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In-line broadband 270° ($3\lambda/4$) chevron four-reflection wave retarders

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The net differential phase shift Δ_t introduced between the orthogonal p and s linear polarizations after four successive total internal reflections inside an in-line chevron dual-Fresnel-rhomb retarder is a function of the first internal angle of incidence φ and prism refractive index n . Retardance of $3\lambda/4$ (i.e., $\Delta_t = 270^\circ$) is achieved with minimum angular sensitivity when $\varphi = 45^\circ$ and $n = 1.900822$. Several optical glasses with this refractive index are identified. For Schott glass SF66 the deviation of Δ_t from 270° is $\leq 4^\circ$ over a wavelength range of $0.55 \leq \lambda \leq 1.1 \mu\text{m}$ in the visible and near-IR spectrum. For a SiC prism, whose totally reflecting surfaces are coated with an optically thick MgF_2 film, $\Delta_t = 270^\circ$ at two wavelengths: $\lambda_1 = 0.707 \mu\text{m}$ and $\lambda_2 = 4.129 \mu\text{m}$. This coated prism has a maximum retardance error of $\approx 5^\circ$ over $>$ three octaves (0.5 to $4.5 \mu\text{m}$) in the visible, near-, and mid-IR spectral range. Another mid-IR $3\lambda/4$ retarder uses a Si prism, which is coated by an optically thick silicon oxynitride film of the proper composition, to achieve retardance that differs from 270° by $<0.5^\circ$ over the $3\text{--}5 \mu\text{m}$ spectral range. © 2008 Optical Society of America

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1. Introduction

Quarter-wave retarders (QWRs) and half-wave retarders are versatile optical elements that find wide application in the control and analysis of polarized light. The most achromatic retarders are those that use one to four total internal reflections (TIRs) at coated or uncoated prism surfaces [1–10]. To provide a higher degree of freedom in polarization state generation and detection, an in-line three- or four-reflection retarder that can be rotated about the light beam is often required. A particularly attractive in-line device uses the compact symmetric chevron dual-Fresnel-rhomb design shown in Fig. 1, as was first proposed in [4]. By use of suitable transparent prism and thin-film coating materials, reasonably achromatic 90° and 180° retarders are obtained [4,10]. Near- 270° chevron retarders that use high-index (near 1.9) uncoated LaSF31 and thin-film-coated SF59 Schott glass prisms for the $400\text{--}700\text{ nm}$ visible range were also presented in [4].

We provide further analysis and design of both coated and uncoated visible and infrared chevron dual-Fresnel-rhomb $3\lambda/4$ retarders of the type shown in Fig. 1. The net differential reflection phase shift between the p and s linear polarizations (parallel and perpendicular to the common plane of incidence) of $\Delta_t = 270^\circ$ is achieved after four successive TIRs at equal angles of incidence of $\varphi = 45^\circ$. These devices function essentially as QWRs (since the additional half-wave retardation is inconsequential) and offer the following advantages: (1) compact design with a minimum (length/aperture) aspect ratio of 2, as compared to longer devices that operate at higher angles of incidence [3]; (2) a small retardance error of $<1^\circ$ within an external (in air) field of view (FOV) of $\pm 2n$ deg, where n is the high refractive index of the prism; and (3) a quasi-achromatic operation over a broad bandwidth in the visible and near- to mid-IR spectrum by judicious choice of prism and coating materials. In the practical implementation of these devices, retardance errors that may be caused by residual stress birefringence should be taken into consideration.

