On the collusive nature of managerial contracts based on comparative performance

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Abstract

We show that managerial delegation based upon comparative performance may generate collusive outcomes observationally equivalent to those typically associated with repeated games or cross ownership. This happens when rivals' profits are positively weighted in the managerial incentive scheme. We also identify the level of time discounting at which a repeated game based upon Nash reversion would achieve the same degree of collusion. Accordingly, such managerial contracts should attract the attention of antitrust authorities.

JEL Codes: L13, L41

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1 Introduction

Since the pioneering contributions by Vickers (1985) and Fershtman (1985), the literature on strategic delegation has been growing significantly and various types of managerial incentives have been put forward. We may roughly group such incentives into three types, depending on whether, in addition to its own profits, a firm's objective function includes also output (Vickers, 1985; Fershtman, 1985) or revenue (Fershtman and Judd, 1987; Sklivas, 1987), market share (Jansen *et al.*, 2007, 2009; Ritz, 2008) or the rival's profits (Salas Fumas, 1992; Lundgren, 1996, Aggarwal and Samwick, 1999; Miller and Pazgal, 2001).

There exists a strand of literature focussing on the emergence of implicit collusion among managerial firms in repeated games (Reitman, 1993; Spagnolo, 2000, 2005; Lambertini and Trombetta, 2002). Moreover, the possibility for cross-ownership to generate collusive outcomes has also been investigated in detail (Reynolds and Snapp, 1986; Malueg, 1992; Reitman, 1994; Gilo *et al.*, 2006).

What we want to illustrate is indeed that another route potentially replicating collusion is represented by the use of delegation contracts based on comparative performance as in Salas Fumas (1992), Lundgren (1996), Aggarwal and Samwick (1999) and Miller and Pazgal (2001). This is the case whenever the weight attached in managerial contracts to rival firms' profits is positive, which obtains at equilibrium under Bertrand competition in a market for substitute goods. The source of this result is the following. As we know from Miller and Pazgal (2001), the adoption of incentive based upon comparative performance yields a unique subgame perfect equilibrium irrespective of the specific market variables being set by managers. Moreover, the resulting profits are somewhere between those associated with the Bertrand

and Cournot equilibria played by entrepreneurial firms. Hence, moving from the pure Bertrand outcome to the managerialised one is equivalent to partially colluding in prices, while it is procompetitive if the departure point is Cournot. This partially collusive outcome also replicates that engendered by systematic cross-ownership by the same amount in the entire industry.

In the remainder of the paper, we briefly reconstruct the basic result in Miller and Pazgal (2001) and then, using the folk theorem based on grim trigger strategies (Friedman, 1971), illustrate the tacitly collusive supergame reproducing the same result. Finally, at the empirical level a few relevant facts highlighted by Aggarwal and Samwick (1999) are worth recollecting. According to their data, a positive weight is attached to rivals' profits also in industries where capacity constraints bite (which typically fall under the Cournot label), in contrast with the theoretical prediction. In summary, all of this should draw the antitrust agencies' attention to industries in which comparative performance is a key component of managerial incentives, as what follows shows that this could be a relatively simple way of implementing collusion without explicit agreements.

2 The model

Here we report the essential elements of Miller and Pazgal's (2001) duopoly model. Inverse and direct market demand functions can be appropriately specified as Singh and Vives (1984),

$$p_i = a - q_i - \sigma q_j \tag{1}$$

$$q_i = \frac{a}{1 + \sigma} - \frac{p_i}{1 - \sigma^2} + \frac{\sigma p_j}{1 - \sigma^2} \tag{2}$$

depending on whether Cournot or Bertrand competition is considered. As in Miller and Pazgal (2001), firms' marginal cost is the same and constant.

