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# Order and Disorder, Entropy in Math, Science, Nature and the Arts

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*Abstract:* Often, science and engineering students have difficult time in viewing certain concepts in their holistic form. An example for such a concept is the concept of Entropy. Although it originated from the field of thermodynamics, the relation between order and disorder is a profound one and manifests itself in various fields, sometimes unrelated, such as math and science, nature and the arts. In the discipline of physics, the amount of disorder in a system has been quantified by the concept of Entropy. In the area of information theory it provides a quantitative measure of the amount of compression that may be achieved in a coding scheme and in nature it poses an intriguing question when we consider biological systems. In the arts, which is an unrelated field to the previously mentioned, entropy can be seen in the balance and the tension between the regular and irregular, the expected and the unexpected themes, which make a piece of art valuable. In this paper we provide some interesting insights into the concept of Entropy and its implications in the fields of physics, information theory, decision making, nature and the arts.

*Keywords*: Entropy, Order and disorder, Evolution, Information theory, Arts, Decision making

## 1 Introduction

The Concept of Entropy in Physics

A familiar simple experiment illustrates the basic idea of Entropy from the field of Thermodynamics. Assume we have a container with two chambers separated by a divider, one containing hot water and the other cold water. If we remove the divider, the water molecules will 'mix' together and after while the temperature of the water will settle to some value between the temperature of the hot and the cold water. On the other hand the opposite scenario seems impossible: A lukewarm container of water at a temperature of 70 degrees is not going to 'rearrange' itself spontaneously, so that the temperature of the water in the right half will be 68 degrees and in the left half 72 degrees.

If we think about this experiment in terms of level of 'order' in a system, in the first experiment, the system moved from a higher level of order to a lower one. In the second case it moved from a lower level of 'order' to a 'higher' level. This is due to the statistical nature of the kinematics of the water molecules and is an implication of the $2^{nd}$ law of Thermodynamics. It states that the level of 'disorder', or the entropy, in any 'closed system' can only increase in time, implying that the arrow of time has a direction and its direction is associated with the increase in the total entropy.

The concept of entropy has its origin from the field of thermodynamics, and it was formulated as the second law of thermodynamics in the mid of the $19^{th}$ century. Though its statement is simple, it has important consequences on the physical world and later found applications in areas such as information theory and game theory. By some accounts, the second law of thermodynamics was discovered by Rudolph Clausius and Lord Kelvin in the mid of the $19^{th}$ century, while by another account, it was Sadi Carnot who discovered it 25 years earlier. In any case, Clausius found that the ratio of heat exchange to absolute temperature was constant in ideal heat cycles. He named this physical quantity 'entropy'. According to the second law, the total entropy cannot decrease in a 'closed' physical system and this has some consequences. As an example, heat cannot move from a cold object to a warm one without additional work and the state of a system cannot change without investing energy. In a refrigerator we use electrical energy to remove heat from the cold interior and move it to the warmer exterior (the room).

Boltzmann, at the end of the $19^{th}$ century looked at the concept of entropy from a different point of view. Rather than analyzing the heat exchange between molecules in a system, he examined the process through a kinematical model and used the laws of probability to show that in such a system, over time, the kinematical parameters of the molecules will be evenly distributed. It can be shown, that heat transfer and this process are related to each other.

The consequence of the second law that the amount of entropy can only increase in time, provides a direction to the arrow of time and poses an interesting question about the invariance of physical laws with respect to time. Newton's laws of mechanics are symmetric with respect to time. One can take a movie, run it backwards and none of Newton's laws will be violated. The second law of thermodynamics, though, will be violated. Physical events in a closed system cannot occur in a direction which decreases the entropy in a closed system. Thus the total entropy in the universe is increasing constantly.

## 2 Entropy and Biological Organisms

Biological organisms pose an interesting challenge to the concept of entropy. Entropy can be examined in the context of biological systems at two levels: at the level of the individual organism, which begins as a single cell and develops to a complex organism and at the level of the evolution of the species. At the first level, from the time of conception (egg and sperm), the organism becomes more and more complex with level of order that increases. In sickness and in aging though, the process is reversed, resulting in destruction of structure and disorganization, but for the first period, the development of the organism seems to defy the second law of thermodynamics.

Entropy can be viewed also at the context of the evolution of the species, in which natural selection creates more complex species in time, starting billion years ago with simple single cell organisms and ending with the complex species we find today. This direction of evolution also seems not be compatible with the second law of thermodynamics.

In the $19^{th}$ century these arguments caused a controversy that was resolved by noting that biological systems are not closed systems and that during their life time they exchange matter, energy and information with the outside world. Due to this exchange of matter, energy and information, although the entropy within the organism decreases, the overall entropy (taking in account the increase in the outside world) increases.

In the context of Entropy, life can be viewed as a ‘pocket’ of lower level of entropy relative to its surroundings, which develops and at some point of time it achieves its lowest level. From that point then, its entropy gradually increases such that eventually (in death) it is in equilibrium with its surroundings.

