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Transmission of *p*- and *s*-polarized light through a prism and the condition of minimum deviation

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The condition of minimum deviation (MD) by a transparent optically isotropic prism is re-derived, and expressions for the intensity transmittances $T_p(\theta)$ and $T_s(\theta)$ of an uncoated prism of refractive index n and prism angle α for incident p- and s-polarized light and their derivatives with respect to the internal angle of refraction θ are obtained. When the MD condition ($\theta=\alpha/2$) is satisfied, T_s is maximum and T_p is maximum or minimum. The transmission ellipsometric parameters ψ_t,Δ_t of a symmetrically coated prism are also shown to be locally stationary with respect to θ at $\theta=\alpha/2$. The constraint on (n,α) for maximally flat transmittance (MFT) of p-polarized light at and near the MD condition is determined. The transmittance T_p of prisms represented by points that lie below the locus (n,α) of MFT exhibits oscillation as a function of θ. No similar behavior is found for the s polarization. Magnitudes and angular positions of the maxima and minima of the oscillatory T_p-versus-θ curves are also calculated as functions of α for a ZnS prism of refractive index $n=2.35$ in the visible. © 2010 Optical Society of America

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1. INTRODUCTION

One of the fundamental contributions of Sir Isaac Newton to optics is his study of the dispersion of a beam of sunlight into its component colors by a prism [1]. This provided the basis of prism spectrometers that are used in many scientific and industrial applications [2–4]. Prisms also serve as wavelength-tuning elements in lasers [5–7]. Still another important function of prisms is for measurement of refractive indices of transparent materials as functions of wavelength as is often done using the minimum-deviation (MD) method [8–12].

Given that light is refracted in and out of a prism at oblique incidence, it is surprising that polarization effects associated with light transmission through prisms have largely been ignored. One exception is the prism spectroscopic ellipsometer described by Azzam *et al.* [13] that can be used for the characterization of transparent or semi-transparent thin films deposited on the prism as a substrate. In this paper interesting new results related to the transmission of p- and s-polarized light through a transparent optically isotropic prism are presented.

In Section 2 a simple and straightforward derivation of the condition of MD by a prism is presented. In Section 3 expressions for the intensity transmittances $T_p(\theta)$ and $T_s(\theta)$ of an uncoated transparent prism of refractive index n and prism angle α and their derivatives as functions of the internal angle of refraction θ are obtained. It is shown that the MD condition ($\theta=\alpha/2$) is also a condition of maximum T_s and maximum or minimum T_p. Section 4 extends the results to the transmission ellipsometric angles ψ_t,Δ_t of a symmetrically coated prism to show that these parameters are also locally stationary with respect to θ at $\theta=\alpha/2$.

In Section 5 the constraint on (n,α) for achieving maximally flat transmittance (MFT) of p-polarized light $T_p(\theta)$ with respect to the angle of refraction θ at and near the MD condition is determined. For points in the (n,α) plane that lie below the locus of MFT $T_p(\theta)$ becomes an oscillatory function of θ. No similar behavior is found with the s polarization. In Section 6 the magnitudes and angular positions of the extrema of the $T_p(\theta)$ oscillatory response are obtained as functions of the prism angle α for a ZnS prism of refractive index $n=2.35$ in the visible. Finally, a brief summary of the paper is given in Section 7.

2. SIMPLE DERIVATION OF THE CONDITION OF MINIMUM DEVIATION

Figure 1 shows the transmission of a ray of monochromatic light through a prism of refractive index n and prism angle α. The angles of incidence and refraction at the entrance and exit faces of the prism are ϕ and θ at point A and Θ and Φ at point B. The directions p and s indicate the linear polarizations parallel and perpendicular to the common plane of incidence which coincides with a principal section of the prism and the plane of the page in Fig. 1. The total angular deviation introduced by the prism is denoted by γ_t.

Standard textbook derivations [14,15] of the MD condition use the external angle of incidence ϕ as the independent variable. However, we find it more convenient to use the internal angle of refraction θ of the ray segment AB inside the prism as our independent variable. This choice is also important for the analysis presented in the remaining sections of this paper.

