i,t)dt, \qquad (7)$$

$$L\left[\left.\frac{\partial \tilde{e}(\rho,t)}{\partial \rho}\right|_{\rho=\rho^i}\right] = \left.\frac{\partial}{\partial \rho}(y_0 - T_d(s)\tilde{r}(\rho,s))\right|_{\rho=\rho^i},$$
$$= T_d(s)\frac{1}{C(\rho^i,s)^2}\left.\frac{\partial C(\rho,s)}{\partial \rho}\right|_{\rho=\rho^i} u_0. \qquad (8)$$

Here note that it is possible to compute the equation (6) offline because the equations (7) and (8) can be computed offline. Thus the iteration in the equation (5) can be computed offline and suboptimal solutions are obtained because the optimization problem in the equation (4) with (3) is nonlinear and non-convex.



(a) The input data $u_0(t)$



(b) The output data $y_0(t)$

Figure 2. A data set $\{u_0, y_0\}(\rho = [15 \quad 15 \quad 15]^T)$

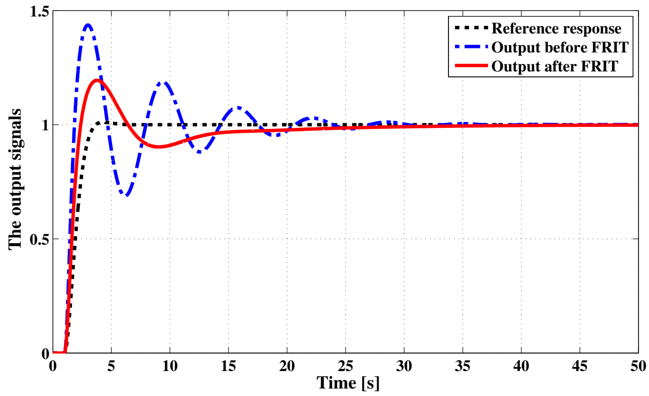


Figure 4. The output signals $\rho = [15 \quad 15 \quad 15]^T$

D. Motivated nnumerical examples

Now we consider the vehicle system considered in the paper [11] and assume that the input and output data $\{u_0, y_0\}$ is obtained in the figure 2 in advance. The PID parameters are given as $\rho = [15 \quad 15 \quad 15]^T$. Based on the paper [4], we consider the following reference model which is described as the 3-order system.

$$T_s(s) = \frac{1}{\frac{1}{256}s^3 + \frac{3}{8}s^2 + s + 1} \tag{9}$$

The maximum number of iteration of FRIT is 1000 and the gain a^i is given adequately in the equation (5). Then we obtain the following PID parameter based on FRIT.

$$\rho^* = [9.45 \quad 0.92 \quad 9.81]^T, \tilde{J}(\rho^*) = 0.017.$$

The figure 4 shows the output signals based on FRIT. The dotted line denotes the reference response $T_d(s)r$, the dashed line denotes the output signal before FRIT and the solid line denotes the output signal after FRIT. From the figure 4, we can calculate the value of the performance index in the equation (2) as

$$J(\rho^*) = 0.206.$$

We can see that the output signals are improved. However it seems that the PID tuning is not enough since the value of performance index is not well improved after the iteration is 100 (See the solid line in the figure 6).

Next the parameters are given as $\rho = [20 \quad 20 \quad 20]^T$. The figure 5 shows the output signals after FRIT. The case 1 means the output using $\rho = [15 \quad 15 \quad 15]^T$ and the case 2 means the output using $\rho = [20 \quad 20 \quad 20]^T$. In case 2, the PID parameters and the value of the performance index are obtained as

$$\rho^* = [12.39 \quad 0.99 \quad 9.35]^T, \tilde{J}(\rho^*) = 0.016.$$

The figure 6 shows the values of the performance index $\tilde{J}(\rho^i)$ in case 1 and case 2. The value in case 2 is smaller than one in case 1 however the output signal in case 2 does not become better. From the figure 5, the value of the performance index in the equation (2) as

$$J(\rho^*) = 0.268.$$

Thus the output signal in case 2 is not improved and high gains does not achieve a better performance.

From the above numerical examples, it seems that the optimal solution is difficult to obtain based on FRIT only and the suboptimal solutions are highly dependent to the initial PID parameters. Moreover the number of iteration is large because the values of performance indexes do not become small after the iteration is 100 in FRIT. To overcome this problem, we propose the FRIT-PSO approach for PID parameter tuning by utilizing the advantage of FRIT. The advantage is offline computation of PID parameters.

