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Experimental and observational geometry

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EXPERIMENTAL AND OBSERVATIONAL GEOMETRY

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A THESIS

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IN PARTIAL FULFILLMENT

OF THE

REQUIREMENTS FOR THE

DEGREE OF MASTER OF ARTS

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By

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APPROVED — J. William Harris
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June, 1928 — Harriet E. Boss, Librarian

To

Friend

PROFESSOR WALTER H. BUXTON

This book is dedicated by

one who owes much to

his teaching in

mathematics

PREFACE

Geometry has the distinction of being one of the oldest subjects given in the high-school.

Its subject-matter was formulated and organized by the Greeks into a fine system of thought before the time of Christ. Since leaving the hands of the Greeks, geometry has received only a few minor changes, and these largely in in recent years.

Heretofore, the study of geometry has been made almost entirely dependent upon memory and reasoning. Geometricians have been slow in adopting the laboratory and observational methods.

This thesis has been written to encourage the student in his work of observing geometrical forms, and in the construction of good designs and geometrical figures, and to obtain a better practical understanding of the figures and principles of geometry through the laboratory and observational work.

A. D. F.

College of the Pacific

Stockton, California, June, 1929

CONTENTS

LABORATORY EQUIPMENT

Much of the work suggested in this outline can be done with a very small amount of equipment; but it is urged where-ever possible to furnish the laboratory with as complete equipment as possible. Added equipment furnishes opportunity for the students to work along lines of their greatest interest.

A. Suggestions for General Equipment:

1. Laboratory tables, chairs, etc.
2. Transit instrument.
3. Plane table and tripod for elementary surveying.
4. Black-board compasses.
5. Pendulum apparatus.
6. Large slide rule (for class reading).
7. Meter sticks.
8. Yard sticks.
9. Carpenter's square.
10. Surveyor's tapes.
11. Architect's scale.
12. Engineer's scale.
13. Large slated globe.
14. Set of geometric solids.
15. Law of lever apparatus.
16. Simple machines (pulleys, etc.).
17. Sphere, cone and cylinder of same size.
18. Stereoscope and slides.
19. Force appliance.

20. Transparent cube for projection.
21. Weighing apparatus.
22. Mirrors.
23. Scissors large and small.
24. Simple apparatus for investigating strength of bridges, and materials.
25. Glue, or paste.
26. Plane, convex, and concave mirrors.
27. Sextant for measuring angles.
28. Simple range finder.
29. The five regular polyhedrons.
30. Drawing boards.

Much of this equipment may be made by the students, and it would furnish them good training to do this.

B. Individual Equipment:

1. Paper and pencil.
2. Set of instruments:
    (a) Compasses;
    (b) Ruling pen;
    (c) Dividers, etc.
3. Protractor.
4. Ruler.
5. Triangles, $30^\circ \times 60^\circ$ and 45°.
6. Ink.
7. Erasers.

C. Books

It is well to have a number of books in the laboratory or recitation room. These books should be on a variety of subjects. They should not only treat on pure and applied mathematics, but also on subjects related to geometry. Occasionally the class period or a part of the supervised study period may be given to the reading of these books, the reports and discussions may be given later.

Books on the following subjects are suggested:

1. Mathematical history.
2. Mathematical theory.
3. Other branches of mathematics:
   - (a) Arithmetic.
   - (b) Algebra.
   - (c) Trigonometry.
   - (d) Analytic geometry.
   - (e) Calculus.
4. Mechanical arts.
5. Engineering.
6. Books on design.
7. Books on shop or industrial design.
8. Books on science, physics, chemistry, astronomy, etc.
9. Historical geometrical problems.

INTRODUCTION

THE USE OF TERMS

This treatise on experimental and observational geometry, or what might be called laboratory and field geometry, is a detailed account of how to conduct experiments dealing in geometrical materials. These experiments may be performed in the geometrical laboratory (a room provided with equipment essential for the experiments) or they may be performed out of doors, according to the nature of the experiment.

The observational work called for, and the experiments done outside the class room, together with trips and excursions, made by the individual student, or by the class as a whole, constitute what is termed field work. The sections treating on observational geometry direct the student in noticing the applications of geometry in his environment. Most of this will consist in observation of geometrical forms in architecture, nature and art, and the use and arrangements of lines, and space relationships wherever found. For a more detailed account of experiments provided, and suggestions for observational work, the reader is referred to the main body of this thesis.

THE PURPOSE OF THIS TREATISE

In the realization of the value of laboratory work in the sciences in providing real present experience, adaptable to the needs of each individual student, this laboratory manual has been written to provide directions for the student's use in the laboratory room, and to furnish suggestions for his observing outside the laboratory. All this has been modeled after the plan of the laboratory manuals used in Physics, which have proved their value for many years.

HOW TO USE THE MANUAL

This manual can be used with any good standard text in geometry. The plan of procedure should be varied depending upon the type of material under consideration, but in general the following order of presentation is urged:

1. LABORATORY APPROACH

The laboratory experiment serves as an introduction to the theorems of geometry. Often the student of chemistry first becomes acquainted with an element in his experiments in the laboratory. In like manner in order to prepare the student for the theoretical or logical proof of the geometry text, some experiment should be given the student. The student will thus become acquainted with the terms used and the principles involved and will understand and better appreciate the theoretical proof when it is given.

2. THEORETICAL PROOF

The theoretical, or general proof is the kind of proof given in the geometry text. This should consist of (a) proof by analysis and then the (b) proof by synthetic method. For more extended discussions of the methods of proof the reader is referred to these topics given under methods of study.

3. NUMBER RELATIONSHIP

As a number of geometry texts give numberous problems emphasizing number relationship, no space will be devoted to this topic here, except to indicate its value as means of testing the student's knowledge of the use of the theorems.

4. OBSERVATIONAL WORK

The work of the student in observing helps him to correlate the geometry of the text with the applications and use of geometry outside the text. This is best given along with or following the theorem or principle to be observed.

5. LIBRARY READING

Students should be given assignments in books on history of geometry, art, architecture, surveying and historical problems; reports and discussion of such reading should be made in class. Each will receive profit by giving the report in class, and also by hearing the reports given by others. Outside reading often helps to awaken interest. For suggestions of books which contain the type of reading material useful for rally days or for assigned readings refer to the Bibliography C.