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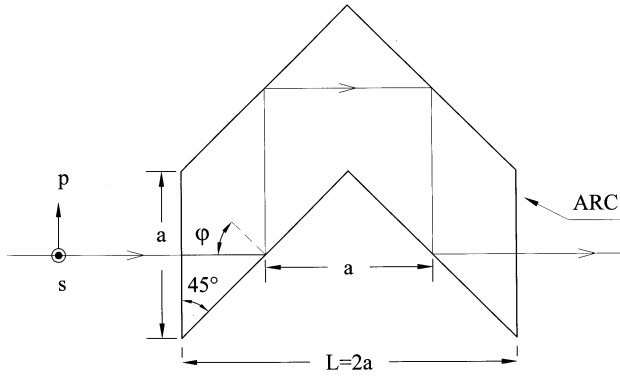


Fig. 1. In-line symmetric chevron dual-Fresnel-rhomb prism retarder that uses four TIRs at the same angle of incidence $\varphi = 45^\circ$. Light enters and leaves the prism normal to its entrance and exit faces, which are ARC. p and s are the linear polarizations parallel and perpendicular to the common plane of incidence, respectively. The length/aperture aspect ratio of this compact prism is $L/a = 2$.

In Section 2 the cumulative retardance $\Delta_t(\varphi)$ is treated as a function of angle of incidence φ of the first TIR and prism refractive index n . The desired retardance, $\Delta_t = 270^\circ$, is obtained with minimum angular sensitivity when $n = 1.900822$ and $\varphi = 45^\circ$.

In Section 3 several optical glasses that have $n = 1.900822$ are identified, and good spectral response is demonstrated for an uncoated SF66 Schott glass prism over a broad (550 to 1100 nm) visible and near-IR spectral range.

In Section 4 a SiC prism whose totally reflecting faces are coated with an optically thick (noninterference) film of MgF_2 is proposed that achieves $\Delta_t = 270^\circ$ at two wavelengths, $\lambda_1 = 0.707 \mu\text{m}$ and $\lambda_2 = 4.129 \mu\text{m}$, and introduces a maximum retardance error of $\approx 5^\circ$ over $>$ three contiguous octaves in the visible to mid-IR spectral range.

In Section 5 another novel $3\lambda/4$ design is presented that uses an IR-transparent Si prism that is coated by an optically thick silicon oxynitride (SiON) film at its totally reflecting faces. Mole fraction x of SiO_2 in the $\text{SiO}_2\text{-Si}_3\text{N}_4$ solid solution that achieves the proper index ratio, $n(\text{Si})/n(\text{SiON}) = 1.900822$, is determined as a function of wavelength λ . (A quarter-wave layer of Si_3N_4 at the entrance and exit faces of the Si prism provides an effective antireflection coating at normal incidence.) Near-achromatic performance with a retardance error of $< 0.5^\circ$ is obtained in the $3\text{-}5 \mu\text{m}$ mid-IR spectral range when $x = 0.3278$.

Section 6 gives a brief summary. Finally a method is described in Appendix A for finding the wavelengths at which the ratio of refractive indices of two transparent optical materials with known (three-term) Sellmeier dispersion relations equals a desired value by solving an equivalent sixth-degree equation in λ^2 .

2. Cumulative Differential Phase Shift in the Chevron Four-Reflection Prism Retarder

Figure 1 shows the propagation of a monochromatic light beam via four successive TIRs at angles of incidence φ (first two reflections) and $90^\circ - \varphi$ (last

two reflections) inside a chevron prism of refractive index n . (The light path in Fig. 1 portrays the case when $\varphi = 45^\circ$.) To achieve high (near 100%) throughput, the entrance and exit faces, where light enters and leaves the prism at or near normal incidence, are antireflection coated (ARC). Cumulative differential phase shift (or retardance) Δ_t introduced after four TIRs inside the prism is given by [10]

$$\Delta_t(\varphi) = 2[\Delta(\varphi) + \Delta(90^\circ - \varphi)], \quad (1)$$

$$\Delta(\varphi) = 2 \tan^{-1}[(n^2 \sin^2 \varphi - 1)^{1/2} / (n \sin \varphi \tan \varphi)]. \quad (2)$$

