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Loop quantum effect and the fate of tachyon field collapse

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Abstract. We study the fate of gravitational collapse of a tachyon field matter. In presence of an inverse square potential a black hole forms. Loop quantum corrections lead to the avoidance of classical singularities, which is followed by an outward flux of energy.

1. Gravitational collapse of a tachyon matter

We discuss the spacetime region inside the collapsing sphere which will contain the chosen matter content. In this region, an homogeneous and isotropic Friedmann- Robertson-Walker (FRW) metric will be considered as the setting for the gravitational collapse in comoving coordinates. However, in order to study the whole spacetime, we must match this interior spacetime to a suitable exterior region. In the model herein, it is convenient to consider a spherically symmetric and inhomogeneous spacetime such as the Schwarzschild or the generalized Vaidya geometries to model the spacetime outside the collapsing sphere (cf. see [1] for a discussion on the whole spacetime structure including the exterior geometry). Let us nevertheless be more precise for the benefit of the reader: There is a particular setting which has been of recent use, namely in investigations of loop effects in gravitational collapse involving a scalar field; it is the marginally bound case ($k = 0$), where the interior spacetime is dynamical and is parameterized by a line element as follows,

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2], \tag{1}$$

where t is the proper time for a falling observer whose geodesic trajectories are labeled by the comoving radial coordinate r, and $d\Omega^2$ being the standard line element on the unit two- sphere; we have set the units $8\pi G = c = 1$. The dynamical evolution from Einstein's field equations for this marginal bound case regarding the interior region, can be presented as [2]

$$\rho = \frac{F_{,r}}{R^2 R_{,r}}, \quad p = -\frac{\dot{F}}{R^2 \dot{R}}, \quad \dot{R}^2 = \frac{F}{R}, \tag{2}$$

where t and r are the proper time and the comoving radial coordinate for a falling observer, respectively; we have set the units $8\pi G = c = 1$. The quantity $F(R)$ is the mass function with $R(t,r) = ra(t)$ being the area radius of the collapsing shell. Furthermore, ρ and p are energy

density and pressure of the collapsing matter, respectively. The notation ",r" and "$\cdot$" denote a derivative with respect to r and t, respectively.

Integration of Eq. (2) gives the following relation for the mass function as $F = \frac{1}{3}\rho(t)R^3$. A spherically symmetric spacetime containing a trapped region is characterized by the corresponding boundary (trapped) surface, extracted from the equation $F = R$. In the region where the mass function satisfies the relation $F > R$, it describes a trapped region and the regions in which the mass function is less than the area radius i.e., $F < R$, are not trapped.

To investigate the gravitational collapse, we will consider herein a homogeneous tachyon field $\phi(t)$, as matter content of the collapsing cloud. Employing A. Sen's effective action, the corresponding energy density and pressure for tachyon field are given by [3]

$$\rho(\phi) = 3H^2 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \qquad p(\phi) = -V(\phi)\sqrt{1 - \dot{\phi}^2}, \tag{3}$$

where $V(\phi)$ is the tachyon potential and is assumed to be of the form $V(\phi) = V_0\phi^{-2}$. Using the equation for the energy conservation, we get the field equation for ϕ as

$$\ddot{\phi} = -(1 - \dot{\phi}^2)\left[3H\dot{\phi} + \frac{V_{,\phi}}{V}\right], \tag{4}$$

where ",ϕ" denotes the derivative with respect to the tachyon field and $H = \dot{a}/a$ is the Hubble rate which for collapsing scenario is $H < 0$. Using Eq. (3) the equation of state becomes $w = p(t)/\rho(t) = \dot{\phi}^2 - 1$.

Let us study the collapse of a tachyon field from a dynamical system perspective. We define a new time variable τ (instead of the proper time t) for our collapsing system as $\tau \equiv -\log(a/a_0)^3$, which is defined in the interval $0 < \tau < \infty$. For any time dependent function $f = f(t)$ we can write $df/d\tau \equiv -\dot{f}/3H$, where a dot over a variable denotes its derivative with respect to time t.

