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# Two Essays in Finance: "Selection Biases and Long-run Abnormal Returns" And

"The Impact of Financialization on the Benefits of Incorporating Commodity Futures in Actively Managed Portfolios"

#### A Dissertation

Submitted to the Graduate Faculty of the University of New Orleans in partial fulfilment of the requirements for the degree of

Doctor of Philosophy in Financial Economics

By

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August, 2015

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#### **DEDICATION**

To my wife, Manu Raut, who suffered the pain of having a Ph.D. student as her husband and to my daughter, Eshika Adhikari, who spent hours every day looking for me. I cannot express how grateful I am to both of you for all of the sacrifices that you've made on my behalf.

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**ABSTRACT** 

This dissertation consists of two essays. First essay investigates the implications of researcher data

requirement on the risk-adjusted returns of firms. Using the monthly CRSP data from 1925 to

2013, we present evidence that firms which survive longer have higher average returns and lower

standard deviation of annualized returns than the firms which do not. I further demonstrate that

there is a positive relation between firms' survival and average performance. In order to account

for the positive correlation between survival and average performance, I model the relation of

survival and pricing errors using a Farlie-Gumbel-Morgenstern joint distribution function and fit

resulting the moment conditions to the data. Our results show that even a low correlation between

firm survival time and pricing errors can lead to a much higher correlation between the survival

time and average pricing errors. Failure to adjust for this data selection biases can result in

over/under estimates of abnormal returns by 5.73 % in studies that require at least five years of

returns data.

Second essay examines diversification benefits of commodity futures portfolios in the light of the

rapid increase in investor participation in commodity futures market since 2000. Many actively

managed portfolios outperform traditional buy and hold portfolios for the sample period from

January, 1986 to October, 2013. The evidence documented through traditional intersection test and

stochastic discount factor based spanning test indicates that financialization has reduced

segmentation of commodity market with equity and bond market and has increased the riskiness

of investing in commodity futures markets. However, diversifying property of commodity

portfolios have not disappeared despite the increased correlation between commodity portfolios

returns and equity index returns.

JEL Classification: G1; G12; G13; G14

Keywords: Survival Bias; Non-Survival Bias; Data Selection Bias, Commodity Futures;

Diversification; Spanning Tests

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## CHAPTER 1

## **Selection Biases and Long-Run Abnormal Returns**

#### 1. Introduction

Most empirical finance studies use a set of historical equity returns data. Almost every asset pricing model tests involves selecting a set of returns data. Researchers construct their sample by including firms which meet their selection criteria and excluding those firms that do not. For example, if the firms do not survive long enough to have a sufficient number of observations for estimation purposes, they are excluded from the sample. Similarly, firms that existed at the start of a study, but were delisted for various reasons before the study period begins are also excluded from the sample. Such data conditioning can introduce a survivor bias in performance measures because the characteristics of the firms that are included in the sample differ systematically from those of the firms which are dropped from the sample. Failing to adjust for survival related biases can lead to a significant distortion in the benchmark used for performance measurement. Benchmarking errors are more serious in longer horizons as these errors accumulate.

Specifically, I argue that ex-post sampling introduces bias in the measures of risk-adjusted returns. These biases could be survivorship bias, non-survivorship bias or truncation bias. Many empirical studies can be subject to the biases I discuss in this study, but I focus on long run performance studies because the performance measures used to detect the long-term impact of corporate events are generally subject to these biases<sup>1</sup>.

Beginning with Fama, Fisher, Jensen and Roll (1969) and followed by Ritter (1991), Ikenberry, Lakonishok and Vermaelen (1995), Kothari and Warner (1997), Rau and Vermaelen (1998), and others, CARs were historically used to examine long term returns following major corporate events. Under the CARs approach, abnormal returns are calculated each month relative to a benchmark and then aggregated over three to five year time periods. The benchmark return could use a portfolio such as CRSP equally weighted index or expected returns calculated using a

<sup>&</sup>lt;sup>1</sup> Kothari and Warner (1997), Barber and Lyon (1997), Lyon, Barber, and Tsai (1999), Mitchell and Stafford (2000), Brav (2000), and Kolari and Pynnone (2010) address other issues encountered in performing long-run event studies and propose some remedies.

well-known factor model. The parameters of the benchmark model are estimated using 24 months of time series data prior to an individual announcement date. The estimated parameters are then used to form model predictions and matched with event firm actual returns during the event period. The CARs approach requires at least five and up to eight years of continuous returns data to evaluate long term performance. If there is a relationship between firm survival and performance then having multi-year data requirement, such as in long term CARs studies, can impart a serious bias in performance measurement.

CARs suffer from the "bad model problem", as coined by Fama (1998). A bad model produces a spurious abnormal average monthly return or alpha, which eventually becomes statistically significant in cumulative monthly abnormal returns. If the parameter estimates of the benchmark model are biased, cumulating monthly abnormal returns over time also makes estimates of long-term abnormal returns biased.

The correlation between survivorship and average performance is one potential source of bad model problems. In CARs studies the firms that comprise the benchmark, on average, do not survive as long as the ones under study. This mortality mismatch between benchmark and event firms affects performance measurement --causing an upward bias in alphas.

Following the work of Ikenberry, Lakonishok and Vermaelen (1995), Barber and Lyon (1997) and Lyon, Barber, and Tsai (1999), the matched-firm buy-and-hold technique to measuring abnormal returns has become the popularly used technique. In this method, buy and hold abnormal returns are calculated as the difference between buy-and-hold returns of event firms and a set of control firms. Most existing studies use a carefully constructed matched reference portfolio as the benchmark to avoid known biases and access statistical significance with a bootstrapping procedure (see Bessembinder and Zhang (2013) and references therein for a review of this methodology). Matched firms are usually selected based on firm characteristics such as industry, size, and book-to-market ratio. Events and matched firms are held in a portfolio for a period of three to five years or until delisting date; whichever comes first. If a matched firm for an event firm is delisted, a new firm is drawn from the original list of candidates.

The matched-firm buy-and-hold abnormal return measuring technique imparts a bias on long-run performance estimates if the performance of both the event and control firms are affected

by survival time. In the buy-and-hold technique, event firms must have a minimum of three years of post-event stock returns to calculate abnormal returns. Matched firms are selected at the end of the year prior to the event firm's year based on the size and book-to-market, and included for a holding period of three to five years. The difference between these two portfolios is often regarded as abnormal or unexpected return. The abnormal return may be driven by performance of event firms or control firms. The proportion of surviving verses non-surviving firms in each of these portfolios can influence measured abnormal returns. Previously methodologies do not directly account for bias caused by survival. This problem is potentially more severe with riskier firms (i.e., firms with small market capitalization and high book-to-market) because these firms are less likely to survive long enough to be included in the sample of long-run studies as pointed out by Kothari, Sabino and Zach (2004).

An alternative methodology to the buy-and-hold matched firm approach is the calendar-time portfolio approach. Loughran and Ritter (1995), Brav and Gompers (1997), Brav, Geczy and Gompers (2000), among other, have used the calendar-time portfolio approach and Fama (1998) favors this approach. It focuses on the mean abnormal time series returns to portfolios of event firms. For each calendar month, event firms' portfolio returns are computed as the equally weighted average return of all firms that have experienced the same event within the previous three to five years. Portfolios are rebalanced monthly and drop all firms that reach the end of the long-horizon study period and add all firms that have just experienced the event. Monthly excess returns of the event portfolio are regressed on the risk factors and the intercept is used as a measure of the average monthly abnormal return. The choice of risk factors is guided by theory.

Although the calendar time portfolio approach is less susceptible to data selection biases, it is not bias free. For instance, suppose the post event period is five years. For each calendar month, a portfolio is constructed comprising all firms that experienced the event within the last five years. Events firms are tracked up to five years or up to the delisting date following the event. Each month, new firms are added and some firms are deleted. Hence, the proportion of non-surviving firms and young firms in the monthly portfolio is time varying. If the majority of months in the period under study is dominated by either young firms or non-surviving firms then the results obtained from the calendar time portfolio approach are biased, and sometimes very odd as documented in Ritter and Welch (2003). This feature of the multifactor regression based tests of

abnormal returns provides some background as to why the long-run event studies' results are sensitive to the exact time period chosen<sup>2</sup>.

The long-run over-performance or under-performance reported in past event studies may be subject to biases from data requirements and sample period selection. Neglecting these biases produces misleading results in long-horizon event studies whether the abnormal returns are measured using CARs, BHARs or Jensen's alpha approach. Hence, the evidence of significant abnormal returns reported in the literature might be the result of benchmarking errors rather than a failure of a particular asset pricing model.

To examine the implications of data requirement biases on the calculation of long-run abnormal returns, I analyze in detail the performance of firms which meet the data requirement criteria and firms which do not meet the data selection criteria. I analyze two samples constructed from the Center for Research into Securities Prices (CRSP) database. The first sample consists all firms which have at least 24 months of continuous returns in the CRSP database over the period December 1925 to December 2013. Further, I also examine performance characteristics of firms by splitting the first sample into end of sample period survivors and non-survivors (active and non-active firms). To see whether there is any data truncation bias, I form second sample consisting of all firms which have at least 24 months of returns in the CRSP data base over the period December 1972 to December 2013. From the first sample, I split firms into firms which meet data selection criteria and firms that do not meet selection criteria and then, I examine the performance and other characteristics of data for these two groups.

In my empirical analysis, I show that the firms which meet data requirement have higher risk-adjusted annualized average returns and lower standard deviation of annualized returns than the firms that do not meet data requirement. The difference in average performance also depends on the calculation method applied. For example, the annualized CAPM alpha and its standard deviation for firms that survive more than five years are 4.15 percent and 19.72 respectively, whereas the same measures for firms that do not survive through five years are -9.61 and 48.30,

<sup>&</sup>lt;sup>2</sup> Boehme and Sorescu (2002) show that the positive post dividend price drift documented by Mitchaely, Thaler and Womack (1995) is confined to the period from 1964 to 1998 and there is no evidence of any abnormal price drift for the period prior to 1964. Ritter and Welch (2002) document that the long-run underperformance of IPOs is sensitive to the time period examined. Similarly, Fu and Huang (2014) argue that the long-run abnormal returns following both repurchases and seasoned equity offerings disappear for the events in the most recent decade.

respectively. The annualized three factor alpha and its standard deviation for firms not surviving more than five years are -6.23 percent and 52.66 respectively but the same measures are 2.22 percent and 21.84 for firms surviving beyond five years.

The analysis of the data also shows that survivors have positive risk adjusted returns from the beginning of their birth time but non-survivors have negative risk-adjusted returns. The average performance of non-survivors is consistent with the IPO under performance literature. The IPO underperformance is not true for survivors. This finding clearly has implication on IPO literature.

In addition, when I examined whether survival is correlated with risk-adjusted performance measures, I estimate parameters of Cox proportional hazard model of firm survival using full sample and sub-sample data. The results indicate that performance is correlated with survival no matter which asset pricing model is used. Further, I find the correlation between survival and average returns is decreasing for surviving firms and increasing for non-surviving firms. I observe similar characteristics of the data as I require survival over longer future periods. This phenomenon is pronounced in sub-sample as well.

To examine the relationship between survival and average pricing errors, I develop a general framework for modelling pricing errors and survival time using the Farlie-Gumbel-Morgenstern (FGM) bivariate distribution of Morgenstern (1956), Farlie (1960) and Gumbel (1958). I then compute conditional moments and apply them to the data to estimate the relationship between survival and pricing errors and conditional expectations and the variance of average pricing errors, given survival time.

In my framework, I assume average pricing errors follow normal distribution and survival time, defined as the number of months from birth (which occurs when I observe returns in CRSP) to death (which occurs when CRSP stops reporting returns), follows an exponential distribution. I use the FGM distribution to model the joint occurrence of survival and average pricing errors and assume that the birth variable is exponentially distributed and independent of average pricing errors. By allowing the birth variable to be independent of average pricing errors I can concentrate on the conditional expectation and dependence properties between survival and average pricing errors. Interestingly, I find a very high correlation between birth and survival. To address this issue, I model birth as a convex combination of survival time and another exponentially distributed variable.

Given this setup, I achieve two main findings. First, despite a low correlation between pricing errors and survival time, the correlation between average pricing errors and survival can be very high. Using my model, I show that pricing errors and survival time have statistically significant low correlation (less than 2 percent) but the correlation between average pricing error and survival time can go up to 14 percent depending on how the data is selected. Further, the correlation is determined by the conditional variance of average pricing errors and covariance between survival and pricing errors. Therefore, to have consistently high correlation, either the variance of the average pricing errors has to be consistently high or conditional covariance between survival and pricing errors has to be high or both. Second, I derive a conditional expectation of average pricing errors which links the average pricing errors to the survival time. When this expectation to the CRSP data, it shows that the average pricing errors for a given survival time increase as the survival time increases. The survival adjusted average risk adjusted returns is always statistically insignificant. My results have important implications for long-run performance studies and market efficiency because these studies often require long history of returns series.

While the intuition as to why survival and the average pricing errors may be correlated is not readily apparent, I postulate that survival time captures certain specific characteristics of a company. One of the main reason is the market discipline in which market rewards those firms which experience positive abnormal returns even when real return is negative and punishes those firms which experience negative returns when true return is positive. Further, firms that perform poorly are less likely to survive longer. Another reason is that survival time confirms stability in business relations, captures differences in the life cycle of a company and controls for differences in the life cycle of a firm. As one would expect firm riskiness may decline with firm age as argued by Faccio, Marchica and Mura (2011) as firms with a longer life span should be able to generate internal financing funds. These factors help older firms realize higher risk-adjusted returns. Therefore, the including only firms that survive in a sample leads to a positive risk-return relationship.

The rest of the paper is organized as follows. Section II provides review of related literature. Section III examines the data and provides a comparative analysis of surviving and non-surviving firms in terms of risk-adjusted returns and correlation between survival and average

pricing errors. Section IV discusses the general framework for modelling the joint behavior of survival and average pricing errors. Section V presents the results from the estimates of my non-linear conditional moments. Finally, section VI summarizes the main findings and implications of the study.

#### 2. Literature Review

Many past studies which have used long-term security prices discuss about the biases that I study here. They include Ball and Watts (1979), Banz and Breen (1986), Brown, Goetzmann and Ross (1995), Kothari and Warner (1997), Barber and Lyon (1997), Li and Xu (2002) and Linnainmaa (2013). Ball and Watts (1979) examine the time series behavior of earning per share for three samples of firms: two samples without any survival requirement and one sample with at least 50 years of survival requirement. They compare the characteristics of the earning per share for these samples. They find no significant difference in the results for these samples and conclude that the effects of survival requirement on different statistics are minimal.

Banz and Breen (1986) evaluate the impact of the ex-post selection bias on the returns of the portfolio. They formed two portfolios on the basis of size and earnings yield using a partially complete COMPUSTAT data base and the unbiased data base which is "the sequentially collected COMPUSTAT file". They find significant difference in these portfolio returns. They also show that the differences in the portfolio returns could be due to survivorship bias.

One of the influential papers that argue that there could be serious survival bias in the long horizon event studies is Brown, Goetzmann and Ross (1995). They examine the implication of the data requirement on measures used to study long-term market behavior. They provide some analysis of the consequences of survival for studies in long-term stock market returns and event studies. They argue that survival criteria related to whether a firm survived can give the appurtenance of abnormal returns around events. They point out biases induced by survival conditioning in the study of cross-sectional cumulative excess return measures that are commonly used in the context of event studies.

Kothari and Warner (1997) is one of the early papers to examine survival related biases in reference to long run abnormal returns. They argue that minimum data requirement impose detectable biases in the mean abnormal returns and the standard deviation of returns for long-

horizon studies. In their calculation, three year CARS is 40.8 percent with no prior data requirement and 45.1 percent with four years of data requirement. Similarly, the BHAR increases from 48.8 percent to 53.7 percent as the prior data availability is increased from zero to four years. Their results strongly suggest that conditioning a sample on prior data availability is associated with higher future mean returns. They also point out that failing to address survival biases in long-horizon tests can potentially affect the specification of test statistics.

In a similar study, Linnainmaa (2013) argues that researchers' effort to correct for survivorship bias effects introduces a bias in the opposite direction which he calls "reverse survivorship bias" and defines it as the gap between the expected alpha and in-sample alpha estimate.

On the contrary, Li and Xu (2002) argue that the survival problem in the current literature are probably exaggerated. In their modelling framework, they derive a mathematical relationship between the ex-ante survival probability and the average survival bias and show that to have high survival bias, the probability of market survival over the long-run has to be extremely small. But existing historical evidence shows high rate of survival. Therefore, survival bias should not have any impact on the U.S. equity premium.

Most of the studies reviewed in this section talk about the data selection, survival and non-survival related biases in different contexts. However, none of these studies examine the presence of ex post data conditioning biases and implications of such biases for calculating risk adjusted returns using CRSP data base. In this study, I try to fill this gap in the literature by examining return patterns of survivors and non-survivors in the CRSP data base and pointing out how statistics used to detect long-term abnormal returns might be affected. My work complements previous research that suggests caution in interpreting the evidence of long-horizon price performance for methodological reasons. For example, Kothari and Warner (1997), Barber and Lyon (1997), Brav and Gompers (1997), Brav et al. (1998), Fama (1998) and others question the economic significance of the evidence of long-horizon studies.

## 3. Survival and Performance Analysis

## 3.1 Data and performance measurement

The initial dataset consists of monthly returns on common equity (share codes 10 or 11) of 21,630 firms from the CRSP database spanning December 1925 to December 2013, excluding utilities companies and firms whose real stock price (in 2000 dollars) drops below \$5 during its database life. To be in the sample, a firm must have be in CRSP and be traded in major exchanges (Exchange Code <5). The initial dataset reduces to 18,876 firms after I require at least 24 months of consecutive non-missing returns to compute rolling alphas. The risk free rate,  $r_f$ , and risk factors MKT, SML, HML, and MOM are obtained from Kenneth R. French's website.

In the framework of this paper, I define firm birth and survival times using firms' listing and/or delisting dates in the CRSP database. The firms birth date, b, is defined as the first date listed in the CRSP database. The firm born earlier gets higher value for b as compared to firms born in the latter dates. The birth dates are used for selecting censored, non-censored, and subsamples. Similarly, firm survival time, s, is defined as the number of months between listing and delisting dates in the CRSP database. For censored firms, survival time is unknown, and therefore, I use the end date of the sample as delisting date to define the survival variable. If a firm has missing returns between two periods that have returns, it is still considered as one firm.

I examine the relationship between survival/non-survival and average pricing errors for firms in six different samples. The first sample is the entire CRSP database over the full sample period. I argue that I may loose some information when I ignore the fact that end of sample period surviving firms and non-surviving firms may systematically behave differently. The parameter estimates will also be bias because ignoring right-censored firms is ignoring firms which have the propensity to be greater than a given value. In this case, the expectation of my estimator is smaller than the real value of surviving time. In order to account for this survival and non-survival biases, I split the first sample into censored and non-censored data. The censored sample includes all those firms which are still alive at the end of sample period whereas the non-censored sample includes all those firms that are already dead by the end of the sample period. Further, CRSP expanded its coverage in July 1962 by including stocks traded on the American Stock Exchange, and in December 1972 by adding NASDAQ traded stocks. To account for the beginning of the CRSP

Nasdaq period and data truncation bias, I also examine a sub-sample of my entire CRSP data covering the period December 1972 to December 2013. In addition, I study the censored and non-censored sample of firms from this sub-sample.

I denote the return during period t on firm i by  $r_{it}$  and the number of time periods as T. I use beta pricing models to describe the return generating process. More specifically, individual firm level performance is measured as average pricing error computed from three standard benchmarks:<sup>3</sup> the Jensen (1968) CAPM,

$$r - r_f = a_1 + b_1 MKT + \epsilon_1, \tag{1}$$

the Fama and French (1993) 3-factor model,

$$r - r_f = a_3 + b_3 MKT + c_3 SML + d_3 HML + \epsilon_3, \tag{2}$$

and the Carhart (1997) four factor model,

$$r - r_f = a_4 + b_4 MKT + c_4 SML + d_4 HML + e_4 MOM + \epsilon_4.$$
 (3)

These models are designed to capture the relation between returns and risk factors used to measure exposures to unknown state variables. If the factor exposures capture all variation in expected returns, the intercepts in models (1) through (3) are zero for all securities and portfolios. The intercepts in (1) through (3) are referred as pricing errors and they measure the deviation of excess returns from what the corresponding returns should be according to the given pricing model. For each firm, I compute the average of pricing errors obtained from a rolling regression and use it as a measure of performance.

The average firm alphas are calculated in two steps. First, I calculate 24-month rolling window alphas for each firm by running a rolling time series regression for models (1), (2), and (3). A full 24-months of returns is required for each estimate. Second, individual firm alphas are computed from each model by averaging its rolling alphas,  $\alpha^s = \frac{\sum_{t=k^*}^{t=s} a_{jt}}{s-k^*+1}$ ,  $j = \{1,3,4\}$ , where  $k^* = 24$  is the length of the estimation window and s is the firm survival time adjusted for any missing observations. Cross-sectional average alphas are calculated as equally-weighted individual firm level alphas estimated from the models above. The cross-sectional mean for firms surviving s

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<sup>&</sup>lt;sup>3</sup> Firm and time subscripts are omitted for clarity.

months, denoted as  $E(\alpha|s)$ , is calculated as equally weighted alphas of all firms that survive s months, i.e.  $E(\alpha|s) = \sum_{i}^{N} \alpha_{i}^{s} / N$ .

To study the effects and characteristics of survival and non-survival biases, I classify firms in each sample into survivors and non-survivors and condition on survival time. The survivors given k (the minimum number of months required for firms to have returns to be included in the sample) include all firms that survive beyond k months. The average performance for k months survivors is calculated by averaging alphas of the firms that live beyond k months,  $E_s(\alpha|s>k)=\sum_{j=1}^N \alpha_j^s/N$  where s>k. Correspondingly, non-survivors given k months include all the firms that survive at most k months. Their cross-sectional mean alpha for k months non-survivors ( $E_{ns}(\alpha|s\leq k)$ ) is calculated as the equally-weighted average of alphas from firms that survive at most s months i. e.  $E_{ns}(\alpha|s\leq k)=\sum_{j=1}^N \alpha_j^s/N$  where  $s\leq k$ . I compute these conditional averages for survivors and non-survivors as I increase the value of k.

In addition to average performance, I also examine the Pearson correlation and rank correlation between survival time and alphas and standard deviation of alphas for survivors and non-survivors. The Pearson correlation coefficient is computed as  $Cov(\alpha_j^s,s)/\sqrt{Var(\alpha_j^s)Var(s)}$ , where s>k for survivors, and  $s\leq k$  for non-survivors. Similarly, the rank correlation coefficient is computed using the same formula using ranks instead of real values of  $\alpha_j^s$  and s. Finally, the cross-sectional standard deviation of risk-adjusted returns is calculated as usual for survivors (s>k) and non-survivors ( $s\leq k$ ).

I begin the presentation of my results with summary statistics. Table I presents summary statistics and correlation between survival and average pricing errors as well as correlation between survival and birth time for different samples. Pcorr and Scorr refers to Pearson correlation and Spearman's rank correlation. The results in Table I shows that the end of sample period surviving firms have higher average risk adjusted returns than firms that do not survive at the end of sample period. Inactive firms also referred as the end of the sample non-survivors have negative risk adjusted mean returns irrespective of model used. Further, I find a higher correlation between CAPM alphas and survival compared to the correlation between survival and alphas computed

Table 1

Descriptive Statistics and Correlation between Survival and Mean Pricing Errors

This table reports summary statistics from different samples. Pcorr and Scorr refers to Pearson and Spearman correlation coefficients between survival and average pricing errors. Panel A shows results for whole period sample which consists of 18876 firms that have at least two years of prior data requirement over the period December 1925 to December 2013. Panel B and Panel C presents the end of sample surviving firma and the end of the sample non-surviving firms. Panel D reports results for sub-sample which consists of 17,596 firms with at least two years of prior data requirement over the sample period December 1972 to December 2013. Panel E and Panel F displays results for sample of firms that are censored and non-censored respectively. Mean returns are annualized in all panels.