From Snell's law, and assuming that light is incident from air or vacuum of refractive index 1, the angular deviation $\phi-\theta$ on first refraction at point A is given by

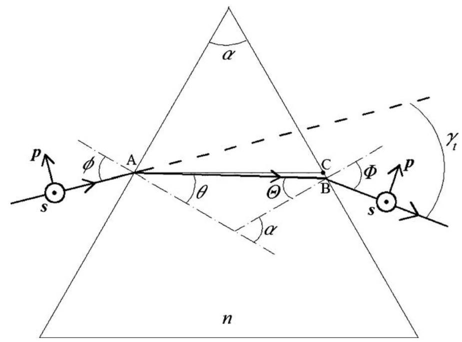

Fig. 1. Ray tracing of p- or s-polarized light as it is transmitted through a prism of refractive index n and prism angle α. The common plane of incidence coincides with the plane of the page which is also a principal section of the prism.

$$\gamma(\theta) = \sin^{-1}(n \sin \theta) - \theta. \tag{1}$$

Given that $\Theta = \alpha - \theta$ from Fig. 1, the total angular deviation γ_t after two successive refractions is expressed as

$$\gamma_t(\theta) = \gamma(\theta) + \gamma(\alpha - \theta). \tag{2}$$

By taking the first derivative of both sides of Eq. (2) with respect to θ one obtains

$$\gamma_t'(\theta) = d\gamma_t/d\theta = \gamma'(\theta) - \gamma'(\alpha - \theta). \tag{3}$$

Equation (3) immediately indicates that $\gamma_t'(\theta)=0$ when $\theta = \Theta = \alpha/2$; therefore the well known MD condition is satisfied when the refracted ray segment inside the prism (shown as segment AC in Fig. 1) is normal to the bisector of the prism angle α or parallel to the base of an isosceles prism. Furthermore, the MD angle is readily obtained from Eqs. (1) and (2) as

$$\gamma_{t\,\min} = 2\{\sin^{-1}[n \sin(\alpha/2)] - (\alpha/2)\}. \tag{4}$$

The widely used MD method of determining the prism refractive index [8–12] is based on solving Eq. (4) for n:

$$n = \frac{\sin[(\gamma_{t\,\min} + \alpha)/2]}{\sin(\alpha/2)}. \tag{5}$$

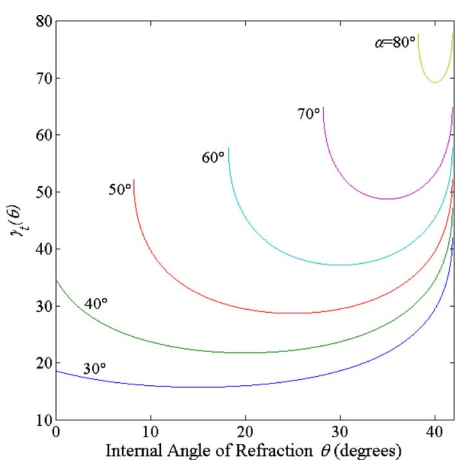

Fig. 2. (Color online) Deflection angle $\gamma_t(\theta)$ calculated from Eqs. (1) and (2) is plotted as a function of the internal angle of refraction θ for glass prism of refractive index $n=1.5$ and different prism angles α from 30° to 80° in steps of 10°.

Figure 2 shows a family of curves of the deflection angle $\gamma_t(\theta)$ calculated from Eqs. (1) and (2) versus θ for a glass prism, $n=1.5$, and prism angles α from 30° to 80° in steps of 10°. For prism angles $\alpha > \theta_c$, where $\theta_c = \sin^{-1}(1/n) = 41.81°$ is the critical angle of total internal reflection, the range of θ is limited to $0 \le \alpha - \theta_c < \theta \le \theta_c$. When $\alpha < \theta_c$ the curves of $\gamma_t(\theta)$ in Fig. 2 (for $\alpha = 30°, 40°$) start at $\theta=0$ but can be extended to also include negative values of θ in the range $\alpha - \theta_c \le \theta \le 0$. Each curve in Fig. 2 is symmetric around its minimum, located at $\theta = \alpha/2$, as expected. By contrast, the curve of $\gamma_t(\phi)$ as a function of the external angle of incidence ϕ is *not* symmetric around the minimum; see, e.g., [15].