Therefore, without further loss of generality, it is set equal to zero. Hence, the profit function of firm i is $\pi_i = p_i q_i$, and the manager of firm i maximises

$$M_i = \pi_i + \theta_i \pi_j \tag{3}$$

The game has a three-stage structure, with owners and managers playing noncooperatively under imperfect, symmetric and complete information at each stage. In the first, owners decide whether to hire managers or not; in the second, any owner who has decided to separate control from ownership sets the delegation contract to maximise profits; in the third, market competition takes place, given the decisions taken at the two former stages. The solution concept is subgame perfection by backward induction.

2.1 The Cournot industry

Firms are quantity-setters, and the profit function of firm j writes

$$\pi_j = \left(a - q_j - \sigma q_i\right) q_j \tag{4}$$

so that the manager of firm i chooses output q_i to maximise

$$\widetilde{M_i} = \pi_i - \theta_i \sigma q_i q_j \tag{5}$$

which generates the following first order condition (FOC):

$$\frac{\partial M_i}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} + \theta_i \cdot \frac{\partial \pi_j}{\partial q_i} = 0 \tag{6}$$

which is equivalent to

$$\frac{\partial \widetilde{M_i}}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} - \theta_i \sigma q_j = 0 \tag{7}$$

Clearly, $\theta_i > 0$ (resp., $\theta_i < 0$) exerts an anticompetitive (procompetitive) effect. That is, in order for proper comparative performance to emerge at

4

equilibrium, θ_i *must be negative*. Otherwise, what is nominally a comparative performance evaluation mechanism is in fact a collusive one.

The resulting best reply function of firm i in the quantity space is

$$q_i^* (q_j) = \frac{a - \sigma q_j (1 + \theta_i)}{2} \tag{8}$$

telling that delegation modifies its slope

$$\frac{\partial q_i^* (q_j)}{\partial q_j} = -\frac{\sigma (1 + \theta_i)}{2} \tag{9}$$

Hence, comparative performance takes the form of a rotation of the reaction function.

Under unilateral delegation, the Cournot-Stackelberg outcome obtains with an optimal contract identified by

$$\theta^N (m, e) = -\frac{\sigma (2 - \sigma)}{4 - \sigma (2 + \sigma)} < 0 \tag{10}$$

and the corresponding firms' equilibrium profits are

$$\pi^{CN} (m, e) = \frac{a^2 (2 - \sigma)^2}{8 (2 - \sigma^2)}; \pi^{CN} (e, m) = \frac{a^2 [4 - \sigma (2 + \sigma)]^2}{16 (2 - \sigma^2)^2} \tag{11}$$

which indeed correspond to the Cournot-Stackelberg profits. If $\sigma = 1$, then $\theta^N (m, e) = -1$. That is, under product homogeneity unilateral delegation illustrates the emergence of pure comparative performance evaluation at the asymmetric equilibrium.

If both firms delegate, the subgame perfect contract is identified by

$$\theta^N (m, m) = -\frac{\sigma}{2 + \sigma} < 0 \tag{12}$$

and per-firm equilibrium output and profits are

$$q^{CN} (m, m) = \frac{a (2 + \sigma)}{4 (1 + \sigma)}$$
$$\pi^{CN} (m, m) = \frac{a^2 (4 - \sigma^2)}{16 (1 + \sigma)} \tag{13}$$

5

As we know from Singh and Vives (1984), the Cournot-Nash profits of the game played by entrepreneurial firms are $\pi^{CN}(e,e) = a^2/(2+\sigma)^2$. Hence, the first stage of the game looks as in Matrix 1.

$$j$$

		m	e
i	m	$\frac{a^2\left(4-\sigma^2\right)}{16(1+\sigma)}$; $\frac{a^2\left(4-\sigma^2\right)}{16(1+\sigma)}$	$\frac{a^2(2-\sigma)^2}{8(2-\sigma^2)}$; $\frac{a^2[4-\sigma(2+\sigma)]^2}{16(2-\sigma^2)^2}$
	e	$\frac{a^2[4-\sigma(2+\sigma)]^2}{16(2-\sigma^2)^2}$; $\frac{a^2(2-\sigma)^2}{8(2-\sigma^2)}$	$\frac{a^2}{(2+\sigma)^2}$; $\frac{a^2}{(2+\sigma)^2}$