-	Mean	t-Stat	Sdtdev	Min	Max	Pcorr	Scorr					
Panel	A: Full Sam	ple (18876)	Firms)									
Ret	13.4422	27.3046	0.0564	-0.9231	1.3000	0.0441	0.0044					
CAPM	1.0666	5.0043	0.0244	-0.1833	0.2224	0.1147	0.1471					
FF3F	0.3259	1.4133	0.0264	-0.1831	0.3899	0.0677	0.0981					
FF4F	0.4958	2.1288	0.0267	-0.1906	0.4020	0.0663	0.0961					
Survival	167.85	151.36	152.36	26	1057	1	1					
Birth	374.5564	239.7445	214.6464	25	1056	0.5911	0.4574					
Panel B	Panel B: Active Firms in Full Sample (3208 Firms)											
Ret	20.3879	42.8347	0.0225	-0.1264	0.6037	-0.0778	-0.0448					
CAPM	6.6866	21.4786	0.0147	-0.0975	0.2072	0.0527	0.0496					
FF3F	6.2401	18.5877	0.0158	-0.1174	0.2460	0.0101	0.0112					
FF4F	5.8931	17.8534	0.0156	-0.1227	0.2018	0.0206	0.0394					
Survival	257.76	75.13	194.32	26	1057	1	1					
Birth	256.7703	74.8412	194.3216	25	1056	0.9999	0.9999					
Panel C:	Inactive Firm	ns in Full Sa	imple (1566	8 Firms)								
Ret	12.0201	20.5683	0.0610	-0.9231	1.3000	0.0482	0.0092					
CAPM	-0.0841	-0.3396	0.0258	-0.1833	0.2224	0.1087	0.1389					
FF3F	-0.8850	-3.2999	0.0280	-0.1831	0.3899	0.0563	0.0776					
FF4F	-0.6093	-2.2446	0.0283	-0.1906	0.4020	0.0554	0.0765					
Survival	149.44	138.50	135.06	26	1043	1	1					
Birth	398.6729	236.9473	210.6065	31	1056	0.6389	0.5239					
Panel D: S	Subsample (	17596 Firms	s)									
Ret	15.1222	28.3619	0.0589	-0.9231	1.3000	0.0547	0.0433					
CAPM	1.3413	5.9014	0.0251	-0.1833	0.2224	0.1227	0.1675					
FF3F	0.4017	1.6306	0.0272	-0.1831	0.3899	0.0643	0.1005					
FF4F	0.5328	2.1465	0.0274	-0.1906	0.4020	0.0657	0.1008					
Survival	166.45	145.18	152.08	26	1057	1	1					
Birth	343.1942	254.5816	178.8215	25	1056	0.6825	0.4952					

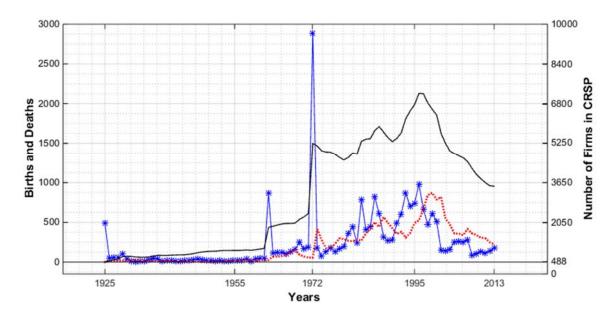
Table I-Continued

Pane E: Active Firms in Sub-sample (3208 Firms)											
Ret	20.5510	43.1820	0.0225	-0.1264	0.6037	-0.0660	-0.0212				
CAPM	6.6778	21.4284	0.0147	-0.0975	0.2072	0.0495	0.0486				
FF3F	6.2046	18.4665	0.0159	-0.1174	0.2460	0.0040	0.0063				
FF4F	5.8714	17.7783	0.0156	-0.1227	0.2018	0.0170	0.0363				
Survival	257.76	75.13	194.32	26	1057	1	1				
Birth	256.7703	74.8412	194.3216	25	1056	0.9999	0.9990				
Panel F: In	nactive Firms	s in Sub-sam	nple (14388	Firms)							
Ret	13.9117	21.6374	0.0643	-0.9231	1.3000	0.0647	0.0607				
CAPM	0.1514	0.5647	0.0268	-0.1833	0.2224	0.1213	0.1657				
FF3F	-0.8921	-3.0687	0.0291	-0.1831	0.3899	0.0525	0.0807				
FF4F	-0.6575	-2.2396	0.0293	-0.1906	0.4020	0.0547	0.0815				
Survival	146.09	132.11	132.64	26	1043	1	1				
Birth	362.4636	256.8485	169.2729	31	1056	0.7320	0.5648				

from three and four factor models. Furthermore, I observe a very high correlation between firm survival and arrival time.

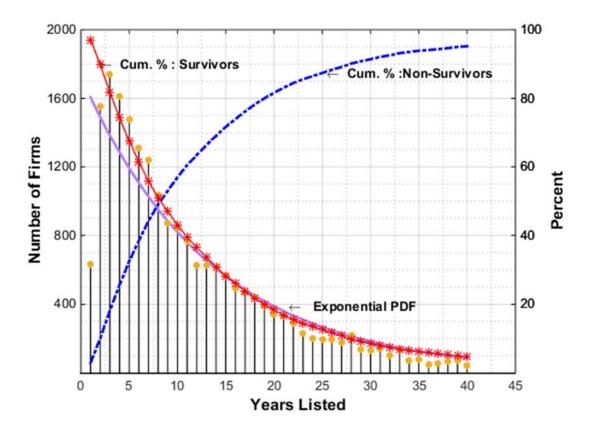
## 3.2 Firm births, deaths and survival analysis

In Figure 1, I plot the number of firms born, the number of firms died and the total number of existing firms in the CRSP data base on a yearly basis. I observe a low number of firm births and deaths prior to December 1972 because the original CRSP file contained only stocks from the New York Stock Exchange (NYSE). I see from the figure that the number of birth and deaths in each year varies substantially over time after 1974 and the number of firm deaths exceeds number of firm births since 1997. Firms may die for various reasons such as a split, merger, liquidation, or dropped but I do not differentiate among these causes of death. My analysis shows that the major cause of death is mergers and acquisitions followed by dropped delisting for various reasons. In my sample, about 42.28 percent of delisting is due to mergers and acquisitions and 34.63 percent is due to dropped delisting.



**Figure 1**. **Number of Firms, Firm Births and Deaths in CRSP by Year.** The figure illustrates number of firms, number of firm births and deaths in CRSP database by year. Y-axis on the left represents number of births and deaths and the y-axis on the right represents number of firms in the CRSP database. The solid line represents number of firms in the CRSP, the line with strikes represents number of firm births and dotted line represents number of firm deaths. The two spikes in the line with strikes correspond to 1962 and 1972 when CRSP expanded its database by adding stocks traded on American Stock Exchange and NASDAQ respectively.

Figure 2 provides information about the number of firms and cumulative percentage of survivors and non-survivors given survival time. It also presents the exponential curve fitted to the survival data. The vertical lines represent the number of firms surviving T years. For instance, there are 1,480 firms that survive five years. The solid line is the exponential curve fitted to the actual survival data. The dotted line depicts the percent of firms that do not survive beyond T years and the line with the stars shows the percent of firms that survive more than T years. For instance, approximately 13 percent of firms do not live beyond two years and approximately 87 percent of firms live more than two years. The percent of survivors decreases to 67.53 percent by year five and 43.06 percent by year ten. Similarly, the number of non-survivors increases to 32.47 percent and 56.94 percent by years five and ten respectively. Overall, the plots of the cumulative percentage of survivors and non-survivors depict the impact of the minimum data requirement on the numbers of firms surviving or non-surviving.



**Figure 2**. **Illustrating Survival of the Firms and Plot of Exponential PDF to the Data.** The figure illustrates the survival and non-survival rate of firms in the CRSP data base and the plot of the exponential probability density function of the data. The vertical bars shows number of firms surviving corresponding number of months. The solid line shows the exponential probability density function fitted to the data. The dotted line shows percent of firms that survive less than or equal to T years. Similarly, the line with star plots percent of firms that survive beyond T years.

## 3.3 Survival and performance

In this section, I look at the influence of increasingly longer period data requirements on firm performance in the context of the overall dataset and a dataset truncated to include only firms living after December 1972. I choose my sub-sample period beginning in December 1972 because CRSP's data coverage expanded its coverage to include domestic common stocks traded on the NASDAQ stock market at that time. Both of these datasets are bifurcated into non-active firms for which mortality date is known and active firms for which mortality date is not known.

Table II reports the average performance of firms from entire dataset grouped by their survival time for my overall sample. The mean returns of firms which live over shorter periods are clearly below those of the firms that live over longer periods. In terms of risk-adjusted performance, almost all average alphas are negative and significantly different from zero for firms that survive six or less than six years. In contrast, the average alphas for firms that live beyond six years are all positive and statistically significant. This might be because poorly performing firms drop out of the sample and well performing firms continue to live longer.

Another important feature of the data is that increasing average returns for a given s starts to decline after a certain number of years. Such return patterns give of a humped shape average return curve, increasing up to twenty years and then dropping. The end of the sample survivors, or active firms, have slightly different return patters than the others. They have higher average returns than those with similar survival times. I do not see increasing average returns for firms as I require firms to survive longer period in the sample of firms that survive to end of the sample period.

Table III and Figure 5 report risk adjusted returns for firms in different samples selected based on data requirement. As I can see in the Figure 5 that the average risk-adjusted returns for firms that meet data requirement and for firms which do not meet data requirement increases as the survival time increases across all models and all samples. Non-surviving firms always underperform surviving firms and experience negative risk-adjusted returns. The significantly different from zero negative average returns indicate that the performance preceding the delisting of a firm is overwhelmingly bad. In contrast, firms in the survival group have positive risk-adjusted average returns irrespective of survival time. The overall risk-adjusted mean return is negative up to six years and positive thereafter.

In order to provide further insights into the performance of the firms that meet minimum data requirement and the firms that do not meet the data requirement, I examine performance of firms splitting them in different groups as in Table III. Panel A in Table II reports risk adjusted averages for firms in entire sample. Similarly, Panel B and Panel C in Table II show risk adjusted averages for active firms and non-active firms respectively. Averages for the firms which meet data requirements are presented in column 2 to column 5 in each panel and averages for firms which do not meet are shown in columns 7 to 10 in each panel.

Table II

Average Performance Based on Survival Time

This table reports cross-sectional average of average pricing errors given the survival time. The sample includes all the 18876 firms that have at least 24 months of continuous returns in the CRSP database over the period 1926 to 2013. Firm survival time is presented in years under the column heading Years. CAPM, FF and FF4 refer to the averages of alphas estimated using the CAPM, the three factor model and four factor model. I repost t-statistics under the column heading t-Stat for testing the null hypothesis that the corresponding average is equal to zero. Panel A shows the results from full sample, Panel B from censored sample and Panel C from non-censored sample.

		All Firms		Ac	ctive Firms		Non-active Firms			
Years	CAPM	FF4F	N	CAPM	FF4F	N	CAPM	FF4F	N	
2.5	-14.97*	-10.12*	380	10.43	18.23**	31	-17.23*	-12.64*	349	
3	-12.91*	-6.07*	821	-3.79	-2.09	58	-13.60*	-6.37*	763	
4	-10.46*	-6.90*	1564	-9.02**	-4.18	116	-10.58*	-7.11*	1448	
5	-5.46*	-2.76**	1465	-13.06*	-9.71**	71	-5.08*	-2.41**	1394	
6	-2.25**	-2.08**	1311	7.53**	7.50**	52	-2.65*	-2.48**	1259	
7	0.77	0.41	1209	4.94**	3.11	140	0.22	0.05	1069	
8	3.06*	1.62	1040	8.92*	6.97*	136	2.18**	0.81	904	
9	3.17*	0.97	855	6.27*	3.83**	122	2.65*	0.49	733	
10	3.20*	1.02	868	2.07	1.02	128	3.40*	1.02	740	
12.5	7.16*	3.25*	325	5.86*	4.94**	32	7.30*	3.07*	293	
15	5.93*	3.66*	575	10.08*	8.41*	127	4.76*	2.31*	448	
20	6.14*	4.25*	328	9.42*	9.31*	89	4.92*	2.37**	239	
25	6.33*	3.39*	200	9.82*	9.32*	27	5.79*	2.47*	173	

Table III

Cross-sectional Average of Mean Pricing Errors for Survivors and Non-survivors

This table reports cross-sectional mean of average firm alphas conditional on a firm surviving time. The data in this table are for the period December 1925 to December 2013. Firm alphas are estimated using the CAPM, the three-factor and the four-factor model in rolling time-series regression setting. A firm that disappears during year T is assigned into the year T "Do not meet data requirement" pool of firms; firms still alive beyond year T are assigned into the year T "Meet data requirement" pool. The first column reports firm's survival time and RET refers to risk unadjusted returns. The columns in this table reports annualized average alphas for firms that either survive (columns (3) to (5)) or do not survive (columns (7) to (9)) based on the CAPM, the three factor model and the four factor-model respectively through the T<sup>th</sup> year of their lives, conditional on survival time. N refers to the total number of firms in each sample.

Pan	ച	Δ.	Δ1	$1  \mathrm{F}$	irms
1 411	C.I.	$\overline{}$	$\neg$	1 1	111115

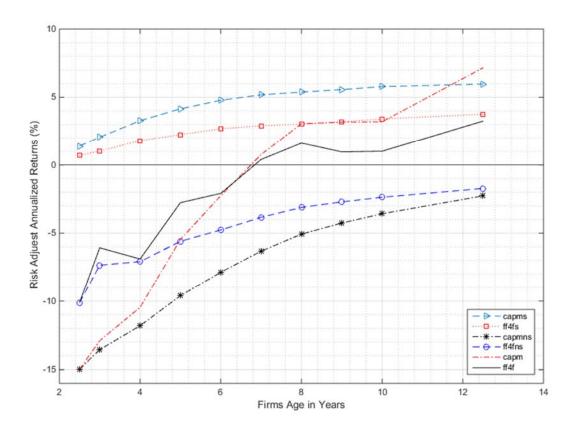
Firm		Meet	data require	ment		Do not meet data requirement						
Age	RET	CAPM	FF3F	FF4F	N	RET	CAPM	FF3F	FF4f	N		
2.5	13.69*	1.40*	0.57**	0.71*	18496	1.32	-14.97*	-11.51*	-10.12*	380		
3	14.07*	2.06*	0.94*	1.03*	17675	4.22	-13.56*	-8.69*	-7.35*	1201		
4	15.07*	3.28*	1.75*	1.80*	16111	3.97	-11.81*	-8.00*	-7.10*	2765		
5	16.02*	4.15*	2.22*	2.25*	14646	4.51**	-9.61*	-6.23*	-5.59*	4230		
6	16.63*	4.78*	2.67*	2.68*	13335	5.78*	-7.87*	-5.30*	-4.76*	5541		
7	16.90*	5.18*	2.91*	2.91*	12126	7.23*	-6.32*	-4.31*	-3.84*	6750		
8	17.05*	5.38*	3.01*	3.03*	11086	8.31*	-5.07*	-3.49*	-3.11*	7790		
9	17.14*	5.56*	3.15*	3.20*	10231	9.07*	-4.26*	-3.02*	-2.71*	8645		
10	17.38*	5.78*	3.32*	3.40*	9363	9.57*	-3.57*	-2.62*	-2.37*	9513		
12.5	17.63*	5.97*	3.68*	3.76*	7634	10.60*	-2.26*	-1.95*	-1.72*	11242		
15	17.83*	6.05*	3.92*	4.02*	6154	11.32*	-1.34*	-1.41*	-1.21*	12722		
20	17.97*	5.73*	3.42*	3.61*	4490	12.03*	-0.39	-0.64**	-0.47*	14386		
25	17.83*	5.52*	3.12*	3.36*	3213	12.54*	0.15	-0.25	-0.09*	15663		

Panel B: Non-active Firms

Firm		Meet	data require	ment		Do not meet data requirement
Age	RET	CAPM	FF3F	FF4F	N	RET CAPM FF3F FF4f N
2.5	12.41*	0.31	-0.60**	-0.34	15319	-4.96 -17.23* -13.52* -12.64* 349
3	12.91*	1.04*	-0.22	-0.02	14556	0.37 -14.74* -9.64* -8.34* 1112
4	14.17*	2.32*	0.59*	0.77*	13108	1.01 -12.39* -8.45* -7.65* 2560
5	15.27*	3.20*	0.96*	1.14*	11714	2.39 -9.81* -6.35* -5.80* 3954
6	15.99*	3.90*	1.40*	1.58*	10455	4.05** -8.08* -5.47* -5.00* 5213
7	16.40*	4.32*	1.59*	1.75*	9386	5.48* -6.67* -4.58* -4.14* 6282
8	16.66*	4.55*	1.68*	1.85*	8482	6.54* -5.56* -3.91* -3.52* 7186
9	16.70*	4.73*	1.79*	1.98*	7749	7.44* -4.80* -3.50* -3.14* 7919
10	16.90*	4.87*	1.85*	2.08*	7009	8.07* -4.10* -3.10* -2.79* 8659
12.5	17.07*	4.84*	1.96*	2.21*	5439	9.34* -2.70* -2.40* -2.11* 10229
15	17.22*	4.79*	2.08*	2.37*	4240	10.09* -1.89* -1.98* -1.71* 11428
20	17.35*	4.59*	1.68*	2.02*	3016	10.75* -1.20* -1.50* -1.24* 12652
25	17.29*	4.38*	1.51*	1.85*	2123	11.19* -0.78* -1.26* -1.00* 13545

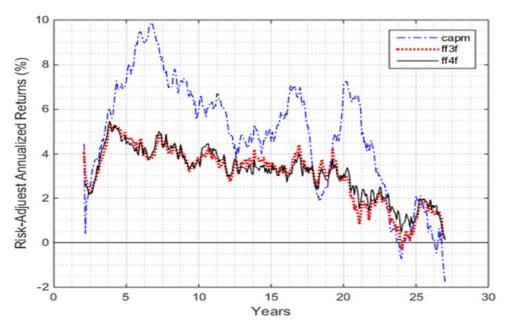
Panel C: Active Firms

Firm		Meet	data require	ement		Ι	o not meet	data requir	ement	
Age	RET	CAPM	FF3F	FF4F	N	RET	CAPM	FF3F	FF4f	N
2.5	19.88*	6.65*	6.19*	5.77*	3177	72.10**	10.43	11.04	18.23**	31
3	19.48*	6.84*	6.33*	5.92*	3119	52.33*	1.16	3.13	4.99	89
4	18.98*	7.46*	6.83*	6.31*	3003	40.96*	-4.60	-2.37	-0.20	205
5	19.03*	7.95*	7.25*	6.70*	2932	34.86*	-6.77*	-4.52	-2.65	276
6	18.92*	7.96*	7.25*	6.68*	2880	33.28*	-4.51**	-2.63	-1.04	328
7	18.64*	8.12*	7.42*	6.86*	2740	30.65*	-1.68	-0.68	0.20	468
8	18.31*	8.07*	7.34*	6.86*	2604	29.33*	0.71	1.51	1.73	604
9	18.53*	8.16*	7.42*	7.01*	2482	26.74*	1.64	2.22	2.08	726
10	18.80*	8.49*	7.70*	7.33*	2354	24.78*	1.71	2.23**	1.92	854
12.5	19.03*	8.77*	7.94*	7.59*	2195	23.32*	2.17*	2.55*	2.22**	1013
15	19.19*	8.84*	8.00*	7.69*	1914	22.16*	3.51*	3.64*	3.23*	1294
20	19.24*	8.08*	6.98*	6.84*	1474	21.36*	5.50*	5.61*	5.09*	1734
25	18.88*	7.74*	6.25*	6.28*	1090	21.16*	6.14*	6.23*	5.69*	2118

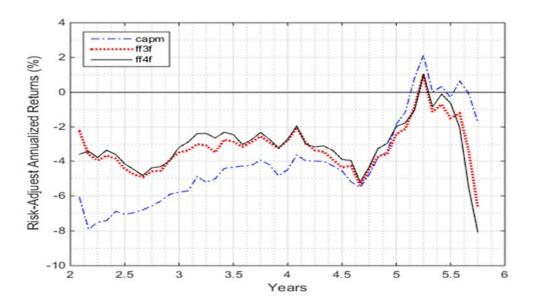


**Figure 3. Risk-Adjusted Annualized Returns of Survivors and Non-survivors**. This figure depicts risk adjusted returns for firms that survive beyond T years (Survivors) and for firms that survive less than or equal to T years (non-survivors). capms, ff3fs and ff4fs refer to the average of pricing errors for survivors based on the CAPM, Fama and French three factor and Carhart four factor models. Similarly, capmns, ff3fns and ff4fns refer to the average of pricing errors for non-survivors based on the CAPM, Fama and French three factor and Carhart four factor models.

Results from Table III, I find that average risk-adjusted returns are higher for firms that live longer. Therefore, as I add firms with longer survival time, the cross-sectional mean risk adjusted returns increases. In other words, performance measures strengthen as I use progressively more data for firms surviving longer periods into the future. These results are robust to choice of factor models. Further, I find that average risk adjusted returns for active firms is higher than non-active firms. For instance, annualized mean risk adjusted return based on CAPM model for the firms which survive beyond five years is 7.95 percent among active firms and 3.20 percent among non-active firms. Similarly, active firms which do not meet data requirement outperform similar non-active firms across all data requirement criteria and models chosen.



**Figure 4. Long Lived Firms' Performance by Age.** This figure illustrates the average performance by age for firms which survive six years or more. The capm, ff3f and ff4f show risk-adjusted average returns based on CAPM model, Fama and French three factor model and Carhart four factor mdoel.



**Figure 5. Short Lived Firms' Performance by Age.** This figure illustrates the average performance by age for firms which survive less than six years. The capm, ff3f and ff4f show risk-adjusted average returns based on CAPM model, Fama and French three factor model and Carhart four factor mdoel.

Further, firms that live longer have positive performance since the beginning of their life and firms with negative performance in their early life tend to die earlier. In order to calculate average performance by their age, I split the whole sample of firms into two groups: firms that survive six or more than six years and firms which live less than six years. Then I compute cross sectional mean for each group by averaging rolling alphas for a specific survival time. For instance, to calculate cross sectional mean for year T, I average of all T<sup>th</sup> year alphas of all firms in each group. I choose sixth year as a cutoff point because I observe in Table II that firms which survive six years or less experience negative risk adjusted returns. Figure 3 plot cross-sectional mean of all firms which survive less than six years. As I can see from Figure 4 cross-sectional mean of all firms which survive less than six years. As I can see from Figure 3 that firms which survive longer period have positive risk adjusted returns since the early period of their life. On the contrary, firms that do not survive beyond six years have negative risk adjusted average risk returns. Further, mean return based on CAPM alpha has the highest absolute value and mean returns based on three and factor models are very similar. For each group, the mean risk-adjusted returns is increasing in the early period of firms' life and it starts to decline after some years.

This finding clearly has implications on calculation of long-run returns. For instance, IPO underperformance phenomenon is clearly not applicable for certain firms which survive over long period of time.

#### 3.4 Cross-sectional variation of performance

Table IV reports the standard deviation of annualized returns. The standard deviation is decreasing for both survivors and non-survivors as survival time increases. This is primarily due to the nature of sample construction. The survivors with greater survival requirements are subsets of the samples of firms with only two and half years of survival requirement. Therefore, as I require firms to survive longer, I systematically exclude smaller and more risky firms. As a result, I observe declining standard deviations for firms that survive longer periods into the future. For the same reason, I notice decreasing variability of alphas for non-surviving firms.

Table IV
Standard Deviation of Cross-Sectional Alphas

This table reports annualized standard deviation of average firm alphas conditional on a firm meet or does not meet data requirement. Firms that meet data requirement include those firms that live beyond T years and firms that do not meet data requirement include those firms that live at most T years. The first column represents firms' age. Columns 2 and 6 shows standard deviations of firms mean returns for firms that meet data requirement criteria and firms that do not meet data requirement criteria. Columns 3 to 5 shows standard deviations of average pricing errors obtained from CAPM, Fama and French three factor and Carhart four factor models for survivors. Columns 7 to 9 presents standard deviation of average pricing errors based on CAPM, Fama and French three factor and Carhart four factor models for non-survivors.