6. OUTLINE FOR RALLY DAYS

Certain special geometry rally days may be appointed, and an informal program be held in the class room instead of the regular class work. A display of drawings and a discussion of interesting problems should be made at this time.

A. Program Suggestions:

1. Reports concerning the lives of famous mathematicians can be made.
2. Engineering feats which demand exact mathematical calculations may be discussed.
3. Applications of geometry to be made by students interested in various lines as:

1. Astronomy	3. Art
2. Carpentry	4. Engineering

4. Oral reports on various topics can be given.
5. Historical topics may be assigned and discussed.

B. Displays May be Made of the Following:

1. Two or three of the best note books.
2. A geometrical design from each student.
3. Constructions from each student.
4. Models to illustrate the theorems or their application.

C. Discussion of Fallacies:

1. Prove all triangles are isosceles.
2. That two perpendiculars can be drawn from an external point to a line.
3. Draw a line which shall have three ends, etc.

7. BENEFITS TO BE DERIVED

The student should be benefitted by a broader understanding of geometry if he follows faithfully the suggestions given above; for he will have a view of the same principle from various angles of approach. The advantages derived from using observational and laboratory work in connection with the regular text book method may be briefly mentioned as follows:

1. Various angles of approach are used.
2. Acquaintance with the principles involved is obtained before a consideration of the theoretical proof.
3. Present experience is furnished for each student.
4. Something is furnished for each student so that all are kept busy.
5. A flexible course is obtained so that a way may be found to adapt instruction to each.

CHAPTER I

GEOMETRY AS THE SCIENCE OF SPACE

HOW GEOMETRY TREATS OF SPACE

"Geometry", say Slaught and Lennes in their text on plane geometry, "is the science which deals with the properties of space in which we live."

As usually treated geometry deals with points, lines, surfaces, and solids. Not much attention is given by the student or teacher to geometry in its more general aspects, that is, geometry as the science of space. The student may become so absorbed in his study of theorems and the formal proof, that he may not recognize or realize that geometry treats of space, and the relation of lines and surfaces to one another.

By considering geometry as the science of space and space relationship, we get a unifying principle which may be carried throughout the study connecting its various parts. As the child has hadintimate and varied experiences in his contact with objects, and their relationship in space; the consideration of geometry as the science of space helps to connect his past experience with the study of geometry.

When in geometry we consider a comparison of shapes; relation of one figure to another, as in congruence, or similarity, the comparison of areas, the relationship of one line to another as is studied in proportion; we study it as an aspect of space. In the consideration of the following, note how space may enter as a means of comparison and unification:

A point is space of zero dimension, or that which has location only;

A line is space in one direction, or space of one dimension only;

A surface is that which has space of two dimensions, length and width;

A solid is space of three dimensions.

It is interesting to think of the limitations and possibilities of an individual who would be adapted to and capable of living in a world of each of the four cases of dimensions mentioned above. A being living in a world of one dimension could think of motion as in a forward or backward direction only. In a world of two dimensions the idea of height and depth would be lacking; and a being of a two dimentional world would think only of space as lines or as a surface.

A number of applications of the principles of geometry in the arts, make use of space relationships, as will be shown in the main body of this thesis, especially under such headings as good proportions in a rectangle, vertical divisions of the primary mass, horizontal divisions of the primary mass, and under applications of proportion.

Geometrical qualities are not concerned with the kind of material in a material body; but only with the number of units in this material, as in finding area, etc; and in its shape, whether triangular, quadrilateral, circular, etc; or with the relationship of one figure to another. In physics and chemistry the properties of the material in the body are studied.

In physics we note the effect of heat, light, or electricity upon the material, while in chemistry we try to find what elements the material contains and how their properties change under chemical manipulation; on the other hand in geometry we study the properties of the space the body occupies separate and apart from the properties of the material body. Thus we note in geometry the size of the object, the kind of shape it possesses, its position, or relative position, the kind of angles on its surface. As the study of gemmetry must be based upon the knowledge of the child it would be well here to consider how the child receives his knowledge of space.

HOW THE INFANT RECEIVES A KNOWLEDGE OF SPACE

It is very certain that at birth the infant has no knowledge of space.

"The field of vision at the beginning of life," says Preyer in the Infant Mind, page 7, "resembles a chart placed close to the eyes upon which the colored, the bright, and dark parts, of the surface blend with one another so that nothing is distinctly apprehended....In reality he sees no object at all, and is very slow in learning to distinguish between above and below, left and right, near and far."

No doubt the child soon gets an idea of mere expanse of space which has no particular form. This would be a conception of space of two dimensions. It is only after much time has elapsed that the infant begins to comprehend the meaning of the third dimension, or is able to estimate distance[1].

This idea of distance comes to the child through much experimentation, such as reaching for things near, or being

1 Titchener Psychology, 1913, page 303.

transported to various objects. The child advances in intelligence largely in proportion as he advances in his process of organizing the sensations received into definite space relationships.

SPACE PERCEPTIONS OF THE BLIND

The blind are known to have developed a well ordered perception of space, as is shown in their ability to go unaided through the streets of our large cities, and also by their ability to find anything needed in their own homes. Their knowledge of space has come through the tactile and kinaesthetic senses.

If a blind person should be given sight he would be unable at once to estimate distances with the eye, or even to name the objects he sees although they would be immediately recognized by touch. The papers have reported the history of a number of such cases, and the experiences of these people in organizing the eye sensations into a system of space relationships give us a hint as to the like experiences of the child. The old Bible story concerning the blind man who after having received his sight saw men as trees walking is not far from the truth psychologically.[1]

[1]Mark 8:23

SUMMARY

A moment of thinking concerning our own experience will convince us that our conception of space is a complex form of experience; resulting from our efforts to arrange the sensations received through the various senses, into a system.

The child before entering school does not proceed very far in his perception of space beyond the recognition of objects, estimation of distances, and the comparison of objects in an elementary and haphazard way. When the child on the other hand takes up the study of geometry, he proceeds in a definite and systematic way, to consider the science of space, and deals with this space in a theoretical as well as a practical way.

CHAPTER II

THE HISTORY OF GEOMETRY AS RELATED TO THIS THESIS

HISTORY

Geometry has had a long and very interesting history. As far back as the records of civilization go, we find that man has used the principles of geometry in his building, designing of his ornaments, making implements of war, and in land surveying.