Still, it is the mystery of life how these ‘pockets’ of low level entropy develop and why at some point there is a process of degradation that gradually equates the entropy of the organism to its surrounding.

## 3 Entropy and Information Theory

Information, whether it is data or images, can be represented in more than one way. In data compression we are interested in a representation that uses a minimal number of numbers, letters, or other symbols to represent certain information. If we allow certain amount of error in the reconstruction of the information, we can achieve a better compaction of information. There are various data compression techniques and all are based on the principle that whenever there is some ‘regularity’ in the information, then there is a more compact representation of that data. In mathematical terminology the operators that change the representations are called transformations. In the area of information theory, it was Claude Shannon, an electrical engineer at Bell Labs, who developed the mathematical foundations of information theory and introduced the concept of entropy in the context of providing the bound for the compaction of a piece of information. He used the probability distribution of the symbols in any representation to define the entropy of that representation and found the theoretical lower limit on the number of symbols that are needed in representing any type of data by using the level of the entropy in the data.

The concept of entropy was later adapted to multivariable data. For a sequence of data a measure of regularity can be provided by the autocorrelation of the data. In multidimensional data it is the covariance of the various coordinates that provide information about the level of correlation between the different coordinates. Correlation implies lower entropy and redundancy. Reducing this redundancy is important as it provides a tool to reduce the dimensionality of certain problems which otherwise are impossible to solve. An example for such mathematical tool in statistics is PCA or Principal Component Analysis, which serves to transform multidimensional correlated data into uncorrelated new data while reducing the dimensionality of the representation. To illustrate the use of Principal

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Component Analysis consider the small percentage of the population which develops certain physical or mental pathologies (e.g. schizophrenia). Scientists would like to have early detection of those who have the potential to develop the sickness so they can be treated earlier. One way to do that is to find certain attributes such as psychological, behavioral, physical or intellectual that differentiate the group of interest from the general population and can be used to predict unknown cases. Unfortunately, in general, the discriminating information is distributed and convoluted in many attributes in such a way that it is impossible to detect the group of interest by simply observing the data. In certain cases, Principal *Component Analysis* can be used to reduce the dimensionality of the problem to a manageable size. PCA takes the discriminating information, which originally is distributed and convoluted in many attributes and compacts it into a fewer set of new attributes. With a small number of attributes it becomes possible to differentiate the group of interest from the general population. When the distribution of the attributes in a multivariate population is normal, PCA provides the highest level of discrimination between the two groups among all possible transformations, i.e. it provides the highest level of compaction of the information and in the context of entropy, it provides the representation of the data with the highest level of entropy and therefore the lowest level of redundancy. PCA decorrelates the data and transforms it into a new uncorrelated representation. Correlation implies regularity and order. When data is decorrelated it implies 'disorder' or higher entropy and it will be closer to ‘random values’ when compared to the correlated one.

## 4 Entropy and Statistics

The concept of entropy can also be examined in the context of statistical distributions. Using the formula for the entropy of a finite scheme, one can calculate the entropy associated with various continuous distributions. It can be shown that among all distributions with same variance, the distribution with the highest level of entropy is the Gaussian distribution. The central limit theorem tells us that phenomena that are determined by uncorrelated random factors will have a normal distribution. This observation can be used to detect whether there is some external factor affecting a process. For example in the production of resistors, there are many factors affecting the resistance of an individual resistor, e.g. the amount of resistive material, its homogeneity, its geometry, etc. Now, if all these factors are random and independent, the distribution of the resistance values would be Gaussian. If we find it is not, this may be evidence that there is non-random factor in the process affecting the deviation, for example a defective cutting machine. Thus in the absence of any directing forces, a system tends to be in a state of maximum entropy.

Can we come up with any meaningful statements relating to a system characterized by random events and seemingly chaos and disorder? The answer to this question is positive and is provided by the fields of probability and statistics which began to be defined only in the seventeenth century.

While we cannot predict who is going to be murdered in the next year, we can with a high level of confidence estimate the number of homicide cases in the country. The ability to provide quantitative estimates relating to random events is of great importance. Casinos cannot predict which customer will win and which will lose but their business is based on the law of large numbers, which guarantees with a high level of confidence that the casino will win a certain percentage (which depends on the game) of total amount being gambled at the end of the day. Insurance companies cannot predict which customer will be involved in an accident but if the insured population is large and diverse, they can predict with a high level of confidence the number of accidents in various categories and assign the premium accordingly. Thus we can come up with meaningful statements about seemingly chaotic system that can be characterized with high level of entropy.

## 5 Entropy as a Measure of the Uncertainty in Decision Making Process

In decision making process, we may face situations in which we have to allocate resources to deal with a situation in which we have only estimates of the likelihood of its states. For example, a military commander faces a possible attack that can come from different locations. He has a set probability values representing his estimate for a possible attack from each location. It would make sense that he should allocate his defensive resources based on these estimations to optimize the chances of defending his unit.

A question that he may ask himself would be: ‘What is the level of confidence I can associate with my decision’.