When $\varphi = 45^\circ$ (optical path length inside the prism $= 4na$, Fig. 1), Eqs. (1) and (2) give

$$\Delta_t(45^\circ) = 8 \tan^{-1}[(n^2 - 2)^{1/2} / n]. \quad (3)$$

From Eq. (3) a net retardance

$$\Delta_t(45^\circ) = 3\pi/2, \quad (4)$$

is obtained if

$$[(n^2 - 2)^{1/2} / n] = \tan(3\pi/16). \quad (5)$$

The solution of Eq. (5) for n can be put in the form

$$n = \frac{(\sqrt{2} + 1)}{[(4 + 2\sqrt{2})^{1/2} - 1]^{1/2}} = 1.900822. \quad (6)$$

In Fig. 2 $\Delta_t(\varphi)$ of Eq. (1) is plotted as a function of φ in the range of $\varphi = 45^\circ \pm 2^\circ$ when $n = 1.900822$. From Fig. 2 it is evident that

$$\Delta_t = 270^\circ, \quad \partial \Delta_t / \partial \varphi = 0, \quad (7)$$

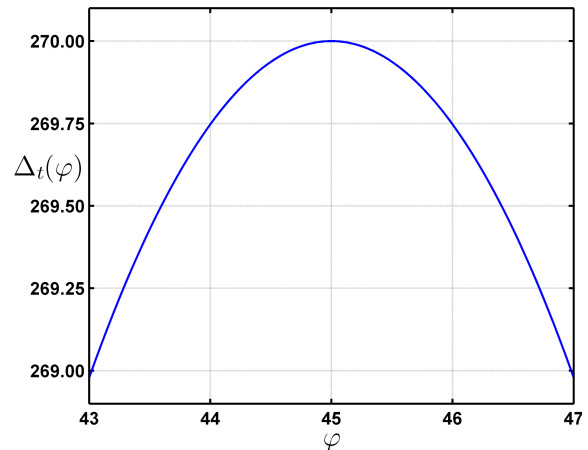


Fig. 2. (Color online) Cumulative retardance $\Delta_t(\varphi)$ [Eq. (1)] plotted versus angle of incidence φ in the range of $43^\circ \leq \varphi \leq 47^\circ$ for a prism with refractive index $n = 1.900822$. Notice that $\Delta_t = 270^\circ$ and $\partial \Delta_t / \partial \varphi = 0$ are satisfied at $\varphi = 45^\circ$.

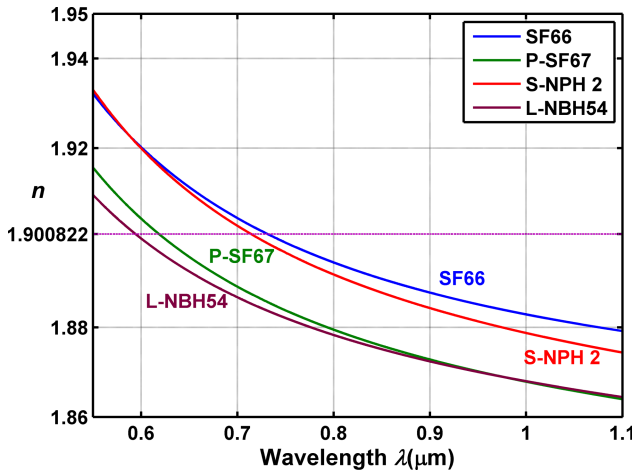


Fig. 3. (Color online) Refractive index $n(\lambda)$ of Schott glasses SF66 and P-SF67 and Ohara glasses S-NPH 2 and L-NBH54 plotted versus wavelength λ . The coefficients of the dispersion formulas of these glasses [Eq. (8)] are listed in Table 1. All curves intersect the line $n = 1.900822$ at wavelengths λ that are also listed in Table 1.

at $\varphi = 45^\circ$, and the maximum retardance error is $\approx 1^\circ$ over an internal FOV of $\pm 2^\circ$. The external FOV (in air) is n times larger, as obtained when Snell's law is applied at the entrance face of the prism.