3. INTENSITY TRANSMITTANCES FOR *p*- AND *s*-POLARIZED LIGHT

For a given polarization ($\nu = p$ or s) the intensity transmittance on single refraction at the air-prism interface is given by

$$\tau_\nu = 1 - R_\nu, \quad \nu = p, s, \tag{6}$$

where R_ν is the Fresnel intensity reflectance (loss) for the ν polarization. From the well known Fresnel reflection coefficients [16] we obtain

$$\tau_p(\theta) = \frac{4n \cos \theta (1 - n^2 \sin^2 \theta)^{1/2}}{(n^2+1) - (n^4+1)\sin^2 \theta + 2n \cos \theta (1 - n^2 \sin^2 \theta)^{1/2}}, \tag{7}$$

$$\tau_s(\theta) = \frac{4n \cos \theta (1 - n^2 \sin^2 \theta)^{1/2}}{1 + n^2 \cos 2\theta + 2n \cos \theta (1 - n^2 \sin^2 \theta)^{1/2}}, \tag{8}$$

as functions of the internal angle of refraction θ.

The overall throughput of the prism for the p or s polarization after two successive refractions (as shown in Fig. 1) is given by the product of intensity transmittances at the entrance and exit faces of the prism:

$$T_\nu = \tau_\nu(\theta)\tau_\nu(\alpha - \theta), \quad \nu = p, s. \tag{9}$$

The first, second, and third derivatives of the transmittances T_ν ($\nu = p$ or s) of Eq. (9) with respect to θ are given by

$$T_\nu'(\theta) = \tau_\nu'(\theta)\tau_\nu(\alpha - \theta) - \tau_\nu(\theta)\tau_\nu'(\alpha - \theta), \tag{10}$$

$$T_\nu''(\theta) = \tau_\nu''(\theta)\tau_\nu(\alpha - \theta) - 2\tau_\nu'(\theta)\tau_\nu'(\alpha - \theta) + \tau_\nu(\theta)\tau_\nu''(\alpha - \theta), \tag{11}$$

$$T_\nu'''(\theta) = \tau_\nu'''(\theta)\tau_\nu(\alpha - \theta) - 3\tau_\nu''(\theta)\tau_\nu'(\alpha - \theta) + 3\tau_\nu'(\theta)\tau_\nu''(\alpha - \theta) - \tau_\nu(\theta)\tau_\nu'''(\alpha - \theta). \tag{12}$$

Under the MD condition, $\theta = \alpha/2$, Eq. (10) gives

$$T_\nu'(\alpha/2) = 0, \quad \nu = p, s. \tag{13}$$

Equation (13) shows that the MD condition also represents a condition of maximum or minimum transmission for p- and s-polarized light. Equation (12) indicates that the third and by induction all higher-order odd-numbered derivatives are also zero at $\theta = \alpha/2$.

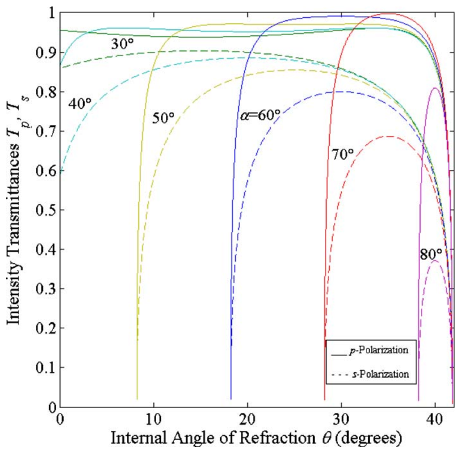

Fig. 3. (Color online) Intensity transmittances of p- and s-polarized light $T_p(\theta)$ and $T_s(\theta)$ of an uncoated prism of refractive index $n=1.5$ are plotted as functions of the internal angle of refraction θ for prism angles α from 30° to 80° in steps of 10°.