The amount of geometry used by the ancient nations, notably the Egyptians, before the Greeks, did not exceed that usually given in a good course in mensurational arithmetic with the addition of the practical applications to land measurements, this practical application being necessary by the annual inundation of the Nile.

However, when geometry was introduced into Greece, the wise men, philosophers and teachers of this country, immediately began the study of geometry in a purely abstract way as the science of space. The other nations carried the study of geometry no farther than was absolutely needed for their practical purposes. But it was very different with the Greeks. As they did not appreciate manual labor, the practical did not appeal to them except as it applied to art, sculpture, architecture, or to argumentation, and rhetoric. Since geometry furnishes aid in the study of these subjects, and gives an opportunity for the use of logic, its study became a passion with them.

"From the moment," Cajori says in his History of Mathematics, page 15, "that Hellenic philosophers applied themselves to the study of Egyptian geometry, this science assumed a radically different aspect."

In less than three hundred years the Greeks discovered and formulated the proofs for most of the theorems given in our text books. Thales, Plato, Aristotle, Archimedes, Euclid, and others made contributions to the subject.

For our purpose in this treatise it is well to notice the methods used in the development of the science of geometry.

1. First the ancient nations were interested in the subject entirely from the standpoint of its applications. By observing and experimenting they became acquainted with the principles of geometry.

2. Then came the Greeks who developed the theoretical proofs, and wrote the first important text book. This book was the result of the combination of two sciences brought to a high state of perfection by them; namely, logic, and the science of space.

3. During the middle ages, and as long as the Aristotelian psychology held the field, students were given formal geometry as a training of the "faculty of reasoning."

4. And during our modern times text book writers have added a number of applications seeking to adapt the subject to our modern life. A number of changes have been made in the text book as will be shown in the next section of this treatise.

5. Other changes recommended for better adapting the material to each student include the use of laboratory exercises, observational work and a more liberal consideration of methods of

study, and a wider use of proof by analysis.

THE EVOLUTION OF THE GEOMETRY TEXT BOOK

The evolution of the geometry text book illustrates very aptly the history of the art of printing and book-making, as well as the progress of the science of geometry. A number of subjects taught in the high school have only a recent history, but the history of geometry goes back to a date when the method of book-making was very different from that used at present. Geometry has been recorded upon clay tablets, stone, metals, skins of animals, parchment, and on paper. A collection of these would show an interesting contrast with our present form of the geometry text.

An important evolution has also taken place in reference to content of the geometry, arrangement of the materials, and appearance of the page. At one time the Greeks wrote their sentences without separating the words. A comparison of geometry text books published about one hundred years ago with our modern texts will reveal a number of important changes. In these older texts the figures were usually poorly drawn, an essay type of proof was given, and often the figure and proof were on different pages.

The more modern texts use what may be called the unit page and unit line. The unit page has the theorem and figure on the same page; the unit line has only one statement and its reason to the line. Wentworth was the first textbook writer to use these principles.

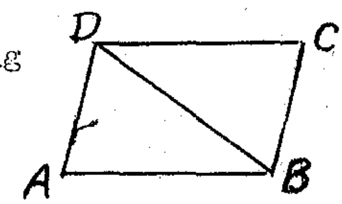
Changes also have been made in the order and arrangement of the theorems, in the kind and number of exercises and the applications. In the next few pages a number of theorems are given which have been reproduced from a few geometry text books as a sample of the method of arranging the proofs and the styles of the pages; to illustrate a few of these changes which have come within the last one hundred years.

Essay type of proof, as given by Davies, published in 1837:

PROPOSITION XXX. THEOREM

IF TWO OPPOSITE SIDES OF A QUADRILATERAL ARE EQUAL AND PARALLEL, THE REMAINING SIDES WILL ALSO BE EQUAL AND PARALLEL, AND THE FIGURE WILL BE A PARALLELOGRAM.

Let A.B.C.D. be a quadrilateral, having the sides A.B.,D.C.,equal and parallel; then will the figure be a parallelogram.



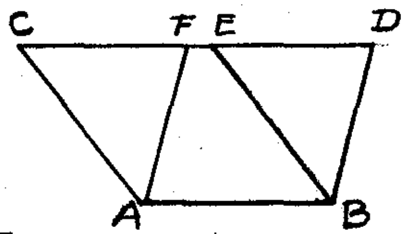
For, draw the diagonal D.B.,dividing the quadrilateral into triangles.

Then, since A.B. is parallel to D.C.,the alternate angles A.B.D.,B.D.C., are equal (Prop. XX. Cor. 2); moreover, the sides D.B.is common, and the side A.B.= D.C; hence the triangle A.B.D. is equal to the triangle D.B.C. (Prop. V); therefore, the side A.D.is equal to B.C.,the angle A.D.B.= D.B.C.,and consequently A.D. is parallel to B.C; hence the figure A.B.C.D. is a parallelogram.

Proof from Robinson's Geometry, published in 1862, also essay type:

THEOREM XXVII.

PARALLELOGRAMS ON THE SAME BASE, AND BETWEEN THE SAME PARALLELS, ARE EQUIVALENT, OR EQUAL IN RESPECT TO AREA OR SURFACE.



Let ABEC and ABDF be two parallelograms on the same base AB, and between the same parallels AB and CD; we are to prove that these two parallelograms are equal.

Now, CE and FD are equal, because they are each equal to AB, (Th. 24); and, if from the whole line CD we take, in succession, CE and FD, there will remain ED=CF, (Ax. 3); but BE=AC, and AF=BD, (Th. 24); hence we have two triangles, CAF and EBD, which ave the three sides of the one equal to the three sides of the other, each to each; therefore, the two triangles are equal, (Th. 21). If, from the whole figure ABDC, we take away the triangle CAF, the parallelogram ABDF will remain; and if from the whole figure we take away the other triangle EBD, the parallelogram ABEC will remain. Therefore, (Ax.3), the parallelogram ABDF= the parallelogram ABEC.

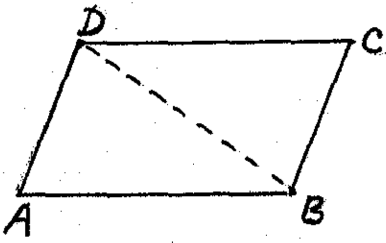
Hence the theorem; PARALLELOGRAMS ON THE SAME BASE, ETC.