3. Optical Glasses for $3\lambda/4$ Four-Reflection Wave Retarders

Figure 3 shows the refractive index $n(\lambda)$ versus wavelength λ for two Schott glasses (SF66 and P-SF67) and two Ohara glasses (S-NPH 2 and L-NBH54) with published dispersion formulas [11,12] of the form

$$n^2 - 1 = \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}. \quad (8)$$

In Fig. 3 all curves intersect the line $n = 1.900822$. Table 1 lists $(B_j, C_j), j = 1, 2, 3$ of Eq. (8) for each optical glass and wavelengths λ in nanometers at which $n = 1.900822$. The wavelengths at which n of Eq. (8) is equal to a desired value are obtained by solving an equivalent cubic equation in λ^2 [10]. Also included in Table 1 are the corresponding values of $\partial n/\partial \lambda$,

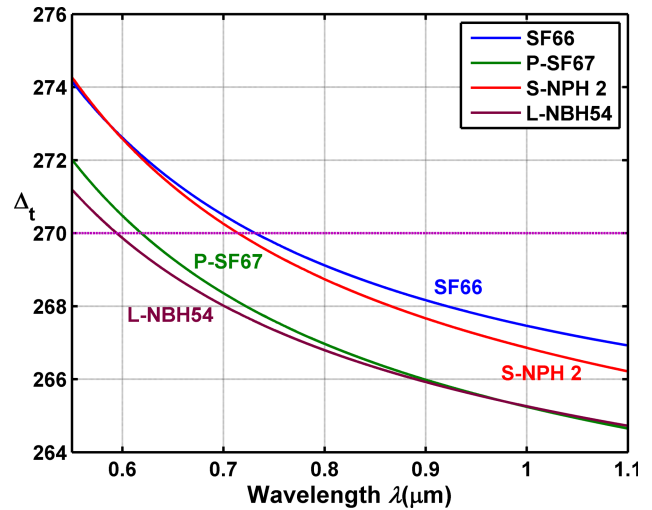


Fig. 4. (Color online) Cumulative retardance $\Delta_t(\lambda)$ plotted versus wavelength λ over the spectral range of $0.55 \leq \lambda \leq 1.1 \mu\text{m}$ for uncoated in-line chevron four-reflection retarders that are made of optical glasses whose properties are summarized in Table 1.

$$\partial n/\partial \lambda = (-\lambda/n) \sum_{i=1}^3 \frac{B_i C_i}{(\lambda^2 - C_i)^2}, \quad (9)$$

which is obtained by the differentiation of Eq. (8). Figure 4 shows $\Delta_t(\lambda)$ versus λ for the four uncoated glass prisms (at $\varphi = 45^\circ$) over the $0.55 \leq \lambda \leq 1.1 \mu\text{m}$ spectral range. For Schott glass SF66 the deviation of Δ_t from 270° is $< 4^\circ$ over one octave in the visible and near-IR spectrum.

4. Broadband $3\lambda/4$ Total Internal Reflection Retarder Using MgF_2 -Coated SiC Prism

The constants of the dispersion relations for SiC [13] and MgF_2 are listed in Table 2. For an assumed polycrystalline MgF_2 film, the average of the ordinary and extraordinary refractive indices n_o and n_e of single-crystal MgF_2 [14] is calculated and fitted with a three-term Sellmeier dispersion relation of the same form as Eq. (8). The residual root-mean-square error of this fit is $< 1 \times 10^{-5}$ over the spectral range of $0.5 \leq \lambda \leq 4.5 \mu\text{m}$. The thickness of the MgF_2 film is assumed to be many times the penetration depth of the evanescent field in the rarer medium (MgF_2) for light reflection at the SiC- MgF_2 interface, so interference in the film is nonexistent or negligible. The phase

Table 1. Constants of Dispersion Relations [Eq. (8)] of Four Optical Glasses [11,12]^a

Glass Type	SF66	P-SF67	S-NPH 2	L-NBH54
B_1	2.07842233	1.97464225	2.03869510	2.00722652
B_2	0.407120032	0.467095921	0.437269641	0.442086773
B_3	1.76711292	2.43154209	2.96711461	3.37287426
C_1	0.0180875134	0.0145772324	0.0170796224	0.0135705124
C_2	0.0679493572	0.0669790359	0.0749254813	0.0591808873
C_3	215.266127	157.444895	174.155354	219.832665
λ (nm)	731.365	618.589	714.287	594.225
$\partial n/\partial \lambda$ ($\mu\text{m})^{-1}$	-0.0840	-0.0763	-0.0795	-0.0752

^a λ is the wavelength at which $n = 1.900822$ for each glass, and $\partial n/\partial \lambda$ is calculated using Eq. (9) at that wavelength.

