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Determination of the optic axis and optical properties of absorbing uniaxial crystals by reflection perpendicular-incidence ellipsometry on wedge samples

R. M. A. Azzam

Given an arbitrarily cut uniaxial crystal wedge, a procedure is described using reflection perpendicular-incidence ellipsometry (PIE) for (1) locating the optic axis, and (2) determining the ordinary (N_o) and extraordinary (N_e) complex refractive indices. The optic axis is located by finding the principal directions of the two wedge faces and subsequently solving three spherical triangles. N_o and N_e are determined by two complex ratios of principal reflection coefficients (of light normally incident on and linearly polarized along the principal directions of each face) as measured by PIE. The solution for N_o and N_e is explicit but requires finding the roots of a sixth-degree algebraic equation in N_o .

1. Introduction

Consider the normal-incidence reflection of a polarized monochromatic (or quasi-monochromatic) light beam by an arbitrary surface of a uniaxial absorbing crystal. Let n be the index of refraction of the transparent medium of incidence, $N_o (= n_o - jk_o)$ and $N_e (= n_e - jk_e)$ be the ordinary and extraordinary complex refractive indices of the crystal, and let θ be the angle between the optic axis and the surface. In Fig. 1, the principal plane, which is defined by the surface normal and the optic axis, is assumed to be in the plane of the page. When the incident light is linearly polarized with its electric vector vibrating either parallel (p) or perpendicular (s) to the principal plane, the reflected light is similarly linearly p and s polarized. Incident states other than the p and s linear eigenpolarizations are changed upon normal-incidence reflection.

If we resolve the electric vectors \mathbf{E}_i and \mathbf{E}_r of the incident and reflected waves into components parallel and perpendicular to the principal plane of complex amplitudes (E_{ip}, E_{is}) and (E_{rp}, E_{rs}), respectively, we can write

$$E_{rp} = R_{pp}E_{ip}, \quad E_{rs} = R_{ss}E_{is}. \quad (1)$$

In Eqs. (1), R_{pp} and R_{ss} are the principal reflection coefficients [the diagonal elements of the diagonal reflection matrix $\mathbf{R} = (R_{ij})$, $i, j = p, s$] and are given by

$$R_{pp} = \frac{n - N_p}{n + N_p}, \quad R_{ss} = \frac{n - N_o}{n + N_o}, \quad (2)$$

where¹

$$N_p^{-2} = N_o^{-2} \sin^2 \theta + N_e^{-2} \cos^2 \theta. \quad (3)$$

The ratio of principal reflection coefficients,

$$\rho = R_{pp}/R_{ss}, \quad (4)$$

can be measured by perpendicular-incidence ellipsometry² (PIE) using instrumentation that operates on null^{3,4} or photometric^{5,6} principles. If Eqs. (2) are substituted into Eq. (4), we obtain

$$\rho = \frac{(n - N_p)(n + N_o)}{(n + N_p)(n - N_o)}. \quad (5)$$

Equation (5) can be solved for N_p in terms of N_o and ρ ; this gives

$$N_p = n \frac{nq + N_o}{n + qN_o}, \quad (6)$$

where

$$q = (1 - \rho)/(1 + \rho). \quad (7)$$

If we eliminate N_p between Eqs. (3) and (6), we obtain

$$N_e^{-2} = \sec^2 \theta [(n + qN_o)/(n^2q + nN_o)]^2 - N_o^{-2} \tan^2 \theta. \quad (8)$$

One of the simplest methods² to determine the complex refractive indices N_o and N_e of a uniaxial

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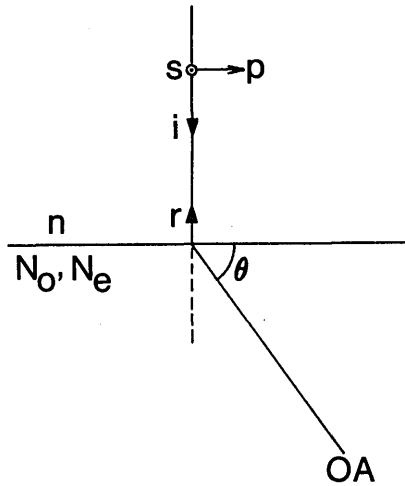


Fig. 1. Normal-incidence reflection of light at the interface between an isotropic ambient of refractive index n and an anisotropic uniaxial crystal (substrate) of complex ordinary and extraordinary refractive indices N_o and N_e . The optic axis OA is in the plane of the page (which, therefore, is a principal plane) and makes an angle θ with the crystal surface. p and s indicate polarization and principal directions parallel and perpendicular to the principal plane, respectively.