Proof from Robinson's Geometry, published in 1862, also essay type:

THEOREM XXV

IF THE OPPOSITE SIDES OF A QUADRILATERAL ARE EQUAL, THEY ARE ALSO PARALLEL, AND THE FIGURE IS A PARALLELOGRAM.

Let A.B.C.D. be any quadrilateral; on the supposition that A.D.= B.C.,and A.B.= D.C.,we are to prove that A.D. is parallel to B.C.,and A.B. parallel to D.C.



Draw the diagonal B.D; we now have two triangles, A.B.D. and B.C.D., which have the side B.D. common, A.D. of the one = B.C. of the other, and A.B. of the one = C.D. of the other; therefore the two triangles are equal (Th. 21), and the angles opposite the equal sides are equal; that is, the angle A.D.B. = the angle C.B.D; but these are alternate angles; hence A.D. is parallel to B.C. (Th. 7, Cor. 1); and because the angle A.B.D. = the angle B.D.C.,A.B. is parallel to C.D., and the figure is a parallelogram.

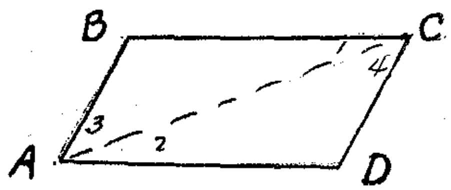
Hence the theorem: IF THE OPPOSITE SIDES OF A QUADRILATERAL, ETC.

COR. This theorem, and also Th. 24 proves that the two triangles which make up the parallelogram are equal; and the same would be true if we drew the diagonal from A. to C; THEREFORE, THE DIAGONAL OF ANY PARALLELOGRAM BISECTS THE PARALLELOGRAM.

A reproduction of a page from Wells and Hart Geometry, a good recent text. This shows the use of the unit line and page, and parts of proof omitted to be supplied by student.

PROPOSITION XXVI. THEOREM

134. IF THE OPPOSITE SIDES OF A QUADRILATERAL ARE EQUAL, THE FIGURE IS A PARALLELOGRAM.



HYPOTHESIS. A.B.= C.D; B.C.= A.D.
Conclusion. A.B.C.D. is a parallelogram.
Prove A.B. // C.D., by proving ∠ 3 = ∠ 4.
Prove B.C. // A.D., by proving ∠2 = ∠ 1.

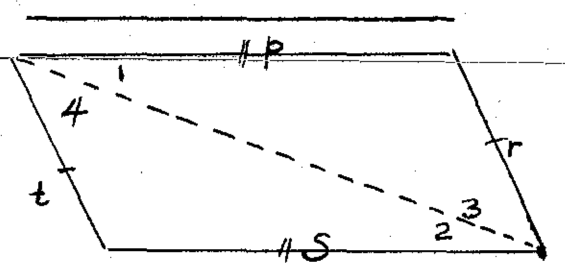
PROOF:

STATEMENTS	REASONS
1. A.B.C.= A.C.D.	1. Give the full proof.
2. ∴ ∠ 3 = ∠ 4	2. Why?
3. ∴ A.B.// C.D.	3. Why?
4. Also B.C. // A.D.	4. Give the full proof.
5. ∴ A.B.C.D. is a ▱	5. Why?

A page from a more recent geometry (1927) by Strader and Rhodes. Only a skeleton of proof is given. The main work is left for the student.

PROPOSITION 20 (C N)

151. THEOREM. A QUADRILATERAL WITH ITS OPPOSITE SIDES EQUAL IS A PARALLELOGRAM.



GIVEN: Quadrilateral prst, p = s, r = t.

TO PROVE: prst is a .

HINT: What is a parallelogram? How are lines proved parallel?

PROOF: Statements

1. Draw diagonal d. In prd and std.	1. By what principle?
2. ? = ?	2. Why?
3. ? = ?	3. Why?
4. ? = ?	4. Why?
5. ? = ?	5. s.s.s. = s.s.s.
6. 1 = 2	6. Why?
7. p // s	7. Why?
8. 3 = 4.	8. Why?
9. R // t.	9. Why?
10. prst is a .	10. (Def) 138.

CHAPTER III

WHAT GEOMETRY SEEKS TO DO TO THE MIND OF THE STUDENT

IT FURNISHES A GENERAL TRAINING WHICH IS TRANSFERABLE TO OTHER ACTIVITIES

Mathematics is often regarded as furnishing a high disciplinary value. "Mathematics," says the old adage, "is the whetstone which sharpens every wit." The knife is not sharpened unless it be properly applied to the whetstone; neither can a person be greatly benefitted, or a positive transfer be expected unless he uses the right methods in his study. He may put in earnest and diligent mental effort, but in a manner which would not be of value even in the special course of mathematics he is pursuing, and hence it could not be expected to furnish a generalized training which would furnish a transfer. In order that a transfer may be realized in the study of geometry there are certain conditions which have to be met besides (1) mere presence in the class (2) or putting in time on the subject. The student will need to:

1. Study the proofs instead of merely memorizing them;
2. See the proof for himself;
3. Get a generalized idea of the principles;
4. See the possibility of a transfer. As an instance of this he should after studying concerning regular polygons in plants, and in linoleum and floor coverings; or after proving that two angles and the included side determine a triangle, the student may figure the distance to an inaccessible object.

Whether a positive transfer shall be shown in the study of geometry depends on a number of things. Chief among these is that the student must see the point in question of transfer in a generalized way.

Whatever knowledge we use, except that which is used in identical way in which it was learned, must be used as a result of a transfer. Inability to transfer leads to formality and to an impractical experience. Ability to transfer in many and varied situations leads to originality, and to a free use of knowledge in many ways. This helps to meet new problems or peculiar situations. It behooves us then to get this generalized idea of the principle when we are learning geometry.

Geometry may further help the student in acquiring neatness in written work, perception of space, and space relationships and proper logical thinking and sequence of topics; proper methods of study, ability to present to others non-memorized material of a difficult nature; as geometry may, aid in more specific ways. This study of geometry should aid very materially in free hand drawing, mechanical drawing, physics, astronomy, art, architecture, landscape gardening, building trades, etc. The student may be helped in any, all, or in none of the ways just mentioned, depending upon the student and also upon the method used by the teacher in his class.