It is also of interest to evaluate the second derivative under the MD condition when $\theta=\alpha/2$. From Eq. (11) we obtain

$$T''_{\nu}(\alpha/2) = 2\tau_{\nu}(\alpha/2)\tau''_{\nu}(\alpha/2) - 2[\tau'_{\nu}(\alpha/2)]^2. \tag{14}$$

The MFT at and near $\theta=\alpha/2$ is achieved when

$$T''_{\nu}(\alpha/2) = 0, \tag{15}$$

or, from Eq. (14),

$$\tau_{\nu}(\alpha/2)\tau''_{\nu}(\alpha/2) - [\tau'_{\nu}(\alpha/2)]^2 = 0. \tag{16}$$

In Section 5 it is shown that the MFT condition of Eq. (16) is only satisfied for the p polarization, but not for the s polarization.

Figure 3 shows the intensity transmittances T_p and T_s of an uncoated glass prism of refractive index $n=1.5$ plotted as functions of the internal angle of refraction θ for prism angles α from 30° to 80° in steps of 10°. T_s is maximum at $\theta=\alpha/2$ for all prism angles α. However, T_p is maximum at $\theta=\alpha/2$ for $\alpha \geq 60°$ but has a shallow minimum at $\theta=\alpha/2$ for $\alpha \leq 50°$. Consequently, MFT for p-polarized light must be achieved at a certain prism angle in the range $50° < \alpha < 60°$. Further analysis of this behavior is considered in Section 5 with reference to a high-index prism of ZnS.

4. STATIONARY ELLIPSOMETRIC PARAMETERS UNDER THE MINIMUM DEVIATION CONDITION

In Sections 2 and 3 it is shown that the intensity transmittances T_p and T_s of a prism for p- or s-polarized light are locally stationary at a maximum or a minimum when the MD condition is satisfied. It readily follows that the intensity ratio T_p/T_s and the transmission ellipsometric parameter [16],

$$\psi_t = \tan^{-1}(T_p/T_s)^{1/2}, \tag{17}$$

are also locally stationary at $\theta=\alpha/2$.

Nontrivial phase shifts (other than 0 or π) occur when light is refracted in and out of a prism which is coated with transparent or semi-transparent thin films at its entrance and exit faces. If the coatings on the two prism faces are identical, the cumulative ellipsometric transmission differential phase shift [16] $\Delta_t = \Delta_{tp} - \Delta_{ts}$ can be written as

$$\Delta_t(\theta) = \Delta_{1t}(\theta) + \Delta_{1t}(\alpha-\theta), \tag{18}$$

where $\Delta_{1t}(\theta)$ is the differential phase shift on refraction at the entrance face. Because Eq. (18) is similar to Eq. (2) for the total deflection angle, an analysis similar to that of Section 2 applies; hence

$$\Delta'_t(\alpha/2) = 0. \tag{19}$$

In conclusion, both ellipsometric parameters ψ_t, Δ_t of light transmission through an uncoated or symmetrically coated prism are locally stationary under the MD condition.

5. MAXIMALLY FLAT TRANSMITTANCE FOR *p*-POLARIZED LIGHT

Substitution of the analytically determined first and second derivatives of the transmittance functions of Eqs. (7) and (8) (not given here) in Eq. (16) gives a complicated transcendental equation that is solved numerically for the prism angle α that achieves MFT of p-polarized light for discrete values of n from 1 to 6 in steps of 0.1. This range of refractive indices covers almost all known visible- and infrared (IR)-transparent materials. The resulting α-versus-n curve that represents the MFT constraint of Eq. (16) is shown by the bottom continuous curve in Fig. 4. The MFT $T_p(\alpha/2)$ decreases monotonically from 1.0 at $n=1$ to 0.8988 at $n=6$ as shown separately in Fig. 5.

No numerical solutions of Eq. (16) could be found for the $\nu=s$ polarization over the entire range of n and α shown in Fig. 4. Therefore, MFT, as we define it here, is not possible with s-polarized light. That MFT is achieved

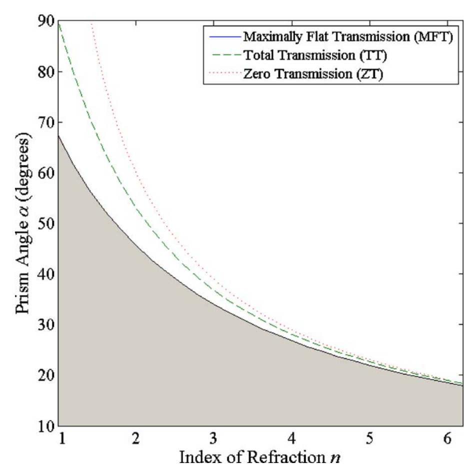

Fig. 4. (Color online) Continuous curve represents the solution of Eq. (16) for the prism angle α as a function of prism refractive index n that achieves MFT of p-polarized light at and near the condition of MD. In the shaded area below the MFT curve the transmittance T_p of p-polarized light is an oscillatory function of the internal angle of refraction θ. The dashed and dotted curves represent conditions of TT and ZT by prisms that satisfy Eqs. (20) and (21), respectively.