crystal is to measure two ratios of principal reflection coefficients ρ_1 and ρ_2 with the crystal immersed in two different ambients (e.g., air and a suitable liquid) of refractive indices n_1 and n_2 . Equating the two values of N_p that result from using Eq. (6) twice yields a quadratic in N_o ,

$$(q_1 n_2 - q_2 n_1) N_o^2 + q_1 q_2 (n_2^2 - n_1^2) N_o + n_1 n_2 (q_2 n_2 - q_1 n_1) = 0, \quad (9)$$

that can be solved for N_o . Equation (8) then gives N_e .

The procedure requires that the direction of the optic axis (hence the angle θ) be known (e.g., the optic axis is parallel to the surface under measurement; $\theta = 0$ and $N_p = N_e$) and that two ambient media be used. The latter requirement can present difficulty in spectroscopic applications, as the ambient must remain transparent over the spectral region used. Furthermore, an inert ambient may not always be readily available. Under these circumstances, it is worthwhile to consider another method in which only one ambient (e.g., vacuum or air) is used and PIE measurements on two crystal surfaces are made. Such a method is described in the following section.

II. Determination of N_o and N_e from PIE Measurements on Two Crystal Surfaces

For simplicity, and without loss of generality, we take $n = 1$. This means that N_o , N_e , (and N_p) are normalized with respect to any ambient refractive index n , so that the ratios N_o/n and N_e/n are being measured.

PIE measurements on two different and arbitrary crystal surfaces in the same ambient yield two ratios of principal reflection coefficients ρ_1 and ρ_2 . The corresponding q_1 and q_2 are determined by Eq. (7). If Eq. (8) is used twice (with $n = 1$), corresponding to the two measurements, and the right-hand sides are equated, we get

$$\sec^2 \theta_1 [(1 + q_1 N_o)/(q_1 + N_o)]^2 - N_o^{-2} \tan^2 \theta_1 = \sec^2 \theta_2 [(1 + q_2 N_o)/(q_2 + N_o)]^2 - N_o^{-2} \tan^2 \theta_2, \quad (10)$$

where θ_1 and θ_2 are the angles between the optic axis and the two surfaces on which measurements are made. Equation (10) can be put in the form of a sixth-degree equation in N_o as follows:

$$A_6 N_o^6 + A_5 N_o^5 + A_4 N_o^4 + A_3 N_o^3 + A_2 N_o^2 + A_1 N_o + A_0 = 0, \quad (11)$$

where

$$\left. \begin{aligned} A_0 &= -\mu^2 \tau \\ A_1 &= -2\mu\sigma\tau \\ A_2 &= (q_1^2 s_2^2 - q_2^2 s_1^2) - \tau(\sigma^2 + 2\mu) \\ A_3 &= 2\nu(q_1 s_2^2 - q_2 s_1^2) - 2\sigma\tau \\ A_4 &= (\nu^2 2\mu)(s_2^2 - s_1^2) - \tau \\ A_5 &= 2\nu(q_2 s_2^2 - q_1 s_1^2) \\ A_6 &= s_2^2 q_2^2 - s_1^2 q_1^2 \end{aligned} \right\}, \quad (12)$$

$$\left. \begin{aligned} \mu &= q_1 q_2 \\ \sigma &= q_1 + q_2 \\ \nu &= 1 + q_1 q_2 \\ s_i &= \sec \theta_i, \quad i = 1, 2 \\ \tau &= (\tan^2 \theta_2 - \tan^2 \theta_1) \end{aligned} \right\}. \quad (13)$$

Equation (11), with the coefficients A_k evaluated using Eqs. (12) and (13), can be solved for N_o . The condition that N_o is confined to the fourth quadrant of the complex plane may be sufficient to exclude all but the correct root of six roots. If necessary, an additional (crude) intensity-reflectance measurement of s -polarized light on one of the two faces can be used for the unambiguous determination of N_o with the help of the second of Eqs. (2). Once N_o has been determined, N_e can be obtained using Eq. (8) with $(\theta, q) = (\theta_1, q_1)$ or (θ_2, q_2) .

The foregoing procedure for the determination of N_o and N_e using PIE measurements on two crystal surfaces in the same ambient is as explicit as that described in Sec. I using PIE measurements on one crystal surface in two ambients. However, the two-ambient method leads to a simple quadratic equation in N_o in contrast with a sixth-degree equation in the two-surface method.