Table 2. Constants of Dispersion Relations [Eq. (8)] of SiC and MgF₂^a

	SiC	MgF ₂ (<i>n_o</i>)	MgF ₂ (<i>n_e</i>)	MgF ₂ (<i>n_a</i>)
<i>B</i> ₁	5.58245	0.48755108	0.41344023	0.4512116386434509
<i>B</i> ₂	2.468516	0.39875031	0.50497499	0.4510220669673775
<i>B</i> ₃	0	2.3120353	2.4904862	1.614987151095338
<i>C</i> ₁	0.02641905655236	0.0018821784	0.0013573786	0.004393354295981150
<i>C</i> ₂	128.9714550336	0.0089518885	0.0082376717	0.004388449893630688
<i>C</i> ₃	0	566.13559131	163.1238562800250	168.0319543854284

^aSiC is the prism material [13]. The refractive index of an optically isotropic polycrystalline thin film of MgF₂ is taken as the average of the ordinary and extraordinary refractive indices of single-crystal MgF₂ [14]: $n_a = (n_o + n_e)/2$. This average index n_a is fitted by a dispersion relation of the same form as that of n_o and n_e [Eq. (8)].

shift on the TIR at the buried SiC–MgF₂ interface is determined by the relative refractive index

$$n = n(\text{SiC})/n(\text{MgF}_2). \quad (10)$$

Figure 5 shows relative index $n(\lambda)$ of Eq. (10) as a function of λ in the $0.5 \leq \lambda \leq 4.5 \mu\text{m}$ spectral range. Note that $n = n(\text{SiC})/n(\text{MgF}_2) = 1.900822$ at two wavelengths: $\lambda_1 = 0.707 \mu\text{m}$ and $\lambda_2 = 4.129 \mu\text{m}$. Figure 6 is a plot of corresponding retardance $\Delta_t(\lambda)$ versus λ . The maximum deviation of Δ_t from 270° is $\approx 5^\circ$ over a spectral range of $>$ three octaves: $0.5 \leq \lambda \leq 4.5 \mu\text{m}$.

For incident linearly polarized light at 45° azimuth from the plane of incidence, the extent to which the transmitted light beam can be considered to be purely circularly polarized is determined by the fraction of power of the output state that resides in the desired circular polarization component. This fraction η is given by [15]

$$\eta = \cos^2[(\Delta_t - 270^\circ)/2]. \quad (11)$$

In the presence of a maximum retardance error of 5° , $\eta = 0.9981$. This indicates effective broadband line-

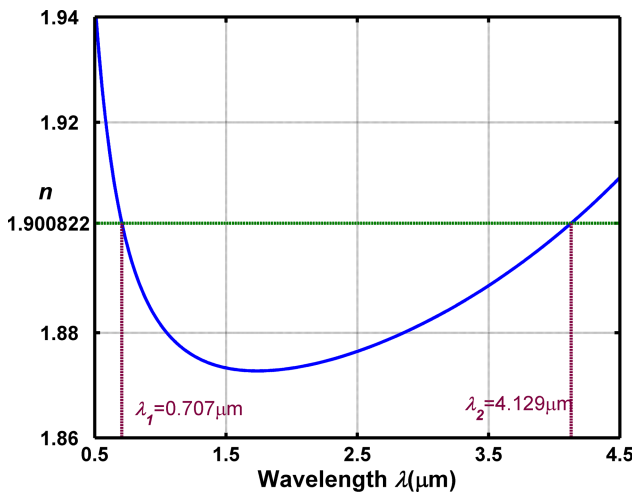


Fig. 5. (Color online) Relative refractive index $n(\lambda) = n(\text{SiC})/n(\text{MgF}_2)$ of MgF₂-coated SiC plotted as a function of wavelength λ over the spectral range of $0.5 \leq \lambda \leq 4.5 \mu\text{m}$. Index ratio $n(\lambda) = 1.900822$ at wavelengths $\lambda_1 = 0.707 \mu\text{m}$ and $\lambda_2 = 4.129 \mu\text{m}$.

ar-to-circular polarization conversion by this MgF₂-coated SiC retarder over a wide spectral range.