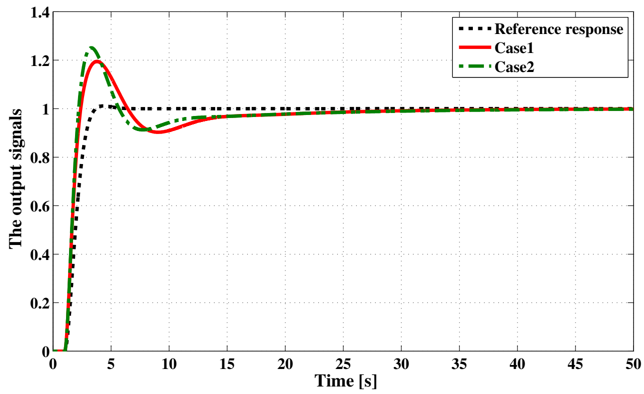


Figure 5. The output signals

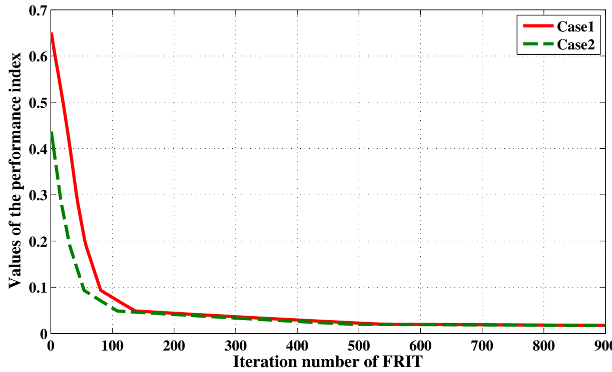


Figure 6. The values of the performance index $\tilde{J}(\rho^i)$ (Case 1 vs Case 2)

Remark 2. In this example, the following 4-order system is used to obtain the one-shot data set $\{u_0, y_0\}$.

$$P(s) = \frac{12s + 8}{20s^4 + 113s^3 + 147s^2 + 62s + 8}.$$

Because the reference model is given as the 3-order system in the equation (9), the optimization problem becomes nonlinear and non-convex. The above examples show that the suboptimal solutions exist. If the reference model is chosen as the higher order system, the optimization problem may become easy but it seems difficult to obtain the optimal solution by using FRIT.

III. PID TUNING BASED ON FRIT-PSO

In this section, the FRIT-PSO approach is proposed. Now we define the considered PID parameter tuning problem as the optimization problem again.

Assuming that the reference model is given as $T_d(s)$, the error signal is defined as the following equation in the same way of the previous section.

$$e(\rho, s) = y - T_d(s)r.$$

where the output signal is y and the reference signal is r in the figure 1.

Problem1. Find the following optimal parameter based on the one-shot data set $\{u_0, y_0\}$ only

$$\rho^* = \arg\min_{\rho} J(\rho),$$
$$J(\rho) = \int_0^{\infty} |e(\rho,t)|^2 dt,$$
$$e(\rho,t) = L^{-1}[e(\rho,s)].$$

A. Optimization and Algorithm of FRIT-PSO

Since the proposed approach is based on PSO, it assumed that the number of particles is n and each particle consists of PID gains. Each particle $\{\rho_1, \rho_2, \cdots, \rho_n\}$ is described as a 3-dimensional vector.

Step 0: Initialization

The initial PID parameter is given as $\rho^0 = [K_P \quad K_I \quad K_D]^T$ and a one-shot data set $\{u_0, y_0\}$ is obtained based on the figure 1. Here assume that the transfer function using ρ^0 is proper. The initial values of particles are randomly assigned as $\{\rho_1(k), \rho_2(k), \cdots, \rho_n(k)\}$ between a suitable range. The parameter k is the number of iteration for FRIT-PSO and the initial number is given as $k=1$.

Step 1: Optimization using FRIT

For each particle ($j = 1,2,\cdots,n$), the fictitious reference signals are defined as

$$\tilde{r}(\rho_j(k), s) = C(\rho_j(k), s)^{-1} u_0 + y_0. \quad (10)$$

The error signal and the performance index are also defined as the following forms. Here assume that the reference model is $T_d(s)$.

$$\tilde{e}(\rho_j(k), s) = y_0 - T_d(s)\tilde{r}(\rho_j(k), s),$$
$$\tilde{J}(\rho_j(k)) = \int_0^{\infty} |\tilde{e}(\rho_j(k), t)|^2 dt, \quad (11)$$
$$\tilde{e}(\rho_j(k), t) = L^{-1}[\tilde{e}(\rho_j(k), s)].$$