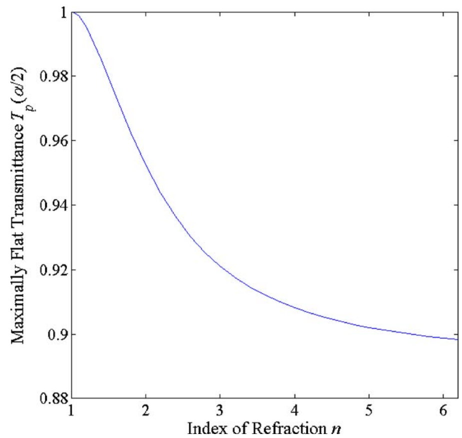

Fig. 5. (Color online) MFT of p-polarized light $T_p(\alpha/2)$ under the MD condition decreases monotonically as a function of prism refractive index n.

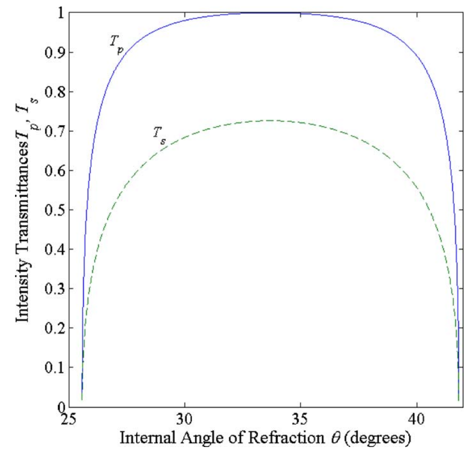

Fig. 6. (Color online) Intensity transmittances $T_p(\theta)$ and $T_s(\theta)$ of p- and s-polarized light are plotted as functions of the internal angle of refraction θ for a Brewster-angle prism with $n=1.5$ and $\alpha=2\tan^{-1}(2/3)=67.38°$.

for the p polarization only can be attributed to the presence of a reflectance minimum at the Brewster angle for p-polarized light.

For reference Table 1 lists α (MFT) and the associated MFT $T_p(\alpha/2)$ for n values from 1.5 to 6 in steps of 0.5. The dashed curve shown in Fig. 4 represents the condition of total transmission (TT) of p-polarized light ($T_p=1$) that occurs when the prism angle α equals double the Brewster angle of internal reflection at the prism-air interface:

$$\alpha = 2\theta_{Bi} = 2\tan^{-1}(1/n). \tag{20}$$

Figure 6 shows the transmittance functions $T_p(\theta)$ and $T_s(\theta)$ as calculated from Eqs. (7)–(9) for a Brewster-angle glass prism with $n=1.5$ and $\alpha=2\tan^{-1}(2/3)=67.38°$. $T_p=1$ at $\theta=\alpha/2=33.69°$ as expected, and the corresponding $T_{s\,\max}=0.726$. Prism angles α for TT obtained from Eq. (20) are included in Table 1.

The limiting case of zero transmission (ZT) of both the p and s polarizations under the MD condition (which is of theoretical interest only) is represented by the topmost dotted curve in Fig. 4. This locus is obtained by setting the prism angle equal to double the critical angle of total internal reflection at the prism-air interface, i.e.,

$$\alpha = 2\theta_c = \sin^{-1}(1/n). \tag{21}$$

The area of the n,α plane above the dotted curve in Fig. 4 represents prisms for which total internal reflection, instead of partial reflection and refraction, occurs at the second prism face for all external angles of incidence ϕ at the entrance face. Prism angles for ZT calculated from Eq. (21) are also listed in Table 1.