III. Determination of the Optic Axis of a Uniaxial Crystal Wedge

The measurements that are required to determine N_o and N_e by the method of Sec. II can be made on a sample in the form of a wedge (or prism). This is advantageous because then only one and the same sample is used, and the two planar wedge surfaces can be prepared under similar conditions. The other alternative would be two surfaces on two different samples of the same crystal.

Furthermore, by determining the principal directions of the two adjacent side surfaces of the wedge, the orientation of (the otherwise unknown) optic axis can be fixed using spherical trigonometry, as will be shown in this section.

The principal directions of an absorbing uniaxial crystal surface are unique orthogonal directions in the

surface along which normally incident light must be linearly polarized so that the reflected light will have the same linear polarization. These directions are denoted by p , which is the line of intersection of the surface with the principal plane (see Sec. I), and s which is, of course, orthogonal to p . These directions can be easily located experimentally (1) from the fact that they are directions of maximum and minimum reflectance for incident linearly polarized light of variable azimuth, or (2) from obtaining extinction along these directions if an arrangement with crossed linear polarizers is used.⁷ Because of the latter property, the principal directions are also often called extinction directions.⁸ For measurements on the two faces of an arbitrarily cut wedge, the p directions, p_1 and p_2 , can be specifically identified as the directions of unequal reflectances. Reflectances in the s_1 and s_2 directions are the same, because they are determined by the same refractive index N_o [see the second of Eqs. (2)].

Figure 2(a) shows the geometry involved with the wedge of two faces 1 and 2. E is the edge of the wedge, and p_1 and p_2 are principal directions in faces 1 and 2 (parallel to the principal plane), determined as explained in the preceding paragraph. p_1 and p_2 intersect E at a common point or origin O . The wedge angle is ω and the angles between p_1 and p_2 and E are α_1 and α_2 . The optic axis OA is uniquely determined as the line of intersection of the two principal planes P_1 and P_2 that are drawn through lines p_1 and p_2 perpendicular to faces 1 and 2 of the wedge.

To determine the angles θ_1 and θ_2 between the optic axis and the wedge faces ($\angle AOp_1$, $\angle AOp_2$), and the angle γ between the optic axis and the wedge edge ($\angle AOE$), spherical trigonometry is applied to the spherical-triangle equivalent to Fig. 2(a) shown in Fig. 2(b) (with O as the center of the sphere). Additional angles β_1 and β_2 and ϵ are defined in Fig. 2(b). The first spherical triangle to be solved is Ep_1p_2 with two known sides α_1 and α_2 and known included angle ω . From Napier's analogies,⁹ the unknown angles β_1 and β_2 are determined as follows:

$$\begin{aligned} \tan \frac{1}{2}(\beta_1 + \beta_2) &= \cot \frac{1}{2}\omega [\cos \frac{1}{2}(\alpha_1 - \alpha_2) / \cos \frac{1}{2}(\alpha_1 + \alpha_2)], \\ \tan \frac{1}{2}(\beta_1 - \beta_2) &= \cot \frac{1}{2}\omega [\sin \frac{1}{2}(\alpha_1 - \alpha_2) / \sin \frac{1}{2}(\alpha_1 + \alpha_2)]. \end{aligned} \quad (14)$$

The law of sines⁹ is then used to obtain ϵ (the angle between the wedge-face principal directions p_1 and p_2):

$$\sin \epsilon = \sin \omega \sin \alpha_1 / \sin \beta_2. \quad (15)$$

The second spherical triangle to be solved is Ap_1p_2 with two known angles $90^\circ - \beta_1$ and $90^\circ - \beta_2$ and known included side ϵ . Napier's analogies determine the unknown angles (sides) θ_1 and θ_2 :

$$\begin{aligned} \tan \frac{1}{2}(\theta_1 + \theta_2) &= \tan \frac{1}{2}\epsilon [\cos \frac{1}{2}(\beta_1 - \beta_2) / \sin \frac{1}{2}(\beta_1 + \beta_2)], \\ \tan \frac{1}{2}(\theta_1 - \theta_2) &= \tan \frac{1}{2}\epsilon [\sin \frac{1}{2}(\beta_1 - \beta_2) / \cos \frac{1}{2}(\beta_1 + \beta_2)]. \end{aligned} \quad (16)$$