The following optimization problems are solved for each particle ($j = 1,2,\cdots,n$) based on the equation (5).

$$\rho_j^*(k) = \arg\min_{\rho_j(k)} \tilde{J}(\rho_j(k))$$

Step 2: Updating particles based on the PSO algorithm

Step 2-a: Updating the local best and the global best

The parameters $\rho_{jL}(k), \rho_g(k)$ are defined as follows,

$$\rho_{jL}(k) = \arg\min_{\substack{\rho_j^*(m) \\ m=1,2,\cdots,k}} \tilde{J}(\rho_j^*(m))$$
$$\rho_g(k) = \arg\min_{\substack{\rho_{jL}(m) \\ j=1,2,\cdots n \\ m=1,2,\cdots,k}} \tilde{J}(\rho_{jL}(m))$$

where the parameter $\rho_{jL}(k)$ is called as the local best of the jth particle and the parameter $\rho_g(k)$ is called as the global best.

Remark 2. If $\tilde{J}(\rho_j(m), T) < \tilde{J}(\rho_{jL}(k-1))$, then the local best $\rho_{jL}(k)$ is updated as $\rho_{jL}(k) = \rho_l(m)$. Otherwise the local best is not updated and the local best is kept as $\rho_{jL}(k) = \rho_{jL}(k-1)$. Moreover if $\tilde{J}(\rho_{jL}(m)) < \tilde{J}(\rho_g(k-1))$, then the global best $\rho_g(k)$ is updated as $\rho_g(k) = \rho_{lL}(m)$. Otherwise the global best is not updated and the global best is kept as $\rho_g(k) = \rho_g(k-1)$.

Step 2-b: Updating the vector and the position

The vector of the jth particle v_j is updated as follows,

$$v_j(k+1) = c_0 v_j(k) + c_1\psi_1\{\rho_{jL}(k) - \rho_j(k)\} + c_2\psi_2\{\rho_g(k) - \rho_j(k)\}.$$

The parameters ψ_1, ψ_2 are random numbers between 0 to 1. The parameters c_0, c_1, c_2 are the weighting factors for stability and performance of PSO. Since the setting of the parameters is based on the paper [12], the parameters which satisfy the equation (11) are used.

$$0 \le c_0 < 1, \ 0 < \frac{c_1 + c_2}{2} < 2c_0 + 2. \quad (12)$$

Then the position of the jth particle ρ_j is updated as follows,

$$\rho_j(k+1) = \rho_j(k) + v_j(k+1),$$

where the position is the PID gain. Thus PID tuning is done by the above equation.

Step 3: Iteration of the PSO algorithm

The iteration of Step 2 is repeated until the iteration number of PSO becomes $k=k_{\max}$. $k_{\max}$ is called as the maximum iteration number of FRIT-PSO in this paper.

For the proposed FRIT-PSO, the following theorems are satisfied.

Theorem 1. For each $\rho_j(k),\ j = 1,2,\cdots,n, k = 1,2,\cdots,k_{\max}$, $J(\rho_j(k)) = 0$ is satisfied, if and only if $\tilde{J}(\rho_j(k)) = 0$ is satisfied. Moreover the optimal solution is given as

$$\rho^* = \rho_g(k_{\max})$$

with the minimum performance index $\tilde{J}(\rho^*)$.

Proof: The fictitious reference signal in the equation (10) can be rewritten as

$$\tilde{r}(\rho_j(k), s) = F(\rho_j(k), s)r,$$
$$F(\rho_j(k), s) = \frac{C(\rho_j(k), s)^{-1} C(\rho^0, s)}{1 + C(\rho^0, s)P(\rho^0, s)} + T(\rho^0, s).$$

On the other hand, the following equation is satisfied.

$$T(\rho_j(k), s)\tilde{r}(\rho_j(k), s) =$$
$$\frac{P(s)C(\rho_j(k), s)}{1 + P(s)C(\rho_j(k), s)}\{C(\rho_j(k), s)^{-1} u_0 + y_0\} = y_0.$$

Then the error signal $\tilde{e}(\rho_j(k), s)$ in the equation (11) is derived as follows,

$$\tilde{e}(\rho_j(k), s) = T(\rho_j(k), s)\tilde{r}(\rho_j(k), s) - T_d(s)\tilde{r}(\rho_j(k), s),$$
$$= F(\rho_j(k), s)(T(\rho_j(k), s) - T_d(s))r,$$
$$= F(\rho_j(k), s)e(\rho_j(k), s). \quad (13)$$

Here note that $\tilde{e}(\rho_j(k), s) = 0$ is satisfied if $\tilde{J}(\rho_j(k)) = 0$ and $e(\rho_j(k), s) = 0$ is satisfied if $J(\rho_j(k)) = 0$. Thus the equation (13) means that $\tilde{J}(\rho_j(k)) = 0$ is a necessary and sufficient condition for $J(\rho_j(k)) = 0$ because $F(\rho_j(k), s)$ is proper. Moreover it is obvious that the optimal parameter ρ^* is given as the global best $\rho_g(k_{\max})$ after the FRIT-PSO algorithm.