6. OSCILLATORY TRANSMITTANCE VERSUS ANGLE FOR *p*-POLARIZED LIGHT

The highlighted area of the n,α plane below the continuous curve of MFT in Fig. 4 represents prisms whose transmittance for p-polarized light $T_p(\theta)$ exhibits oscillation (or a saddle point) as a function of θ. To illustrate this clearly we consider a high-index prism of ZnS which is transparent over a broad visible and IR spectral range from 0.4 to 13 μm [17].

For a ZnS prism of refractive index $n=2.35$ at a wavelength of $\lambda=0.633$ μm, the prism angle for MFT, obtained by solving Eq. (16), is $\alpha=40.9144°$. Figure 7 shows $T_p(\theta)$ as a function of θ for ZnS prisms with three different prism angles α and $n=2.35$. The MFT is achieved when $\alpha=40.9144°$, a response with a saddle point appears for $\alpha=37°$, and a transmittance curve with a single well defined peak is obtained at $\alpha=45°$.

Figure 8 shows the transmittance curves $T_p(\theta)$ for a ZnS prism with $\alpha=40.9144°$ at three different wavelengths [17]. For $\lambda=0.633$ μm, $n=2.35$, and MFT is obtained; at $\lambda=10$ μm, $n=2.2$, and this leads to an oscillatory response; and at $\lambda=0.467$ μm, $n=2.5$, a transmittance curve with a single well defined peak is obtained. Salient features of the oscillatory transmittance of p-polarized light include the angular positions $\theta_{\min},\theta_{\max}$ of the minimum and maximum and the associated minimum and maximum transmittances $T_{p\,\min},T_{p\,\max}$ and their ratio $T_{p\,\min}/T_{p\,\max}$ as functions of the prism angle α.

Figure 9 shows a family of $T_p(\theta)$-versus-θ curves for a ZnS prism with an index $n=2.35$ and prism angles α from

Table 1. Prism Angles α in Degrees that Lead to MFT of p-Polarized Light, TT of p-Polarized Light, and ZT of the p and s Polarizations Listed Versus Prism Refractive Index n[a]

n	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
α(MFT)	54.533	45.690	39.100	34.034	30.006	26.766	24.196	21.924	20.107	18.514
$T_p(\alpha/2)$	0.980	0.952	0.933	0.921	0.913	0.908	0.905	0.902	0.900	0.899
α(TT)	67.380	53.130	43.603	36.870	31.891	28.072	25.058	22.620	20.610	18.925
α(ZT)	83.621	60.000	47.156	38.942	33.203	28.955	25.679	23.074	20.951	19.188

[a]Values of the MFT $T_p(\alpha/2)$ are also included.

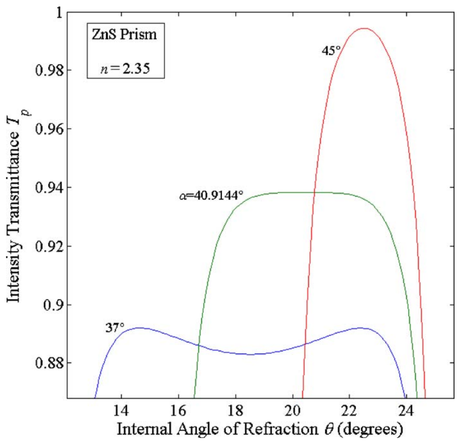

Fig. 7. (Color online) Intensity transmittance of p-polarized light $T_p(\theta)$ is plotted as a function of θ for ZnS prisms of the same refractive index $n=2.35$ but three different prism angles. MFT is achieved when $\alpha=40.9144°$, an oscillatory response appears for $\alpha=37°$, and a transmittance curve with a single well defined peak is obtained for $\alpha=45°$.

5° to 45° in steps of 4°. Figure 9 indicates that the depth of the minimum increases as the prism angle decreases. Although negative values of θ are physically meaningful, we choose not to extend the curves in Fig. 9 over that range, given that each transmittance curve is symmetric around $\theta=\alpha/2$.