Introducing the new set of dynamical variables $x \equiv \dot{\phi}$ and $y \equiv V/3H^2$, the Friedmann constraint equation in (3) becomes $y/\sqrt{1 - x^2} = 1$. From Eqs. (3) and (4) the corresponding autonomous system then reads

$$\frac{dx}{d\tau} = \left(\frac{\lambda}{\sqrt{3}}\sqrt{y} + x\right)(1 - x^2), \qquad \frac{dy}{d\tau} = -xy\left(\frac{\lambda}{\sqrt{3}}\sqrt{y} + x\right), \tag{5}$$

where $\lambda \equiv -V_{,\phi}/V^{3/2} = 2/\sqrt{V_0} = constant$. The critical points (x_c, y_c) of the autonomous system can be obtained easily as: (a) at $(1, 0)$, (b) at $(-1, 0)$, and (c) at $(-\lambda\sqrt{y_0/3}, y_0)$, where $y_0 = -\lambda^2/6 + \sqrt{(\lambda^2/6)^2 + 1} > 0$. From the eigenvalues of the Jacobi matrix, the stability of the system can be determined;

- Fixed points (a) and (b) are asymptotically *stable* nodes (attractor) allowing to select a class of solutions in which, asymptotic time dependence of the tachyon field can be estimated as $\phi(t) \simeq \pm t + \phi_0$, where '+' and '−' denotes fixed points (a) and (b), respectively. From Eq. (3) it is seen that, the effective pressure asymptotically vanishes. This suggests a dust-like behavior where $p \simeq 0$ [4]. Then, the energy density of collapse can be obtained as $\rho = \rho_\phi \propto 1/a^3$; this expression indicates, therefore that, ρ_ϕ diverges as $a \to 0$.

- The critical point (c) is asymptotically *unstable* (source); it can not be considered as a physically relevant solution for the collapse, and hence we ignore its studying here.

The occurrence of spacetime singularities is proved by the existence of incomplete future or past directed non-spacelike geodesics. Then, it is required that the Kretschmann invariant $K = R_{abcd}R^{abcd} = 12[(\ddot{a}/a)^2 + (\dot{a}/a)^4]$ gets unboundedly large values as the singularity is approached along the non-spacelike geodesics terminating there. If such a condition is satisfied,

then the physical system should be considered to exhibit a curvature singularity [2]. Therein one can observe that, this quantity diverges as the physical area radius vanishes, consequently signaling the occurrence of a curvature singularity; the collapse ends up with a black hole formation [1].

2. Semiclassical tachyon field collapse

In this section we will study the loop quantum effects (of the inverse triad type) on tachyon field collapse. To investigate the fate of the collapse we will consider the modified dynamical equation of the system in the semiclassical limit. The modified Friedmann and the Raychaudhury equations for the tachyon field in the semiclassical regime $a_i < a \ll a_*$ are given, respectively by [3]

$$\rho_\phi^{loop} = 3H^2 = \frac{V(\phi)}{\sqrt{1 - A^{-1}q^{-15}\dot{\phi}^2}}, \qquad p_\phi^{loop} = 2\dot{H} + 3H^2 = -\left(1 + 4\frac{\dot{\phi}^2}{Aq^{15}}\right)\rho_\phi^{loop}. \qquad (6)$$

Then, using the energy conservation equation, the modified equation of motion in the semiclassical limit becomes

$$\ddot{\phi} - 12H\dot{\phi}\left(\frac{7}{2} - \frac{\dot{\phi}^2}{Aq^{15}}\right) + \left(Aq^{15} - \dot{\phi}^2\right)\frac{V_{,\phi}}{V} = 0, \qquad (7)$$

where $A \equiv (12/7)^{12}$. Also $a_* = \sqrt{j/3}a_i$ is a critical scale at which the eigenvalue of the inverse scale factor has a power-law dependance on the scale factor; j is the half-integer free quantization parameter, $a_i \equiv \sqrt{\gamma}\ell_P$ is the scale above which a classical continuous spacetime can be defined and below which the spacetime is discrete, and ℓ_P is the Planck length [5]. The modified mass function within this regime ($a \ll a_*$) takes the form $F^{loop} = 1/3\rho_\phi^{loop}R^3$. The behavior of this effective mass function determines whether trapped surfaces will form during the collapse procedure, within the semiclassical regime.

To study the collapsing model using a dynamical system description we introduce a new time variable σ given by $\sigma \equiv -\log q^{3/2}$, where $q = (a/a_*)^2$, being defined in the interval $0 < \sigma < \infty$; the limit $\sigma \to 0$ corresponds to the initial condition of the collapsing system ($a \to a_*$) and the limit $\sigma \to \infty$ corresponds to $a \to 0$. For any time dependent function f, we write $df/d\sigma \equiv -\dot{f}/3H$, where dot denotes the derivative with respect to t.