A. GENERAL TRAINING GIVEN IN GEOMETRY

1. A Training in Neatness

The training given the student in the geometry class in requiring him to write a neat well arranged paper may be transferred as a general habit, so that the student will always produce neat papers, in his other school work, or elsewhere. Whether the student does this or only acquires the habit of writing neat geometry papers, depends on whether he gets the generalized idea of neatness which may be carried over to other subjects.

The history of an experiment by Carrie Squire and a repetition of the same experiment with modifications by Ruediger, illustrates and proves this point. In the first experiment students were required to produce neat papers in arithmetic, but no intimation was given concerning the general idea of neatness which might be used in other classes. Little or no improvement was noticed in English and spelling papers, although the arithmetic papers were greatly improved. In the experiment as repeated by Ruediger, the general practice of neatness in daily life was stressed to the class. Improvement of neatness was observed in the class, where the experiment was tried, and nearly as much improvement was noticed in other classes, where nothing was mentioned about neatness.[1]

It does not require much argument to show that geometry furnishes an opportunity for training of neatness. As most every teacher and geometry text stress neatly written proofs. But most of this training may be confined to the study itself, unless the students are shown, or see for themselves that the same method of procedure could be used in other lines of work.

[1] Starch Educational Psychology, 1927, page 261.

2. Logical Thihking

Geometry exemplifies the ideas of logical thinking better than any other subject taught in the high school. Why should not the student then be a better reasoner and more logical thinker because he has studied geometry? In reference to reasoning habits, students of geometry may be placed in three classes. In the first class belong those who do not learn to reason even in geometrical concepts. In the second class are those who can reason well concerning geometrical theorems and originals, but who seemingly are unable to use the ideas of logical order learned in geometry outside the geometry text. In the third class are those who learn to apply the methods used in the geometry class in other lines of thinking.

It would be well to require every one to take geometry if every student could secure this valuable method which he could apply in other lihes as well. A larger number of students will be able to receive this ability of transfer if they can be taught to use a better method of study. If in conjunction with the regular logical proofs given in the text; logical reasoning, orderly arrangements and proper sequence of thoughts are applied to topics and subjects other than geometric the student will see the possible use of the reasoning method in other lines.

3. Judgment.

A judgment is formed as the result of comparing two simple apprehensions. A large part of the work in geometry is devoted to comparing the parts of a figure, or one figure with another. Reasoning consists in comparing judgment with judgment until a desired conclusion has been reached. The student is checked in his tendency to form hasty, or untrue, statements in geometry because after each statement he must also give a valid reason for its truthfulness. We can judge that a theorem has been proved correct, if:

1. No assumptions have been made;

2. Each statement follows from the preceding statement;

3. Proper sequence has been used;

4. Only true statements have been made;

5. All arguments are based on known theorems.

The student has made some progress if he has learned not to form hasty, untrue judgments when reasoning with geometrical ideas. His progress, however, has been much greater and of more lasting benefit if he learns, because of his study of geometry, how to use the same care and accuracy in forming judgments in all his thinking as well as that which pertains to geometry primarily.

4. Regular attendance

If a student in the geometry class observes that in being absent he has missed the logical sequence, and that his absence from the geometry class makes a difference in his work in that class, and hence absence from other classes must also be avoided, he has learned a valuable lesson as the result of a transfer from geometry.

5. Hard Study

It requires hard study in order to master geometry. But this study is not without its rewards. After the student has spent faithful effort to obtain the solution of a difficult problem, he obtains great joy when he has found the solution. The thrill of difficulties surmounted acts as a stimulus to attack other problems. The student will learn a valuable lesson when he realizes that hard study accomplishes results in his study of geometry, and that since he can master for himself the difficult problems in geometry, he can also master difficulties wherever they are to be met.

6. Argumentation

Training in giving oral proof before a class - not memorized, but logical proof - should result in added ability in argumentation. and it can do so, if the student has seen the possibility of using the type of argument encountered in geometry to argument in general.

B. SPECIFIC ABILITIES

The training received in geometry may transfer in the case of a number of specific abilities. In the subjects listed below, geometry may serve a two-fold purpose, or it may not, according to the student. In the first place, the training received from studying geometry may aid the student in drawing and perceiving a different type of figure from the regular geometrical figures, or in observing the principles of geometry used in a new way. In a second place, these subjects may be easier to the student of geometry because of the actual use of material they contain which is identical with that used in the geometry.

1. ART

Both art and geometry are concerned with the perception of figures. Geometry with its consideration of regular figures lays a foundation for the study of art. Such sections as symmetry, proportion, the construction of figures, the relation of figure to figure, should furnish aid to art students.

2. MECHANICAL DRAWING

This subject also furnishes a number of direct uses of geometry, and also a number of implied uses. The student who has had a good course in observational and constructional geometry should be able to progress nicely in mechanical drawing.

3. PHYSICS

Applications of geometry may be observed in the study of parallelogram of forces, simple and complex machines, the reflection of light, the path of a ray of light through a lens, and in the understanding of any of the figures and diagrams given. The geometrical forms, figures, and a few of the elementary principles of geometry, are used as an alphabet in presenting to us the physical sciences.

4. ASTRONOMY

In order to present to us the relative positions of the planets, their size, distances, orbits, surface markings, and other facts which we cannot see with the unaided eye, the principles of geometry are used. It would be difficult to convey to the human mind a knowledge of these things without their presentation to us in the geometrical form.

5. ARCHITECTURE

The pleasing forms of various kinds of architecture rest upon the proper use of geometrical principles. The building is substantial and adapted to the use it was built for, because of certain physical and geometrical principles used in the structure. The form is pleasing to us because of the proper use of certain principles of geometry; such as proper ratio of its parts, the use of pleasing symmetry and the best use of unity, exphasis, etc.

6. CARPENTRY

The carpenter who understands geometry should be able to use it in a number of ways in his work. When confronted with a new situation, he should be better able to solve his problem because of his study of geometry which has made him free from the need of following the blind rules for special cases.

7. MISCELLANEOUS

Landscape gardening, navigation, surveying and designing, and a number of other subjects, are based on the principles of geometry.