Firm		Meet data r	equiremen	t	Do not meet data requirement				
Age	RET	CAPM	FF3F	FF4F	RET	CAPM	FF3F	FF4	
2.5	54.43	28.10	30.36	30.60	288.29	62.51	69.92	71.90	
3	42.92	25.93	28.19	28.25	211.52	57.92	63.24	65.45	
4	32.01	22.45	24.38	24.43	158.63	52.18	57.54	58.69	
5	26.81	19.72	21.84	21.88	133.52	48.30	52.66	53.51	
6	23.33	17.66	20.09	20.11	119.14	45.36	49.03	49.76	
7	21.21	16.16	18.52	18.60	109.21	42.94	46.46	47.04	
8	19.12	14.81	17.05	17.17	102.57	41.25	44.66	45.16	
9	17.16	13.80	16.02	16.06	98.01	39.93	43.22	43.72	
10	16.29	13.14	15.36	15.42	93.74	38.57	41.74	42.20	
12.5	14.19	11.88	13.97	14.00	86.75	36.28	39.25	39.67	
15	12.54	10.99	12.95	13.09	81.84	34.58	37.41	37.78	
20	11.24	9.11	10.45	10.51	77.17	33.02	35.77	36.12	
25	11.11	8.79	9.64	9.78	74.05	31.82	34.48	34.82	

I also see that the standard deviation of pricing errors for non-surviving firms is larger than surviving firms, which indicates that non-surviving firms are generally risker.

Overall, the above results indicate that surviving firms differ from non-surviving firms across different characteristics. This suggests that results reported in long-horizon studies may be erroneous and such studies need additional sensitivity tests before concluding abnormal price performance following major events.

## 3.5 Correlation between performance and survival

To obtain further economic insight into the nature of relationship between survival and returns conditional on survival, I also analyze whether survival is correlated with performance using Pearson and Spearman's rank correlations. The results are presented in Table V. I find a declining value of the correlation coefficient between firm performance and survival time for the firms that meet data requirement and an increasing value of the coefficient for firms that do not meet data requirement as minimum number of months for prior data requirement increases for all both samples considered. Interestingly, the correlation coefficient goes from positive to negative for firms that meet data requirement and from negative to positive for firms that do not meet data requirement as the survival time increases. All these correlation coefficients are significantly different from zero. The results are consistent across all samples whether I measure the relation between survival and performance using rank correlation or Pearson correlation.

I also investigate why the correlation coefficient turns negative as I require firms to have longer period of prior data. I find that the covariance can be negative without a negative correlation between pricing error and survival time. Since the variance of pricing error is function of survival time, it increases for samples with increasing values of data requirement but starts to decline after certain time.

#### 3.6 Cox proportional hazards model for firm survival

To examine the correlation between firm survival and performance further, I estimate the Cox proportional hazard regression model parameters using my data. I estimate the following basic form which does not include time-dependent covariates or non-proportional hazards:

$$h_i(t|alpha_i) = h_0(t) e^{\beta alpha} \tag{4}$$

where  $h_i(t|alpha_i)$  is the expected hazard function for i<sup>th</sup> firm,  $h_0(t)$  is an arbitrary and unspecified baseline hazard function for continuous time T,  $\beta$  is coefficient vector and X is a set of explanatory variable which is risk-adjusted returns in my case. The parameters of model (4) are estimated using a maximum partial likelihood procedure, based on the conditional probability that a firm dies at time t.

Table V

Correlation between Survival and Average Pricing Errors

This table presents information about correlation between average pricing errors and firms' survival time. Firms that meet data requirement include those firms that live beyond T years and firms that do not meet data requirement include those firms that live at most T years. RET refers to the correlation between risk unadjusted firm level mean returns and firms' survival time. CAPM, FF3F and FF4F represent correlation between survival and firms' average pricing errors obtained from CAPM, Fama and French three factor and Carhart four factor models. Censored firms include those firms that are still alive at the end of the sample period. Non-censored sample include all firms excluding censored firms. Numbers in the brackets represent t-statistics for testing the hypothesis that the population correlation coefficient is zero.

Firm	Meet data requirement		Do not m	Do not meet data requirement		
Age	CAPM	FF3F	FF4F	CAPM	FF3F	FF4F
2.5	0.148*	0.099*	0.097*	0.003	-0.007	-0.005
3	0.144*	0.102*	0.101*	-0.030	-0.019	-0.015
4	0.114*	0.084*	0.085*	0.005	-0.011	-0.011
5	0.084*	0.073*	0.076*	0.041*	0.027	0.024
6	0.057*	0.061*	0.065*	0.069*	0.037*	0.032*
7	0.028*	0.049*	0.053*	0.092*	0.051*	0.046*
8	0.013	0.044*	0.049*	0.113*	0.064*	0.058*
9	-0.001	0.036*	0.038*	0.123*	0.068*	0.060*
10	-0.031*	0.018	0.021**	0.127*	0.070*	0.062*
12.5	-0.072*	-0.031*	-0.023**	0.145*	0.076*	0.069*
15	-0.120*	-0.097*	-0.082*	0.155*	0.084*	0.078*
20	-0.140*	-0.107*	-0.092*	0.166*	0.103*	0.097*
25	-0.160*	-0.118*	-0.112*	0.167*	0.109*	0.103*

How sensitive firm survival is on performance is typically based on comparisons of hazard ratios. Given estimates of the regression parameters in the model (4), the hazard ratio if risk-adjusted return is incremented by one unit is given by

Hazard Ratio (HR) = 
$$\frac{h_0(t) e^{\beta (alpha+1)}}{h_0(t) e^{\beta alpha}} = e^{\beta}$$
 (5)

Using the equation (5), I interpret 100\*(HR-1) as the percentage change in the hazard rate with a 1-unit change in the risk-adjusted return.

Table VI

Parameter Estimates and Hazard Ratios for Cox Proportional Hazard Model of Firm
Survival

This table presents maximum likelihood parameter estimates and hazard ratios for Cox proportional hazard model of firm survival. Full and sub-sample periods refer to December 1925 to December 2013 and December 1972 to December 2013, respectively. The full sample consists of 18,876 observations and the sub-sample has 17,596 observations. The CAPM, FF3F and FF4F refer to risk-adjusted returns estimated using the CAPM, the Fama and French three factor model, and the Carhart four factor model. Panel A reports regression estimates using entire period alpha estimates. Panel B reports regression estimates using sub-sample alpha estimates. The generalized R<sup>2</sup> is computed as  $(1 - \exp(-likelihood ratio^2/sample size))$ . Alpha estimates are multiplied by 100.

Panel A: Current alphas as covariates: full period							
Predictor	Hazard ratio	Estimates	Std. error	Chi-square	Generalized R <sup>2</sup>		
CAPM	0.827	-0.190	0.005	1399.253	0.023		
FF3F	0.883	-0.125	0.005	609.992	0.24		
FF4F	0.889	-0.118	0.005	550.301	0.286		

Panel B: Current alphas as covariate-subsample							
Predictor	Hazard ratio	Estimates	Std. error	Chi-square	Generalized R2		
CAPM	0.831	-0.185	0.005	1344.527	0.165		
FF3F	0.894	-0.112	0.005	513.552	0.462		
FF4F	0.896	-0.110	0.005	490.752	0.470		

Table V presents the maximum likelihood parameter estimates and the hazard ratios. The Cox regression estimates are similar across all asset pricing in both panels. As seen from the table that the probability a firm disappears decreases significantly as the firm's alpha estimate increases. The hazard ratio of 0.827 in Panel A for CAPM alpha suggests that one additional unit increases in risk-adjusted return would lower firm's mortality by about 17.3 percent; this suggests very strong relationship between survival and performance. Results in the Table VI also indicate that Carhart four factor model alphas have the post predictive power of firm disappearances as measured by the generalized R2. Overall, the Cox regression estimates indicate that, no matter which asset pricing model I choose, performance correlates significantly with firm survival.

## 4. Measuring Relation Between Survival and Performance and Their Conditional Expectations

I assume that pricing errors ( $\epsilon$ ) and survival times (s) are not independent and the mean value of the pricing error depends on survival time. Therefore, the relationship between conditional expectation of the pricing error and the fixed value of survival time is of special interest. Moreover, the dependence properties between the pricing error and survival time is also very important. In the regression of pricing errors on survival time, I choose the Farlie-Gumbel-Morgenstern (FGM) bivariate distribution of Morgenstern (1956), Farlie (1960) and Gumbel (1958). It provides a very simple and direct method of constructing a bivariate distribution given its marginal distributions and the correlation between the variates. I choose FGM distribution for its simplicity and applicability when the association between variables is relatively low. The details of the derivation of conditional probabilities and moments are presented in Appendix A.

The FGM bivariate distribution has a joint cumulative distribution function of the form

$$H(\varepsilon, s) = F(\varepsilon)G(s) \left[ 1 + \alpha \left( 1 - F(\varepsilon) \right) \left( 1 - G(s) \right) \right] \quad (|\alpha| < 1)$$

where  $H(\varepsilon, s)$  is the joint cumulative distribution function of  $\varepsilon$  and s, and  $F(\varepsilon)$  and G(s) are arbitrary distribution functions on random variates  $\varepsilon$  and s with the degree of association  $\alpha$ . Lai (1978) shows that the parameter  $\alpha$  is directly proportional to the correlation coefficient. For absolutely continuous marginal distributions, I need  $|\alpha| < 1$ . It is easy to verify that  $\varepsilon$  and s are positively quadrant dependent if  $\alpha > 0$ .

I assume that each firm's alpha or pricing error is drawn from a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . It is denoted as  $\varepsilon \sim N(\mu, \sigma^2)$  with its density function denoted by  $(\varepsilon)$ . Further, I assume that survival time is exponentially distributed. It is denoted as  $s \sim \exp(\lambda)$  with its density function denoted by g(s). As shown in the Appendix B, under this formulation  $\alpha = 3 \rho$ , where  $\rho$  is correlation structure in FGM distribution. Correlation modeled under the distribution is limited to  $\rho \leq \left|\frac{1}{3}\right|$ , but it provides a simple closed form distribution linking random

variables of dissimilar distributions. From equation (4), the density function  $h(\varepsilon, s)$  can be explicitly written as

$$h(\varepsilon, s) = \frac{\partial H(\varepsilon, s)}{\partial \varepsilon \partial s} = \frac{e^{-2s\lambda - \frac{(\varepsilon - \mu)^2}{2\sigma^2}} \lambda (e^{s\lambda} - 3(-2 + e^{s\lambda})\rho \operatorname{Erf}[\frac{-\varepsilon + \mu}{\sqrt{2}\sigma}])}{\sqrt{2\pi}\sigma}$$
(5)

where the error function (Erf) of x is defined as  $\frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$ .

Generally, I assume that firm birth month and survival rates are independent. But I observe in the CRSP data base that these two variables are highly correlated (rank correlation between these two variable is 0.46 over the full period sample, 0.99 in the censored full period sample and 0.52 in the non-censored full period sample). Therefore, I assume the birth variable to be a convex combination of the survival rate and another exponentially distributed random variable u with mean  $1/\theta$ . Therefore,  $t = \omega s + (1 - \omega) u$ . Assuming s and u are independent and  $u = (t - \omega s)/(1 - \omega)$ , joint probability density function of s and t with change of limits can be written as

$$\phi(s,t) = \frac{e^{-\frac{(t-s\,\omega)\theta}{1-\omega} - s\lambda}\theta\lambda}{1-\omega}$$
(6)

Now, the joint density function of the average pricing error, survival rate and birth rates,  $h(\varepsilon, s, t)$ , becomes

$$l(\varepsilon, s, t) = h(\varepsilon, s) \phi(s, t) \tag{7}$$

Equation (7) is used to calculate conditional moments of the average pricing error. From equation (7) the conditional density function of  $\varepsilon_i$ , given  $s_i$  and  $t_i$ , is

$$l(\varepsilon_{i}|s_{i},t_{i}) = \frac{e^{-s\lambda - \frac{(\varepsilon - \mu)^{2}}{2\sigma^{2}}}(e^{s\lambda} - 3(-2 + e^{s\lambda})\rho \operatorname{Erf}[\frac{-\varepsilon + \mu}{\sqrt{2}\sigma}])}{\sqrt{2\pi}\sigma}$$
(8)

The FGM distribution has the advantage property that the variables can be separated as in equation (5). Given this, the conditional expectation of  $\varepsilon$  given s and t, denoted by  $\alpha(\varepsilon|s,t)$  is obtained directly from equation (8) as

$$\alpha(\varepsilon|s,t) = E(\varepsilon|s,t) = \int_{-\infty}^{\infty} \varepsilon \ l(\varepsilon|s,t) \, d\varepsilon = \mu + \frac{6\rho\sigma}{\sqrt{\pi}} \left( G(s) - \frac{1}{2} \right) \tag{9}$$

The conditional expectation increases for  $\rho > 0$  and decreases for  $\rho < 0$  with increasing values of s. The framework for modelling survival and pricing errors described by equations (4) and (9) matches the observed distribution of the average pricing errors- average pricing errors are low for firms that disappear early on, and increase monotonically as firms live longer.

To calculate the variance of the pricing errors, I need the expectation of the squared pricing errors, which can be calculated as:

$$E(\varepsilon^{2}|s,t) = \int_{-\infty}^{\infty} \epsilon^{2} \ l(\varepsilon|s,t) \, d\varepsilon = \mu^{2} + \sigma^{2} + \frac{6\rho\mu\sigma}{\sqrt{\pi}} (2G(s) - 1)$$
 (10)

Given the expectation of the pricing error in equation (9) and the squared pricing errors in equation (10), the conditional variance,  $\sigma^2(\alpha|s,t)$ , of  $\alpha$  can be computed as

$$\sigma^{2}(\alpha|s,t) = E(\varepsilon^{2}|s,t) - \{E(\varepsilon|s,t)\}^{2} = \sigma^{2} - \left(\frac{6\rho\sigma}{\sqrt{\pi}}(G(s) - 0.5)\right)^{2}$$
(11)

The conditional variance also matches the volatility of the distribution of estimated alphas. Further, the conditional covariance,  $Cov(\alpha, s|\Omega)$  as a function of s is

$$Cov(\alpha, s|\Omega) = E[(\varepsilon - E(\varepsilon|\Omega)), (s - E(s|\Omega))]$$
(12)

Since the restrictions are on the survival and arrival times and not on the distribution of  $\alpha$ , I use equations (9), (11), and (12) to compute the conditional expectation, variance, and covariance of the average pricing errors respectively. Explicitly,

$$E(\alpha|\Omega_{m}) = \int \int E(\alpha|s)\phi(s,t) dt ds$$
 (13)

$$Var(\alpha|\Omega_{m}) = \int \int \sigma^{2}(\alpha|s) \,\phi(s,t) \,dt \,ds \tag{14}$$

$$Cov(\alpha, s | \Omega_{m}) = \int \int Cov(\alpha, s | \Omega_{m}) \phi(s, t) dt ds$$
 (15)

In calculating the conditional expectation of the survival and birth time, I can simply use the bivariate distribution of s and t because the effect of the normal variate simply integrates out. Therefore, the conditional expectation of survival time, birth time, and the covariance between survival and arrival times can be computed as

$$E(s|\Omega)) = \int \int s \,\phi(s|t) \,dt \,ds \tag{16}$$

$$E(t|\Omega_{ss}) = \int \int t \,\phi(s|t) \,dt \,ds \tag{17}$$

$$Cov(s,t|\Omega) = E[(s - E(s|\Omega))(t - E(t|\Omega))]$$
(18)

The moments in equation (13) and equation (14) are the mean and standard deviation of the estimated alpha distribution. The moment in equation (15) is the conditional covariance between survival and average pricing errors. This moment condition is a consistent estimator of the correlation coefficient. The moments in equations (16) and (17) are the expectations of my exponentially distributed survival time and arrival time. The moment in equation (18) is the conditional covariance between arrival and survival time. In addition to these conditional expectations, I have percent survived, percent censored, and percent non-censored. Thus, I have a total of seven moment conditions. I use these seven moment conditions to identify six structural parameters  $(\mu, \sigma, \rho, \lambda, \theta, \omega)$  for each information set.

The parameters of my system of equations are estimated by using the conditional moment approach to generalized methods of moments (GMM) estimation technique. GMM, established by Hansen (1982), is quite useful for estimation of nonlinear equation systems. The main advantage of the GMM estimator is that it provides efficient estimates of parameters in the presence of heteroscedastic errors and when the form of the heteroscedasticity is unknown. If I let Z be a random vector that includes both endogenous and explanatory variables, and  $\varphi(Z,\beta)$  be a vector of conditional moment functions, the conditional moment estimation principle rests on the assumption that the probability distribution of Z satisfies the following conditional moment restrictions

$$E(\varphi(Z, \beta_0)|X) = 0 \tag{19}$$

where  $\beta_0$  denotes the unknown parameter vector  $(\mu, \sigma, \rho, \lambda, \theta, \omega)$  and X is a vector of conditioning variables or instruments. If I denote the matrix of instruments by A(X), then a vector of unconditional moment functions  $\gamma(Z, \beta)$  can be obtained from the product

$$\omega(Z,\beta) = A(X)\gamma(Z,\beta) \tag{20}$$

By applying the law of iterated expectations and using equation (19), equation (20) can be evaluated such that the true parameter vector satisfies the following condition

$$E(\gamma(Z, \beta_0)) = 0 \tag{21}$$

The orthogonality conditions defined in equation (21) can be used for the estimation of the unknown parameter vector  $\beta_0$ . The GMM estimator  $\hat{\beta}$  of unknown parameter vector  $\beta_0$  is defined as the vector minimizing the objective function

$$\hat{\beta} = agr \min_{\beta} (\gamma(Z, \beta))' W(\gamma(Z, \beta))$$
 (22)

where  $\beta$  is a five-by-one vector of the parameters of interest, and W is an arbitrary positive-definite weighting matrix. I utilize the inverse of the estimated covariance of moments as a weighting matrix. I use Hansen's J-statistic as a specification test to examine whether the data are consistent with the model.

#### 5. Estimation Results

The results of the six parameters estimates from the moment conditions are presented in Table VII. Panel A and Panel B report the estimated results for full dataset and sub-sample respectively. In both panels, the first parameter,  $\rho$ , measures the correlation between survival time and average pricing errors. The second and third parameters,  $\sigma$  (the scale parameter) and  $\mu$  (the location parameter), describe the shape of the distribution of alphas. The fourth and fifth parameters,  $\lambda$  and  $\theta$ , describe the hazard rates of survival time and the other exponential variable used to model birth rates. The last parameter,  $\omega$ , measures what proportion of the birth time is being explained by the survival time.

Panel A in Table VII presents parameter estimates using entire data set. Columns (2) to (5) of Panel A in Table VII reports results for raw returns, alphas from CAPM, three factor and four factor models respectively and without censoring. Similarly, columns (6) to (9) of Panel A in Table

VII show parameter estimates with censoring and using raw returns, alphas from CAPM, three factor and four factor model. When the parameters of the model are fitted to raw return data, I find annualized mean return returns of 12.41 percent without censoring and 12.51 percent with censoring. But, the estimates of the mean of the distribution of average pricing errors show that the survival-adjusted mean return is statistically insignificant. This finding does not depend on the asset pricing model used to compute risk-adjusted returns. This result is very much consistent with assumption of the model and efficient market hypothesis.

The volatility of the distribution of average pricing error is similar for the CAPM, Fama and French three factor, and Carhart four factor alphas with and without censoring. All estimates are statistically significant.

One of the major focuses of this paper is to look at the relationship between pricing errors and survival times. The estimates of the  $\rho$ , in Table VII, show that the average pricing errors and survival times always have a positive and statistically significant relation for all performance measures. However, the absolute value of the parameter is very low. For instance, the relation between the pricing error based on CAPM model and survival time is 1.93 percent and between the pricing error based on four factor model and survival time is 1.10 percent. I get similar results with and without censoring.

The finding of low correlation between pricing error and survival time is consistent with my assumptions of the model and it has important implication on the relationship between average pricing error and survival time. Even if there is low correlation, such as less than two percent, between survival and pricing error, the correlation between average pricing error and survival can go up to 14.8 percent or can go down to -16.7 percent based on requirement for survival. As the results in Table V indicate that the correlation between average pricing error and survival time is always higher than 2 percent and statistically significant.

The estimates of the parameter  $\lambda$  are significant and very similar across all samples and for all measures of performance. The estimates of  $\omega$  are also all significant. The parameter  $\omega$  sheds some light on the relationship between survival time and birth rates. There is a stronger relationship between survival time and birth rate for end of the sample surviving firms than non-survivors.

Table VII
Estimates of the Conditional Moments: Full Sample

This table presents the parameter estimates of my model. I estimate the model parameters using moments and conditional moments computed from survival time, arrival time, mean firm level returns and alphas generated from CAPM, Fama and French three factor and Carhart four factor models. The five parameters of my model are the mean and variance of the distribution of average pricing errors,  $\mu$  and  $\sigma^2$ ; the correlation coefficient between firms' survival time and their average pricing errors,  $\rho$ ; and the hazard rate parameters  $\lambda$  and  $\theta$  for survival and birth variables. These parameters are estimated using Generalized Methods of Moments (GMM) estimation technique. The estimates of  $\mu$  are annualized in this table. Panel A reports parameter estimates for entire dataset, and Panel B show estimates for sub-sample. The entire sample includes all firms that have at least 24 months of continuous returns over the period December 1925 to December 2013. The total number of firms included in this sample is 18876. The sub-sample consists of 17,596 firms which have at least 24 months of returns data over the period December 1972 to December 2013. The estimate of the model parameters using each model are presented under its name. t-Statistics are reported parentheses.

Panel A: Full Sample

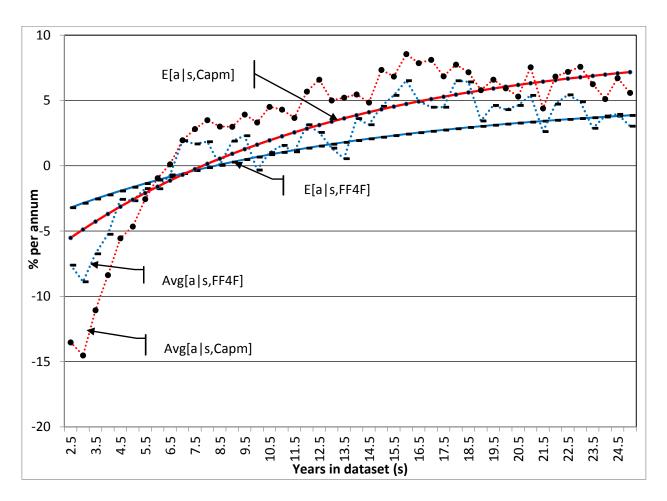
	1		With Censoring						
Parameter	RET	CAPM	FF3F	FF4F	-	RET	CAPM	FF3F	FF4F
ρ	0.0073	0.0193	0.0113	0.0110	-	0.0074	0.0195	0.0114	0.0111
	(6.84)	(20.04)	(11.68)	(11.36)		(6.83)	(20.00)	(11.65)	(11.33)
σ	0.4859	0.2074	0.2268	0.2292		0.4855	0.2075	0.2268	0.2292
	(33.51)	(92.02)	(70.04)	(64.63)		(33.45)	(92.30)	(70.08)	(64.65)
μ	12.4176	-0.0888	-0.4116	-0.2328		12.5184	0.0252	-0.3384	-0.1620
	(19.66)	(-0.33)	(-1.44)	(-0.81)		(20.27)	(0.09)	(-1.21)	(-0.57)
λ	0.0070	0.0070	0.0070	0.0070		0.0066	0.0066	0.0066	0.0066
	(120.98)	(120.98)	(120.98)	(120.98)		(118.14)	(118.14)	(118.14)	(118.14)
θ	na	na	na	na		0.0011	0.0011	0.0011	0.0011
						(53.12)	(53.12)	(53.12)	(53.12)
ω	na	na	na	na		0.7110	0.7110	0.7110	0.7110
						(128.21)	(128.21)	(128.21)	(128.21)

Table VII - Continued

Panel B: Sub-Sample

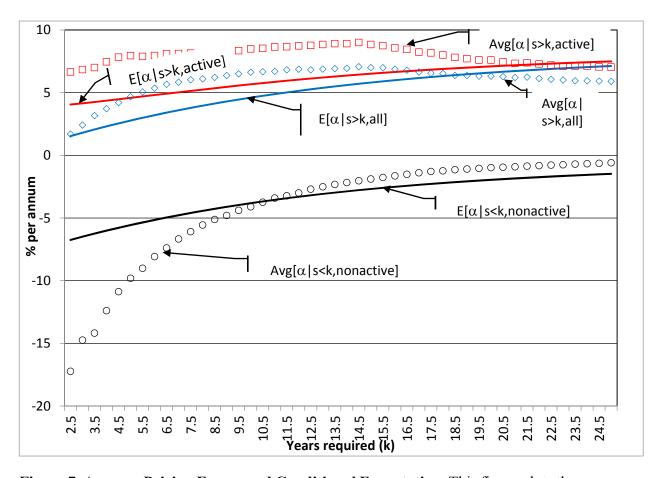
	Without (	Censoring						
Parameter	RET	CAPM	FF3F	FF4F	RET	CAPM	FF3F	FF4F
ρ	0.0092	0.0212	0.0109	0.0112	 0.0097	0.0224	0.0115	0.0118
	(7.98)	(20.23)	(10.48)	(10.67)	(7.98)	(20.17)	(10.43)	(10.63)
σ	0.5015	0.2125	0.2335	0.2353	0.5010	0.2125	0.2335	0.2353
	(33.66)	(90.85)	(69.61)	63.92)	(33.58)	(91.06)	(69.60)	(63.90)
μ	13.2288	-0.2100	-0.4920	-0.3876	13.1352	-0.3024	-0.5424	-0.4404
	(18.58)	(-0.71)	(-1.54	(-1.21)	(18.24)	(-1.01)	(-1.67)	(-1.35)
λ	0.0074	0.0074	0.0074	0.0074	0.0072	0.0072	0.0072	0.0072
	(111.14)	(111.14)	(111.14)	(111.14)	(119.25)	(119.25)	(119.25)	(119.25)
θ	0.0004	0.0004	0.0004	0.0004	0.0001	0.0001	0.0001	0.0001
	(25.18)	(25.18)	(25.18)	(25.18)	(2.33)	(2.33)	(2.33)	(2.33)
ω	0.6870	0.6870	0.6870	0.6870	0.3794	0.3794	0.3794	0.3794
	(48.83)	(48.83)	(48.83)	(48.83)	(16.02)	(16.02)	(16.02)	(16.02)

To account for the effect of truncation, I ran the analysis on both the entire dataset and its subsample covering the period 1972 to 2013. The estimated parameters of the model using the subsample data is presented in Panel B of Table VIII. A comparison between the estimates from the entire sample and sub-period shows that the parameter estimates are very similar. The estimate of  $\mu$  based on risk-unadjusted returns is positive and statistically significant for both with censoring and without censoring. Similarly, estimates of  $\mu$  based on risk-adjusted returns are all statistically insignificant. Estimates of  $\sigma$  are similar across all models whether the estimates are based on with censoring and without censoring. Likewise, there is a positive correlation between survival time and pricing errors and the absolute value of correlation coefficients is less than 2 percent. Taken together, these conditional moment estimates indicate that survival adjusted-returns are statistically significant. These returns depend on data selection and also on the choice of model used to compute risk-adjusted returns. Additionally, I find that the average pricing errors and survival rates have a positive and statistically significant relation.