Finally, the angle γ between the optic axis and wedge edge is derived by solving the right-angled triangle Ep_1A :

$$\cos \gamma = \cos \alpha_1 \cos \theta_1. \quad (17)$$

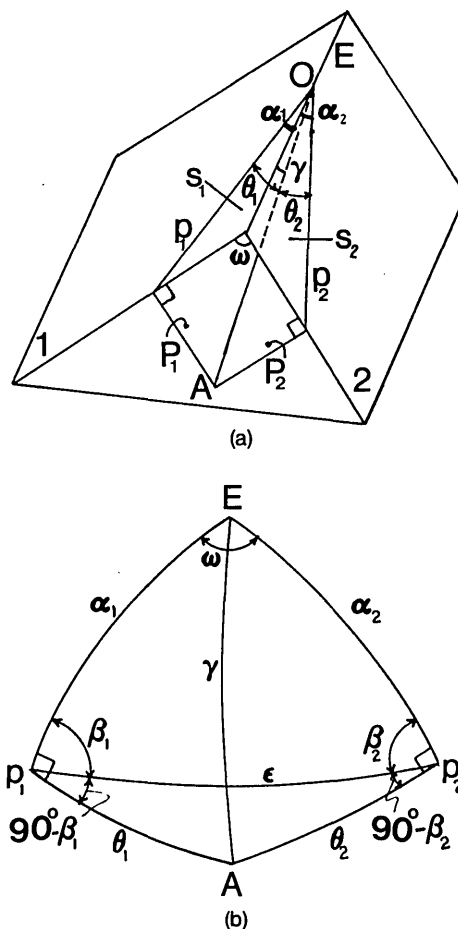


Fig. 2. (a) Geometry of a crystal wedge showing directions and angles that specify the optic axis. (b) The equivalent spherical triangles. See text.

The quadrants of angles calculated using Eqs. (14)–(17) are determined by following well-defined rules.⁹

IV. Summary

We have described a new method for determining the optic axis and optical properties of a uniaxial absorbing crystal from reflection measurements on two surfaces of an arbitrarily cut wedge-shaped sample. By locating the principal directions of the wedge surfaces, it is possible to locate exactly the optic axis using spherical trigonometry, as explained in Sec. III. In particular, the unknown angles θ_1 and θ_2 between the optic axis and the wedge surfaces can be found. Perpendicular-incidence ellipsometry (PIE) is then used to measure the complex ratios of principal reflection coefficients ρ_1 and ρ_2 of the two crystal surfaces, with the wedge in an ambient of known refractive index n (e.g., air or vacuum, $n = 1$). ρ_1 and ρ_2 and θ_1 and θ_2 determine the coefficients [using Eqs. (7), (12), and (13)] of a sixth-degree polynomial equation, Eq. (10), that can be solved for the complex ordinary refractive index N_o . N_e is then obtained by Eq. (8), as explained in Sec. II. These complex refractive indices are normalized with respect to, hence should be multiplied by, the ambient index n to get the true crystal indices.

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6. R. M. A. Azzam, *J. Opt. (Paris)* **9**, 131 (1978).
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9. See, for example, S. M. Selby, Ed., *Standard Mathematical Tables* (Chemical Rubber Company, Cleveland, Ohio, 1972), pp. 198-201.

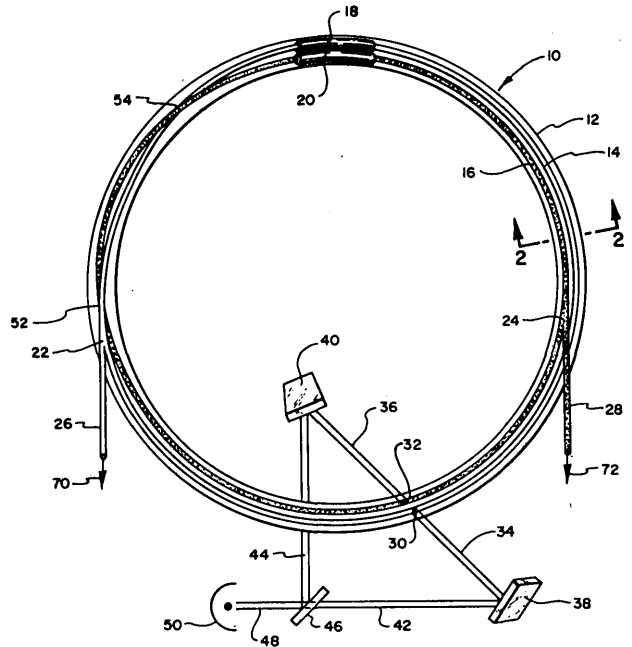
4,120,587

17 Oct. 1978 (Cl. 356-106 LR)

Double optical fiber waveguide ring laser gyroscope.