C. TRAINING IN SPACE PERCEPTION

One important thing often overlooked in studying geometry is the study of spacial relationships, space perception, form, etc. This is often true is the main emphasis is entirely placed on the comparison of theoretical figures of the text. Judd makes this point very clear in his book on Psychology of Secondary Education, 1915, page 60

"Geometricians often neglect space and emphasize logic. The results are that in many cases, the student is trained in the observation of lines and figures, but chiefly in the methods of making logical comparisons. It is possible to find students of geometry quite unobserving of form. This situation will be understood wherever it is found, that the primary emphasis is on Axioms."

Without neglecting the logical form, the pupil may be taught to appreciate numerous applications of the geometry in lines, figures, ratio of shapes, etc. The methods suggested to be used in this training are given in much detail in the main body of this thesis, to which the reader is referred, and will be only very briefly mentioned here in outline form:

1. How distances to inaccessible objects may be found;
2. How the size of the earth is determined;
3. Our position in the solar system, and in the universe;
4. Good proportions in a rectangle;
5. The proper placing of picture in the frame;
6. Unity: the relation of like curves;
7. How lines of trimming pleats, etc. on the dress effect the appearance;

8. The law of the margin;

9. Proper divisions for a vertical mass;

10. Proper divisions for a horizontal mass;

11. Geometry of space in balance, proportion, harmony, emphasis, etc.

12. How to properly place a picture on a wall;

13. Numerous loci problems given in the geometry text;

14. Comparisons of the generation of lines, planes, and solids.

CHAPTER IV

METHODS FOR STUDYING AND TEACHING GEOMETRY

The business of the teacher is not merely to see that the student learns the lesson so the student can reproduce it. He should be more anxious to know, what methods the student is using to prepare his work. Teaching does not consist in hearing of lessons, for directing his mental efforts, in the special field under consideration.

In all classes more time should be taken for the discussion of proper methods of studying than is usually given. The method used in learning a fact is often of more value than the knowledge of the fact. The student needs to be directed, as well as to have opportunity for self expression. If directed properly, students will more readily learn to get along without the immediate aid of the teacher. This should be the aim in all teaching.

The fact that the student on entering the geometry class finds it treats of strange methods and devices, and the further fact, that the student may find it easier for the present to memorize the proof instead of doing logical thinking, furnish the teacher of geometry added reasons for guiding the student in correct methods of studying geometry.

Thousands of students have read over the proofs as given in the geometry text, and have reproduced these proofs, in a formal way, without really understanding why certain steps were taken, and consequently were prepared to know how to attack an original in the proper way.

The three ways by which American workers learn their trade, illustrates very aptly the methods students use in the study of geometry. Statistics reveal the fact that five per cent. of America's 70,000,000 workers learn their trade by the apprenticeship method, and five per cent. by the vocational school method, while the remaining ninety per cent. learn their trade by the "pick-up method." The pick-up method spoils dozens to every one brought to vocational efficiency.

(1) The vocational school method may be compared to the geometry class in which proper methods of study are considered for each student; and consideration and discussion of these methods are given in the class, so that every one has opportunity to know and practice the best methods.

(2) The apprenticeship method of teaching is like that method used in a class in which some consideration is given to methods of studying geometry, but the methods to be used are not discussed in as general or as thorough manner as in the first class mentioned. These discussions are given as occasions arise as shown by the need, or mistakes of the student.

(3) The pick-up method must be used by the student in obtaining the proper method for studying geometry, if the teacher places all his efforts on the student's reproduction of the theorem, without thought on how the student prepares the lesson. If the student does not use the best methods, he may become discouraged and fall behind in his work before he finds a satisfactory method.

TYPES OF STUDY

Attention is called to the following types of studying as listed in a number of books on psychology:

1. The Reading Type

This type of studying is used, when the material is so simple that it can be easily assimilated by a hasty reading. The mere reading should not complete the lesson. Evaluation of topics, organization of the material, comparison to personal experience, and applications should also be used, etc., in completing the study.

2. Memory Type

Sometimes in studying we need to memorize the material verbatim. Very little of this type of studying is needed in geometry.

3. Laboratory Type

During modern times, learning by doing has to a large extent replaced the method of memorizing used during the middle ages. The use of the laboratory method in geometry is advocated in this thesis.

4. Reasoning Type

One of the main reasons for studying geometry is to teach pupils to think and to reason properly and not simply to learn a few geometrical facts, as most of the work in geometry should consist of the laboratory and reasoning types of studying.

METHODS RECOMMENDED FOR THE TEACHING OF GEOMETRY

1. Give a gradual introduction to the methods of geometry by using a number of simple exercises before starting on book one

of the geometry. Two or three weeks may be profitably used in work of this kind, together with a consideration of methods of study and assigned reading.

2. Devote some time to general study methods as discussed in such books as McMurry's *How to Study* and Book's *Learning How to Study and Work Effectively*. Study methods especially applicable to geometry should be stressed. Much material from a ~~good logic text book will furnish help on types of reasoning,~~ kinds of judgment, etc. A consideration of these will help in making applications to every-day life.

3. Make use of the laboratory method as an introductory exercise to acquaint the student with the principles involved in the theorem and an understanding of the figure.

4. The student should anticipate the author, in a study of the theorems given in the text. By thinking ahead, just what steps will be necessary to finish the proof, as well as observing the method used, and the reason for their use in each step of the proof, the student will later be able to think out methods of proof when attacking an original.

5. The best training which the student can receive in order to prepare him for the solution of the originals, as well as for a better understanding of the theorems of the text, is the training given in presenting oral proofs by analysis. This helps connect the proved theorems with the problems to be solved. If the student can learn the general method for attacking the originals, their solution will be a greater pleasure than learning the theorems which are proved in the text. In giving a proof by analysis, he learns this method of attack, since the plan of

proof must be given with the proof. In proving theorems by synthesis, we begin with simple parts (points, lines, angles, or surfaces) and combine these into a more complex form. Houses are made by a process of systhesis - one piece of lumber added to another until the final house is completed. By analogy to a house in the process of analysis we would start with the house already built, and take it apart to learn the method used in its construction. Originals and theorems are like houses already built - they must be analyzed to find the method of proof.

The method used by the discoverer is that of analysis. We must first analyze an original to learn how it can be proved, before we can write the proof by synthesis. Because the student does not know how to give a proof by analysis, makes him weak in devising proofs. A sample of proof by analysis is given on page 31, to indicate the method. Observe what is the minor premise in one line becomes the major premise in the next.