**Figure 6. Average Risk-Adjusted Returns Conditional on Survival and Conditional Expectation Given Survival Time.** This figure shows average performance of firms in the entire sample by their age. The Avg[a|s, Capm] and Avg[a|s, FF4F] represent cross-sectional average of risk-adjusted returns of firms from CAPM, and Carhart four factor model, respectively. Similarly, he solid line represents the regression curve (conditional expectation of average pricing error given survival time) fitted to the data. The E[a|s, Capm] and E[a|s, FF4F] denotes regression line fitted to CAPM alphas and FF4F alphas.

Figure 6 depicts how well the model is fitted to the data. In figure 6, I plot cross-sectional mean pricing errors computed from CAPM and four factor models along with regression lines fitted to alpha distribution obtained from each of these models. My model fits well to the data. It explains 39.66 percent of the CAPM alphas and 47.17 percent of the four factor alphas. My model shows that short-lived firms have negative alphas whereas long-lived firms have positive alphas implying that long-lived firms outperform short-lived firms.



**Figure 7. Average Pricing Errors and Conditional Expectation.** This figure plots the average pricing errors computed from CAPM model and expected pricing errors for active firms, non-active firms and all firms conditional on data requirement. Active firms include 3208 end of sample surviving firms, non-active firms include 15668 end of sample non-surviving and all firms include both types of firms. Dots represent cross-sectional averages for samples obtained by increasing minimum survival time. The solid lines represent expectation of average pricing errors given survival time.

Figure 7 shows mean pricing errors computed from CAPM model and average pricing errors computed from my model for active, non-active and all firms. The samples are selected based on whether firms have k number of months of continuous returns or not.  $\text{Avg}[\alpha|s > k]$  refers to cross-sectional sample mean for firms that live beyond k number of months and  $\text{Avg}[\alpha|s < k]$  represents cross-sectional mean for firms that live at most k number of months. From figure 7 we can see that use of my model can help to correct for survival bias. For instance, the difference in average performance between the non-active firms which live at most five years and all firms which live beyond five years is 14.49 percent per year based on data but it is 8.76 based on my

model. Thus, the use of my model helps us to correct the discrepancy of performance by 5.73 percent. Further, Figure 1 shows that use of my model help to correct for survivorship and non-survivorship bias. Generally, the use of alpha distribution for measuring abnormal returns for firms which survive less than five years under-estimates average alphas and over estimates average alphas for firms that survive more than five years. By using my model, both of these estimates can be corrected.

## 6. Conclusions

Survival or non-survival has serious potential effects on empirical financial research that use historical data. I analyze return characteristics of different samples of firms conditional on survival time and look at the relationship between firms' survival and average pricing errors. I measure firm performance using average pricing errors and document that short-lived firms underperform long-lived firms both in terms of riskiness and risk-adjusted returns. In the CRSP data base, cross-sectional average performance of firms that survive long time period (more than six years) begins to increase as I exclude firms which survive short period (less than six years). Similarly, average performance of firms which survive only short period starts to increase as I start to add surviving firms into the sample. The main finding from the analysis of the performance of long-lived firms and short lived firms is that short-lived firms significantly under-perform long lived firms. Further, active firms or end of sample period surviving firms out-perform non-active or end of sample period non-surviving firms.

The firms which perform badly in early period of their life are likely to drop out of the data base. Likewise, the firms which survive long period of time, beyond six years, experience positive risk-adjusted returns from the first listing date in CRSP data base. The positive risk adjusted return pattern of long time surviving firms is against the well documented long-run underperformance of IPOs. I observe underperformance of IPOs only if firms which survive less than six years are included in the sample. If I exclude all the firms which survive less than six years, the average performance of surviving firms is positive and statistically significant.

One of the main findings of this paper is that I show a positive and statistically significant correlation between survival and average pricing errors. To examine the relation between survival and average pricing errors, I develop a model using the bivariate FGM distribution function and

fit its moment conditions to the sample data. I find that, survival-adjusted mean returns are no different from zero for both samples. The estimates are very similar with and without censoring.

I also find a very low correlation between pricing errors and survival time but very high correlation between average pricing errors and firm survival time. Using my model, I show that even a low correlation between firms' survival time and pricing errors can result in high correlation between average pricing errors and survival time.

My result may have been influenced by the equal weighting scheme for aggregating individual firm returns to cross-sectional average returns. My results may change if I evaluate average performance of firms using a value weighted average because non-survivors are generally smaller and their value is overemphasized with equal-weighting. Also, as I find a positive relation between survival and performance, value-weighting could be more appropriate to reflect firm performance.

The findings of this paper have implications for tests of market efficiency. One of the popular tests of market efficiency is to examine post-event price performance following some kind of major corporate event. These studies often report persistence of positive or negative abnormal returns by comparing performance of events experiencing firms against event non-experiencing firms. These studies never consider survival time in making reference portfolios. As I have shown in this paper, the performance characteristics of long-lived firms and shot-lived firms are different. If these differences are not accounted properly in calculating abnormal returns, it can lead to significant distortions in the performance figures reported in the long-run event studies. Additionally, cross-sectional regression tests of market efficiency could also be affected by survival and non-survival biases examined in this paper.

The findings of the paper also has implication for portfolio management. I can utilize the return characteristics of young and old active firms to form portfolio. Since young firms underperform old firms, I can make long-short portfolio to generate higher returns.

It is difficult to precisely assess the effect of survival or non-survival bias on the results of past studies that exclude firms that do not survive a specific number of years. It is always suggestive to report proportion of survival and non-survival and the potential effect if the non-survivors would have been included in the sample. Furthermore, I also observe differences

between the performance measures that depend on the choice of asset pricing model. Choosing the appropriate model can help to reduce the survival and non-survival biases in empirical studies. In addition, attention must also be given to matching firms based on survival time while comparing returns of event and controlling firms.

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# **CHAPTER 2**

# The Impact of Financialization on the Benefits of Incorporating Commodity Futures in Actively Managed Portfolios

# 1. Introduction

Over years, investors have viewed commodities as a separate asset class that can offer high returns, provides diversification benefits and protects against inflation. Early studies by Bodie and Rosansky (1980) and Fama and French (1987) both document that investors can reduce their risk without having to sacrifice returns by simply adding commodities to a portfolio of stocks. Similarly, Bessembinder (1992) and Roon et al. (2000) showed that commodity provided risk premium for idiosyncratic commodity price risk. Studies by Gorton and Rouwenhorst (2006) and Erb and Harvey (2006) provided some evidence of low correlation between equity and commodity and equity like returns of tactical commodity portfolios. The Standard portfolio choice theory also states that adding an alternative asset that has low correlation with traditional assets to the investor's investment universe would reduce the risk of the portfolio.

However, recent studies, such as Irwin and Sanders (2011), Singleton (2013), Tang and Xiong (2012), Silvennoinen and Thorp (2013), and Henderson et al. (2012) examine different aspects of the "financialization" of the commodity futures market—that is, how increased investors' participation via large flows from speculators and other market participants have impacted the price dynamics of investing in commodity futures. In general, these and related papers find higher correlations among individual commodity futures returns as well as between commodity futures returns and more traditional asset returns, in particular equities.

I also collected 29 commodity futures prices, volume, open interests and total money invested in the commodity index. The results are presented in Table 1. The results in the Table 1 show that there has been tremendous growth in open interest, volume of commodity futures traded and in the correlation between commodity and equity markets since 2000. This could be the result of financial innovation such as introduction of commodity index products and

commodity based exchange traded funds which reduced the transaction costs and eased the access to commodity futures markets. All these changes has made the commodity market more liquid, more risky and less segmented with the equity market.

In this paper, I examine the impact of increased investors' participation in commodity market on the return performance and diversification benefits of six buy-and-hold and twentyseven tactical portfolios of commodity futures given the large inflow of capital to the commodity market after 2000. Traditionally, only those people who were directly or indirectly involved in commodity production or consumption were participating in commodity futures market in order to limit the risk of losing commodity prices change. Changes in the commodity futures market in the last decade provided a low cost vehicle for investors to include commodities as part of a well-diversified portfolios. At the same time, I experience weak stock market performance and a boom in the commodity. All these factors encouraged other people such as institutional investors and large traders who have no interest in the workings of the underlying market to participate in the commodity futures markets. As a results, there was a significant increase of capital in commodity futures market invested by institutional investors and large traders. The increased investors participation in both equity and commodity futures market may have an integrating effect reducing segmentation and equating prices of risks or increased exposure of futures markets to other financial markets. Therefore, it is question of interest to all to ask whether financialization has impacted benefits of investing in commodity futures markets.

In order to address the question, I form six traditional buy-and-hold portfolios and 27 tactical portfolios and examine their return characteristics and diversification benefits when combined with other traditional assets for my whole sample period (January 1986 to October 2013) and two sub-sample periods (January 1986 to December 2000 and January 2001 to October 2013). The buy and hold portfolios include five buy-and-hold sector-based portfolios (e.g. foods and fibers, grains and oilseeds, livestock, energy, and precious metals) as well as one equally-weighted portfolio which encompasses all of the aforementioned sectors. These buy-and-hold portfolios serve as a benchmark in which to compare my other strategies against. The sub-sample analysis affords us the ability to more accurately evaluate the evolution of the

<sup>&</sup>lt;sup>1</sup> The literature on commodity futures lacks a complete consensus on when the financialization period began; however, there is a general agreement that it occurred in the early 2000's. Given this, I analyze individual trading volumes of commodity futures and find January 2001 to be a reasonable estimate in which to split the full sample.

diversification benefits of commodity futures given the financialization of the commodities market. To a lesser extent, the sub-samples allow us to observe any changes in potential return benefits from the various commodity portfolio strategies employed.

In an effort to dig deeper into the potential tactical opportunities of commodity futures I exploit information based on basis, net speculation, mathematical, momentum and term structure signals to create twenty-seven commodity portfolios. The three basis portfolios are utilized based on the findings of Gorton et al. (2013) and the three net speculation portfolios follow the idea from Bessembinder (1992) and DeRoon et al. (2000). Similarly, nine momentum portfolios are constructed as outlined in Miffre and Rallis (2007) and nine term structure signals portfolios are based on DeGroot et al. (2014). Finally, three mathematical portfolios utilize mathematical techniques to maximize the risk-return tradeoff at periodic intervals (see Markowitz (1952) for mean-variance optimization technique, Konno and Yamazaki (1991) for mean absolute deviation optimization technique and Rockafellar and Uryasev (2000) for conditional value at risk optimization method).

In order to examine the diversification benefits of commodity portfolios, I combine my various buy-and-hold and tactical commodity futures portfolios with four different kinds of investor portfolios (i.e. benchmark portfolios) and evaluate whether addition of commodity portfolio to benchmark portfolio provides diversification benefits. I utilize the stochastic discount factor (SDF) based spanning tests to examine the diversification properties of the benchmark and commodity portfolios. I utilize two US domestic portfolios to examine the diversification characteristics of commodity futures in the US markets. The first benchmark is a buy-and-hold portfolio consisting of CRSP value-weighted market index returns, comprised of all NYSE/AMEX/NASDAQ stocks, and the Barclays Capital US Aggregate Bond Index returns. The second benchmark is an actively managed equity portfolio based on the Fama-French monthly size and momentum factors. The other two benchmark portfolios I employ examine the diversification characteristics of commodity futures in an international context. In the international setting, the first benchmark employed is a buy-and-hold portfolio of seven countries' equity index-level returns. The second international benchmark is an actively managed

<sup>&</sup>lt;sup>2</sup> The Barclays Capital US Aggregate Bond Index is a commonly used benchmark by both passive and active investors to measure the portfolio performance of the US dollar-denominated investment grade fixed-rate taxable bond market.

equity portfolio comprised of 23 countries' index-level returns based on the Fama-French monthly size and momentum factors. Similar to the examination of portfolio returns, I evaluate the diversification results over the full and sub-sample periods for both the US domestic and international benchmark portfolios.

The main results of this paper can be summarized as follows. First, many of the tactical portfolios outperform buy and hold portfolios. Information obtained from basis, hedging pressure, momentum and term structures is useful in making well-performing portfolios. But, purely mathematical technique which use past returns to maximize risk-return tradeoff is not very usual. Second, increased capital flows to the commodity market has no impact on the risk adjusted returns pattern of commodity futures portfolios. Risk unadjusted returns of commodity portfolios seem much higher in the second half as compared to the first half. This is just opposite of what I expect with financialization. However, change in alpha tests indicate that no change in risk adjusted alpha is significant.

Third, despite the increased correlation between commodity and equity returns as a result of financialization of commodity markets, diversification benefits of commodity portfolios have largely remained unchanged except for international buy and hold reference portfolio. For investors who hold international buy and hold equity portfolio, commodity portfolios do not provide diversification benefits in the second half of the sample period for the most cases. I argue that financialization may have increased exposure of futures markets to equity market as reflected by change in beta coefficients and dynamic correlation coefficients, diversification benefits of including commodity portfolios in the traditional portfolio has not disappeared. Finally, there is general change in the portfolio frontier after 2000, regardless of reference assets. But, I have not examined the whether the change in portfolio frontiers are economically meaningful.

The remainder of this paper is organized as follows. In section 2, I provide brief review of literature. Section 3 discusses my dataset and the creation of the commodity futures return series. Section 4 explains the construction of the various buy-and-hold and tactical commodity futures portfolios, and provides an analysis of the various portfolio returns. Section 5 discusses the methodology used to evaluate if adding portfolios of commodity futures to an investor's

overall portfolio, whether that be a US domestic or international portfolio, provides any diversification benefits, as well as summarizes the empirical findings. Section 6 offers concluding remarks.

#### 2. Review of Literature

In this section, I briefly review two strands of commodity futures literature: the first one looks at diversification benefits of commodity futures portfolios and the second looks at return characteristics of commodity futures/futures portfolios.

The early studies that access the diversification benefits of commodities include Bodie and Rosansky (1980), Fama and French (1987) and Anson (1999). Bodie and Rosansky (1980) and Fama and French (1987) document that investors can reduce their risk without having to sacrifice returns by simply adding commodities to a portfolio of stocks. Similarly, for the period studied 1974-1997, Anson (1999) demonstrate that an investment in commodities futures index offers diversification benefits for long-term investors.

More recent papers that examine the diversification potential of commodities are Scherer and He (2008), Galvani and Plourde (2010), Conover et al. (2010), Daskalaki and Skiadopoulos (2011), Belousova and Dorfleitner (2012) and Huang and Zhong (2013). Scherer and He (2008) point out the strategic value of commodities by showing improved risk return trade-off when commodities are added to a portfolio of U.S. bonds and equities. Conover et al. (2010) show that adding commodity futures to an equity portfolio substantially reduces the risk of the portfolio and a significant diversification benefit is driven no matter what equity investment style is employed. Daskalaki and Skiadopoulos (2011) examine diversification benefits of commodities under in-sample and out of sample setting. They find that commodities are beneficial only to non-mean-variance investors under in-sample setting and commodity futures are non-beneficial to all types of investors under the out-of-sample setting.

Similarly, Belousova and Dorfleitner (2012) show that the diversification contribution of individual commodities varies greatly (among the different sectors), particularly during bull and bear markets, but that commodities, overall, are valuable diversification tools. These studies provide an interesting analysis of commodity futures as diversification tools; however, they do

not explicitly examine how the diversification properties of commodities have evolved over the last decade and a half in the face of an era characterized by a substantial increase in investor participation in the commodity futures market.

The studies that provide evidence of higher returns for tactical portfolios of commodity futures include Gorton and Rouwenhorst (2006), Erb and Harvey (2006), Miffre and Raillis (2007), Shen et al. (2007), Szakmary et al. (2010), Asness et al. (2013) and DeGroot et al. (2014). In their seminal paper, Gorton and Rouwenhorst measure the characteristics of a basket of 36 commodity futures over the period 1959-2004. They observe that the basket of futures offers the same return and risk premiums as equities, is negatively correlated with equity and bond returns, and acts as a hedge against both expected and unexpected inflation. In a follow-up study, Erb and Harvey (2006) bolster the findings of Gorton and Rouwenhorst, but poignantly note that, "[historically] the average annualized excess return of the average individual commodity future has been approximately zero and that commodity futures returns have been largely uncorrelated with one another. However, the annualized excess return of a rebalanced portfolio of commodity futures can be 'equity-like.'"

Studies by Miffre and Rallis (2007) and Asness et al. (2013) document highly significant positive returns for different rank and holding periods of up to 12 months (i.e. momentum profits). In particular, Asness et al. (2013) report returns of 0.7% for low return momentum portfolios and 13.1% for high return momentum portfolios. Miffre and Rallis (2007) identify 13 profitable momentum strategies in the commodity futures markets which generate an average return of 9.38% per year by tactically allocating wealth towards the best performing commodities and away from the worst performing ones. Moreover, Fuertes et al. (2010) and DeGroot et al. (2014) propose novel tactical strategies which incorporate term structure information (in addition to momentum strategies in some cases) to reap large returns. Fuertes et al. (2010) report annualized alphas of 10.14% and 12.66% for momentum and term structure strategies individually. However, a double-sort strategy which exploits both components generates a return of approximately 21.02%.

Although the previous literature has addressed the diversification properties and return characteristics of commodities and commodity portfolios, no recent work (to the best of my

knowledge) has contemporaneously evaluated these properties of commodities given the changing market landscape. In this paper, I try to fill this gap.

# 3. Data and Commodity Futures Return Construction

The futures prices of the 29 commodity futures used in this study are all obtained from the Commodity Research Bureau (CRB). I extract price series information from January 1, 1986 to October 31, 2013.<sup>3</sup> The breakdown of the commodities by sector is as follows: six are from foods and fibers, nine are from grains and oilseeds, four are from livestock, five are from energy, and five are from precious metals. Table 1 in appendix B provides a detailed list of the individual commodity futures, their respective sectors, exchange, and start date of the prices.

In order to construct the futures return series I follow the procedure outlined by Miffre and Rallis (2007) whereby, for each particular commodity, I roll the daily futures prices of the nearby contract over to the next-nearby contract one month prior to the maturity of the nearby contract. This procedure is done for entire dataset of commodity futures to generate the continuous series of futures prices. I denote daily price of futures expiring in T period by  $(F_{d,T})$ . I compute the daily return series  $(r_d)$ , for each commodity future, by taking the log difference of the daily futures prices on two consecutive trading days  $(r_d = \text{Log}(F_{d,T}) - \text{Log}(F_{d-1,T}))$ . To facilitate my analysis I convert the daily returns into a monthly series. Specifically, following the work of Asness et al. (2013) and Moskowitz et al. (2012) I compound the daily returns into a cumulative monthly returns,  $R_{it} = \sum_{d=1}^{eom} r_{id}$ , where eom refers to end of the month. These return series are then used to create and evaluate the various types of commodity portfolios.

The equity return data used to construct the buy-and-hold US domestic reference portfolio is extracted from CRSP. The equity returns are based on a value-weighted index of all NYSE/AMEX/NASDAQ stocks. The bond index return data used in both the buy-and-hold US domestic reference portfolio and buy-and-hold international reference portfolio is obtained from Bloomberg. The bond index returns are those calculated by Barclays Capital. The equity return data used to compute the buy-and-hold international portfolio is also extracted from Bloomberg.

<sup>&</sup>lt;sup>3</sup> The sample period is selected based on data availability. This particular time frame allows for the most commodity futures to be used which possess continuous return and net speculation data.

The portfolio includes the index returns of seven developed countries: Australia, Canada, France, Germany, Japan, the UK, and the US. The return data for the actively managed US domestic equity and actively managed international equity portfolios, based on the Fama-French monthly size and momentum factors, are taken from Ken French's website.<sup>4</sup> The international portfolio includes the returns of 23 countries from four regions: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Switzerland, Sweden, the UK and the US.

# 4. Commodity Futures Portfolio Returns Performance

# 4.1. Portfolio construction

Following the construction of the futures return series I create 33 different portfolios of commodity futures based on style and performance. Six of the portfolios consist of a buy-and-hold strategy. Equally weighted monthly rebalanced portfolio returns series ( $R_{pt}$ ) by averaging monthly return series, i.e.  $R_{pt} = \sum_{i}^{n} R_{it}/n$ . Five of those six portfolios are equally-weighted commodity sectors—foods and fibers, grains and oilseeds, livestock, energy, and precious metals—which represent the commodity futures specific to that group. The remaining portfolio is an equally-weighted composite of the five aforementioned commodity sectors. The sector-based portfolios help to unveil the heterogeneous nature of how commodity futures returns behave. The sector-based portfolios highlight the fact that each commodity underlying the futures contract, and each sector for that matter, have very unique characteristics in relation to diversification and risk management. This potentially makes some commodity futures groups better diversification tools than others and/or more profitable investment strategies than others. The remaining 27 portfolios are tactical portfolios which are actively rebalanced, 24 of these are rebalanced on a monthly basis, while the three net speculation portfolios are uniquely rebalanced on a weekly frequency and then compounded to a monthly horizon to facilitate further analysis.<sup>5</sup>

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<sup>&</sup>lt;sup>4</sup> http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data library.html

<sup>&</sup>lt;sup>5</sup> Speculation data is reported on a Tuesday-Tuesday basis to the US Commodity Futures Trading Commission (CFTC), and made publically available the following Friday on their website.