Figure 10 gives a plot of the angles $\theta_{\max}, \theta_{\min}$ and their difference $\theta_{\max}-\theta_{\min}$ in degrees as functions of α in the range $0° \leq \alpha \leq 50°$ for a ZnS prism with refractive index $n=2.35$. The limiting case of $\alpha=0$ is that of a parallel slab. In Fig. 10 $\theta_{\min}=\alpha/2$ for $0 \leq \alpha \leq 40.9144°$ and the continuation of this straight line beyond $\alpha=40.9144°$ represents $\theta_{\max}$. Notice that $\theta_{\max}$ is nearly constant ($\approx \theta_{Bi} \approx 23°$) over much of the range of α except for a few degrees below the limiting prism angle of $\alpha=40.9144°$ at which $\theta_{\min}=\theta_{\max}$ and that it increases linearly with α beyond that angle.

Figure 11 shows $T_{p\,\min}, T_{p\,\max}$ and their ratio $T_{p\,\min}/T_{p\,\max}$ as functions of the prism angle α for a ZnS

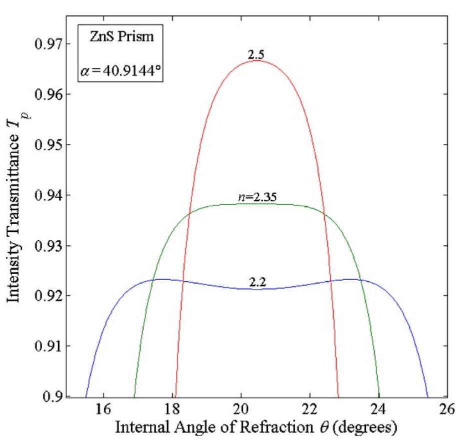

Fig. 8. (Color online) Intensity transmittance of p-polarized light $T_p(\theta)$ is plotted as a function of θ for a ZnS prism with prism angle of $\alpha=40.9144°$ and different refractive indices at three different wavelengths. For $n=2.35$ at 0.633 μm wavelength, a MFT is obtained; $n=2.2$ at 10 μm leads to an oscillatory response; and for $n=2.5$ at 0.467 μm a curve with a single well defined peak is obtained.

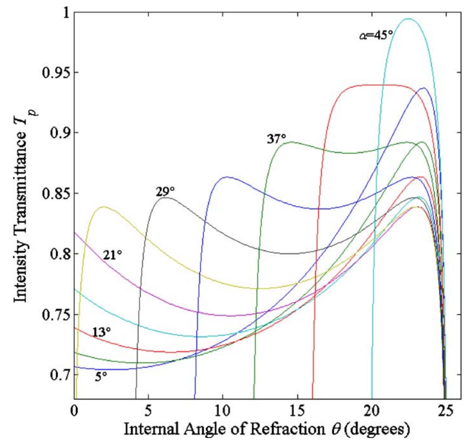

Fig. 9. (Color online) Family of $T_p(\theta)$-versus-θ curves for ZnS prism with refractive index $n=2.35$ and prism angles α from 5° to 45° in steps of 4°.

prism with $n=2.35$. For a parallel slab ($\alpha=0$) of refractive index n, minimum one-way transmission of p-polarized light at normal incidence is given by

$$T_{p\,\min}(0) = 16n^2/(n+1)^4. \tag{22}$$

For $n=2.35$ Eq. (22) gives $T_{p\,\min}(0)=0.7016$. For the parallel slab, TT of p-polarized light occurs at the Brewster angle $[\theta_{Bi}=\tan^{-1}(1/n)]$, and $T_{p\,\max}(\theta_{Bi})=1$ at point A in Fig. 11. TT of p-polarized light is also achieved for a ZnS Brewster-angle prism with $\alpha=2\theta_{Bi}=2\tan^{-1}(1/n)=46.103°$ as represented by point B in Fig. 11. The curves of $T_{p\,\min}, T_{p\,\max}$ merge at point C at which $T_{p\,\min}=T_{p\,\max}$ and $\alpha=40.9144°$.

Another interesting special case corresponds to point D in Fig. 11 where $T_{p\,\max}$ reaches a minimum. Point D represents the transmission of p-polarized light through half of a Brewster-angle prism [18] as shown in Fig. 12. The minimum value of $T_{p\,\max}$ at point D (0.8376) equals the square root of the transmittance of a parallel slab at normal incidence given by Eq. (22).