5. Mid-IR $3\lambda/4$ Four-Reflection Achromatic Retarder Using SiON-Coated Si Prism

The refractive index of vacuum-deposited SiON film can be tuned by controlling mole fraction x of SiO₂ in the SiO₂–Si₃N₄ solid solution of target material [16] such that index ratio

$$n = n(\text{Si})/n(\text{SiON}), \quad (12)$$

is equal to 1.900822 at a desired wavelength. To a good approximation [16] the refractive index of SiON is expressed as a linear function of x :

$$n(\text{SiON}) = xn(\text{SiO}_2) + (1 - x)n(\text{Si}_3\text{N}_4). \quad (13)$$

The refractive indices of SiO₂ and Si₃N₄ in Eq. (13) are calculated using a single-term Sellmeier disper-

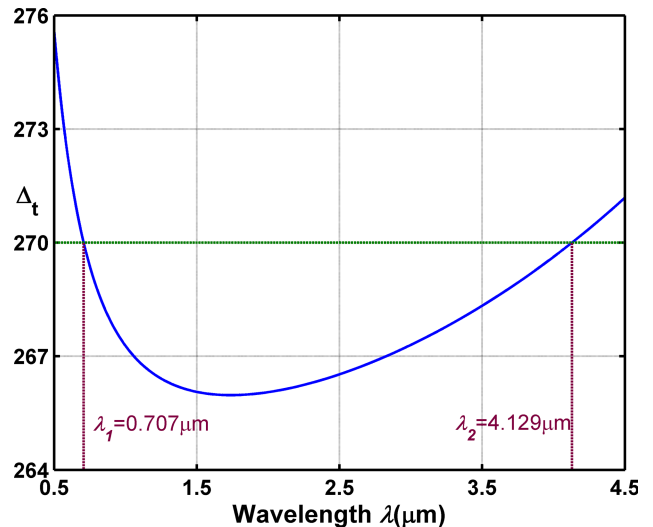


Fig. 6. (Color online) Cumulative retardance $\Delta_t(\lambda)$ of chevron four-reflection MgF₂-coated SiC retarder plotted versus wavelength λ in the spectral range of $0.5 \leq \lambda \leq 4.5 \mu\text{m}$. Exact retardance, $\Delta_t = 270^\circ$, is achieved at wavelengths $\lambda_1 = 0.707 \mu\text{m}$ and $\lambda_2 = 4.129 \mu\text{m}$.

sion relation [16] with constants given by

$$\begin{aligned}
 \text{SiO}_2 : B_1 &= 1.09877, \\
 C_1 &= (0.0924317)^2, \\
 \text{Si}_3\text{N}_4 : B_1 &= 2.8939, \\
 C_1 &= (0.13967)^2, \\
 B_2 = B_3 &= 0.
 \end{aligned} \tag{14}$$

From Eqs. (12)–(14) and the known dispersion relation of Si [15], index ratio n of Eq. (12) is determined as a function of $\lambda(\mu\text{m})$ at discrete values of the SiO_2 molar fraction x from $x = 0.1$ to 0.5 in equal steps of 0.1 , and the results are plotted in Fig. 7. Index ratio $n = 1.900822$ is obtained within a narrow range of x : $0.2434 \leq x \leq 0.3306$. At a given wavelength, molar fraction x that makes $n(\text{Si})/n(\text{SiON}) = 1.900822$ is calculated from

$$x = \frac{n(\text{Si}_3\text{N}_4) - 0.526088n(\text{Si})}{n(\text{Si}_3\text{N}_4) - n(\text{SiO}_2)}. \tag{15}$$

In Fig. 8 x , at which $n = 1900822$, is plotted as a function of wavelength λ in the range of $1.2 \leq \lambda \leq 5.0 \mu\text{m}$.