Matrix 1: Cournot competition

The inspection of Matrix 1 implies that strategy m is dominant, and therefore (m,m) is the unique equilibrium in pure strategies. Additionally, Matrix 1 reproduces the prisoners' dilemma structure, since $\pi^{CN}(m,m) < \pi^{CN}(e,e)$ for all $\sigma \in (0,1]$.

2.2   The Bertrand industry

Assume now that firms choose prices, the relevant individual demand function being (2). In the asymmetric case in which firm i is managerial and firm j is entrepreneurial, equilibrium prices at the third stage are

$$p_i^{BN} = \frac{a(1-\sigma)[2+\sigma(1+\theta_i)]}{4-\sigma^2(1+\theta_i)} \; ; \; p_j^{BN} = \frac{a(1-\sigma)(2+\sigma)}{4-\sigma^2(1+\theta_i)} \tag{14}$$

The optimal contract chosen by the owner of firm i is

$$\theta_i^N = \frac{\sigma(2+\sigma)}{4+\sigma(2-\sigma)} > 0 \tag{15}$$

and the resulting profits are

$$\pi^{BN}(m,e) = \frac{a^2(1-\sigma)(2+\sigma)^2}{8(1+\sigma)(2-\sigma^2)} ; \pi^{BN}(e,m) = \frac{a^2(1-\sigma)[\sigma-\sigma(2-\sigma)]^2}{16(1+\sigma)(2-\sigma^2)^2} \tag{16}$$

When both firms are managerial, one can solve for optimal prices and then find out the optimal symmetric delegation contract at the second stage:

$$\theta^N (m, m) = \frac{\sigma}{2 - \sigma} > 0 \tag{17}$$

which reveals that, under price competition, the optimal contract has an anticompetitive flavour. As established by Miller and Pazgal (2001), the equilibrium outcome of this case is observationally equivalent to that generated, all else equal, under Cournot competition:

$$q^{BN} (m, m) = q^{CN} (m, m) = \frac{a (2 + \sigma)}{4 (1 + \sigma)}$$
$$\pi^{BN} (m, m) = \pi^{CN} (m, m) = \frac{a^2 (4 - \sigma^2)}{16 (1 + \sigma)} \tag{18}$$

The equilibrium level of the market price is

$$p^{BN} (m, m) = \frac{a (2 - \sigma)}{4} \tag{19}$$

The remaining case in which both firms are entrepreneurial yields the following equilibrium profits:

$$\pi^{BN} (e, e) = \frac{a^2 (1 - \sigma)}{(1 + \sigma) (2 - \sigma)^2} \tag{20}$$

A few relevant remarks are in order:

- Under Bertrand behaviour, the first stage of the game (see Matrix 2) produces (m, m) as the unique equilibrium, and the upstream stage is not a prisoners' dilemma, as $\pi^{BN} (e, e) < \pi^{BN} (m, m)$.

$$j$$

		m	e
i	m	$\frac{a^2 \left(4 - \sigma^2\right)}{16(1+\sigma)}$; $\frac{a^2 \left(4 - \sigma^2\right)}{16(1+\sigma)}$	$\frac{a^2(1-\sigma)(2+\sigma)^2}{8(1+\sigma)(2-\sigma^2)}$; $\frac{a^2(1-\sigma)[\sigma-\sigma(2-\sigma)]^2}{16(1+\sigma)(2-\sigma^2)^2}$
	e	$\frac{a^2(1-\sigma)[\sigma-\sigma(2-\sigma)]^2}{16(1+\sigma)(2-\sigma^2)^2}$; $\frac{a^2(1-\sigma)(2+\sigma)^2}{8(1+\sigma)(2-\sigma^2)}$	$\frac{a^2(1-\sigma)}{(1+\sigma)(2-\sigma)^2}$; $\frac{a^2(1-\sigma)}{(1+\sigma)(2-\sigma)^2}$

Matrix 2: Bertrand competition

- The fact that delegation turns out to be a collusive instrument also arises from the delegation schemes used by Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987), as well as in vertical relations based on two-part tariffs as in Bonanno and Vickers (1988).