Theorem 2. For each $\rho_j(k),\ j = 1,2,\cdots,n,\ k = 1,2,\cdots,k_{\max}$, there exists a positive scalar $\beta > 0$ which satisfies the following condition.

$$J(\rho_j(k)) \geq \beta\tilde{J}(\rho_j(k)). \quad (14)$$

Proof : From the equation (13), the following condition is satisfied.

$$\|\tilde{e}(\rho_j(k))\|_2 \leq \|F(\rho_j(k),s)\|_\infty \|e(\rho_j(k))\|_2 \quad (15)$$

where $\|x\|_2$ is a 2-norm of a signal $x(t)$ and $\|X(s)\|_\infty$ is an infinity-norm of a transfer function. Moreover because

$$\tilde{J}(\rho_j(k)) = \|\tilde{e}(\rho_j(k))\|_2^2,\ J(\rho_j(k)) = \|e(\rho_j(k))\|_2^2,$$

are satisfied, then the inequality (15) can be described as

$$\tilde{J}(\rho_j(k)) \leq \|F(\rho_j(k),s)\|_\infty^2 J(\rho_j(k)).$$

Thus the condition (14) is satisfied for the following positive scalar.

$$\beta = \frac{1}{\|F(\rho_j(k),s)\|_\infty^2} > 0.$$

B. A numerical exmaple of FRIT-PSO

Considering the result of Theorem 2, it is clear that the small value of the fictitious performance index is not equivalent to the small value of the real performance index. Thus the gain a^i is important in the equation (5) of the gauss-newton method. Moreover the number of iteration of FRIT $i_{\max}$ can be small because the important problem is how to set the suitable initial PID gains for FRIT. The proposed FRIT-PSO can solve this problem.

Here we demonstrate the proposed approach in comparison with the result shown in the figure 4. The same data set in the previous section is used. The parameters are summarized as follows,

- Data set : $\{u_0, y_0\}$ in the figure 2
- The reference model: the equation (9)
- The number of particles : $n = 50$
- The initial range of particles in FRIT-PSO: $1 < K_P < 5,\ 1 < K_I < 20,\ 1 < K_D < 20$.
  The advantage of the proposed FRIT-PSO method is to be able to select PID gains considering the forecasted information about the gains.
- Parameters of PSO : $c_0 = 0.3, c_1 = 0.5, c_2 = 0.5$ which satisfy the condition (12)
- The maximum number of iteration in FRIT: $i_{\max} = 10$
  This value is decided based on the figure 6.
- The maximum number of iteration in FRIT-PSO: $k_{\max} = 10$

By using the proposed FRIT-PSO, the following PID parameters are obtained.

$$\rho^* = \rho_g(10) = [4.71 \quad 1.20 \quad 18.43]^T$$
$$\tilde{J}(\rho^*) = 0.416$$

The output signals are shown in the figure 6. In this figure, the dotted line denotes the reference response, the dashed line denotes the output signal using FRIT and the solid line denotes the output signal using FRIT-PSO. Moreover the value of the performance index is calculated as

$$J(\rho^*) = 0.170$$

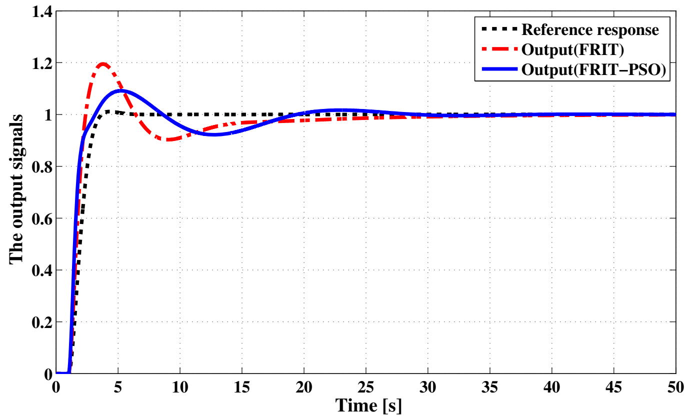


Figure 6. The output signals (FRIT vs FRIT-PSO)