V. VALI, R. W. SHORTHILL, R. GOLDSTEIN, and R. S. KROGSTAD. Assigned to University of Utah Research Institute. Filed 24 Nov. 1975.

Two ring lasers are constructed of optical fibers (14) and (16) so that their lengths are identical. Gain media (18) and (20) are neodymium-doped fibers. Signal is extracted at junction splices (31), (32) so that the beat frequency can be determined. Since the two circular lasers are independent, there is no mode-pulling or lock-in that allows determination of much lower rotation frequencies than in a conventional ring laser gyroscope. Whereas a conventional laser gyroscope runs into these problems when the beat frequency is < 500 Hz, the present patent suggests that 10-Hz beat frequencies will be readily measurable. E.D.P.



Patents continued from page 3067

4,063,800

20 Dec. 1977 (Cl. 350-184)

Zoom lens for a projector.

Y. IIZUKA and S. NAKAMURA. Assigned to Nippon Kogaku K.K. Filed 19 Dec. 1975. In Japan 28 Dec. 1974.

A complex zoom projection lens is described covering a focal-length range of 1.75:1 at $f/1.0$. The front cemented doublet and the rear eight-element double-Gauss lens are fixed, while the intermediate cemented doublet negative component is movable along the axis. Good aberration correction is claimed over a field angle that ranges from $\pm 12.2^\circ$ to $\pm 6.9^\circ$ at this high aperture. R.K.

4,063,801

20 Dec. 1977 (Cl. 350-216)

Telephoto type objective.

H. YOKOTA. Assigned to Canon K.K. Filed 26 July 1976. In Japan 5 Aug. 1975.

Two examples are given of 280-mm $f/3.5$ telephoto lenses that are focused by a movement of the rear negative member. The front positive member consists of an airspaced triplet, while the rear member consists of two negative components that move at different rates for focusing. R.K.

4,148,587

10 Apr. 1979 (Cl. 356-356)

Laser gauge for measuring changes in the surface contour of a moving part.

J. C. ERDMANN, R. I. GELLERT, and R. L. SKAUGSET. Assigned to The Boeing Co. Filed 3 Oct 1977.

A visible laser beam is focused on the surface of a rotating cylinder or laterally moving plate. The speckle pattern formed by light scattered directly back through the focusing lens is detected by two detectors separated by a few millimeters to determine the speckle-rate variation and the speckle-size variation. The signals are analyzed by a correlator that develops the cross-correlation function between the two signals and determines when they are coincident. The focus lens is moved to peak the focus and the correlation signal. E.D.P.

4,158,176

12 June 1979 (Cl. 330-4.3)

Spatial filter system is an optical relay line.

J. T. HUNT and P. A. RENARD. Assigned to U.S.A. as represented by U.S. Department of Energy. Filed 6 Sept. 1977.

A low-power laser beam is sent through a series of lenses and apertures located in a train of amplifiers to produce an amplified beam with minimal phase distortion and beam-intensity variation. It is stated that while a previous glass laser system used in fusion research produces 1×10^9 neutrons/target burst, this invention will allow 5×10^9 neutrons/target burst. E.D.P.

4,182,995

8 Jan. 1980 (Cl. 331-94.5 H)

Laser diode with thermal conducting, current confining film.

F. A. HAWRYLO. Assigned to RCA Corp. Filed 16 Mar. 1978.

A diode laser is constructed of an n -type InP layer and a p -type InP layer that sandwich a p -type InGaAsP layer. A stripe metal electrode deposited on top of the p -type InP is covered with a Ge film that covers the rest of the p -type InP as well. This Ge film acts as a good heat conductor and forms a blocking junction with the p -type InP region, so as to confine the current across the p - n -junction away from the sides of the structure. E.D.P.

4,190,811

26 Feb. 1980 (Cl. 331-94.5 M)

Laser controlled optical switching in semiconductors.

A. J. ALCOCK, P. B. CORKUM, and D. J. JAMES. Assigned to Canadian Patents and Development, Ltd. Filed 11 Nov. 1977.

A pulsed CO_2 IR laser beam of 200-nsec time duration and 10-MW/cm² intensity is reflected off a slab of polycrystalline n -type Ge at Brewster's angle. When a pulsed ruby laser with a 2-nsec pulse duration illuminates the sample surface with intensity in the 15-MW/cm² range, the free-carrier density is increased rapidly so that the plasma-reflection effect occurs at 10.5 μm , thus turning on the reflected beam. Variations of this approach produce ultrashort IR pulses of <100 psec duration. E.D.P.

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