As a specific design a dual-Fresnel-rhomb SiON-coated Si retarder that achieves $\Delta_t = 270^\circ$ at a center wavelength of $4 \mu\text{m}$ is considered. Figure 9 shows $n(\lambda)$: $\Delta_t(\lambda)$ versus λ in the mid-IR spectral range of $3 \leq \lambda \leq 5 \mu\text{m}$ (of atmospheric transparency [17]). Notice that the deviation of Δ_t from 270° is $< 0.5^\circ$, hence $\eta \geq 0.99998$ [Eq. (11)], over a $2 \mu\text{m}$ bandwidth, which indicates excellent achromatic performance.

6. Summary

Cumulative differential phase shift $\Delta_t = 270^\circ$ between the p and s linear polarizations is achieved with minimum angular sensitivity after four TIRs

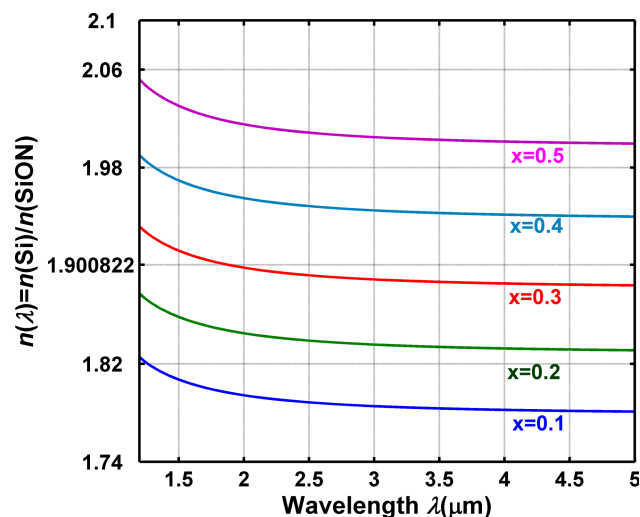


Fig. 7. (Color online) Index ratio, $n = n(\text{Si})/n(\text{SiON})$, calculated as function of wavelength $\lambda(\mu\text{m})$ for discrete values of $x = 0.1$ to 0.5 in equal steps of 0.1 , where x is the mole fraction of SiO_2 in the SiON film.

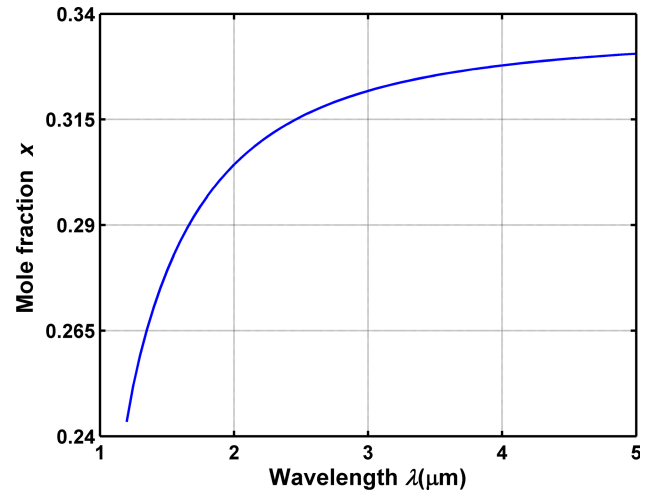


Fig. 8. (Color online) Mole fraction x of SiO_2 in SiON film calculated using Eq. (15), at which $n = 1.900822$ is plotted versus wavelength λ in the spectral range of $1.2 \leq \lambda \leq 5.0 \mu\text{m}$.