The concept of entropy as it is defined in information theory, can provide an answer to that question. Assume we are given a set of n mutually exclusive events of a complete system (that say represent the n different locations the attack can come from), with their probability values. The events together with their associated probabilities create what is called a finite scheme. The concept of entropy can be used to describe the *uncertainty* of the finite scheme and it is calculated as the negative of the sum of the probability products by their logarithm base 2. We can examine two extreme scenarios:

In the first, there are n events, all with zero probability except one event that has a probability of 1. Calculating the entropy for this case we find its value to be 0.

In the second scenario, all n events have equal probability, in which case, the calculated entropy is the logarithm of n base 2.

It can be shown that the entropy associated with the first scenario (=0) is the lowest among all possible scenarios and the entropy associated with the second scenario is the highest possible among all scenarios with n events. In the first case, the level of uncertainty is 0, and we have the highest level of confidence in our decision as we can predict with full confidence what would happen. The second scenario on the other hand represents the highest level of uncertainty, as all events are likely to occur with same probability.

## 6 Entropy and the Arts

In the arts, order and disorder play an important role in the artistic value of artistic works. Order and disorder have to be used in the 'right' balance to attract our attention. A monotonous tone represents the highest level of order, but it can hardly get any appreciation from us while a random noise that represents the highest level of disorder, would not be considered to have any artistic value. It is the equilibrium, the waving of these two vectors that stimulate our interest. It is the unexpected note, the unexpected change in the theme, the rhythm or the scale that although seem to be out of place at first perception, it triggers us to try and make sense of it in the unconscious or conscious digestion process of the piece of art.

The role of order and structure in the art has its footprints in its history. An example is the evolution of the dodecaphonic system in music. Most western music is tonal music and it was only at the beginning of the $20^{th}$ century that there was a significant work done using a different musical framework, namely atonal music developed by composers such as Arnold Schoenberg and his pupil Alban Berg. Tonal music is based on the Major and Minor scales which have distinct structures. The Major scale is composed of 7 tones out of the 12 semi tones of the full octave. The intervals between the tones are given by the pattern 221222. It can shown that three of the notes, the tonic, dominant and the mediant (the first, fifth and third notes) in this scale are harmonically related to each other. There is an overlap of the upper harmonics of these notes, and therefore when sound together they create a pleasant sense of consonance while other tones, their harmonics do not overlap creating a sense of dissonance. Tonal music is based on this triad of notes. Their importance is not only at the chord level but also at the higher level of musical sentences. If a piece is written in C Major, then the tones C,G,E will carry more weight than other tones in the scale. For example, they may appear more frequently or be used more frequently as the ending note in a musical sentence. Thus tonal music provides an environment with restricting harmonic rules, of lower entropy, in which the composer has to work with. In this context, the evolution of atonal music introduced an environment with different and perhaps less restricting rules. Removing the tonal restrictions provides the artist more choices, and using information terminology, he can introduce more artistic information. Compare the first prelude from the well tempered clavier by Bach and a piano piece by Arnold Schoenberg. Both are based on structure but in different ways. Bach's prelude is based on variants of a single musical phrase where the changes are in a single note or scale, all elegant and pleasant to the ear. Schoenberg piece is based on a structure of a mathematical nature (the dodecaphonic scale), but here order and disorder are convoluted in a very complex form which makes it very hard for the untrained ear.

There are some attempts to apply concepts from information theory by, for example, quantifying the amount of artistic information in a musical sentence. Music is a good candidate for discussion about art and information theory as it is abstract and is not for example, subject to restriction of human language as in literature. In this respect one can treat musical parameters such as pitch, duration, pitch transition etc. as the data to which the concept of entropy can be applied to. One can calculate the amount of entropy for a musical score using the probabilities of pitch, pitch transition, and other musical parameters, and then compare the entropy values for pieces written by different composers. These quantitative measures can be an interesting exercise but of course cannot capture the structure of an artistic work. The artistic value cannot be formulated, or presented in quantitative for or else we would be able to mechanically compose art (of course there are mechanical methods using for example statistical musical parameters to produce music in certain styles, but what about the coherency of the piece?

Still if one calculate the entropy for tonal vs. atonal music, Mozart or Beethoven vs. Bach he may find differences.

Why is the combination of order and disorder, the expected and the unexpected, brings with it artistic pleasure? It is probably because it is associated with tension reduction. When we are introduced with an irregularity, a piece of information that is out of place, we try to make sense of it, and when we succeed, we have a feeling of pleasure associated with the tension reduction. Why this mechanism of tension reduction brings pleasure? from a point of view of order and disorder, music can be regarded as a constant change in the level of entropy at the local level, stimulating the mind to resolve those points when the level of entropy (disorder, unexpected) increases, by making 'sense' of it or finding a presentation that has lower level of entropy. Is this mechanism can be explained by evolution? by the fact that the mind was trained in many years of evolution to look for order and regularity in an unpredictable physical environment, to look for 'pockets' of low entropy as that increased the chances of survival. Is artistic pleasure simply an abstraction of this process?

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