6. Adjust and adopt material to the individual student.

7. Reference to theorems by number or page is useless as well as all endeavors to memorize the theorems without an understanding of their meaning.

8. Review the theorems daily by using a card to cover every thing but the theorem. After this has been read, draw the figure including all auxiliary lines needed in the proof. Now slip the card down exposing only the figure, and make comparisons with your drawing. The proof can be written out and compared in the same manner. Theorems may also be reviewed this way orally.

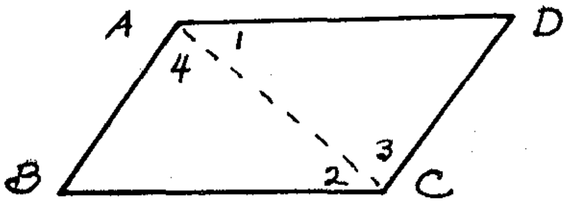
9. Two students may work at the board, one writing the steps of proof, and the other the reasons.

10. If a number of students are working at the board on different review problems and theorems, at a given signal each student may be asked to move over one place to the right, and continue the proof of the other student; some one who was seated can take the place of the first student at the left.

11. Contests may be held in the class. This can be conducted by choosing sides, and having one side challenge the other as to theorems, definitions, originals; or one side may give a statement in the proof and the other side the reason.

PROOF OF A THEOREM BY ANALYSIS DISCOVERY METHOD

If the opposite sides of a quadrilateral are equal, the figure is a parallelogram.



Given A D = B C

and A B = D C

To prove A B C D a ▱

Draw diagonal A C

Major premise	Minor premise	Reason
A B C D is a ▱	if A D // B C and A B // D C	opp.sides //
A D // B C and A B // D C	if ∠ 1 = ∠ 2 if ∠ 3 = ∠ 4	alt.int. ∠s " " "
∠ 1 = ∠ 2 and ∠ 3 = ∠ 4	if ∠ ACD = A B C	corresp. parts
∠ A C D = A B C	if 3 sides are =	

Now

A D = B C		given
A B = D C		"
A C = A C		construction

Since all the conditions for A B C D being a ▱ have been proved the theorem stands Q. E. D.

Proofs should be discovered by this method, but afterward they may be recorded by the synthetic method.

STUDENT'S STUDY-CARD OR SUMMARY OF STUDY HELPS

1. Provide the necessary working tools: paper, pencils, set of drawing instruments and triangles.

2. Learn the proper use of the text, including the proper use of the index and tables, etc.

3. Keep a small assignment note-book.

4. Observe applications of geometry in your excursions or on the way to and from school (record in note-book).

5. Learn the proper use of time. Don't just put in time as you receive benefit from the results and indirectly proportionally to the time. Economize by greater effort and less distractions.

6. Consider the method used in proving the theorems.

7. In learning a proof, list the methods or steps, but do not memorize the wording of the theorem.

8. Use the card system as previously mentioned in reviewing

9. Prove theorems anatylically as well as synthetically.

10. Assume personal responsibility for mastering the course Do not leave this to the teacher.

11. Get the figure clearly in mind.

12. Find out what is wanted:

(a) Recall the various ways you have had for proving the idea wanted;

(b) Choose the best way for doing this for your problem

(c) Review in your mind the general summary for proving lines and angles equal, lines parallel, triangles equal, etc.,etc.

13. Grapple with the problem until a solution is found.

14. Adopt a problem solving attitude.

THE KNOWLEDGE STUDENTS SHOULD OBTAIN IN GEOMETRY

Students who have been in the same class will differ in their knowledge of geometry. And they will differ to a still greater extent in their ability to use this knowledge in new situations. This is to be expected, as students differ in ability, and working spirit; but each student should be allowed some latitude in his chosen line of development, and should be given as much geometry as he is capable.

The best students may do only medium work in one of the lines indicated below, and the poorest student may excel in a few of these; but in general a student who has completed geometry should possess the following:

1. A knowledge of the fundamental geometrical facts and how to prove them;

2. Ability to make neatly and accurately all needed constructions;

3. The ability to work originals unaided;

4. A knowledge of the uses and application of geometry;

5. The habit of accuracy and precision of statement of a problem dealing in geometrical material, or otherwise;

6. The habit of logical organization of ideas;

7. A clear discrimination between inductive and deductive reasoning, and how to use either in a line of argument;

8. The habit of observing the beautiful and correct use of geometrical lines, proportions, etc. in art, architecture, and

nature, etc;

9. Sufficient geometry for advanced work in the lines to be pursued by student;

10. The ability to solve the geometrical problems likely to occur in every-day life.

DESCRIPTION OF THE EXPERIMENT

My first teaching was done in the department of mathematics. Later in teaching physics, chemistry, and astronomy, and a number of other laboratory sciences, I learned to appreciate the value of the laboratory method of instruction, as a means for giving to each student present experiences, and a ready means for connecting the text book work to that of the practical.

This experience of science teaching has largely influenced me in devising and writing this laboratory manual to accompany a geometry text. In a similar way that machines, and apparatus for the study of physics can be collected in an appropriate room, so geometrical materials may be brought together for experimentation, and observation. This laboratory manual is modelled along lines similar to those which so long have proved their success in physics, with the addition of suggestions for observational work.

A few sample pages of student work are added here in order to show the character of the student's work. The exercises furnished something which all could do. The more capable students were given a large number, and more difficult exercises; and a better character of work was required of them.

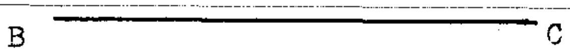
The work of a number of students began to show signs of improvement after the instruction of the laboratory work. Since these students began to understand the geometry through the exercises given, they have expressed appreciation of the methods of experimentation, and observation. Each pupil has a right to as much individual instruction as we can give him. The laboratory plan, and especially the observational work, can be easily adapted to the individual needs and interests of the students.

1. Statement of Problem:
   a. Given; Two sides and an angle opposite one of the sides.
   b. To construct: A triangle using what is given.

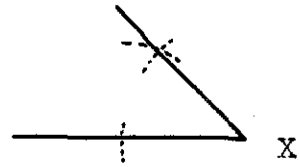
2. Figure:
   a. Parts given: AB = 2 in., BC = $2\frac{1}{2}$ in., Angle X.