- However, while in all of the aforementioned literature all profits become nil at $\sigma = 1$, in the Miller and Pazgal (2001) model this does not happen, which means that the anticompetitive effect of comparative performance-based managerial incentives under Bertrand competition is stronger than that associated with output or revenues and survives to product homogeneity.

3 The mixed case

The scenario in which firm i is a quantity-setter and firm j is a price-setter can be quickly dealt with. The Nash equilibrium profits accruing to entrepreneurial firms are

$$\pi^{qN}(e,e) = \frac{a^2 (2-\sigma)^2 (1-\sigma^2)}{(4-3\sigma^2)^2} \; ; \; \pi^{pN}(e,e) = \frac{a^2 [2-\sigma(1-\sigma)]^2}{(4-3\sigma^2)^2} \qquad (21)$$

In both cases, at the subgame perfect equilibrium firms delegate control to managers, as we know from Miller and Pazgal (2001). Comparing $\pi^{qN}(e,e)$ and $\pi^{pN}(e,e)$ with $\pi^{BN}(m,m) = \pi^{CN}(m,m)$, one finds $\pi^{pN}(e,e) > \pi^{BN}(m,m) = \pi^{CN}(m,m)$ everywhere, whereas $\pi^{qN}(e,e) \gtreqless \pi^{BN}(m,m) = \pi^{CN}(m,m)$ for all $\sigma \lesseqgtr 0.838$. This implies that the first stage of the game is not a prisoners' dilemma for either firm as soon as product substitutability is high enough.

4 Ranking and assessment of equilibria

Leaving aside the mixed case, we may look at the positions of the relevant equilibria in the profit space, as in Figure 1, where Π is monopoly profit and points B and C identify the Bertrand and Cournot equilibria without managers, while point M locates the equilibrium reached when both firms delegate control to managers. This illustrates intuitively that moving from C to M makes the industry more competitive, while moving from B to M looks collusive or replicates the outcome of cross-ownership.
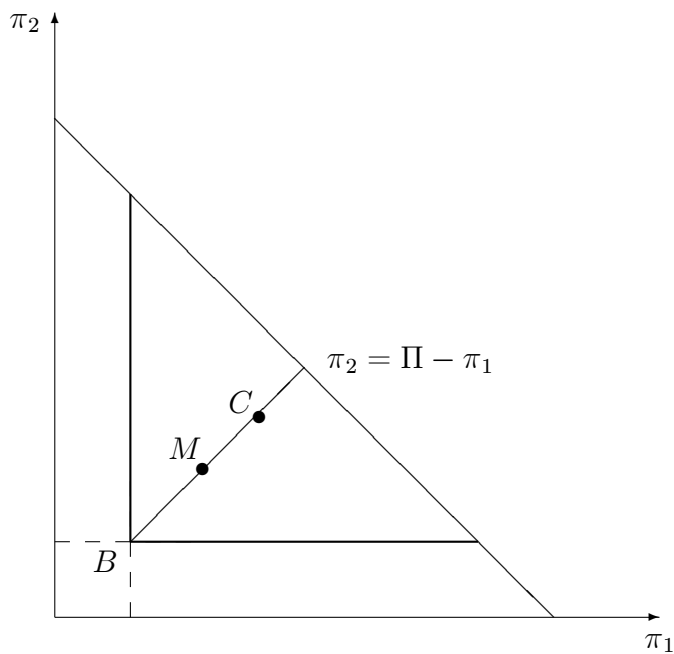

Figure 1 The frontier of industry profits and equilibria

Therefore, managerialisation under Bertrand competition is a shortcut to implement an equilibrium that, otherwise, would be reachable in a supergame between entrepreneurial firms, provided a very specific relationship between time preferences and product differentiation holds. To see this, it suffices

9

to figure out the supergame in prices governed by Friedman's (1971) grim trigger strategies.