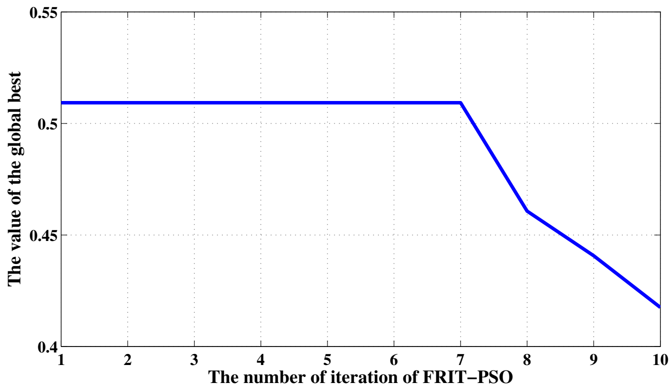


Figure 7. The value of the performance index in FRIT-PSO

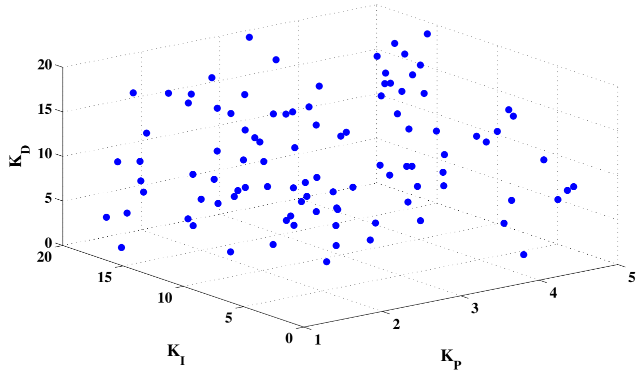


Figure 8. The particle positions before FRIT-PSO

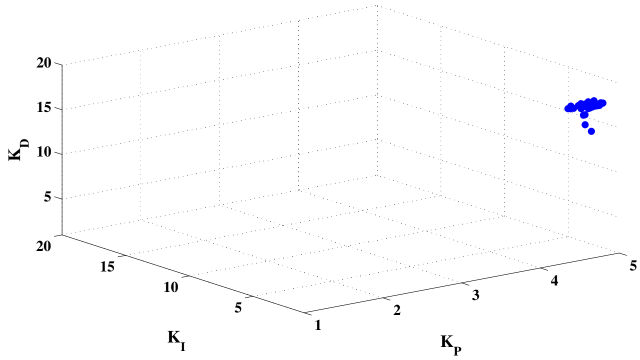


Figure 9. The particle positions after FRIT-PSO

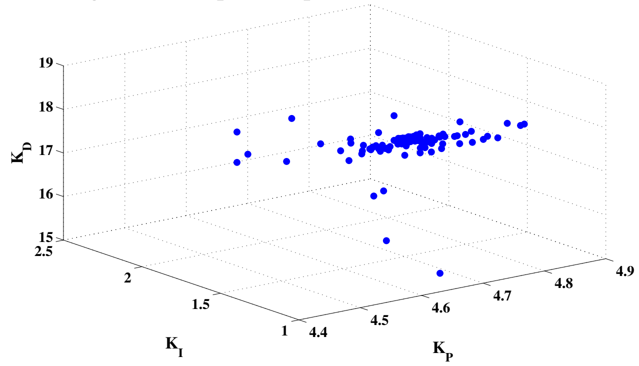


Figure 10. The particle positions after FRIT-PSO(closeup)

The figure 7 shows the value of the performance index for the global best. After the number of iteration $k=7$, the value decreases almost linearly. This means that FRIT-PSO finds a new global best and the updating of each particle performs well after $k=7$. The figure 8 , 9 and 10 show the particle position before and after FRIT-PSO. Before FRIT-PSO, each particle is distributed in a wide space. After FRIT-PSO, the global best is found out and each particle is concentrating at the global best. It is suggested that FRIT-PSO is a better PID tuning approach than FRIT and FRIT-PSO can avoid to go to local minimums. Moreover it is possible to design an optimal PID controller by using FRIT-PSO based on one-shot data without the mathematical model of the control object.

IV. Conclusions

The FRIT-PSO approach is proposed for PID tuning. Because the proposed approach is based on FRIT, offline PID tuning is possible. Moreover the proposed approach can avoid to obtain the local solution since the PSO method is also applied. The performance of the proposed FRIT-PSO is demonstrated by comparing with the FRIT method in the numerical examples.

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