The choice of monthly rebalancing (i.e. monthly holding periods) is dictated by the fact that both the momentum and term structure strategies are most profitable at this particular horizon.<sup>6</sup>

The nine momentum portfolios are formed as follows: at the end of (L) months (i.e. the look-back period) all commodities in the sample are ranked in descending order based on the past (L) month's average return. The commodity futures in the top 33% are assigned to a "High" return portfolio, the commodity futures in the middle 33% are assigned to a "Med" return portfolio, and those commodity futures in the bottom 33% are assigned to a "Low" return portfolio. The portfolios are then held for (H) months (i.e. the holding period). I analyze and report results for 1, 3, and 12 month look-back periods in combination with one month holding periods. Following the approach of Asness et al. (2013), Miffre and Rallis (2007), Shen et al. (2007), and Jegadeesh and Titman (1993, 2001), I evaluate the performance of the High, Med, and Low portfolios over the (H) subsequent months without a time period lag following the ranking (i.e. look-back) period. To reduce the effect of non-synchronous trading and the bid-ask bounce, Jegadeesh and Titman (1993) suggest measuring returns on the portfolios of futures two months after the initial ranking period (L). However, Asness et al. (2013) report that in case of commodity futures whether one lags the ranking period or not, it does not significantly alter the results. Therefore, following Asness et al. (2013) I do not measure portfolio returns with a lag following the ranking period. I derive a single time-series of monthly returns for each actively managed trading strategy in this manner.

The nine term structure portfolios follow an alternative formulation to that of DeGroot et al. (2014). These strategies, as originally motivated by Erb and Harvey (2006) and Gorton and Rouwenhorst (2006), seek to exploit the term structure of commodity futures prices. The term structure measures stem from the work of Samuelson (1965) who argues that the volatility of futures returns decreases as the maturity of contracts increases. Thus, the prices of the front contracts react most heavily to supply, demand, and news shocks, while prices further along the curve are influenced significantly less. Furthermore, as noted by DeGroot et al. (2014), even contracts on the same commodity with different maturities can exhibit large differences in returns and risks. Hence, non-front contracts which are further down the futures curve may

<sup>&</sup>lt;sup>6</sup> Fuertes et al. (2010) document similar findings. I examine momentum and term structure portfolios with look-back periods of 1, 3, and 12 months and holding periods of 1, 3, and 12 months.

behave differently and represent different investment opportunities. I calculate the term structure measures as follows:

$$TS_{i_{-}j} = F_{it} - F_{jt} \tag{1}$$

where  $F_{it}$  is the futures price of the nearby contract i at time t and  $F_{jt}$  is the futures price of the other nearby contract j at time t. The construction of the commodity portfolios based on the term structure is similar to the procedure for the return momentum portfolios. For each individual commodity I utilize equation (1), at various contract horizons, to obtain a daily difference series. Then to facilitate my analysis I average the daily series into a monthly one, whereby the series is then sorted in descending order and the commodity futures in the top 33% are assigned to a "High" term structure (TS) portfolio, the commodity futures in the middle 33% are assigned to a "Med" TS portfolio, and those commodity futures in the bottom 33% are assigned to a "Low" TS portfolio. The portfolios are then held for one (H) month and rebalanced. For each of the portfolios the returns for the month, t+1, are calculated using equal weights for all the futures contracts contained within their respective portfolio. This process is repeated to obtain a continuous time series of returns for the portfolios based on term structure.

The remaining nine portfolios are created from tactical strategies based on commodity futures basis, net speculation, and maximization of the Sharpe Ratios (i.e. the mathematical portfolios), respectively. Specifically, I analyze three portfolios sorted on the futures basis (spot price - futures price) of the commodities in the sample. I rank the commodity futures in descending order based on the past one month's basis, similar to the procedure for the momentum and term structure portfolios, and then form the portfolios. The commodity futures in the top 33% are assigned to a "High" basis portfolio, the commodity futures in the middle 33% are assigned to a "Med" basis portfolio, and those commodity futures in the bottom 33% are assigned to a "Low" basis portfolio. As with the previous portfolios the basis portfolios are also rebalanced monthly. More formally, the basis portfolios are constructed as follows: at the end of each month, t, I calculate the basis for each of the 29 commodity futures. Following Gorton et al. (2013), the basis for each commodity, i, is calculated as:

$$Basis_{i} = \left(\frac{F_{1t}}{F_{2t}} - 1\right) \times \frac{365}{D_{2t} - D_{1t}} \tag{2}$$

where  $F_{1t}$  is the price of nearest futures contract,  $F_{2t}$  is the price of the next-nearby futures contract, and  $D_{1t}$  and  $D_{2t}$  are the number of days before the futures contracts  $F_{1t}$  and  $F_{2t}$  expire, respectively. For each of the portfolios the returns for the month, t+1, are calculated using equal weights for all the futures contracts contained within their respective portfolio. This process is repeated to obtain a continuous time series of returns for the portfolios based on basis.

In order to construct portfolios based on net speculators' positions I utilize the position of trader's data given in the US Commodity Futures Trading Commission's (CFTC's) weekly reports. For each commodity futures contract, i, I compute the variable  $h_{i,t}$ , which is based on the aggregated weekly positions of non-commercial hedgers in all traded markets at time t, and is given as:

$$h_{i,t} = \frac{\text{agg. short hedge positions} - \text{agg. long hedge positions}}{\text{total number of hedge positions}}$$
(3)

Following my prior procedure, each week, t, I rank the commodity futures in descending order based on the past one month's net speculation positions ( $h_{i,t}$ ) and again divide them into three groups. The commodity futures in the top 33% are assigned to a "High" net speculation portfolio, the commodity futures in the middle 33% are assigned to a "Med" net speculation portfolio, and those commodity futures in the bottom 33% are assigned to a "Low" net speculation portfolio. The portfolio returns for week, t+1, are calculated using equal weights for all the futures contracts contained within the respective portfolio. Since the CFTC hedging data is only available on a weekly occurrence, I first calculate the portfolio returns by rebalancing via a weekly frequency, and then convert these weekly returns into monthly returns by compounding them into a cumulative index.

Finally, the mathematical portfolios which utilize the concept of portfolio optimization include Markowitz's (1952) mean variance portfolio, a conditional value at risk portfolio, and a mean absolute deviation portfolio. These portfolios are motivated by practices in the financial industry and utilize mathematical constructs to "optimize" an investor's risk-return tradeoff. The mean-variance portfolio of Markowitz (1952) uses the variance of portfolio returns as the risk proxy. I use the past 250 daily returns of the commodity futures and obtain weights, for each

commodity in the sample, which maximize the Sharp Ratio. Formally, the maximization problem is defined as:

$$\max_{\omega} \mu^T \omega \qquad s.t. \, 1' \omega = 1 \ \ and \, \omega^T \Sigma \, \omega \, \leq \sigma_{max}^2 \tag{4}$$

where,  $\mu$  is the mean return of the commodity futures,  $\Sigma$  is the variance of the commodity returns, and  $\omega$  are the portfolio weights. The weights obtained by maximizing the Sharpe Ratio using the past 250 daily returns are used to invest for the next one month period. The mean-variance portfolio is rebalanced monthly. The conditional value-at-risk (CVaR) portfolio measures risk under portfolio optimization as in Rockafellar and Uryasev (2000, 2002). In this approach, I use the conditional value-at-risk (CVaR) of the portfolio returns as the risk proxy instead of variance of the portfolio returns as in equation (4). The conditional value-at-risk for a portfolio is defined as:

$$CVaR_{\alpha}(x) = \frac{1}{(1-\alpha)} \int_{f(x,r) \ge VaR_{\alpha}(x)} f(x,r)p(r)dr$$
(5)

where,  $\alpha$  is the probability level, f(x,r) is the loss function for a portfolio x and asset returns r, p(r) is the probability density function for asset returns r, and  $VaR_{\alpha}$  is the value-at-risk of portfolio x at probability level  $\alpha$ . To construct the CVaR portfolio returns series, I compute the weights that maximize the ratio of the mean portfolio return to the CVaR using the past 250 daily returns of commodity futures, and then use these weights to invest for the next one month period. The CVaR portfolio is rebalanced monthly. Lastly, the mean-absolute-deviation (MAD) portfolio utilizes the optimization technique of Konno and Yamazaki (1991). The MAD portfolio optimization is similar to the mean-variance technique of Markowitz (1952). However, I utilize Konno and Yamazaki's (1991) redefined risk measure called MAD, which is given as:

$$\frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{n} (r_{it} - \bar{r}_i) \omega_i \right| \tag{6}$$

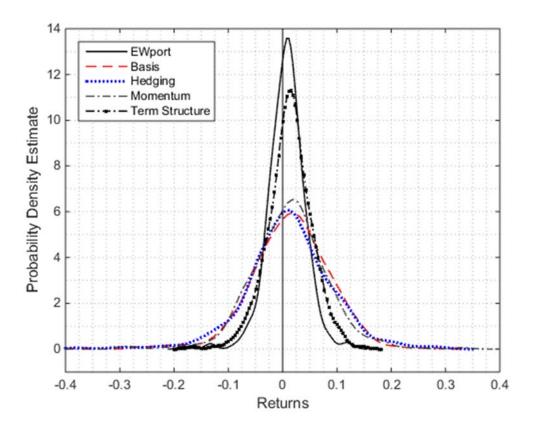
where, T is the length of time horizon, n is the total number of commodities,  $r_{it}$  is the return on the i<sup>th</sup> commodity over the time horizon, t, where, t = 1, 2, ... T,  $\bar{r}_i$  is the mean of i<sup>th</sup> commodity return, and  $\omega_i$  are the portfolio weights. In order to obtain the MAD portfolio return series, I

replace the risk proxy in equation (4) by equation (6), and then solve equation (4) using the past 250 daily returns of commodity futures to obtain the appropriate weights. These weights are then used to allocate funds to invest for the next one month period. The MAD portfolio is similarly rebalanced on a monthly frequency.

# 4.2. Buy-and-hold and actively rebalanced commodity portfolio performance

Figure 1 provides a snapshot of the average annualized futures returns performance of selected portfolios. It shows that buy and hold portfolio returns are more concentrated around zero compared to other actively managed portfolio. Further, actively managed portfolios are riskier than buy and hold portfolios. Among all portfolios, portfolios formed based on hedging and basis information have negative returns less often than other portfolios.

Table 1 presents return pattern of the buy-and-hold and actively managed commodity portfolios. I report the results for both the full sample period and two sub-sample periods, along with the P-values, standard deviations, and Sharpe Ratios of each respective portfolio examined. Panel A summarizes the returns of the futures portfolios formed using traditional buy-and-hold strategies. Over the full sample period (January 1986 to October 2013) the energy sector has the highest annualized mean return of all five groups at 14.32%, this is followed by the precious metals sector with an average return of 8.63%. Upon examining the two sub-sample periods an interesting feature emerges, the average returns of the buy-and-hold portfolios, in general, tend to be higher in the second period (January 2001 to October 2013) when compared to the first period (January 1986 to December 2000). Furthermore, the average annualized return performance of most buy-and-hold portfolios over the first sub-sample sample period are not significantly different from zero at conventional significance levels of 10%, whereas in the second sub-sample period this trend is reversed. The overall results of the first sub-sample period are largely consistent with Erb and Harvey (2006) who find that the average annualized excess return of the average individual commodity future over the period 1982-2004 has been approximately zero. However, findings over the second sub-sample period seem to tell a much different story.



**Figure 1.** Return Distributions. This figure plots the return distributions of selected commodity portfolios. EWport refers to equally weighted portfolio of commodity futures, and Basis, Hedging, Momentum and Term Structure refer to commodity portfolios formed based on information on basis, hedging, momentum and term structure signals.

Panels B, C, and D display the performance of the tactical basis, speculation, and mathematical portfolios, respectively. Over the full sample period the High basis portfolio generates the largest annualized mean return of all portfolios at 17.92%. The sub-sample analysis shows that much of the large return over the full sample period is due to the tremendous performance of the portfolio in the second sub-sample period with a mean return of 22.51%. In general, the other two basis portfolios seem to earn a return commensurate with the more traditional buy-and-hold commodity portfolios. The average return of the High speculation portfolio over the full sample period (14.26%) also ranks it as one of the top performing portfolios. Interestingly, an examination of the sub-sample periods shows that the High speculation portfolio significantly outperforms the Low and Med portfolios by a wide margin in

**Table 1**Return Performance of Commodity Portfolios

Teetain i cironnance of Co	Period: 01/31/1986 to					Period: 01/31/1986 to					Period: 01/31/2001 to				
		10/31	/2013		_	12/31/2000					10/31/2013				
Portfolios	Mean	P-val	SD	SR		Mean	P-val	SD	SR		Mean	P-val	SD	SR	
Panel A: Buy-and-Hold															
Foods & Fibers	6.49	0.04	0.57	0.11		3.13	0.39	0.47	0.07		10.14	0.06	0.66	0.15	
Grains & Oilseeds	7.73	0.04	0.67	0.12		5.03	0.25	0.57	0.09		10.68	0.09	0.77	0.14	
Livestock	6.09	0.08	0.62	0.10		6.37	0.22	0.67	0.10		5.79	0.20	0.56	0.10	
Energy	14.32	0.02	1.05	0.14		14.37	0.07	1.02	0.14		14.26	0.11	1.09	0.13	
P. Metals	8.63	0.01	0.63	0.14		5.98	0.11	0.48	0.13		11.52	0.06	0.77	0.15	
Ewport	8.49	0.00	0.42	0.20		6.57	0.00	0.30	0.22	_	10.58	0.01	0.52	0.20	
Panel B: Basis															
Lowbasis	7.83	0.01	0.50	0.16		6.61	0.04	0.41	0.16		9.17	0.05	0.58	0.16	
Medbasis	10.61	0.00	0.47	0.22		8.98	0.00	0.33	0.27		12.39	0.01	0.59	0.21	
Highbasis	17.92	0.00	0.89	0.20		13.70	0.01	0.67	0.21	_	22.51	0.01	1.08	0.21	
Panel C:														_	
Speculation															
Lowspec	11.71	0.00	0.50	0.23		5.59	0.06	0.38	0.15		18.38	0.00	0.60	0.31	
Medspec	9.02	0.00	0.56	0.16		8.39	0.03	0.50	0.17		9.71	0.05	0.61	0.16	
Highspec	14.26	0.01	0.98	0.15		14.36	0.02	0.77	0.19		14.14	0.14	1.18	0.12	
Panel D: Mathematical										_					
Portmv	1.97	0.50	0.53	0.04		-0.89	0.79	0.43	-0.02		5.09	0.31	0.62	0.08	
Portcvar	0.11	0.97	0.51	0.00		-1.23	0.69	0.41	-0.03		1.58	0.74	0.60	0.03	
Portmad	1.57	0.60	0.53	0.03		-0.90	0.79	0.44	-0.02	_	4.27	0.39	0.62	0.07	

Table 1 (Cont.)

Period: 01/2 10/31/2							Period: 01/31/1986 to 12/31/2000					Period: 01/31/2001 to 10/31/2013			
Portfolios	Mean	P-val	SD	SR	Me	an	P-val	SD	SR	Mea	an	P-val	SD	SR	
Panel E: Momentum															
LowL1H1	12.92	0.00	0.52	0.25	9.9	95	0.01	0.46	0.21	16.1	15	0.00	0.58	0.28	
MedL1H1	7.49	0.01	0.50	0.15	3.3	36	0.24	0.37	0.09	11.9	99	0.02	0.61	0.20	
HighL1H1	12.70	0.01	0.91	0.14	10.	23	0.05	0.66	0.15	15.4	40	0.09	1.12	0.14	
LowL3H1	11.88	0.00	0.51	0.23	9.7	72	0.00	0.41	0.24	14.2	23	0.00	0.60	0.24	
MedL3H1	6.39	0.02	0.48	0.13	2.7	79	0.31	0.35	0.08	10.3	31	0.03	0.58	0.18	
HighL3H1	13.84	0.01	0.91	0.15	10.	53	0.05	0.68	0.16	17.4	15	0.05	1.12	0.16	
LowL12H1	11.06	0.00	0.50	0.22	6.9	92	0.03	0.41	0.17	15.5	57	0.00	0.58	0.27	
MedL12H1	8.59	0.04	0.74	0.12	8.3	33	0.14	0.73	0.11	8.8	6	0.15	0.76	0.12	
HighL12H1	14.66	0.01	0.94	0.16	13.	44	0.02	0.72	0.19	15.9	99	0.08	1.13	0.14	
Panel F: Term Structure															
LowTS1_2	15.81	0.00	0.49	0.32	15.	22	0.00	0.42	0.36	16.4	16	0.00	0.56	0.29	
MedTS1_2	9.95	0.00	0.49	0.20	8.8	39	0.00	0.37	0.24	11.0	)9	0.02	0.59	0.19	
HighTS1_2	9.75	0.06	0.91	0.11	4.8	38	0.35	0.67	0.07	15.0	)7	0.10	1.11	0.14	
LowTS1_3	13.07	0.00	0.50	0.26	12.	73	0.00	0.45	0.28	13.4	13	0.00	0.56	0.24	
MedTS1_3	9.28	0.00	0.47	0.20	7.7	72	0.00	0.33	0.23	10.9	98	0.02	0.59	0.19	
HighTS1_3	12.93	0.01	0.90	0.14	7.3	39	0.15	0.65	0.11	18.9	98	0.04	1.11	0.17	
LowTS1_4	11.50	0.00	0.51	0.22	8.6	63	0.01	0.45	0.19	14.6	52	0.00	0.57	0.26	
MedTS1_4	8.52	0.00	0.46	0.19	8.8	34	0.00	0.35	0.25	8.1	8	0.07	0.55	0.15	
HighTS1 4	14.41	0.00	0.91	0.16	10.	47	0.05	0.69	0.15	18.7	70	0.04	1.10	0.17	

This table provides the return performance of the various styles of commodity futures portfolios over the full sample period (January 31, 1986 to October 13, 2013) and two sub-sample periods (January 31, 1986 to December 31, 2000 and January 1, 2001 to October 31, 2013). "Mean" represents the average annualized return of the commodity portfolio in percent, "P-val" is the P-value based on two-tailed significance tests for testing the hypothesis of whether the mean return is equal to zero, "SD" is the standard deviation of the portfolio, and "SR" is the Sharpe Ratio.

the first period, but in the second period the Low speculation portfolio average return surpasses that of the High by an average margin of about 4.00%. Finally, the mathematical portfolios, which are based on the financial theory of maximizing portfolio Sharpe Ratios using commodity weights, consistently yield the worst return performance of all commodity portfolios considered. In fact, all returns for the portfolios in Panel D are always statistically insignificant.

Panel E presents the return performance of the tactical momentum portfolios where I consider three different look-back periods and a one month holding period. For instance, the first portfolio of panel E, LowL1H1, represents the annualized mean return of an equally-weighted portfolio holding the lowest return futures—the bottom 33% of commodity futures when sorted on past returns—based on a one month return look-back period (L) and one month holding period (H). All other momentum portfolios follow a similar interpretation. Over the whole sample period the annualized mean return of the HighL12H1 portfolio (14.66%) outperforms all other momentum portfolios examined. The next highest momentum portfolio return strategy is the HighL3H1 portfolio with a mean return of 13.84%. I find that the High momentum portfolios, in general, tend to outperform their similar Low momentum portfolio counterparts. An analysis of the sub-sample periods yields similar conclusions. The HighL1H1 and Low1H1 portfolios present a minor exception to this finding with very similar returns of 12.70% and 12.92%, respectively. Prior work by Miffre and Rallis (2007) and Asness et al. (2013) similarly find higher return performance for the High (recent winner) momentum groups.

Panel F presents the return performance of the tactical term structure portfolios where I consider three different horizons. For instance, the first portfolio of Panel F, LowTS1\_2, represents the annualized mean return of an equally-weighted portfolio holding the smallest differenced futures contracts—the bottom 33% of commodity futures when sorted on the difference between nearby and next-nearby contracts—based on a one month look-back period and one month holding period. All other term structure portfolios follow a similar interpretation. Over the whole sample period the annualized mean return of the LowTS1\_2 portfolio (15.81%) outperforms all other term structure portfolios examined. The next highest term structure portfolio return strategy is the HighTS1\_4 portfolio with a mean return of 14.41%. The results of the term structure portfolios are a bit more varied than those observed in the momentum section. The Low TS portfolios formed on the difference between the nearby contracts and the contracts

at the shorter horizons (i.e. next-nearby and next-next-nearby) display the highest return performance amongst their Med and High counterparts. However, this finding is not true of the longer horizon difference contracts (TS1\_4) where the High TS portfolio return performance is superior. Sub-sample analysis of the return performance yields consistent findings for the LowTS1\_2 and HighTS1\_4 portfolios. Contrastingly, the LowTS1\_3 portfolio maintains the high return performance (12.73%) over the first sub-sample period, but the HighTS1\_3 portfolio obtains the highest return performance over the second sub-period (18.98%). Overall, these findings are in line with the argument of DeGroot et al. (2014) that contracts on the same commodity with different maturities can exhibit large differences in returns and risks.

Focusing solely on buy-and-hold strategies, investment in the energy sector offers by far the greatest return potential. Energy sector investment is commensurate with many of the high performing tactical strategies based on speculation, momentum, and term structure over the full sample period. Regarding the tactical strategies, I see that the average annualized returns formed on basis tend to outpace all other tactical (and buy-and-hold) strategies over the full sample period. Looking at the latter sub-sample period, which is characterized by the financialization of the commodity market, the High basis portfolio again offers the highest return strategy, followed by the Low speculation portfolio, and strategies based on High momentum and High term structure.

#### 4.3. Risk-adjusted commodity portfolio performance

The results in Table 1 provide a broad summary of the return performance of both buyand-hold and various styles of tactical commodity portfolios. Moreover, it provides an analysis
of how the returns of such strategies have evolved over time, with a particular emphasis on the
last decade. This change is interesting given the prominent strand of literature which documents
the financialization of the unique asset class over the last decade. Tang and Xiong (2012)
highlight the impact of this change by recognizing the unique characteristics of the commodity
futures market that precipitated the rapid growth of the commodity index investment. Prior to the
early 2000's commodity prices largely provided a risk premium for idiosyncratic price risk (see
Bessembinder, 1992; DeRoon et al., 2000) and had little or no correlation with more traditional
asset markets. These features bear a sharp contrast to the price dynamics of typical financial

assets which are well-known for solely carrying a premium for systematic risk and generally are highly correlated with each other. The fundamental process of financialization resulted in an increase of the correlations among the returns of the different types of futures and an increase in the correlations between more traditional assets, thus altering the pricing dynamics of the commodity futures. I posit that the changes in futures returns in the latter sub-sample period are a reflection of this process. In order to more comprehensively evaluate the impact of financialization on the futures returns in the commodities market I utilize risk-adjusted measures of performance. As such, I calculate risk-adjusted returns for the different portfolios of commodity futures using the regression model of the following form:

$$R_{pt} = \alpha_p + \alpha_D I_t + \beta_E (R_{Et} - R_{rft}) + \beta_D (R_{Et} - R_{rft}) * I_t + \beta_B (R_{Bt} - R_{rft}) + u_{pt}$$
 (7)

where,  $R_{pt}$  is a commodity portfolio return time series,  $I_t$  is an indicator variable such that  $I_t$  is 0 for period 1986 to 2000 and 1 otherwise,  $R_{Et}$  is returns on CRSP value-weighted market index,  $R_{Bt}$  is Barclays Capital monthly bond index return series and  $u_{pt}$  is a series disturbance terms. The risk-adjusted return results from the model in (7) are presented in Table 4.

The most interesting feature of Table 2 is that 32 of the 33 coefficients on the indicator variable which measure whether change in risk adjusted alpha is significant or not are not significant. On the other hand, the coefficient on the interaction term between the indicator variable and stock returns is significant for 32 portfolios. The coefficient on the interaction term measures the exposure of commodity market to the equity market. This results in the Table 4 show that commodity market has been more integrated with equity market but there is no impact of increased financial flows to the risk adjusted return pattern of the commodity portfolios.

