Public Debt Cyclicality and Long-Run Growth

Vladimir Kuhl Teles[*]

Milene Takasago[**]

Fernando Antônio Ribeiro Soares[***]

Abstract: This paper investigates the relationship between public debt cyclicality and economic growth. It expands the classical Barro (1990) model by relaxing the balanced budget hypothesis, and by introducing the existence of public spending adjustments costs. Our main findings are that the optimal volatility of public debt and consequently of output is different from zero. In other words there is an optimal volatility of public debt that policy makers should achieve to maximize growth.

Keywords: Hopf Bifurcation Theorem, Limit Cycles, Fiscal Policy, Economic Growth.

Resumo: Este artigo busca investigar a relação entre ciclicalidade da dívida pública e o crescimento econômico. O artigo de Barro (1990) é expandido por se relaxar a hipótese de orçamento equilibrado do governo e introduzindo custos de ajustamento aos gastos públicos. Os principais resultados são que a volatilidade ótima da dívida pública e consequentemente do produto são diferentes de zero. Em outras palavras isso significa que há uma volatilidade da dívida pública ótima que os "policy makers" devem alcançar para maximizar o crescimento e o bem estar.

Palavras-Chave: Teorema de Bifurcação de Hopf, Ciclos Limite, Política Fiscal, Crescimento Econômico.

JEL Classification: 023, 041.

1 Introduction

Economists argue that macroeconomic policies should affect primarily the shortrun while economic structural characteristics affect the long-run. Despite that, endogenous growth models have been recently suggesting that macroeconomic policies can also have consequences in the long run.

Ramey and Ramey (1995) opened an interesting discussion in this theme estimating a significant relation between macroeconomic

[*] Escola de Economia de São Paulo (EESP) da Fundação Getúlio Vargas (FGV). E-mail: vladimir.teles@fgv.br.

[**] Departamento de Economia da Universidade de Brasília (UnB). E-mail: mtakasago@uol.com.br.

[***] Secretaria de Aviação Civil do Ministério da Defesa. E-mail: fernandosoares@unb.br.

volatility and long-run growth. A lot of papers followed in the same vein [e.g. KALAITZIDAKIS et al (2000), FATÁS and MIHOV (2003), AGHION, et al (2005) and AGHION and MARINESCU (2006)] and they argue that stabilization policies should rise the long-run growth. Other researchers go to the opposite direction, that is, the short-run volatility cannot affect economic growth [e.g. ACEMOGLU et al (2003) and EASTERLY (2005)].

In this paper we address this issue theoretically looking for the cyclicality of public debt growth and its relation with GDP growth. It expands the classical Barro (1990) model by relaxing the balanced budget hypothesis and by introducing the existence of public spending adjustments costs.

Our contribution in this paper is two-fold. First of all it studies the optimality of balanced budgets. And secondly, it examines the influence of such optimality on the relationship between output volatility and long run growth. Our main findings are that policy makers maximize growth by keeping public debt volatility in an optimal level. And that public debt optimal volatility, and conse-quently output optimal volatility, are different from zero.

2 The model

The model we are proposing follows a tradition of endogenous growth models with government, à la Barro (1990). Thus, the Barro (1990) model is expanded by relaxing the balanced budget hypo-thesis, and by introducing the existence of public spending adjust-ments costs.

In order to relax the balanced budget hypothesis, an equation for public debt dynamics is introduced, which is given by,

$$\dot{B} = G - \tau Y + rB \qquad (1)$$

where B represents the public debt, G government spending, τ the tax burden, Y income, and r the rate of interest.

At the same time, and in accordance with Barro (1990), the firm's production function is given by,

$$Y_i = AL_i^{1-\alpha} K_i^{\alpha} G^{1-\alpha} \qquad (2)$$

where $0 < \alpha < 1$, Y_i is the output of firm i; A is a technological parameter; L_i and K_i are, respectively, the labor and capital stock available to firm i. It must also be pointed out that the exponent of G

is equal to 1-α so that constant returns regarding variables K_i and G may be obtained, wich consequently implicates that the economy is presenting endogenous growth.

Considering a closed economy without depreciation, the equation of motion for capital shall be,

$$\dot{K} = Y - C - \tau Y \tag{3}$$

where C is consumption.