We are about to identify the critical level of the discount factor δ such that, if firms have exactly that discount factor in mind, the supergame ruled by the infinite Nash reversion after any deviation from the collusive path enables them to get exactly the same per-period profits as those associated to the Bertrand equilibrium under bilateral managerialisation.

The generic individual collusive profits are $\pi^c = p^c (a - p^c) / (1 + \sigma)$. If firm i sticks to the collusive price p^c while the other deviates, this can happen in two ways.[1] The first is a deviation that brings the cheated firm's quantity to zero:

$$q_i \left(p^c, p^d \right) = 0 \Leftrightarrow p_0^d = \frac{p^c - a \left(1 - \sigma \right)}{\sigma} \tag{22}$$

yielding

$$\pi_0^d = \frac{\left(a - p^c \right) \left[p^c - a \left(1 - \sigma \right) \right]}{\sigma^2} \tag{23}$$

This holds for low levels of product differentiation. The second type of deviation, which becomes relevant provided differentiation is high enough, takes place along the cheating firm's best reply, at the following price:

$$p_{br}^d = \frac{a \left(1 - \sigma \right) + \sigma p^c}{2} \tag{24}$$

which yields

$$\pi_{br}^d = \frac{\left[a \left(1 - \sigma \right) + \sigma p^c \right]^2}{4 \left(1 - \sigma^2 \right)} \tag{25}$$

The stability condition is

$$\frac{\pi^c}{1 - \delta} \geq \pi_k^d + \frac{\delta \pi^{BN} \left(e, e \right)}{1 - \delta} \, ; \, k = 0, br \tag{26}$$

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[1] The baerings of product differentiation on the optimal deviation from collusive pricing has been extensively debated. See Deneckere (1983); Majerus (1988); Ross (1992); Lambertini (1997); Albæk and Lambertini (1998, 2004), *inter alia*.

Henceforth, we pose $p^c = p^{BN}(m, m) = p^{CN}(m, m)$ and solve the two versions of (26) w.r.t. δ.

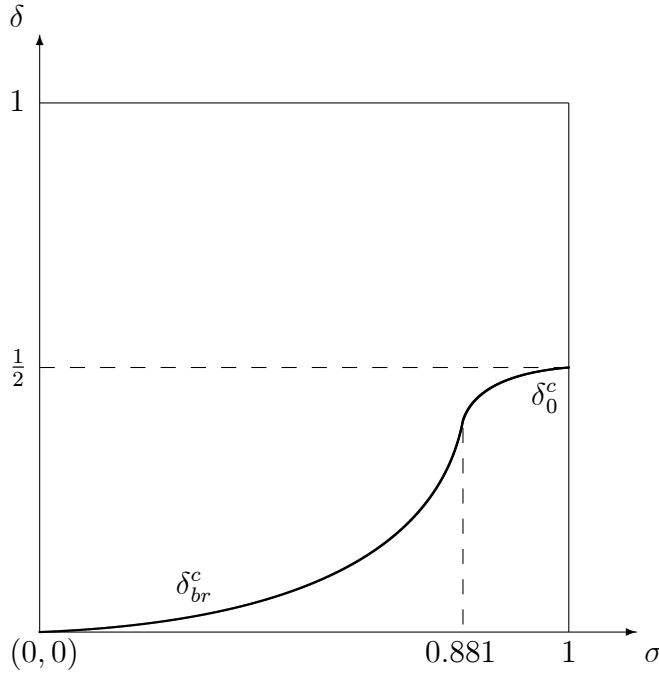If $k = 0$, (26) delivers the following critical threshold of the discount factor:

$$\delta_0^c = \frac{(2 - \sigma)^2 \left[4 - 3\sigma^2 (1 + \sigma) - \sigma^4\right]}{16(1 - \sigma) + \sigma^2 \left[\sigma^2 (5 - 3\sigma) - 8\right]} \tag{27}$$

while if $k = br$, we have

$$\delta_{br}^c = \frac{\sigma (2 - \delta)^2}{16(1 - \sigma) + \sigma^3} \tag{28}$$

