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Letters to the Editor

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Explicit determination of thickness of a transparent film on a transparent substrate from angles of incidence of equal *p* and *s* reflectivities

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Reflectivity matching (RM) provides the basis for simple and accurate determination of the refractive index and thickness of transparent films because no absolute photometry is required and only angle-of-incidence measurements are made. The most well-known RM method is due to Abelès. Abelès's method[1–3] determines the refractive index of a transparent film from the angle of incidence at which the reflectivities for incident p-polarized light of two adjacent regions of a substrate coated and uncoated by the film are matched. Recently, other polarization-dependent and independent RM techniques[4,5] have been described for the determination of refractive index and thickness of transparent films. These newer methods also require a substrate with film-coated and uncoated areas (or with a thickness step in the film).

A distinct RM technique that dispenses with the aforementioned special sample requirement is based on matching the reflectivities of the film–substrate system for the p and s polarizations of incident light. Angles of incidence of equal p and s reflectivities are easily detected using, for example, a single-element rotating-polarizer ellipsometer[6] (SERPE). In SERPE, synchronous rotation at speed ω of a polarizer in the incident beam modulates the reflected light flux at 2ω with the modulation component disappearing when p and s reflectivity matching (PSRM) takes place. Conditions for PSRM are exactly those of the film–substrate system acting as a reflection retarder; as such, they have been explored before[7] and will not be repeated here. An important conclusion from Ref. 7 is that PSRM is possible at one or more angles of incidence for all film thicknesses above a certain minimum value. Below this minimum, PSRM is still possible but only within a few thickness bands. The characteristic thickness band structure depends on the film and substrate optical properties and is given in Ref. 7 for the SiO_2–Si system at wavelength $\lambda = 6328$ Å.

In principle, PSRM at several angles of incidence can be used to determine all the optical properties of a given film–substrate system and film thickness. However, the inversion problem is implicit and will in general require an iterative computer program.

In this Letter we consider a special case of interest that permits an explicit solution, namely, the determination by PSRM of thickness of a transparent film on a transparent substrate of known refractive indices.

The ratio of p and s reflection coefficients, $\rho = R_p/R_s$, is given by[8]

$$\rho = \frac{A + BX + CX^2}{D + EX + FX^2}, \tag{1}$$

where

$$\begin{aligned} &A = r_{01p}, \quad B = r_{12p} + r_{01p}r_{01s}r_{12s}, \quad C = r_{01s}r_{12p}r_{12s}, \\ &D = r_{01s}, \quad E = r_{12s} + r_{01p}r_{02s}r_{12p}, \quad F = r_{01p}r_{12p}r_{12s}; \end{aligned} \tag{2}$$

$$X = \exp[-j2\,\pi(d/D_\phi)], \tag{3}$$

$$D_\phi = \frac{\lambda}{2}(N_1^2 - N_0^2 \sin^{-2}\phi)^{-1/2}. \tag{4}$$

In Eq. (4) N_0 and N_1 are the refractive indices of the ambient and film, and λ is the vacuum wavelength of light. We take $N_0 = 1$ to correspond to the most common case of an air ambient.

For a transparent film on a transparent substrate, the Fresnel interface reflection coefficients $r_{ij\nu}$ are real at all angles of incidence and so are the coefficients A, B, C, D, E, and F that appear in Eq. (1) and are given by Eqs. (2). [Because $N_0 = 1$, total internal reflection does not occur at either the ambient–film (01) or film–substrate (12) interface.]

The condition of PSRM corresponds to

$$|\rho| = 1. \tag{5}$$

If we rewrite X of Eq. (3) as $X = \exp(-j\theta)$, where $\theta = 2\pi(d/D_\phi)$, Eq. (1) becomes

$$\rho = \frac{(A + B\cos\theta + C\cos 2\theta) - j(B\sin\theta + C\sin 2\theta)}{(D + E\cos\theta + F\cos 2\theta) - j(E\sin\theta + F\sin 2\theta)}. \tag{6}$$

The condition that $|\rho| = 1$ reduces to the following simple quadratic equation in $\cos\theta$:

$$\alpha_2\cos^2\theta + \alpha_1\cos\theta + \alpha_0 = 0, \tag{7}$$

where

$$\begin{aligned} \alpha_0 &= (A^2 + B^2 + C^2) - (D^2 + E^2 + F^2) - 2(AC - DF), \\ \alpha_1 &= 2B(A + C) - 2E(D + F), \\ \alpha_2 &= 4(AC - DF). \end{aligned} \tag{8}$$

From Eq. (7) we find

$$\cos\theta = [-\alpha_1 \pm (\alpha_1^2 - 4\alpha_0\alpha_2)^{1/2}]/2\alpha_2. \tag{9}$$

For a given film–substrate system at a given ϕ of PSRM, $r_{ij\nu}$; $A,B,\ldots,F$; and $\alpha_0,\alpha_1,\alpha_2$ can be evaluated in that order. Then Eq. (9) can be solved for θ, from which d is determined by

$$d = (\theta/2\pi)D_\phi, \tag{10}$$

where the film-thickness period D_ϕ is given by Eq. (4).