At the same time, defining the following relations,

$$k = (K / B) \tag{4}$$

$$c = (C / B) \tag{5}$$

and,

$$g = (G / B) \tag{6}$$

and substituting (2)[1] in (1) and (3), we may then synthesize (1) and (3) in a single state equation, given by,

$$\dot{k} = (1 - \tau + \tau k)(AL^{1-\alpha} k^{\alpha} g^{1-\alpha}) - c - (g + r)k \tag{7}$$

Hence, the maximization of a utility function subject to equation (7) and to the non-Ponzi condition, translated by way of a transversality condition, would generate a linear result with a constant public debt, similar to Barro (1990). However, the insertion of government spending adjustment costs may implicate an optimal choice of cyclical changes in the public debt. Thus, the expansion of the public debt aiming at productive investments in order to generate externalities may be considered optimal, since the externalities effect would mean that the impact of public spending on the product would be non-null.

Therefore, since the government does not distribute its spending continually over time, initially it would increase public spending, thus increasing public debt, while at a later stage, the government would be confronted with a non-Ponzi condition, and it would reduce

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[1] Following the original Barro model we can use the Samuelson (1954) hypothesis that G is nonrival and nonexcludable. Hence, each firm makes use of all of G, and one firm's use of the public good does not diminish the quantity available to others. In this case we can say that the production function of the economy is $Y = AL^{1-\alpha} K^{\alpha} G^{1-\alpha}$.

public debt, configuring a cyclical relation with regard to public spending, which would then implicate a cyclical relation of the output. Thus, according to our approach, the government spending adjustment costs would then be the source of economic cycles. In order to introduce this cyclical relation in the model, the Hopf Bifurcation Theorem is applied, following the methodology proposed by Feichtinger et al. (1994), and especially by Greiner and Semmler (1996) and Wirl (2002), that describes the modeling of limit cycles for endogenous growth models.

Policy makers are thus faced with the following maximization problem,

$$\max \int_0^\infty \left[\left(\frac{c^{1-\sigma}-1}{1-\sigma} \right) + \upsilon_0 g - \frac{\gamma}{2} \Phi^2 \right] e^{-\rho t} dt \tag{8}$$

$$s..t. \quad \dot{k} = (1-\tau+\tau k)(AL^{1-\alpha}k^\alpha g^{1-\alpha}) - c - (g+r)k$$

$$\dot{g} = \Phi$$

where the above utility function combines the commonly used constant intertemporal substitution elasticity with two other terms: $\upsilon_0 g$, which represents the stock externality of government spending; and $(\gamma/2)\Phi^2$, the government spending adjustment costs. It symbolizes the disutility of the social planner to do changes in the productive government spending.[2]

In this sense, the Hamiltonian current value shall be given by,

$$H = \left[\left(\frac{c^{1-\sigma}-1}{1-\sigma} \right) + \upsilon_0 g - \frac{\gamma}{2} \Phi^2 \right] +$$

$$\lambda_k \left[(1-\tau+\tau k)(AL^{1-\alpha}k^\alpha g^{1-\alpha}) - c - (g+r)k \right] + \lambda_g \Phi \tag{9}$$

Hence, by applying the first order conditions and substituting in the movement equations of k and g, we obtain the following canonic equations:

$$\dot{k} = (1-\tau+\tau k)(AL^{1-\alpha}k^\alpha g^{1-\alpha}) - c - (g+r)k \tag{10}$$

$$\dot{g} = \frac{\lambda_g}{\gamma} \tag{11}$$

[2] We can argue that it disutility represents some rigidity in the government spending and that it is hard to implement fiscal reforms which may imply political costs.

$$\dot{\lambda}_k = \lambda_k(\rho - \tau AL^{1-\alpha}k^\alpha g^{1-\alpha} - (1-\tau+\tau k)\alpha AL^{1-\alpha}k^{\alpha-1}g^{1-\alpha} + (g+r)) \quad (12)$$

$$\dot{\lambda}_g = \rho\lambda_g - \upsilon_0 - (1-\alpha)(1-\tau+\tau k)AL^{1-\alpha}k^\alpha g^{-\alpha}\lambda_k \quad (13)$$