We herein make use of a *new* set of convenient dynamical variables: $\tilde{x} \equiv A^{-1/2}q^{-15/2}\dot{\phi}$, $\tilde{y} \equiv V/3H^2$, $\tilde{z} \equiv A^{1/2}q^{15/2}$. Then, Eqs. (6)-(7) in terms of $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are presented as

$$\frac{d\tilde{x}}{d\sigma} = -\frac{\xi}{\sqrt{3}}\tilde{z}\sqrt{\tilde{y}}\left(1 - \tilde{x}^2\right) + \tilde{x}\left(4\tilde{x}^2 - 9\right),$$

$$\frac{d\tilde{y}}{d\sigma} = -\frac{\xi}{\sqrt{3}}\tilde{z}\sqrt{\tilde{y}}\left(1 - \tilde{x}^2\right) + \tilde{x}\left(4\tilde{x}^2 - 9\right), \qquad \frac{d\tilde{z}}{d\sigma} = -5\tilde{z}, \qquad (8)$$

where $V(\phi) = \beta\phi^{-2}$, which brings $\xi \equiv -V_{,\phi}/V^{3/2} = 2/\sqrt{\beta}$ as a constant. The Hamiltonian constraint equation (6) in terms of new variables takes the form $\tilde{y}/\sqrt{1 - \tilde{x}^2} = 1$. The properties of each critical point will be determined by the eigenvalues of the (3×3)-Jacobi matrix. Finally, for the dynamical system (8), there is, under the constraints, just one physical fixed point for the physical system: (p_o) as $(0, 1, 0)$. In order to discuss the stability of (p_o), we have to determine the eigenvalues (and eigenvectors) of then Jacobi matrix. This analysis shows that, all eigenvalues are real but one is zero, the rest being negative implying that, the autonomous system is nonlinear with a non-hyperbolic point. The asymptotic properties cannot be simply

determined by linearization and we need to resort to other method: the center manifold theorem. Using the center manifold analysis one can show that, (p_o) is an asymptotically *stable* critical point (for a more detailed and complete discussion on the above elements, the reader may consult ref. [1]).

The solution for the effective energy density, the effective pressure in the neighborhood of the critical point (p_o) can be easily obtained that

$$\rho_\phi^{loop} \simeq V(\phi)\left(1 + \frac{1}{2}\frac{\dot{\phi}^2}{Aq^{15}}\right), \quad p_\phi^{loop} \simeq -V(\phi)\left(1 + \frac{9}{2}\frac{\dot{\phi}^2}{Aq^{15}}\right). \quad (9)$$

For a sufficient choice of initial condition, this fixed point leads to a solution for the scale factor of the collapsing system as $a^{42}(\phi) = 42\sqrt{\beta/3}\ln|\phi_s/\phi|$, where ϕ_s is a constant of integration. Then, it is seen that, when the tachyon field approaches the finite value as $\phi \to \phi_s$, the scale factor vanishes as $a \to 0$. On the other hand, the potential of tachyon field decreases from its initial value, and approaches a finite value at $a = 0$. Furthermore, the effective energy density which is given by $\rho_\phi^{loop} \approx V(\phi) = \beta/\phi^2$ does not blow up, becoming instead small and remaining finite. On the other hand, the effective mass function is $F^{loop}/R \approx (\beta/3\phi^2)r^2a^2$ and vanishes as $a \to 0$; that is, no trapped surfaces form for collapsing shells. Moreover, the effective pressure in Eq. (9) evolves such that $p_\phi^{loop} \approx -\rho_\phi^{loop} = -\beta/\phi^2$, remaining finite and negative, giving rise to an outward flux of energy in semiclassical regime.

Semiclassical analysis for the Kretschmann scalar shows that, it reaches a local maximum as the singularity is approached and then converges to finite values. Consequently this behavior can be interpreted as an effective singularity avoidance.

3. Conclusion

In this paper we considered the marginally bound FRW as a spherically symmetric model for an homogeneous interior spacetime of a gravitational collapsing system constituted by a tachyon matter field with the potential of an inverse square form. We investigated the classical and semiclassical regimes which the latter characterized by inverse triad corrections in order to establish the corresponding final state. Within the classical setting, we found analytically a situation where the tachyon evolved towards a dust-like asymptotic behavior; this suggested a black hole forming. In semiclassical regime, loop quantum corrections (an inverse triad type), pointed to the regularity of the spacetime geometry and the energy density of the system, where classical trapped surfaces were absent therein. These conditions indicated that, the spacetime in semiclassical collapse is singularity-free, hence there is neither a naked singularity nor a black hole forming as final state of collapse. In other words, loop quantum corrections of the inverse triad type seem to induce an outward flux of energy at the final state of the collapse.

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5. References

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