Another interesting results of the Table 2 is that 18 portfolios out of 33 have significantly different form zero risk adjusted returns. All mathematical portfolios have negative but insignificant risk adjusted returns. The High basis and High speculation portfolios both maintain significant risk-adjusted returns. All High momentum portfolios outperform other momentum portfolios. Similarly, the Low TS portfolios have significant risk-adjusted returns. In some cases, the Med TS portfolios are significant but the returns are lower than those of the Low TS

**Table 2**Risk-Adjusted Return Performance of Commodity Portfolios

Portfolios	$\alpha_p$	P-val	$\alpha_D$	P-val	$eta_D$	P-val
Panel A: Buy-and-Hold			<del>_</del>		· -	
Foods & Fibers	3.6728	0.3150	4.8636	0.4309	0.4369	0.0003
Grains & Oilseeds	3.8136	0.3914	4.2153	0.5729	0.3654	0.0112
Livestock	5.9380	0.2577	1.2804	0.8533	-0.0353	0.8094
Energy	15.9576	0.0460	-3.7821	0.7474	0.7015	0.0061
P. Metals	5.9474	0.1099	3.9226	0.5755	0.5273	0.0003
Ewport	6.5154	0.0060	2.4512	0.5964	0.4145	0.0004
Panel B: Basis						
Lowbasis	6.1685	0.0547	0.7803	0.8854	0.3902	0.0010
Medbasis	9.3683	0.0004	1.3923	0.7909	0.4776	0.0001
Highbasis	14.0225	0.0077	6.0003	0.5486	0.8217	0.0011
Panel C: Speculation						
Lowspec	6.1568	0.0379	10.7371	0.0498	0.4617	0.0001
Medspec	8.9292	0.0196	-2.0944	0.7257	0.5126	0.0000
Highspec	14.2798	0.0152	-5.4193	0.6125	0.8761	0.0000
Panel D: Mathematical						
Portmv	-2.0948	0.5269	5.1827	0.3804	0.2751	0.0271
Portevar	-1.7050	0.5799	2.0298	0.7156	0.2892	0.0110
Portmad	-1.9463	0.5652	3.8661	0.5122	0.3404	0.0054
Panel E: Momentum			_			
LowL1H1	9.9276	0.0054	6.5256	0.2491	0.2435	0.0507
MedL1H1	2.9696	0.3092	6.0601	0.2838	0.4905	0.0032
HighL1H1	10.1623	0.0648	0.1720	0.9867	0.9773	0.0002
LowL3H1	9.1877	0.0049	3.5983	0.5140	0.3554	0.0038
MedL3H1	2.8480	0.3023	5.9882	0.2825	0.3470	0.0343
HighL3H1	10.9699	0.0485	3.2173	0.7574	0.8554	0.0013
LowL12H1	7.5322	0.0148	6.7221	0.2157	0.4112	0.0008
MedL12H1	8.7839	0.1493	-0.9754	0.9069	0.4753	0.0094
HighL12H1	12.6970	0.0236	-0.1131	0.9914	0.8009	0.0013

Table 2 (Cont.)

Portfolios	$\alpha_p$	P-val	$\alpha_D$	P-val	$\beta_D$	P-val
Panel F: Term Structure	Ρ					
LowTS1_2	14.8198	0.0000	-0.3019	0.9555	0.3661	0.0025
MedTS1_2	9.2724	0.0018	0.5250	0.9227	0.4037	0.0003
HighTS1_2	5.1589	0.3337	7.1289	0.4838	0.8466	0.0007
LowTS1_3	12.8208	0.0004	-0.9840	0.8582	0.3941	0.0007
MedTS1_3	7.4934	0.0037	1.9507	0.7093	0.3519	0.0016
HighTS1_3	7.1468	0.1647	8.6624	0.3936	0.8121	0.0017
LowTS1_4	9.0111	0.0115	4.3053	0.4448	0.4082	0.0009
MedTS1_4	8.9462	0.0011	-2.0555	0.6826	0.3259	0.0014
_HighTS1_4	10.0176	0.0645	5.1985	0.6086	0.7880	0.0021

This table provides estimates form the following regression equation:

$$R_{pt} = \alpha_p + \alpha_D I_t + \beta_E (R_{Et} - R_{rf}) + \beta_D (R_{Et} - R_{rf}) * I_t + \beta_B (R_{Bt} - R_{rf}) + u_{pt},$$

where  $R_E = \text{CRSP}$  value-weighted index,  $R_B = \text{Barclays}$  Capital monthly bond index returns,  $I_t = \text{Index} = 1$  if date>=2000, and zero otherwise. Annualized value of  $\alpha_p$  is presented in the table. "P-val" refers to the P-value based on two-tailed significance tests for testing the hypothesis of whether the estimated coefficients are zeros.

portfolios. Thus, for a relatively small subset of my overall commodity portfolios, a tactical strategy based on, High basis, High speculation, High momentum, and Low term structure can be a profitable return tactic in an era characterized by financialization.

Taken together, the results of Table 1 and 2 support the idea that the increase in equity-commodity comovement has subsequently changed the price dynamics of commodity futures. The contrast in returns and risk-adjusted alphas, as shown by the two sub-sample periods, lends credence to the argument that commodity markets were more segmented from outside financial markets prior to 2001. In the era characterized by the financialization of the commodity markets the ability of many types of buy-and-hold and tactical portfolios to earn higher returns (and risk-adjusted returns) has largely been diminished for many portfolios.

# 5. Diversification Benefits of Commodity Futures

Tables 1 and 2 evaluate the return performance of both buy-and-hold and tactical commodity portfolios. However, one of the overriding reasons investors include commodity futures in their portfolio is for the diversification benefits as documented in prior literature. Yet, given the changing landscape of the commodity futures market via financialization, do commodity portfolios provide diversification benefits today? Furthermore, how have these diversification benefits performed over recent economic recession periods when they were desired most?

### 5.1. Testing methodology

To examine the diversification properties of my commodity futures portfolios, I use the stochastic discount factor (SDF) frontier based spanning test<sup>7</sup> introduced by Hansen and Jagannathan (1991), and later developed by DeSantis (1995), Bekaert and Urias (1996), and Maroney and Protopapadakis (2002). The spanning test help us to determine whether a set of new assets improve the investment opportunity set relative to a benchmark asset. In this approach, I construct a frontier of benchmark assets and ascertain whether that benchmark remains unchanged after increasing the number of assets in the portfolio. If the two frontiers coincide then there is spanning. In this case, there is no diversification benefit from adding new assets to the benchmark asset. However, if adding a new set of assets leads to a significant shift of the frontier, relative to the frontier of benchmark assets, then there is no spanning. In this case, the new set of assets provides diversification benefits.

For expositional purpose, I let  $R_t = [R'_{pt}, R'_{qt}]^t$  represent returns on n = p + q risky assets at time t, where  $R_{pt}$  and  $R_{qt}$  represent returns on p benchmark assets and returns on the q test assets, respectively. Further, I let  $m_t$  be the investor's marginal rate of substitution or discount factor. The main question I try to address here is how the region of the admissible discount factors changes when a group of test assets are added to the benchmark set of securities. Under the assumption that there are no transaction costs and the Law of One Price holds, the general unconditional asset pricing model can be written as:

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<sup>&</sup>lt;sup>7</sup> DeRon and Nijiman (2001) provide a comprehensive survey of this literature.

$$E(R_t m_t) + E(m_t) = 1_n \tag{8}$$

Hansen and Jagannathan (1991) show that the linear projection of  $m_t$  onto the set of returns being priced has the minimum variance which satisfies equation (8), this means the lower bound of the discount factor that satisfies equation (8) is as follows:

$$m_{ct} = c + [R_t - E(R_t)]'\beta_c + \varepsilon \tag{9}$$

where,  $\varepsilon$  is the error of the regression. Substituting the value of  $m_t$  from equation (9) into equation (8) I get the following:

$$E(R_t m_{ct}) + E(m_{ct}) - 1_n = 0 (10)$$

Equation (10) can be used to examine whether or not a subset of the assets in  $R_{\rm pt}$  price all of the assets in  $R_{\rm t}$ . In order to implement the tests based on the SDF frontier, DeSantis (1995) proposes pre-specifying two values of risk-free rates,  $m_{\rm c_1}$  and  $m_{\rm c_2}$ . Then I can specify the following system of orthogonality conditions for the spanning test:

$$R_t - E(R_t | m_{c_1}) = \varepsilon_{1t}, \quad E(\varepsilon_{1t}) = 0$$

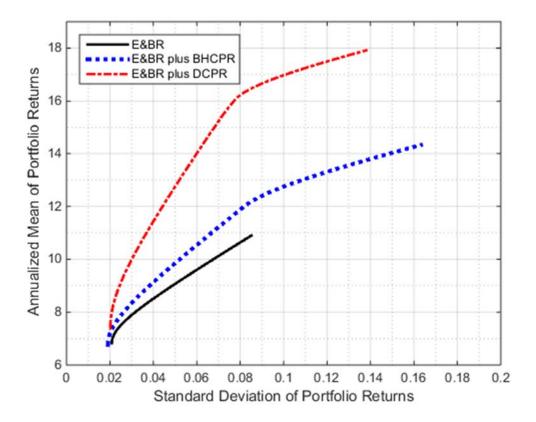
$$R_t - E(R_t | m_{c_2}) = \varepsilon_{2t}, \quad E(\varepsilon_{2t}) = 0$$

$$\text{where, } E(R_t | m_{c_j}) = \frac{1 - cov(R_t, m_{c_j})}{E(m_{c_j})}, \quad m_{c_j} = E(m_{c_j}) + r'_{pj}\beta_{pj} + r'_{qj}\beta_{qj}, \quad j = 1, 2$$

$$\text{and } r_t = R_t - E(R_t)$$

Since there are n assets, there are 2 n orthogonality conditions. The system is just identified without restrictions and is linear with coefficients  $\{\beta_{p1}, \beta_{q1}, \beta_{p2}, \beta_{q2}\}$ . Spanning implies 2q overidentifying conditions that state there is no need to include test assets in the construction of the SDFs:  $\{\beta_{q1}, \beta_{q2} = 0\}$ . Under the null hypothesis of spanning, the Hansen J-Statistic (Hansen, 1982; Hansen and Singleton, 1982) can be used to evaluate the over-identifying conditions implied by spanning. It has an asymptotic chi-square distribution with  $2 \times n_2$  degree of freedom.

Figure 2 helps us to understand testing methodology. From Figure 1 we see that there is shift in portfolio frontier as we add commodity portfolio to bench mark assets. The spanning tests help us to detect whether the shift in the portfolio frontier is statistically significant or not.



**Figure 2.** Efficient Portfolio Frontiers with and without Commodity Portfolio Returns. E&BR refers to CRSP value weighted stock index and Barclays Capital monthly bond index returns. BHCPR stands for buy and hold commodity portfolio returns and DCPR means dynamic commodity portfolio returns. Solid line represents the portfolio frontier of portfolio that includes returns on equity and bond only. The dotted line is the portfolio frontier of the portfolio that consists of returns on equity, bond and five sectoral commodity portfolios. The line with the dash shows the plot of the efficient frontier of the portfolio that consists of returns on equity, bond, commodity portfolios formed based on basis, hedging, momentum and term structure.

In order to examine the impact of financialization, I split the sample in two parts: pre 2000 era and post 2000 era. I modify the spanning test framework to capture the before and after 2000 shits in frontiers as:

$$R_{t} - E(R_{t}|m_{c_{1}}(I_{t})) = \varepsilon_{1t}, \quad E(\varepsilon_{1t}|I_{t}) = 0$$

$$R_{t} - E(R_{t}|m_{c_{2}}(I_{t})) = \varepsilon_{2t}, \quad E(\varepsilon_{2t}|I_{t}) = 0$$
where, 
$$E(R_{t}|m_{c_{j}}(I_{t})) = \frac{1 - cov(R_{t}m_{c_{j}}(I_{t}))}{E(m_{c_{j}}(I_{t}))},$$
(12)

$$m_{c_i}(I_t) = E(m_{c_i}(I_t)) + r'_{pj}[\delta_{pj} + \phi_{pj}I_t] + r'_{pj}[\delta_{qj} + \phi_{qj}I_t]$$
 and  $r_t = R_t - E(R_{t|I_t})$ 

The new coefficients are  $\{\delta_{pj}, \delta_{qj}, \phi_{pj}, \phi_{qj}\}$ .  $I_t$  is an indicator variable that is zero before 2000 and one therefore after. The covariances of returns and the SDFs are now allowed to change to fit an additional set of average returns.

# 5.2. Spanning test results

The following sections report the results of the spanning tests using the US domestic buy-and-hold, US actively managed, international buy-and-hold, and international actively managed portfolios as benchmark assets. The work of Bekaert and Urias (1996) show that the power of the spanning tests is extremely sensitive to the number of benchmark assets; therefore, I limit the number of assets in each case. I sequentially test whether adding portfolio of commodity futures to the benchmark asset provide any diversification benefits. I utilize all buy-and-hold and tactical commodity portfolios examined in Tables 1 and 2.

## 5.2.1. US domestic buy-and-hold benchmark portfolio

Table 3 presents the spanning test results using a US domestic buy-and-hold portfolio, which consists of the CRSP value-weighted market index returns and the Barclays Capital US Aggregate Bond Index returns, as the benchmark asset. Each column provides J-stat P-values of the spanning test from adding the portfolio of commodity futures to the benchmark portfolio. The second column reports the J-stat p-values for testing the hypothesis that reference assets span the test asset for the whole period. These are the p-values obtained from placing restriction on equation (11) that  $\beta_{qj} = 0$ . These tests are unconditional in the sense that the SDFs in equation (11) use the unconditional moments of my data. Results for the whole sample period show that the null hypothesis of spanning is rejected at the (more stringent) significance level of 5% for almost all commodity portfolios. This means that investors can improve their investment diversification opportunities by adding the respective portfolio of commodities to the benchmark asset.

<sup>&</sup>lt;sup>8</sup> Based on prior literature, I follow both Maroney and Protopapadakis (2002) and Errunza et al. (1999) who predominately use significance levels of 5% when evaluating spanning test results.

The column 3 to column 6 presents J-stat p-values for spanning tests that focus on restrictions on equation (12). Restrictions on equation (12) allow for a change in the mean variance frontier after 2000. The restrictions  $\delta_{qj} = \phi_{aj} = 0$  put on equation (12) examine the contribution of commodity portfolios to frontiers conditional on sub-periods. Results indicate that all other commodity portfolios except Grains and Oilseeds, Energy and MedL12H1 are important to both frontiers. This means that only for the buy-and-hold grains and oilseeds commodity portfolio, Energy portfolio and the dynamic MedL12H1 portfolio do I fail to reject the null hypothesis, implying that the portfolio provides no diversification benefits when combined with the benchmark asset.

Columns 4 and column 5 test the hypothesis that whether commodity provides diversification benefit in pre-2000 period ( $\delta_{qj}=0$ )and post-2000 period ( $\phi_{qj}=0$ ). All commodity portfolios provide diversification benefit before 2000. However, evaluating the spanning test results over the second half of the sample period reveals that ten out of 33 commodity portfolios no longer provide any diversification benefits. Specifically, out of all buyand-hold portfolios considered, only half of the portfolios provide any diversification benefit. If I turn my attention to the tactical commodity portfolios I find that 26 out of 33 portfolios continue to provide diversification benefits for the US domestic buy-and-hold benchmark. It is very interesting that seven of the 33 commodity portfolios considered do not provide any diversification benefits in the post-2000 era, whereas in the prior 14 years all commodity portfolios provided diversification benefits. Hence, in the decade notably marked by a dramatic increase in commodity market participants, resulting in increasing equity-commodity return correlations, I document that the salient diversification feature which characterized the commodity market has otherwise been weakened for both buy-and-hold and tactical portfolios of commodities when combined with a traditional buy-and-hold portfolio of US equities and bonds. The last column in Table 3 reports J-stat p-values that examine coefficients for a general change in frontier after 2000 ( $\phi_{pj}=0$ ). Results indicate that frontiers defined in the second sub-period are indeed different.

 Table 3

 Diversification Properties of Commodity Portfolios: US Domestic Buy-and-Hold Reference Portfolio

Restrictions on equation					
	Unconditional	Pre- vs Post	Pre-	Post-	Difference in
		Period	Period	Period	Frontiers
Portfolios	$\beta_{qj} = 0$	$S_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$
	ryj	<i>qj</i>	- 4)	' q j	' pj
Panel A: Buy-and-Hold	0.000	0.000	0.000	0.000	0.000
Foods & Fibers	0.000	0.000	0.000	0.029	0.002
Grains & Oilseeds	0.031	0.102	0.033	0.069	0.004
Livestock	0.000	0.000	0.000	0.776	0.054
Energy	0.030	0.056	0.030	0.684	0.031
P. Metals	0.000	0.000	0.000	0.002	0.000
Ewport	0.000	0.000	0.000	0.002	0.000
Panel B: Basis					
Lowbasis	0.002	0.000	0.000	0.043	0.001
Medbasis	0.000	0.000	0.000	0.002	0.000
Highbasis	0.000	0.000	0.000	0.032	0.001
Panel C:					
Speculation					
Lowspec	0.000	0.000	0.000	0.000	0.000
Medspec	0.006	0.002	0.001	0.096	0.006
Highspec	0.004	0.026	0.009	0.045	0.002
Panel D: Mathematical					
Portmv	0.001	0.000	0.000	0.002	0.000
Portevar	0.000	0.000	0.000	0.009	0.000
Portmad	0.001	0.000	0.000	0.001	0.000
Panel E: Momentum					
LowL1H1	0.000	0.000	0.000	0.067	0.010
MedL1H1	0.000	0.000	0.000	0.002	0.000
HighL1H1	0.002	0.007	0.003	0.022	0.000
LowL3H1	0.000	0.000	0.000	0.013	0.001
MedL3H1	0.000	0.000	0.000	0.025	0.000
HighL3H1	0.003	0.000	0.000	0.055	0.001
LowL12H1	0.000	0.000	0.000	0.011	0.001
MedL12H1	0.027	0.063	0.046	0.625	0.035
HighL12H1	0.001	0.001	0.001	0.074	0.001

Table 3 (Cont.)

Restrictions on equation						
	Unconditional	Unconditional Pre- vs Post		Post-	Difference in	
		Period	Period	Period	Frontiers	
Portfolios	$\beta_{qj}=0$	$\delta_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$	
Panel F: Term Structure						
LowTS1_2	0.000	0.000	0.000	0.108	0.003	
MedTS1_2	0.000	0.000	0.000	0.006	0.001	
HighTS1_2	0.004	0.000	0.000	0.014	0.001	
LowTS1_3	0.000	0.000	0.000	0.080	0.002	
MedTS1_3	0.000	0.000	0.000	0.001	0.000	
HighTS1_3	0.002	0.000	0.000	0.017	0.001	
LowTS1_4	0.000	0.000	0.000	0.043	0.002	
MedTS1_4	0.000	0.000	0.000	0.003	0.001	
HighTS1_4	0.003	0.000	0.000	0.044	0.001	

This table presents the spanning test results using a US domestic buy-and-hold portfolio, which consists of the CRSP value-weighted market index returns and the Barclays Capital U.S. Aggregate Bond Index returns, as the benchmark asset. The reported numbers J-stat p values from tests of the restriction on Equation (3). The null hypothesis of all tests is spanning; that is, adding a portfolio of commodity futures to the benchmark assets that include "returns on the CRSP value—weighted index" and "returns on Barclays Capital monthly bond index" provides no diversification benefits. The restriction  $\beta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier for whole period;  $\delta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the pre-period;  $\phi_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the pre-period; and tests  $\phi_{pj}$ =0 tests the hypothesis that frontiers are the same in pre- and post- periods.

# 5.2.2. US domestic actively managed benchmark portfolio

I also investigate the potential for diversification benefits using an actively managed equity-based portfolio as the benchmark asset. Table 4 presents the spanning test results using a US domestic actively managed portfolio, which consists of returns on six US equity portfolios formed on the Fama-French monthly size and momentum factors, as the benchmark asset. In all cases the various portfolios of commodity futures examined exhibit that commodity portfolios always provide diversification benefits irrespective of time period considered. The significance of the sub sample period results are stronger than what I document over the same time period in Table 3.

**Table 4**Diversification Properties of Commodity Portfolios: US Domestic Actively Managed Reference Portfolio

PORTION						
		on equation				
	Unconditional	Pre- vs Post	Pre-	Post-	Difference in	
		Period	Period	Period	Frontiers	
Portfolios	$\beta_{qj} = 0$ $\delta$	$S_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$	
Panel A: Buy-and-						
Hold						
Foods & Fibers	0.000	0.000	0.000	0.002	0.000	
Grains &	0.000	0.000	0.000	0.002	0.000	
Oilseeds						
Livestock	0.000	0.000	0.000	0.217	0.000	
Energy	0.000	0.000	0.000	0.035	0.000	
P. Metals	0.000	0.000	0.000	0.000	0.000	
Ewport	0.000	0.000	0.000	0.000	0.000	
Panel B: Basis					_	
Lowbasis	0.000	0.000	0.000	0.011	0.000	
Medbasis	0.000	0.000	0.000	0.000	0.000	
Highbasis	0.000	0.000	0.000	0.000	0.000	
Panel C:					_	
Speculation						
Lowspec	0.000	0.000	0.000	0.000	0.000	
Medspec	0.000	0.000	0.000	0.002	0.000	
Highspec	0.000	0.000	0.000	0.000	0.000	
Panel D:					_	
Mathematical						
Portmv	0.000	0.000	0.000	0.008	0.000	
Portcvar	0.000	0.000	0.000	0.007	0.000	
Portmad	0.000	0.000	0.000	0.009	0.000	
Panel E: Momentum					_	
LowL1H1	0.000	0.000	0.000	0.007	0.000	
MedL1H1	0.000	0.000	0.000	0.000	0.000	
HighL1H1	0.000	0.000	0.000	0.001	0.000	
LowL3H1	0.000	0.000	0.000	0.000	0.000	
MedL3H1	0.000	0.000	0.000	0.003	0.000	
HighL3H1	0.000	0.000	0.000	0.000	0.000	
LowL12H1	0.000	0.000	0.000	0.001	0.000	
MedL12H1	0.000	0.000	0.000	0.235	0.000	
HighL12H1	0.000	0.000	0.000	0.004	0.000	

Table 4 (Cont.)

	Restrictions on equation						
_	Unconditiona	nconditional Pre- vs Post		Post-	Difference in		
_		Period	Period	Period	Frontiers		
Portfolios	$\beta_{qj} = 0$	$\delta_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$		
Panel F: Term Structure							
LowTS1_2	0.000	0.000	0.000	.053	0.000		
MedTS1_2	0.000	0.000	0.000	.000	0.000		
HighTS1_2	0.000	0.000	0.000	.000	0.000		
LowTS1_3	0.000	0.000	0.000	.047	0.000		
MedTS1_3	0.000	0.000	0.000	.000	0.000		
HighTS1_3	0.000	0.000	0.000	.000	0.000		
LowTS1_4	0.000	0.000	0.000	.027	0.000		
MedTS1_4	0.000	0.000	0.000	.000	0.000		
HighTS1_4	0.000	0.000	0.000	.000	0.000		

This table presents the spanning test results using a US domestic buy-and-hold portfolio, which consists of returns on six US equity portfolios formed on the Fama-French monthly size and momentum factors, as the benchmark asset. The reported numbers J-stat p values from tests of the restriction on Equation (3). The null hypothesis of all tests is spanning; that is, adding a portfolio of commodity futures to the benchmark assets that include "returns on the CRSP value—weighted index" and "returns on Barclays Capital monthly bond index" provides no diversification benefits. The restriction  $\beta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier for whole period;  $\delta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the pre-period;  $\phi_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the post-period; and tests  $\phi_{pj}$ =0 tests the hypothesis that frontiers are the same in pre-and post- periods.

However, in contrast to that of Table 3, I find that over both sub-sample periods the significance of the results are much more strongly preserved. Only the buy-and-hold Livestock sector portfolio and a couple of tactical portfolios (i.e. MedL12H1 and LowTS1\_2) in the latter half of the sample period provide no additional diversification benefits when added to the actively managed benchmark asset.