Since, for $p^c = p^{BN}(m, m) = p^{CN}(m, m)$, $p_{br}^d = p_0^d$ at $\sigma = 0.881$, it turns out that also $\delta_{br}^c = \delta_0^c$ at the same level of product substitutability. The resulting picture appears in Figure 2.

Figure 2 The critical theshold of δ

The inspection of Figure 2 tells that, for all δ at least as high as the envelope of δ_{br}^c and δ_0^c, entrepreneurial Bertrand firms could stabilise partial collusion in correspondence of profits at least as large as $\pi^{BN}(m,m) = \pi^{CN}(m,m)$. If this is not the case, they may use strategic delegation based upon this peculiar form of comparative performance evaluation to locate themselves along the envelope, for any given value of σ.

Aggarwal and Samwick (1999) use the same theoretical model as in Miller and Pazgal (2001) in both Bertrand and Cournot settings,[2] and the resulting predictions are then tested using a large data set covering a wide range of different industries. While the model predicts a negative (positive) relation between a manager's remuneration and the rival firms' profits under Cournot (Bertrand) competition, the data seem to suggest persistently a positive link, disconfirming the presence of comparative performance-based contracts, independently of the specific nature of the industry. For instance, Aggarwal and Samwick (1999, p. 2002) find "... that both own- and rival-firm pay-performance sensitivities are positive for total compensation". If this applies fairly systematically in industries ranging from food and tobacco (resembling Bertrand) to chemicals, petroleum and machinery (Cournot), their findings seem to suggest that the firms involved use delegation in a pro-collusive way even when this is not literally driven by Bertrand competition as the theoretical model would predict. Hence, any empirical evidence of this kind emerging in industries where capacity constraints matter, should attract the attention of antitrust agencies.

It is worth stressing two aspects: the first is that this class of models requires managerial contracts to be observable (see, e.g., Fershtman *et al.*, 1991); the second is that they usually are, in view of the rules imposed to

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[2]But their calculations are incorrect as they do not invert the demand system and just replace prices with quantities without manipulating parametric coefficients.

large corporations. Hence, the information about the structure of managerial incentives would reveal the intention to mimic collusion or cross ownership if a positive weight is assigned to rivals' profits. Moreover, in a Cournot industry, managers should react by expanding capacity to increase output levels. In Bertrand industries, they would instead either leave some capacity idle or dismantle portions of installed capacity. Hence, observing variations in installed capacity would be in itself suggestive of the presence of a possible problem connected with anti-competitive practices.

References

[1] Aggarwal, R.K. and A.A. Samwick (1999), "Executive Compensation, Strategic Competition, and Relative Performance Evaluation: Theory and Evidence", *Journal of Finance*, **54**, 1999-2043.

[2] Albæk, S., and L. Lambertini (1998), "Collusion in Differentiated Duopolies Revisited", *Economics Letters*, **59**, 305-8.

[3] Albæk, S., and L. Lambertini (2004), "Price vs Quantity in Duopoly Supergames with Close Substitutes", *Annals of Regional Science*, **38**, 567-77.

[4] Bonanno, G. and J. Vickers (1988), "Vertical Separation", *Journal of Industrial Economics*, **36**, 257-65.

[5] Deneckere, R. (1983), "Duopoly Supergames with Product Differentiation", *Economics Letters*, **11**, 37-42.