at the same angle of incidence, $\varphi = 45^\circ$, inside a compact chevron dual-Fresnel-rhomb prism (Fig. 1) of refractive index $n = 1.900822$. A $3\lambda/4$ retarder made of uncoated SF66 Schott glass exhibits a retardance error of $\leq 4^\circ$ over a spectral range of one octave: $0.55 \leq \lambda \leq 1.1 \mu\text{m}$. A SiC prism that is coated by an optically thick MgF_2 film achieves $\Delta_t = 270^\circ$ at two wavelengths, $\lambda_1 = 0.707 \mu\text{m}$ and $\lambda_2 = 4.129 \mu\text{m}$, and has a maximum retardance error of $\approx 5^\circ$ over the extended spectral range of $0.5 < \lambda \leq 4.5 \mu\text{m}$. Finally a SiON-coated Si chevron prism is described that exhibits a retardance error of $< 0.5^\circ$ in the $3\text{--}5 \mu\text{m}$ mid-IR spectral range.

Appendix A

The Fresnel reflection coefficients of p - and s -polarized light at the interface between two transparent materials of refractive indices n and n' are determined by

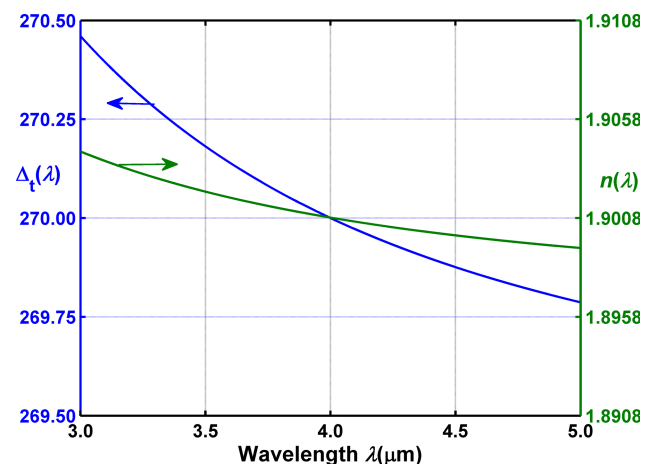


Fig. 9. (Color online) Relative refractive index, $n = n(\text{Si})/n(\text{SiON})$, and cumulative retardance $\Delta_t(\lambda)$ plotted versus wavelength λ , in the spectral range of $3.0 \leq \lambda \leq 5.0 \mu\text{m}$, for a chevron four-reflection SiON-coated Si retarder. $\Delta_t = 270^\circ$ is achieved at center wavelength $\lambda = 4.0 \mu\text{m}$ with retardance error of $< 0.5^\circ$ over the $3\text{--}5 \mu\text{m}$ mid-IR spectral range.

the index ratio n/n' and the angle of incidence. Let n and n' be expressed in terms of three-term Sellmeier dispersion relations [Eq. (8)] with coefficients (B_i, C_i) and (B_i', C_i') , respectively, and let

$$\gamma = (n/n')^2, \quad x = \lambda^2. \quad (\text{A1})$$

The value of $x = \lambda^2$, at which a given value of $\gamma = (n/n')^2$ is obtained, is determined by solving the following sixth-degree equation in x :

$$(\gamma - 1)Q_0Q_0' + \gamma Q_0 \sum_{i=1}^3 Q_i' - Q_0' \sum_{i=1}^3 Q_i = 0. \quad (\text{A2})$$

In Eq. (A2) $Q_i, Q_i', i = 0, 1, 2, 3$ are cubic expressions of x that are given by

$$\begin{aligned} Q_0 &= (x - C_1)(x - C_2)(x - C_3), \\ Q_1 &= B_1x(x - C_2)(x - C_3), \\ Q_2 &= B_2x(x - C_1)(x - C_3), \\ Q_3 &= B_3x(x - C_1)(x - C_2). \end{aligned} \quad (\text{A3})$$

The corresponding expressions for Q_0', Q_1', Q_2', Q_3' are obtained by replacing (B_i, C_i) with (B_i', C_i') ; $i = 1, 2$, and 3 in Eq. (A3).

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