Thus, in order to apply the Hopf Bifurcation Theorem, the Jacobian for (10) to (13) must be obtained, which is given by,

$$J = \begin{pmatrix} X & (1-\tau+\tau k)Z - k & 0 & 0 \\ 0 & 0 & 0 & 1/\gamma \\ V & \lambda_k\left[1 - Z\left(\frac{\alpha}{k}(1-\tau) + (1+\alpha)\tau\right)\right] & \rho - X & 0 \\ \lambda_k\left[1 - Z\left(\tau + \frac{(1-\tau+\tau k)\alpha}{k}\right)\right] & \dfrac{\lambda_k(1-\tau+\tau k)\alpha Z}{g} & \lambda_k - (1-\tau+\tau k)Z & \rho \end{pmatrix}$$

$$(14)$$

where,

$$Z = (1-\alpha)AL^{1-\alpha}k^\alpha g^{-\alpha}$$

$$V = \frac{\alpha\lambda_k gZ}{(\alpha-1)k}\left(\tau(1+\alpha) + \frac{(1-\tau)(\alpha-1)}{k}\right) \quad and$$

$$X = AL^{1-\alpha}k^\alpha g^{1-\alpha}\left(\frac{\alpha}{k} + \tau(1+\alpha - \frac{\alpha}{k})\right) - g - r$$

Therefore, following Dockner and Feichtinger (1991), the eigenvalues of a Jacobian of type (14) are given by,

$$\theta_{1\,2}^{3\,4} = \rho/2 \pm \sqrt{(\rho/2)^2 - W/2 \pm (1/2)\sqrt{W^2 - 4\det(J)}} \quad (15)$$

where W is the sum of the determinants,

$$\begin{vmatrix} X & 0 \\ V & \rho - X \end{vmatrix} + \begin{vmatrix} 0 & 1/\gamma \\ \frac{\lambda_k(1-\tau+\tau k)}{g} & \rho \end{vmatrix} + 2\begin{vmatrix} (1-\tau+\tau k)Z - k & 0 \\ \lambda_k\left[1 - Z\left(\frac{\alpha}{k}(1-\tau) + (1+\alpha)\tau\right)\right] & 0 \end{vmatrix} \quad (16)$$

At the same time, the Jacobian has a purely imaginary pair of eigenvalues if, and only if, the conditions,

$$W^2 + 2\rho^2 W = 4\det(J) \quad (17)$$

and

$$W > 0 \quad (18)$$

are met.

For our model, the value for W is,

$$W = X(\rho - X) - \frac{1}{\gamma g}(1 - \tau + \tau k)\alpha Z \qquad (19)$$

Thus, by applying the bifurcation condition of (17) in (19) and choosing γ as a bifurcation parameter, it is then possible to obtain the critical value γ_{crit} that guarantee the validity of (17).[3] It must be noted that the steady-state trajectories for the state and co-state variables do not depend explicitly on the bifurcation parameter. These results thus allow us to formalize a proposition regarding the behavior of government spending and the product, as follows.

Proposition 1: Considering optimal control problem (8), and the equilibrium trajectories (10) to (13), then the Hopf Bifurcation Theorem, using γ as a bifurcation parameter, assuming the validity of (17) and (18), implies the existence of limit cycles.

Proof: Given the choice of the model's critical values, and considering the validity of conditions (17) and (18), the critical value of the bifurcation parameter may be calculated. In this case, the Jacobian obtained around equilibrium assumes a pair of purely imaginary eigenvalues, with a non-null intersection velocity, producing periodic solutions for values greater or less than the critical value.

Hence, the above proposition establishes that government spending adjustment costs implicate cyclical behavior for this costs, and, as a result, for the public debt as well. Thus, since public spending is considered productive spending in this study, by increasing private sector productivity by way of positive externalities, the cyclical behavior of productive government spending implicates cyclical behavior for the economy's productivity, generating economic cycles in an optimal environment.

3 Final Considerations

In this paper we have analyzed the dynamics of the cyclicality of public debt on an expansion of Barro (1990) model, and the relationship between public debt cyclicality, productivity, and economic growth. Our central contribution is the demonstration of the

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[3] To it be true it is only necessary that $\gamma_{crit} > 0$, following Feichtinger et al (1994). It condition is a direct consequence of (18), because we know that Z, the marginal return of the government spending, is positive. Thus, using a simple algebraic manipulations in (19), we can conclude that any value of γ ensures that the condition (18) will be positive.

optimality of economic cycles caused by variations in government spending. In this way, we show that in the presence of public expenditures externalities and adjustment costs, the balanced budgetary policy is not a maximizing growth strategy.

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