[6] Fershtman, C. (1985), "Managerial Incentives as a Strategic Variable in Duopolistic Environment", *International Journal of Industrial Organization*, **3**, 245-53.

[7] Fershtman, C. and K. Judd (1987), "Equilibrium Incentives in Oligopoly", *American Economic Review*, **77**, 927-40.

[8] Fershtman, C., K. Judd and E. Kalai (1991), "Observable Contracts: Strategic Delegation and Cooperation", *International Economic Review*, **32**, 551-59.

[9] Friedman, J.W. (1971), "A Noncooperative Equilibrium for Supergames", *Review of Economic Studies*, **38**, 1-12.

[10] Gilo, D., Y. Moshe and Y. Spiegel (2006), "Partial Cross Ownership and Tacit Collusion", *RAND Journal of Economics*, **37**, 81-99.

[11] Jansen, T., A. van Lier and A. van Witteloostuijn (2007), "A Note on Strategic Delegation: The Market Share Case", *International Journal of Industrial Organization*, **25**, 531-39.

[12] Jansen, T., A. van Lier and A. van Witteloostuijn (2009), "On the Impact of Managerial Bonus Systems on Firm Profit and Market Competition: The Cases of Pure Profit, Sales, Market Share and Relative Profits Compared", *Managerial and Decision Economics*, **30**, 141-53.

[13] Lambertini, L. (1997), "Prisoners' Dilemma in Duopoly (Super)Games", *Journal of Economic Theory*, **77**, 181-91.

[14] Lambertini, L. and M. Trombetta (2002), "Delegation and Firms' Ability to Collude", *Journal of Economic Behavior and Organization*, **47**, 359-73.

[15] Lundgren, C. (1996), "Using Relative Profit Incentives to Prevent Collusion", *Review of Industrial Organization*, **11**, 533-50.

[16] Malueg, D. (1992), "Collusive Behavior and Partial Ownership of Rivals", *International Journal of Industrial Organization*, **10**, 27-34.

[17] Majerus, D. (1988), "Price vs Quantity Competition in Oligopoly Supergames", *Economics Letters*, **27**, 293-7.

[18] Miller, N.H. and A.I. Pazgal (2001), "The Equivalence of Price and Quantity Competition with Delegation", *RAND Journal of Economics*, **32**, 284-301.

[19] Reitman, D. (1993), "Stock Options, and the Strategic Use of Managerial Incentives", *American Economic Review*, **83**, 513-24.

[20] Reitman, D. (1994), "Partial Ownership Arrangements and the Potential for Collusion", *Journal of Industrial Economics*, **42**, 313-22.

[21] Reynolds, R. and B. Snapp (1986), "The Competitive Effects of Partial Equity Interests and Joint Ventures", *International Journal of Industrial Organization*, **4**, 141-53.

[22] Ritz, R.A. (2008), "Strategic Incentives for Market Share", *International Journal of Industrial Organization*, **26**, 586-97.

[23] Ross, T.W. (1992), "Cartel Stability and Product Differentiation", *International Journal of Industrial Organization*, **10**, 1-13.

[24] Salas Fumas, V. (1992), "Relative Performance Evaluation of Management: The Effects on Industrial Competition and Risk Sharing", *International Journal of Industrial Organization*, **10**, 473-89.

[25] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, **15**, 546-54.

[26] Sklivas, S.D. (1987), "The Strategic Choice of Managerial Incentives", *RAND Journal of Economics*, **18**, 452-58.

[27] Spagnolo, G. (2000), "Stock-Related Compensation and Product-Market Competition", *RAND Journal of Economics*, **31**, 22-42.

[28] Spagnolo, G. (2005), "Managerial Incentives and Collusive Behavior", *European Economic Review*, **49**, 1501-23.

[29] Vickers, J. (1985), "Delegation and the Theory of the Firm", *Economic Journal*, **95** (Conference Papers), 138-47.