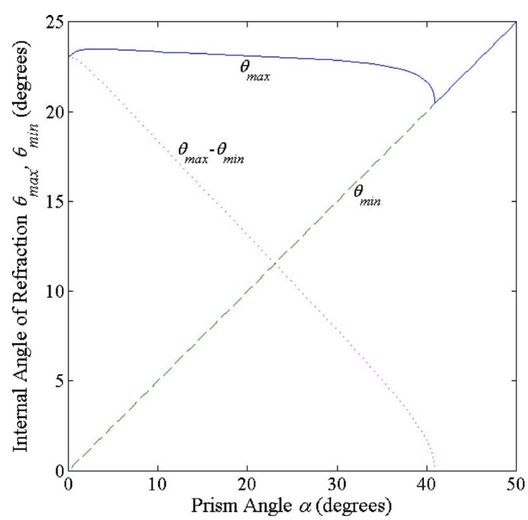

Fig. 10. (Color online) Internal angles of refraction of maximum and minimum transmittances of p-polarized light $\theta_{\max}, \theta_{\min}$ and their difference $\theta_{\max}-\theta_{\min}$ in degrees are plotted as functions of prism angle α in the range $0° \leq \alpha \leq 50°$ for a ZnS prism with refractive index $n=2.35$.

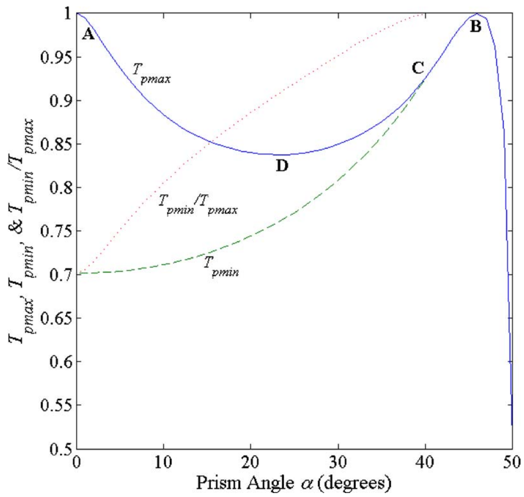

Fig. 11. (Color online) Minimum and maximum transmittances of p-polarized light $T_{p\,\min}, T_{p\,\max}$ and their ratio $T_{p\,\min}/T_{p\,\max}$ are plotted as functions of prism angle α for ZnS prism with $n=2.35$. The significance of points A, B, C, and D is discussed in the text.

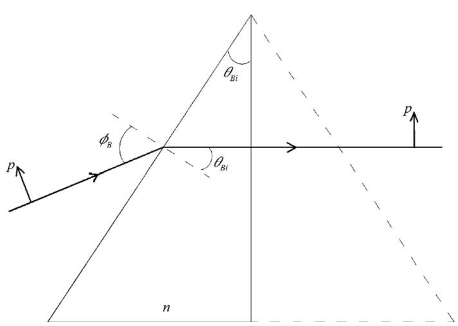

Fig. 12. Transmission of p-polarized light through half of a Brewster-angle prism with prism angle $\alpha=2\theta_{Bi}=2\tan^{-1}(1/n)$.

7. SUMMARY

A simple and direct derivation of the condition of minimum deviation (MD) by a transparent optically isotropic prism is presented. Expressions for the intensity transmittances $T_p(\theta)$ and $T_s(\theta)$ of an uncoated prism of refractive index n and prism angle α for incident p- and s-polarized light and their derivatives with respect to the internal angle of refraction θ are obtained. The MD condition ($\theta=\alpha/2$) is accompanied by maximum transmittance T_s and maximum or minimum transmittance T_p. Likewise, the transmission ellipsometric parameters ψ_t, Δ_t of a symmetrically coated prism are locally stationary with respect to θ at $\theta=\alpha/2$.

The locus of (n,α) for prisms with maximally flat transmittance (MFT) of p-polarized light at and near the MD condition is determined. Prisms represented by points (n,α) that lie below the MFT locus have transmittance T_p that oscillates with respect to θ. No similar behavior is found for the s polarization. Magnitudes and angular positions of the extrema of the oscillatory T_p-versus-θ curve are also calculated as functions of α for a ZnS prism of refractive index $n=2.35$ in the visible.

ACKNOWLEDGMENT

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