The differing results of Tables 3 and 4 may be explained by the additional risk inherent in the frequently rebalanced equity portfolios based on size and momentum factors when compared to the traditional buy-and-hold portfolios. When the actively managed risky portfolio is

Table 5
Diversification Properties of Commodity Portfolios: International Buy-and-Hold Reference
Portfolio

Restrictions on equation						
	Unconditional	Pre- vs Post	Pre-	Post-	Difference in	
	Officoliditional	Period	Period	Period	Frontiers	
Portfolios	$\phantom{aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa$					
FOILIOHOS	$\beta_{qj} = 0$ $\delta$	$S_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$	
Panel A: Buy-and-						
Hold						
Foods & Fibers	0.001	0.006	0.003	0.159	0.001	
Grains & Oilseeds	0.000	0.000	0.000	0.371	0.000	
Livestock	0.000	0.001	0.027	0.315	0.001	
Energy	0.000	0.004	0.031	0.022	0.000	
P. Metals	0.000	0.062	0.055	0.206	0.002	
Ewport	0.000	0.000	0.000	0.007	0.000	
Panel B: Basis						
Lowbasis	0.000	0.000	0.000	0.123	0.000	
Medbasis	0.000	0.000	0.000	0.000	0.000	
Highbasis	0.000	0.000	0.002	0.119	0.001	
Panel C: Speculation						
Lowspec	0.000	0.000	0.000	0.078	0.000	
Medspec	0.000	0.001	0.000	0.068	0.000	
Highspec	0.000	0.001	0.005	0.008	0.000	
Panel D: Mathematical						
Portmv	0.002	0.055	0.012	0.155	0.001	
Portcvar	0.001	0.008	0.001	0.143	0.001	
Portmad	0.003	0.075	0.015	0.303	0.001	
Panel E:						
Momentum						
LowL1H1	0.000	0.000	0.001	0.665	0.002	
MedL1H1	0.000	0.000	0.001	0.013	0.000	
HighL1H1	0.000	0.005	0.015	0.029	0.000	
LowL3H1	0.000	0.000	0.000	0.336	0.001	
MedL3H1	0.001	0.002	0.001	0.025	0.001	
HighL3H1	0.000	0.003	0.034	0.068	0.001	
LowL12H1	0.000	0.000	0.000	0.086	0.000	
MedL12H1	0.001	0.023	0.007	0.797	0.001	
HighL12H1	0.000	0.003	0.010	0.074	0.001	

Table 5 (Cont.)

Restrictions on equation						
	Unconditional	Unconditional Pre- vs Post		Post-	Difference in	
_		Period	Period	Period	Frontiers	
Portfolios	$\beta_{qj} = 0$ $\delta$	$G_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$	
Panel F: Term Structure						
LowTS1_2	0.000	0.000	0.000	0.102	0.000	
MedTS1_2	0.000	0.000	0.000	0.000	0.000	
HighTS1_2	0.000	0.043	0.244	0.240	0.003	
LowTS1_3	0.000	0.000	0.000	0.137	0.000	
MedTS1_3	0.000	0.000	0.000	0.000	0.000	
HighTS1_3	0.000	0.007	0.093	0.273	0.001	
LowTS1_4	0.000	0.000	0.000	0.043	0.000	
MedTS1_4	0.000	0.000	0.000	0.009	0.000	
HighTS1_4	0.000	0.009	0.035	0.276	0.001	

This table presents the spanning test results using an international buy-and-hold portfolio, which consists of returns on seven developed nation's equity indices and the Barclays Capital U.S. Aggregate Bond Index returns, as the benchmark asset. These nations include: Australia, Canada, France, Germany, Japan, the UK, and the US. This table presents the spanning test results using a US domestic buy-and-hold portfolio, which consists of returns on six US equity portfolios formed on the Fama-French monthly size and momentum factors, as the benchmark asset. The reported numbers J-stat p values from tests of the restriction on Equation (3). The null hypothesis of all tests is spanning; that is, adding a portfolio of commodity futures to the benchmark assets that include "returns on the CRSP value—weighted index" and "returns on Barclays Capital monthly bond index" provides no diversification benefits. The restriction  $\beta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier for whole period;  $\delta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the pre-period;  $\phi_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the post-period; and tests  $\phi_{pj}$ =0 tests the hypothesis that frontiers are the same in pre- and post- periods.

augmented with different styles of commodity portfolios risk is subsequently reduced. Overall, the diversification findings observed over all sample periods are strongly consistent and show that if an investor is willing to take on the additional risk of an actively managed benchmark portfolio, the majority of commodity portfolios in both a buy-and-hold and tactical setting can provide substantial diversification benefits.

# 5.2.3. International buy-and-hold benchmark portfolio

The results of Tables 3 and 4 solely focus on the US domestic case as the benchmark asset. However, commodities are global products and investors who seek diversification opportunities generally hold securities from numerous different nations and not just the US, thus it seems rather intuitive to investigate the diversification properties of commodities on an international stage as well. Table 5 presents the spanning test results using an international buyand-hold portfolio, which consists of returns on seven developed nation's equity indices and the Barclays Capital US Aggregate Bond Index returns, as the benchmark asset. The interpretation of results follows exactly from the prior sections. The full sample results of Table 5 show very similar results when compared to the US domestic case. However, all of the buy-and-hold and tactical portfolios now become highly significant under the international benchmark portfolio. This result is of particular merit since it is the tactical portfolios which generally provide greater return potential.

An examination of the two sub-sample periods shows a somewhat trend to what was observed in Table 3, but with much weaker diversification benefits preserved in the latter subsample period. Over the first half of the full sample period the vast majority of commodity portfolios provide exceptional diversification benefits when combined with a buy-and-hold international portfolio of equities and bonds, just as in the US domestic case. However, over the latter half of the sample period the spanning test results show that numerous commodity portfolios do not provide same form of diversification. While it is readily apparent that several commodity portfolios have lost their power as diversification tools in moving from the first subsample period to the second, the overall findings bear a sharp contrast to what was observed in the US domestic analysis. The latter sub-sample results are somewhat mixed, particularly when compared to the US domestic case, but overall findings suggest international diversification opportunities in the post-2000 era using commodity portfolios have been diminished. Hence, while the diversification properties of commodity portfolios have been substantially reduced in the last decade for an international buy-and-hold portfolio of equities and bonds, just as in the buy-and-hold US domestic case, it seems to be to a much more degree. Nonetheless, this evidence points towards the financialization of the commodity market as weakening (to varying degrees) the diversification opportunities for all types of buy-and-hold investors.

## 5.2.4. International actively managed benchmark portfolio

Table 6 presents the spanning test results using an international actively managed portfolio, which consists of returns on six international equity portfolios formed on the Fama-French monthly size and momentum factors from 23 developed nation's equity indices, as the benchmark asset. Interestingly, the full sample results are the same when compared to the US domestic case in Table 4. All commodity portfolios are significant diversifiers when combined with dynamic international portfolio. However, in moving to the sub-sample analysis the results are strikingly similar to those of Table 4. Virtually all commodity portfolios provide substantial diversification benefits in the first period, but in the second period several of these portfolios lose their significance. It seems that the addition of commodity portfolios to an actively managed benchmark asset, whether it be a US or international portfolio, offers the same diversification opportunities.

Comparing the international actively managed benchmark results to the international buyand-hold benchmark findings shows only marginal diversification gains for the actively managed
reference portfolio. Specifically, only two more of the buy-and-hold commodity portfolios and
the whole subset of mathematical portfolios become significant for post-2000 period in Table 6
versus the results of Table 4. The tactical portfolio provide stronger diversification benefits when
combined with dynamic international portfolio as compared to buy and hold international
portfolio. This is particularly interesting given the strong contrast between the results of the US
domestic buy-and-hold and actively managed reference portfolios. Thus, in the international
portfolio setting the diversification gains from using an actively managed benchmark portfolio
versus a traditional buy-and-hold approach provides much stronger diversification gains to the
investor.

**Table 6**Diversification Properties of Commodity Portfolios: International Actively Managed Reference Portfolio

PORTIOIIO						
		s on equation				
	Unconditiona		Pre-	Post-	Difference in	
		Period	Period	Period	Frontiers	
Portfolios	$\beta_{qj} = 0$	$\delta_{qj} = \phi_{qj} = 0$	$\delta_{qj}=0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$	
Panel A: Buy-and-Hold	l					
Foods & Fibers	0.000	0.000	0.000	0.000	0.001	
Grains & Oilseeds	0.000	0.000	0.000	0.000	0.000	
Livestock	0.000	0.000	0.000	0.436	0.011	
Energy	0.001	0.203	0.080	0.619	0.003	
P. Metals	0.000	0.000	0.000	0.002	0.001	
Ewport	0.000	0.000	0.000	0.000	0.000	
Panel B: Basis						
Lowbasis	0.000	0.000	0.000	0.014	0.001	
Medbasis	0.000	0.000	0.000	0.000	0.000	
Highbasis	0.002	0.035	0.011	0.035	0.002	
Panel C: Speculation						
Lowspec	0.000	0.000	0.000	0.000	0.000	
Medspec	0.000	0.000	0.000	0.041	0.001	
Highspec	0.000	0.015	0.003	0.010	0.000	
Panel D: Mathematical	!					
Portmv	0.000	0.000	0.000	0.039	0.004	
Portcvar	0.000	0.000	0.000	0.020	0.002	
Portmad	0.000	0.000	0.000	0.042	0.004	
Panel E: Momentum						
LowL1H1	0.000	0.000	0.000	0.122	0.001	
MedL1H1	0.000	0.000	0.000	0.000	0.000	
HighL1H1	0.000	0.000	0.000	0.001	0.000	
LowL3H1	0.000	0.000	0.000	0.004	0.000	
MedL3H1	0.000	0.000	0.000	0.000	0.000	
HighL3H1	0.006	0.059	0.012	0.030	0.003	
LowL12H1	0.000	0.000	0.000	0.000	0.000	
MedL12H1	0.000	0.000	0.001	0.199	0.002	
HighL12H1	0.005	0.041	0.007	0.051	0.002	

Table 6 (Cont.)

	Restrictions on equation						
	Unconditional	Unconditional Pre- vs Post		Post-	Difference in		
		Period	Period	Period	Frontiers		
Portfolios	$\beta_{qj}=0$	$S_{qj} = \phi_{qj} = 0$	$\delta_{qj} = 0$	$\phi_{qj} = 0$	$\phi_{pj} = 0$		
Panel F: Term Structure							
LowTS1_2	0.000	0.000	0.000	0.117	0.002		
MedTS1_2	0.000	0.000	0.000	0.000	0.000		
HighTS1_2	0.001	0.010	0.002	0.004	0.001		
LowTS1_3	0.000	0.000	0.000	0.096	0.001		
MedTS1_3	0.000	0.000	0.000	0.000	0.000		
HighTS1_3	0.003	0.026	0.006	0.012	0.002		
LowTS1_4	0.000	0.000	0.000	0.027	0.000		
MedTS1_4	0.000	0.000	0.000	0.000	0.000		
HighTS1_4	0.004	0.090	0.027	0.055	0.002		

This table presents the spanning test results using an international actively managed portfolio, which consists of returns on six international equity portfolios formed on the Fama-French monthly size and momentum factors from 23 developed nation's equity indices, as the benchmark asset. The nations include: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Switzerland, Sweden, the UK, and the US. The reported numbers J-stat p values from tests of the restriction on Equation (3). The null hypothesis of all tests is spanning; that is, adding a portfolio of commodity futures to the benchmark assets that include "returns on the CRSP value—weighted index" and "returns on Barclays Capital monthly bond index" provides no diversification benefits. The restriction  $\beta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier for whole period;  $\delta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the pre-period;  $\delta_{qj}$ =0 tests the hypothesis that adding commodity does not change frontier in the post-period; and tests  $\delta_{pj}$ =0 tests the hypothesis that frontiers are the same in pre- and post- periods.

# 6. Concluding Remarks

This paper examines the impact of surge in investments in commodity futures on the risk adjusted returns and diversification benefits of commodity portfolios. Many recent studies which examine the effects of the financialization of the commodity futures market argue that the highly touted benefits, such as "equity-like" returns and diversification properties, may be eroding due to the increasing co-movement between commodity futures and traditional assets like equity and bonds. Given these suppositions, I form six buy and hold and twenty-seven tactical commodity

portfolios and examine returns properties and diversifying characteristics of these portfolios over the full sample and sub-samples.

Of all the portfolios considered, buy and hold Energy sector portfolio, High basis portfolio, High speculation portfolio, the High momentum and Low term structure tactical portfolios generally exhibit higher risk adjusted returns. Although many commodity portfolios seemingly exhibit higher risk unadjusted returns in the post-2000 era, there is no significant change in the average performance of commodity portfolios after accounting for risk. Analysis of my subsample show that there is a general change in the beta coefficient of equity return indicating that commodity market has been more exposed in the latter sample period.

Furthermore, I implement stochastic discount factor based spanning test to access whether financialization has impacted the diversification benefits of commodity portfolios when combined with portfolio of traditional assets. I test each commodity portfolio against four bench marks: one domestic and one international buy and hold traditional portfolios, and one domestic and one international dynamic traditional portfolios. For the overall sample period, I find the evidence that commodity portfolios provide significant diversification benefit regardless of portfolio tested against any reference assets. Further, commodity portfolios provide similar diversification benefits for most part.

Analysis of diversification benefits in the sub-sample period provide dissimilar results when commodity portfolios are combined with buy and hold reference portfolios and with dynamic reference portfolio. The diversifying ability of commodity portfolio gets weaker in the second half period when buy and hold reference portfolio is considered. But, results are pretty much remains the same both in the pre-2000 period and post-2000 period when dynamic reference portfolio is considered. My tests also reveals that the two sub-period frontiers are different.

Overall, evidence suggests that commodity market has been more integrated with the equity market but risk adjusted returns pattern has not changed much and diversification of incorporating commodity portfolio into traditional portfolio has not disappeared. Changes in stock market betas suggest that maintaining portfolio composition over long time horizons is sub-optimal as asset correlations invariably change in the light of market condition. Further,

diversification tests based on long samples can lead to incorrect inferences. My method allows for a one-time change in asset correlations and shows the importance of permitting time variation. Incorporating richer asset correlation dynamics in the context of my model could be a topic of further research.

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#### APPENDICES

# Appendix A

# 1. Derivation of conditional expectation

In this appendix, I derive the moments characterizing the joint behavior of firm performance and firm survival accounting for sample selection biases arising from ways in which researchers truncate datasets and from the censoring that inevitably occurs in a finite dataset.

In my general modelling framework, I model the entry and survival of firms from a dataset using two exponentially distributed variates t and s, where t determines when firms are born into a dataset and s determines on how long firms survive. I then examine the restriction on these random variables when a researcher requires firms in her dataset to have continuous return histories, selects a sub-sample of data with continuous return histories, and conducts cross-sectional analysis in a sub-sample with a continuous return history requirement. As the censoring of survival time is always a feature of the data that contains a current set of firms, I consider censoring effects with my data truncation criterion. Therefore, I examine six information sets related to entire dataset, sub-sample and their censored and non-censored samples. In the following sections, I introduce notations and conditional probabilities and moments that I use in this paper.

### 1.1 Notations

For expositional purposes and to develop my model, I define the following variables. Let s, t and  $\varepsilon$  be three random variables representing firm's survival time, firm's birth data and average pricing errors such that  $s \sim EXP(\lambda)$  with probability density function (PDF)  $g(s) = e^{-\lambda s} \lambda$  and cumulative distribution function (CDF)  $= G(s) = 1 - e^{-\lambda s} t \sim EXP(\delta)$  with PDF  $= b(t) = e^{-\delta s} \delta$  and CDF  $= B(t) = 1 - e^{-\delta s}$  and  $\varepsilon \sim N(\mu, \sigma^2)$  with PDF  $= f(\varepsilon) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\varepsilon - \mu}{\sigma}\right)^2}$  and CDF  $= F(\varepsilon) = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{x - \mu}{\rho\sqrt{2}}\right)\right)$ , where  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$ .

I also need some additional notations to make different samples. I introduce T as data length, D as mortality month, h as first month of in a sub-sample, q as sub-sample length,  $k^*$  as the minimum number of months required for estimating rolling alphas and k as the minimum number of months a researcher requires a firm to survive to be included in the sample.

Now I can define different samples with different restrictions in terms of these notations. My first sample is a simple  $k^*$  months continuous returns requirement in the entire dataset,  $\Omega_0 = \{t \geq k^*, s \geq k^*\}$ . The second and third samples include censored and non-censored firms form the first sample. For censored firms, the mortality date is not observed because they survive to the end of the dataset, i.e.  $D > T \Rightarrow T - t + s > T \Rightarrow s > t$  and for the non-censored firms  $s \leq t$ . The information criteria for second and third samples can be denoted as  $\Omega_{0c} = \{s > t \geq k^*, s \geq k^*\}$  and  $\Omega_{0nc} = \{t \geq k^*, k^* \leq s < t\}$ .

In order to examine truncation bias, I also examine sub-sample of my entire dataset. For firms to be included in the sub-sample, it must have at least  $k^*$  months of data after the start of the sample,  $D \ge h + k^* \Rightarrow T - t + s \ge T - q + k^*$  or  $t \le s + q - k^*$  or  $t \le s + a$ , where  $a = (q - k^*) > 0$ . Therefore, the information criteria for my sub-sample is  $\Omega_1 = \{k^* \le t \le s + a, s \ge k^*\}$ . Again, for censored sub-sample, s > t and non-censored sub-sample  $s \le t$ . Their selection criteria should be  $\Omega_{1c} = \{s > t \ge k^*, s \ge k^*\}$  and  $\Omega_{1nc} = \{s \le t \le s + a, s \ge k^*\}$  respectively.

# 1.2 Distribution preliminaries

The survival and arrival of firms into a dataset would not be important to pricing unless pricing errors were correlated with firm entry or survival time. Therefore, I allow for correlation between pricing errors, ε and survival times, s but assume pricing errors are uncorrelated to birth dates t. Assuming that the joint behavior of average pricing errors and survival can be characterized by Farlie-Gumbel-Morgenstern type bivariate distribution function, the joint cumulative distribution functions in terms of above notation can be written as

$$H(\varepsilon, s) = F(\varepsilon)G(s)[1 + 3\rho(1 - F(\varepsilon))(1 - G(s))] \tag{A.1}$$

where  $\rho$  measures the degree of association between survival and average pricing errors. Therefore, the joint probability density function of survival and pricing errors can be written as

$$h(\varepsilon, s) = \frac{\partial H(\varepsilon, s)}{\partial \varepsilon \partial s} = \frac{e^{-2s\lambda - \frac{(\varepsilon - \mu)^2}{2\sigma^2}} \lambda (e^{s\lambda} - 3(-2 + e^{s\lambda})\rho \operatorname{Erf}[\frac{-\varepsilon + \mu}{\sqrt{2}\sigma}])}{\sqrt{2\pi}\sigma}$$
 A.2

I find high positive correlation between survival and birth variables in the CRSP data base. To address this issue, I assume birth and survival variables are not independent. Therefore, I model birth rate as a convex combination of survival rate and another independent exponentially distributed random variable u such that  $u \sim EXP(\vartheta)$  with PDF =  $i(u) = e^{-\theta u} \lambda$  and  $CDF = I(u) = 1 - e^{-\theta u}$ . Then, the joint PDF of s and u is  $\theta e^{-\theta u} \lambda e^{-\lambda s}$ . Since I assume  $t = \omega s + (1 - \omega) u$ , the joint PDF s and t can be written as

$$\phi(s,t) = \frac{e^{-\frac{(t-s\,\omega)\theta}{1-\omega} - s\lambda}\theta\lambda}{1-\omega}$$

Hence, the joint density function for pricing errors, survival and birth times is

$$l(\varepsilon, s, t) = h(\varepsilon, s) \phi(s, t)$$
 A.4

# 1.3 Conditional Probabilities, Expectations and Variance

Given the joint PDF in equation (A.4), the conditional probabilities for given different information sets can be computed as

$$P(\Omega_{...}) = \iiint_{-\infty}^{\infty} l(\varepsilon, s, t) \ d\varepsilon \ dt \ ds = \iiint_{\infty} g(s) \ \phi(s, t) \ dt \ ds$$

because the effect of normal variate simply integrates out. Equation (A.5) is used to calculate conditional probabilities for given information sets. The results are presented in Table A1.

Since the restrictions are on the survival and arrival times and not the distribution of pricing errors, the conditional moments for the mean and variance of pricing errors can be computed without setting limits on survival and arrival time. The expectation of pricing errors leaving the limits on *s* and *t* indefinite and the information set generic is,

$$E[\varepsilon|\Omega_{...}] = \frac{1}{P(\Omega_{...})} \iiint_{-\infty}^{\infty} \varepsilon f(\varepsilon)g(s) \left[1 + 3\rho \left[\left(1 - 2F(\varepsilon)\right)\left(1 - 2G(s)\right)\right]\right] \phi(s,t) \ d\varepsilon dt ds$$

and distributing the integral w.r.t.  $\varepsilon$  I get,

$$[\varepsilon|\Omega_{\dots}] = \frac{1}{P(\Omega_{\dots})} \iint \phi(s,t) \left[ \int_{-\infty}^{\infty} \varepsilon f(\varepsilon) d\varepsilon + 3\rho(1 - 2G(s)) \left[ \int_{-\infty}^{\infty} \varepsilon f(\varepsilon) d\varepsilon - 2 \int_{-\infty}^{\infty} \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon \right] \right] dt ds$$

The integrands in this expression are standard (i.e., equal to the mean: $\mu$ ) except for the term  $\int_{-\infty}^{\infty} \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon = \frac{1}{2} \mu + \frac{\sigma}{2\sqrt{\pi}},$  which involves the cumulative normal distribution and whose derivation is shown in Appendix B. After evaluating the integrals the expectation becomes,

$$E[\varepsilon|\Omega_{...}] = \frac{1}{P(\Omega_{...})} \iint \phi(s,t) \left[\mu - 3\rho \frac{\sigma}{\sqrt{\pi}} (1 - 2G(s))\right] dtds$$
 A.6

Equation (A.6) is used with six information sets to calculate conditional expectation and the results are presented in Table A4.

In calculating the variance of pricing errors, I need the expectation of squared pricing errors which is,

$$E[\varepsilon^{2}|\Omega_{...}] = \frac{1}{P(\Omega_{...})} \iiint_{-\infty}^{\infty} \varepsilon^{2} f(\varepsilon) g(s) \left[ 1 + 3\rho \left[ \left( 1 - 2F(\varepsilon) \right) \left( 1 - 2G(s) \right) \right] \right] \phi(s,t) \ d\varepsilon dt ds$$

Distributing the integral w.r.t.  $\varepsilon$  I get the expression,

$$[\varepsilon^2 | \Omega_{...}] =$$

$$\frac{1}{P(\Omega_{...})} \iint g(s) \, \phi(s,t) \left[ \int_{-\infty}^{\infty} \varepsilon^2 \, f(\varepsilon) d\varepsilon + 3\rho (1 - 2G(s)) \left[ \int_{-\infty}^{\infty} \varepsilon^2 f(\varepsilon) d\varepsilon - 2 \int_{-\infty}^{\infty} \varepsilon^2 f(\varepsilon) \, F(\varepsilon) d\varepsilon \right] \right] dt ds$$

where the first two integrands are standard (i.e., equal to  $\sigma^2 + \mu^2$ ) and the solution to the third integrand,  $\int_{-\infty}^{\infty} \varepsilon^2 f(\varepsilon) F(\varepsilon) d\varepsilon = \frac{1}{2} (\sigma^2 + \mu^2) + \frac{\mu \sigma}{\sqrt{\pi}}$ , shown in Appendix B again involves the cumulative normal distribution. After evaluating integrals, a similar expression to that of the mean is given,

$$E[\varepsilon^{2}|\Omega_{...}] = \frac{1}{P(\Omega)} \iint \phi(s,t) \left[ \sigma^{2} + \mu^{2} - 6\rho \frac{\mu\sigma}{\sqrt{\pi}} (1 - 2G(s)) \right] dtds$$
 A.7

Using the equations (A.6) and (A.7) I can compute the variance as

$$Var[\varepsilon|\Omega_{...}] = \frac{1}{P(\Omega_{...})} \iint \phi(s,t) \left[ \sigma^2 - \left( \frac{6\rho\sigma}{\sqrt{\pi}} (G(s) - \frac{1}{2}) \right)^2 \right] dtds$$

which is of the form used with the six information sets to calculate conditional variance shown in Table A4.

In calculating conditional survival time, the effect of the normal variate simply integrates out leaving the joint pdf of birth date and survival time. To show this, consider expected survival time:

$$E[s|\Omega_{...}] = \frac{1}{P(\Omega_{...})} \iiint_{-\infty}^{\infty} sf(\varepsilon)g(s) \left[1 + 3\rho \left[\left(1 - 2F(\varepsilon)\right)\left(1 - 2G(s)\right)\right]\right] \phi(s,t) \ d\varepsilon dt ds$$

Distributing the integrals w.r.t.  $\varepsilon$  gives,

$$E[s|\Omega_{...}] =$$

$$\frac{1}{P(\Omega_{...})}\iint s \,\phi(s,t) \left[ \int_{-\infty}^{\infty} f(\varepsilon)d\varepsilon + 3\rho(1-2G(s)) \left[ \int_{-\infty}^{\infty} f(\varepsilon)d\varepsilon - 2 \int_{-\infty}^{\infty} f(\varepsilon) F(\varepsilon)d\varepsilon \right] \right] dtds$$

where the first two integrands are equal to one and the non-standard term is  $\int_{-\infty}^{\infty} f(\varepsilon)F(\varepsilon)d\varepsilon = \int_{-\infty}^{\infty} F(\varepsilon)'F(\varepsilon)d\varepsilon = \frac{[F(\varepsilon)]^2}{2}\Big|_{-\infty}^{\infty} = \frac{1}{2}$ . The result,

$$E[s|\Omega_{...}] = \frac{1}{P(\Omega_{...})} \iint s \,\phi(s,t) \,dtds$$

demonstrates that the conditional expected survival time does not depend on pricing errors, because pricing errors are unrestricted and the Morgenstern distribution has constant marginal. Equation (A.9) with different information sets is used to calculate conditional expectation of survival and arrival time.