Equation (9) always provides one acceptable solution for $\cos\theta$, leading to two acceptable solutions for θ in the range 0

$< \theta < 2\pi$ that give two film thicknesses d_1 and d_2, where $d_2 > d_1$, $d_2 = D_\phi - d_1$. This is verified graphically if we recall[9] that the locus of ρ, as d is varied at a constant ϕ, is a closed contour in the complex plane that encloses either one of the two points $\rho = -1$ or $\rho = +1$. Such locus is symmetrical with respect to the real axis[10] and intersects the unit circle at two points $\rho = \exp(\pm j\Delta)$ with associated film thicknesses d_1, d_2 so that $d_1 + d_2 = D_\phi$.

Addition of integral multiples mD_ϕ of the film-thickness period D_ϕ to d_1 and d_2 leads to other acceptable solutions for film thickness. If we assume that the period order number m is known *a priori*, e.g., if d is known to be within a certain range, it is possible to distinguish between the two solutions if the handedness of the reflected polarization is determined[11] for incident linearly polarized light. Right-handedness (Δ positive) corresponds to the smaller of the two thicknesses, $d_1 + mD_\phi$; left-handedness (Δ negative) to the larger thickness, $d_2 + mD_\phi$. Alternatively, if more than one angle of incidence of PSRM is available (as is the case for films of thickness of the order of λ or greater), unambiguous determination of thickness is possible. At each angle we have two arithmetic series of film-thickness solutions. Such series will differ from angle to angle, and only one unique film thickness, the correct thickness, would be common to series at different angles.

The explicit method for film-thickness measurement by PSRM should continue to hold sufficiently accurately for transparent films on weakly absorbing or semiconducting substrates (with extinction coefficient/refractive index ratio $\kappa \lesssim 0.1$). This can be checked by calculating the Fresnel reflection coefficients r_{12p} and r_{12s} at the film–substrate interface at the measured angles of incidence of PSRM and verifying that these coefficients are essentially real (with negligible imaginary part).

The method discussed in this Letter may also be useful as a basis of an iterative procedure to measure thickness and refractive indices of a transparent film on a transparent or weakly absorbing substrate from several angles of incidence of PSRM. Initial values of the refractive indices of the film and substrate are used to compute film thicknesses at each angle. Iteration on refractive indices is begun with the aim of reaching a unique solution for film thickness that is common to all angles.

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8. See, for example, R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1977), Sec. 4.4.
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10. Exact mirror-image symmetry with respect to the real axis occurs only if the substrate is transparent. Substrate absorption causes asymmetry.
11. For example, by use of circular polarization filters (e.g., of the sheet type) of known handedness.

Extinction ratio of germanium wedge-plate infrared polarizers

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Transmitting-type polarizers and analyzers for the far IR region of the optical spectrum can be constructed from plates mounted at the Brewster angle rather than Glan, Rochon, or Wollaston configurations. This constraint is caused mainly by the fact that at long wavelengths the required angle between the crystal optical axis and the crystal cut exceeds the critical angle of total internal reflection.

The degree of polarization P of the transmitted light when the incident light impinges on a pile of plates mounted at the Brewster angle is given by[1]

$$P = \frac{I_p - I_s}{I_p + I_s} = \frac{m}{m + \left(\dfrac{2n}{1-n^2}\right)^2}, \tag{1}$$

where m = number of plates,
n = index of refraction, and
I_p, I_s = intensities of parallel and perpendicular polarization components.

Equation (1) assumes no material absorption but takes into account multiple reflections inside the plates. From Eq. (1) it can be easily seen that for a given number of plates, the higher the index of refraction the larger the value of the degree of polarization for the transmitted light. When used with a high-power laser, for example, as part of an electrooptic modulator system, internal reflection in flat parallel plates may produce excessive heat causing deterioration of the degree of polarization that otherwise can be obtained at low-power levels.

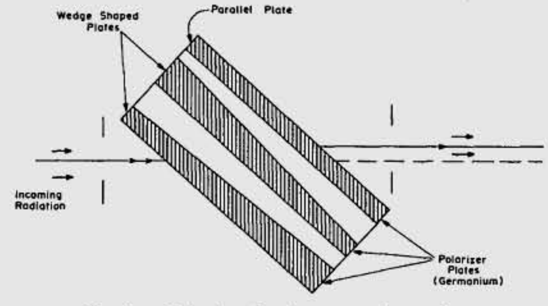

Fig. 1. Polarizer/analyzer configuration.

To reduce the number of internal reflections, we designed two sets of three germanium Brewster angle plates for which two out of the three plates were constructed to have 0.5° wedge. The two wedged plates were mounted opposite to each other as shown in Fig. 1. The third plate was designed to have parallel surfaces. The deviation of a single plate having a 0.5° wedge is ~7° for light incident at the Brewster angle. Therefore, to obtain a beam with reasonable angular deviation, wedges were placed opposite to each other as shown in Fig. 1. It should be pointed out that for the test setup two identical sets of polarizers were constructed.