### 1.4 Conditional Covariance

The conditional covariance between the pricing error and survival time, denoted by  $Cov(\varepsilon, s|\Omega_m)$  is defined as

$$Cov(\varepsilon, s|\Omega_{m}) = E[(\varepsilon - E(\varepsilon|\Omega_{m}))(s - E(s|\Omega_{m}))]$$
A.10

Similarly, the conditional covariance between the survival and arrival time, denoted by  $Cov(s, t|\Omega_{m})$ , is defined as

$$Cov(s,t|\Omega_{...}) = E[(s - E(s|\Omega_{...}))(t - E(t|\Omega_{...}))]$$
A.11

The equation (A.10) and (A.11) with different information criteria are used to calculate conditional covariance between pricing error and survival and between survival and birth rate. The results are presented in Table A.3 and Table A.5.

Table A. Probabilities, expectations and moment conditions

A1. Conditional Survival (s) and Birth Time (t) Probabilities and Limits by sample type

	conditional survival (s) and shall time (v) trobustities and similar type						
Sample	Without censored	Non-Censored survival					
	Survival	[t > s]	[s > t, t > k]				
Data requirement	$P[\Omega_0] =$	$P[\Omega_{0,nc}] =$	$P[\Omega_{0,c}] =$				
[s > k]	$e^{-k\lambda}$	$\frac{e^{-k(\theta+\lambda)}\lambda}{\theta+\lambda}$	$\frac{\theta e^{-\frac{k\lambda}{w}}w - \theta P[\Omega_{0,nc}]}{w(\theta + \lambda) - \lambda}$				
ds =	(1, 00)		` ,				
	{ <i>k</i> ,∞}	$\{k,\infty\}$	$\{t,t/w\}$				
dt =	n.a.	{ <i>s</i> ,∞}	{ <i>k</i> ,∞}				
Data and start	$P[\Omega_1] =$	$P[\Omega_{1,nc}] =$	$P[\Omega_{1,c}] =$				
date selection $[s > k, t < s + a]$	$P[\Omega_0] - e^{\frac{a\theta}{-1+w}} P[\Omega_{0,nc}]$	$\left(1-e^{\frac{a\theta}{-1+w}}\right)P[\Omega_{0,nc}]$	$Pigl[\Omega_{0,c}igr]$				
ds =	{ <i>k</i> ,∞}	$\{k,\infty\}$	$\{t,t/w\}$				
dt =	$\{ws, s+a\}$	$\{s,s+a\}$	$\{k,\infty\}$				

A2. Orthogonality conditions without censoring

Data requirement only	Data requirement and sub-sample
$\Omega_0 = [s > k]$	$\Omega_1 = [s\&t > k, t < s + a]$
$m_1[\Omega_0] = a - E[a s]$	$m_1[\Omega_1] = a - E[a s]$
$m_2[\Omega_0] = s - E[s \Omega_0]$	$m_2[\Omega_1] = s - E[s \Omega_1]$
$m_3[\Omega_0] = (a - E[a s])^2 - V[e s]R_c[S_k]$	$m_3[\Omega_1] = (a - E[a s])^2 - V[e s]R_c[S_k]$
$m_4[\Omega_0] = (a - E[a s])s$	$m_4[\Omega_1] = (a - E[a s])s$
	$m_5[\Omega_1] = t - E[t \Omega_1, s]$
	$m_6[\Omega_1] = (t - E[t \Omega_1, s])s$
Conditions Jointly Estimated	Conditions Jointly Estimated
$E\big[\varepsilon_l[\Omega_0]\big] = 0, l = \{1,4\}$	$E[m_l[\Omega_1]] = 0, l = \{1,6\}$
Parameters: $\{\mu, \lambda, \rho, \sigma\}$	Parameters: $\{\mu, \lambda, \rho, \sigma, \theta, w\}$

#### Orthogonality conditions with censored survival, [s > t, s = t] and restrictions $\Omega_0$ or $\Omega_1$ A3.

Non-censored survival $s < t$ , $d_{cs} = 0$	Censored survival $s > t, d_{cs} = 1$
$m_1[\Omega_{j,nc}] = a - E[a s]$	$m_1[\Omega_{j,c}] = a - E[a \Omega_{j,c}]$
$m_2[\Omega_{j,nc}] = s - E[s \Omega_{j,nc}]$	$m_2[\Omega_{j,c}] = s - E[t \Omega_{j,c}]$
$m_3[\Omega_{j,nc}] = (a - E[a s])^2 - V[e s]R_c[S_k]$	$m_3[\Omega_{j,c}] = \frac{(a - E[a \Omega_{j,c}])^2}{-V[e \Omega_{j,c}]R_c[S_k]}$
$m_4[\Omega_{j,nc}] = (a - E[a s])s$	$m_4[\Omega_{j,c}] = (a - E[a \Omega_{j,c}])t - C[a,t \Omega_{j,c}]$
$m_5[\Omega_{j,nc}] = t - E[t \Omega_{j,nc}, s]$	$m_5[\Omega_{j,c}] = t - E[t \Omega_{j,c}]$
$m_6[\Omega_{j,nc}] = (t - E[t \Omega_{j,nc},s])s$	$m_6[\Omega_{j,c}] = (t - E[t \Omega_{j,c}])^2 - V[t \Omega_{j,c}]$

# Conditions Jointly Estimated

Conditions Jointly Estimated 
$$E[m_l[\Omega_{j,nc}](1-d_{cs}) + m_l[\Omega_{j,c}]d_{cs}] = 0, l = \{1,6\}, \text{ for each } j = \{0, \text{ or } 1\}$$

Parameters estimated:  $[\mu, \lambda, \rho, \sigma, \theta, w]$ 

Table B. Expression for expectations

$$\begin{split} \overline{E[s|\Omega_0]} &= k + 1/\lambda \\ \overline{E[s|\Omega_{0,nc}]} &= k + 1/\lambda \\ \overline{E[s|\Omega_{0,nc}]} &= E[s|\Omega_{1,nc}] = k + \frac{1}{\theta + \lambda} \\ E[s|\Omega_c] &= \left[ (w-1)(w(\theta + \lambda) - \lambda) \left[ \lambda^2 e^{\frac{k(w(\theta + 2\lambda) - \lambda)}{(w-1)w}} (\theta^2 k w + \theta(2w-1)(k\lambda + 1) + \lambda) \right] \right] \\ \lambda(w-1)(k\lambda + 2)) - (\theta + \lambda)^2 e^{\frac{kw(\theta + \lambda)}{w-1}} \times \\ \left[ (-k\lambda^2 + \lambda w(k(\theta + \lambda) - 2) + w^2(\theta + 2\lambda)) \right] e^{\left[ -\frac{k(-\lambda + w^2(\theta + \lambda) + \lambda w)}{(w-1)w} \right]} \\ \lambda(1 - w)(\theta + \lambda)^2 (\lambda - w(\theta + \lambda))^2 \left( we^{-\frac{k\lambda}{w}} - \frac{\lambda e^{-k(\theta + \lambda)}}{\theta + \lambda} \right) \\ E[s|\Omega_1] &= k + \frac{1}{\theta + \lambda} + \frac{e^{k\theta}}{\lambda(-e^{-1+\omega}\lambda + e^{k\theta}(\theta + \lambda))} \\ E[s|\Omega_{1,nc}] &= E[s|\Omega_{0,nc}] \end{split}$$

# **B.2** Arrival time

$$\begin{split} \overline{E[t|\Omega_{0,nc}]} &= k + \frac{1-w}{\theta} + \frac{1}{\theta+\lambda} \\ E[t|\Omega_{0},s] &= sw + (1-w)\frac{1}{\theta} \\ E[t|\Omega_{0,nc},s] &= s + \frac{1}{\theta}(1-w) \\ E[t|\Omega_{0,nc}] &= \frac{\lambda(\theta+\lambda)}{e^{\frac{k\lambda}{w}}\lambda - e^{k(\theta+\lambda)}w(\theta+\lambda)} \left[\frac{e^{\frac{k\lambda}{w}}(1+k(\theta+\lambda))}{(\theta+\lambda)^2} - \frac{e^{k(\theta+\lambda)}w(w+k\lambda)}{\lambda^2}\right] \end{split}$$

$$E[t|\Omega_{1},s] = \frac{(s(1-w)+a)e^{\frac{\theta(a+s)}{w-1}}}{e^{\frac{\theta(a+s)}{w-1}}-e^{\frac{\theta sw}{w-1}}} + sw + (1-w)\frac{1}{\theta}$$

$$E[t|\Omega_{1,\text{nc}},s] = s + (1-w)\frac{1}{\theta} + \frac{a}{1-e^{\left(\frac{a\theta}{1-w}\right)}}$$

$$V[t|\Omega_{0,c}] = \begin{bmatrix} w^{5}(\theta+\lambda)^{5}e^{3k(\theta+\lambda)} + \lambda^{2}w(\theta+\lambda)[3\lambda^{2}+2w^{2}(\theta+\lambda)^{2}-2k^{2}(\theta+\lambda)^{2}-2k^{2}(\theta+\lambda)]e^{k(\theta+\lambda+\frac{2\lambda}{w})} - \lambda w^{2}(\theta+\lambda)^{2}[2\lambda^{2}+3w^{2}(\theta+\lambda)^{2}-2k^{2}(\theta+\lambda)]e^{k(\theta+\lambda)}e^{k(\theta+\lambda)} - \lambda^{5}e^{\frac{3k\lambda}{w}} \end{bmatrix}$$

$$\lambda^{2}(\theta+\lambda)^{2}(w(\theta+\lambda)e^{k(\theta+\lambda)} - \lambda e^{\frac{k\lambda}{w}})^{3}$$

$$\begin{split} E\big[t\big|\Omega_{1,nc}\big] = & \ a + \frac{a}{-1 + e^{\frac{a\,\theta}{-1 + w}}} + E\big[t\big|\Omega_{0,nc}\big] \\ E\big[t|\Omega_1\big] = & \ \frac{e^{-k(\theta + \lambda)}}{p[\Omega_1]\theta\lambda} \left[ \frac{e^{-\frac{k\lambda}{w}} \left(\theta^2 w e^{k(\theta + \lambda)} (k\lambda + w) + \lambda^2 (w - 1)(\theta k - w + 1) e^{\frac{k\lambda}{w}}\right)}{w(\theta + \lambda) - \lambda} - \frac{\lambda^2 e^{\frac{as\theta}{w - 1}} (\theta^2 (as + k) + \theta(\lambda(as + k) - w + 2) + \lambda - \lambda w)}{(\theta + \lambda)^2} \right] \end{split}$$

# **B.3** Conditional expectations and variance of pricing errors

$$E[a|s] = \mu + A[.5 - e^{-s\lambda}]$$

$$V[a|s] = \sigma^2 - A^2[.5 - e^{-s\lambda}]^2$$

$$E[a|\Omega_0] = \mu + .5A(1 - e^{-k\lambda})$$

$$E[a|\Omega_{0,nc}] = \mu + A\left[\frac{1}{2} - \frac{e^{-k\lambda}(\theta + \lambda)}{(\theta + 2\lambda)}\right]$$

$$E\left[a\middle|\Omega_{0,c}\right] = \begin{cases} \mu + A(.5 - E_c), \text{ where} \\ E_c = e^{-\frac{2k\lambda}{W}\theta} \left[\frac{w(\theta + 2\lambda) - 2\lambda e^{k(-\theta + 2(\frac{1}{W} - 1)\lambda)}}{P[\Omega_c]2(\theta + 2\lambda)(w\theta + 2(w - 1)\lambda)}\right] \end{cases}$$

$$E\left[a\middle|\Omega_1\right] = E\left[a\middle|\Omega_{1,nc}\right] = \mu + A\left[\frac{1}{2} - \frac{e^{-2k\lambda}}{P[\Omega_{1,nc}]2(\theta + 2\lambda)}\left(\theta - 2\lambda\left(e^{\frac{\theta(as - kw + k)}{W - 1}} - 1\right)\right)\right] \rbrace$$

$$E\left[a\middle|\Omega_{1,c}\right] = E\left[a\middle|\Omega_{0,c}\right] \rbrace$$

$$V\left[e\middle|\Omega_c\right] = \sigma^2 + A^2\left(E_c(1 - E_c) - \frac{1}{4}\right) \rbrace$$

$$V\left[e\middle|\Omega_{0,nc}\right] = \frac{1}{4}A^2\left(-1 + 4e^{-2k\lambda}(\theta + \lambda)\left(\frac{e^{k\lambda}}{\theta + 2\lambda} - \frac{1}{\theta + 3\lambda}\right)\right) + \sigma^2 \rbrace$$

# **B.4** Covariance between arrival time and pricing errors

$$C[a,t|\Omega_{0,c}] = \frac{AE_c E[t|\Omega_{0,c}]}{P[\Omega_{0,c}]} - \frac{A\theta e^{-k\left(\theta + \frac{2\lambda(w+1)}{w}\right)} \left(w(\theta + 2\lambda)^2 e^{k(\theta + 2\lambda)}(2k\lambda + w) - 4\lambda^2(k(\theta + 2\lambda) + 1)e^{\frac{2k\lambda}{w}}\right)}{P[\Omega_c]\left(4\lambda(\theta + 2\lambda)^2 \left(\theta w + 2\lambda(w - 1)\right)\right)}$$

$$C[a, t | \Omega_{1,c}] = C[a, t | \Omega_{0,c}]$$

Variance adjustment factor

$$R_c = [1 + 3kS_k - k^2]/[3kS_k^2]$$
, where  $S_k = s - k + 1$ 

# 2. Derivation of non-standard integrals

In this Appendix I show the derivations of the non-standard integrals used in computing expectations in Appendix A which are,  $\int_{-\infty}^{\infty} f(\varepsilon) F(\varepsilon) d\varepsilon$ ,  $\int_{-\infty}^{\infty} \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon$ , and  $\int_{-\infty}^{\infty} \varepsilon^2 f(\varepsilon) F(\varepsilon) d\varepsilon$ . First is useful to rewrite the normal distribution function in terms of an error function:

$$F(\varepsilon) = \frac{1}{2} \left( 1 + \operatorname{erf}\left(\frac{-\varepsilon + \mu}{\sqrt{2}\sigma}\right) \right)$$

Next change variables  $z = \frac{\varepsilon - \mu}{\sigma \sqrt{2}}$ ,  $dz = \frac{1}{\sigma \sqrt{2}}$  in the distribution function and density to get,

$$f(z)F(z) = \frac{e^{-z^2}}{2\sqrt{\pi}} (1 + eft(z))$$

The new variable has E(z) = 0 and  $Var(z) = \frac{1}{2}$ . The integrals that aid in computations are:

$$\int_{-\infty}^{\infty} e^{-z^2} dz = \frac{\sqrt{\pi}}{2} \operatorname{erf}(z) \Big|_{-\infty}^{\infty} = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} e^{-z^2} \operatorname{erf}(z) dz = \frac{\sqrt{\pi}}{4} \operatorname{erf}(z)^2 \Big|_{-\infty}^{\infty} = 0$$

$$\int_{-\infty}^{\infty} z e^{-z^2} \operatorname{erf}(z) dz = \frac{\operatorname{erf}(z\sqrt{2})}{2\sqrt{2}} \Big|_{-\infty}^{\infty} - \frac{e^{-z^2} \operatorname{erf}(z)}{2} \Big|_{-\infty}^{\infty} = \frac{1}{\sqrt{2}}$$

$$\int_{-\infty}^{\infty} z^2 e^{-z^2} \operatorname{erf}(z) dz = \frac{1}{8} \left[ \sqrt{\pi} \operatorname{erf}(z)^2 - 4 e^{-z^2} \operatorname{erf}(z) - \frac{2 e^{-2z^2}}{\sqrt{\pi}} \right] \Big|_{-\infty}^{\infty} = 0$$

# **II.1** Solution for $\int_{-\infty}^{\infty} f(\varepsilon) F(\varepsilon) d\varepsilon$

$$\int_{-\infty}^{\infty} f(\varepsilon) F(\varepsilon) dz = \int_{-\infty}^{\infty} \frac{e^{-z^2}}{2\sqrt{\pi}} (1 + eft(z)) dz = \int_{-\infty}^{\infty} \frac{e^{-z^2}}{2\sqrt{\pi}} dz + \int_{-\infty}^{\infty} \frac{e^{-z^2}eft(z)}{2\sqrt{\pi}} dz = \frac{1}{2}$$

# **2.2** Solution for $\int_{-\infty}^{\infty} \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon$

$$\int_{-\infty}^{\infty} \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon = \int_{-\infty}^{\infty} (\mu + z \, \sigma \sqrt{2}) \frac{e^{-z^2}}{2\sqrt{\pi}} \left( 1 + eft(z) \right) dz$$

$$= \mu \int_{-\infty}^{\infty} \frac{e^{-z^2}}{2\sqrt{\pi}} dz + \frac{\sigma \sqrt{2}}{2\sqrt{\pi}} \int_{-\infty}^{\infty} z \, e^{-z^2} \, dz + \mu \int_{-\infty}^{\infty} \frac{e^{-z^2} eft(z)}{2\sqrt{\pi}} \, dz + \frac{\sigma \sqrt{2}}{2\sqrt{\pi}} \int_{-\infty}^{\infty} z \, e^{-z^2} \, erf(z) \, dz$$

$$= \frac{\mu}{2} + \frac{\sigma}{2\sqrt{\pi}}$$

# 2.3 Solution for $\int_{-\infty}^{\infty} \varepsilon^2 f(\varepsilon) F(\varepsilon) d\varepsilon$

$$\varepsilon^{2} = 2 z^{2} \sigma^{2} + \mu^{2} + 2\sqrt{2} z \mu \sigma$$

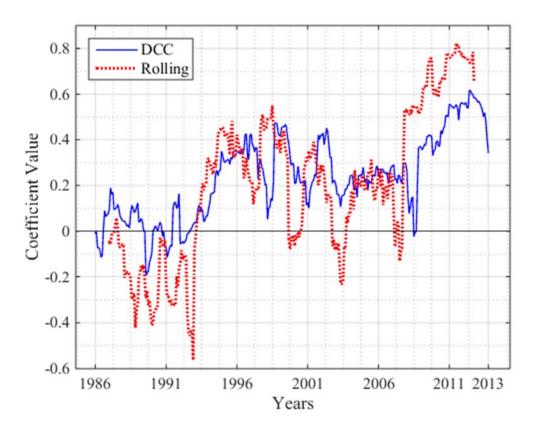
$$\therefore \int_{-\infty}^{\infty} \varepsilon^{2} f(\varepsilon) F(\varepsilon) d\varepsilon = \int_{-\infty}^{\infty} (2 z^{2} \sigma^{2} + \mu^{2} + 2\sqrt{2} z \mu \sigma) \frac{e^{-z^{2}}}{2\sqrt{\pi}} (1 + eft(z)) dz$$

$$= \frac{\mu^{2}}{2} + \frac{\sigma^{2}}{2} + \int_{-\infty}^{\infty} (2 z^{2} \sigma^{2} \frac{e^{-z^{2}}}{2\sqrt{\pi}} \operatorname{erf}(z) + \mu^{2} \frac{e^{-z^{2}}}{2\sqrt{\pi}} \operatorname{erf}(z) + 2\sqrt{2} z \mu \sigma \frac{e^{-z^{2}}}{2\sqrt{\pi}} \operatorname{erf}(z)) dz$$

$$= \frac{\mu^{2}}{2} + \frac{\sigma^{2}}{2} + \frac{2 \sigma^{2}}{2\sqrt{\pi}} \int_{-\infty}^{\infty} z^{2} e^{-z^{2}} \operatorname{erf}(z) dz + \frac{\mu^{2}}{2\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^{2}} \operatorname{erf}(z) dz$$

$$+ \frac{\sqrt{2} \mu \sigma}{\sqrt{\pi}} \int_{-\infty}^{\infty} z e^{-z^{2}} \operatorname{erf}(z) dz$$

$$= \frac{1}{2} (\sigma^{2} + \mu^{2}) + \frac{\mu \sigma}{\sqrt{\pi}}$$



**Figure 1.** Correlation between equity and commodity. This figure plots the correlation between equally weighted commodity index returns and CRSP value weighted stock index returns over time. The dotted line represents 24 months rolling correlation and the solid line shows the dynamic conditional correlation.

**Table 1**Sample of Commodity Futures

Sample of Commod Sector	Exchange	Commodity	Exchange	Futures Start Date
	Symbol	3	S	
	CC	Cocoa	ICE	July, 1959
	KC	Coffee	ICE	August, 1972
Foods & Fibers	JO	Orange Juice	ICE	February, 1967
	SB	Sugar #11	ICE	January, 1961
	CT	Cotton	ICE	July, 1964
	LB	Lumber	CME	October, 1969
	WA	Barley	WCE	May, 1989
	WC	Canola	WCE	September, 1974
	C_	Corn #2	CBOT	July, 1959
	O_	Oats	CBOT	July, 1959
Grains &	RR	Rough Rice #2	CBOT	August, 1986
Oilseeds				
	S_	Soybeans	CBOT	July, 1959
	SM	Soybean Meal	CBOT	July, 1959
	ВО	Soybean Oil	CBOT	July, 1959
	W_	Wheat	CBOT	July, 1959
	FC	Feeder Cattle	CME	November, 1971
Livestock	LC	Live Cattle	CME	November, 1964
	LH	Lean Hogs	CME	February, 1966
	PB	Pork Bellies	CME	September, 1961
	$\operatorname{CL}$	Crude Oil	NYMEX	March, 1983
	НО	Heating Oil #2	NYMEX	December, 1984
Energy	HU	Unleaded Gas	NYMEX	November, 1978
	NG	Natural Gas	<b>NYMEX</b>	April,1990
	PN	Propane	NYMEX	August, 1987
	HG	Copper	NYMEX	July, 1959
	GC	Gold	NYMEX	December, 1974
Precious Metals	PA	Palladium	NYMEX	January, 1977
	PL	Platinum	NYMEX	March, 1968
	SI	Silver	NYMEX	June, 1963

This table provides the individual commodity futures examined, the respective sectors to which the commodity futures belong, as well as futures exchange information and start dates.

**Table 2**Commodity Futures Market Over Time

			Sectors					
Descriptions	Periods	Foods &	Grains &	Livestock	Energy	Precious Metals	All Sectors	
		Fibers	Oilseeds				Combined	
Volume (in '000)	1986- 2000	45	155	26	148	65	439	
,	2001- 2013	128	483	64	564	176	1414	
	Percent Higher	1.82	2.13	1.47	2.82	1.7	2.23	
Open Interest (in '000)	1986- 2000	309	703	133	525	322	1992	
(	2001- 2013	944	2186	395	1485	602	5612	
	Percent Higher	2.06	2.11	1.98	1.83	0.87	1.82	
Correlation Between Equity	1986- 2000	0.0581	0.1303	0.07	0.0434	0.0432	0.0851	
and Commodity	2001- 2013	0.2825	0.2989	0.0723	0.2468	0.3552	0.3608	
	Percent Higher	3.86	1.29	0.03	6.68	7.22	3.24	
Total index investment		\$13b in 2003	255.7b in April 2011 (All time high)			197.5b in October, 2013		

This table reports what has happened the volume, open interest and correlation between commodity market and equity in each sector and combined commodity market.

# **VITA**

The author, Ramesh K. Adhikari, was born in Nepal. He obtained Bachelor's Degree in Mathematics and Economics in 1999 and Master's Degree in Economics in 2003 from Tribhuvan University, Nepal. He also received a Master's degree in Economics from California State University Sacramento (CSUS) in 2009. He received his Ph.D. in Financial Economics from University of New Orleans in 2015.