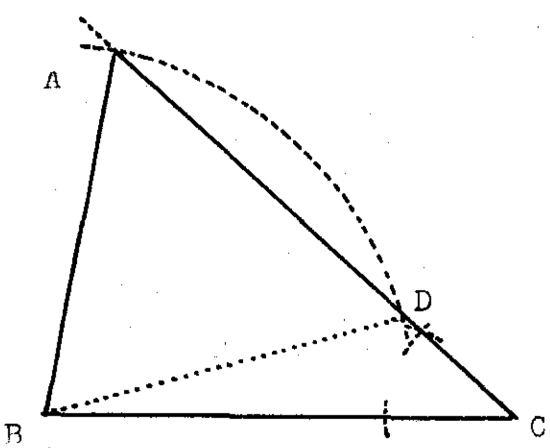
   b. Construction of figure.

~~3. Method:~~
   a. Make a base of BC.



   b. Using BC as a leg and C as center, construct an angle equal to angle X.





   c. Using AB as radius, draw an arc, intercepting AC at A.

   d. Draw AB.

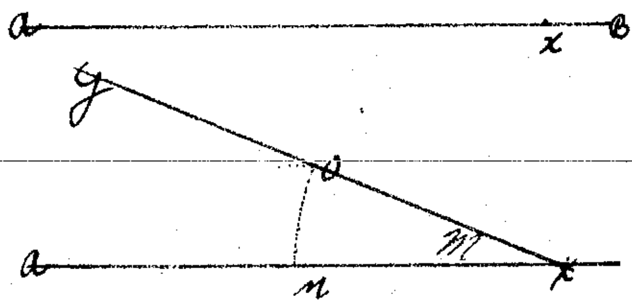
4. Proof:

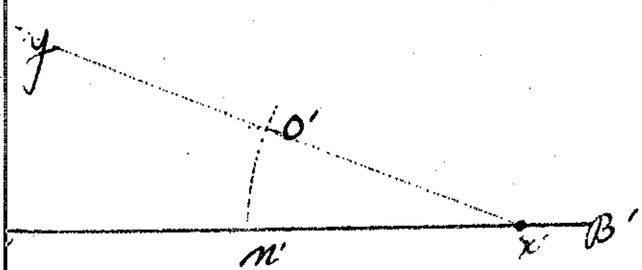
a.	Figure ABC is a triangle.	a.	Three sides const.
b.	AB and BC and Angle C are equal to AB, BC, and angle X.	b.	By const.
c.	Therefore, triangle ABC is const. with two given sides and an opposite angle.	c.	Since it is const. with given sides and opposite angle.

5. Discussion:
   a. Since the arc made using AB as radius will intercept AC at D also, a second triangle, triangle DBC, may be drawn with 2 given sides and opposite angle.

Gordon Jones.

tement of problem:
Given: a point on a line and an ∠.
To const. an ∠ at a given point in a line = to a given ∠
gure
Parts given: ab; point x; ∠m
onstruction of figure.
ethod:
Draw a'b'
Place point x on ab
Using x as the center
trike an arch on ∠m
d an equal one on
b'.
Using n as the center on
ne and n' as the center
n the other strike equal
rcs intersecting O on no.
Draw yx through O and
. thus finishing the ∠.



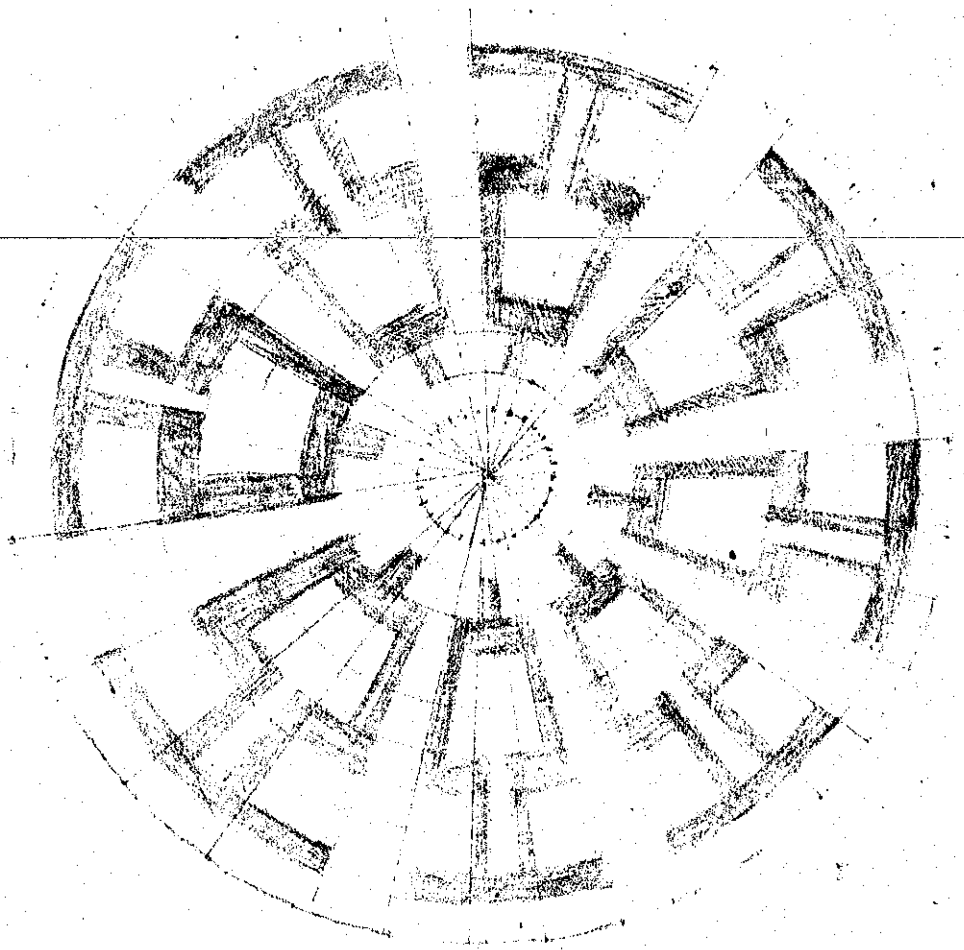


Proof

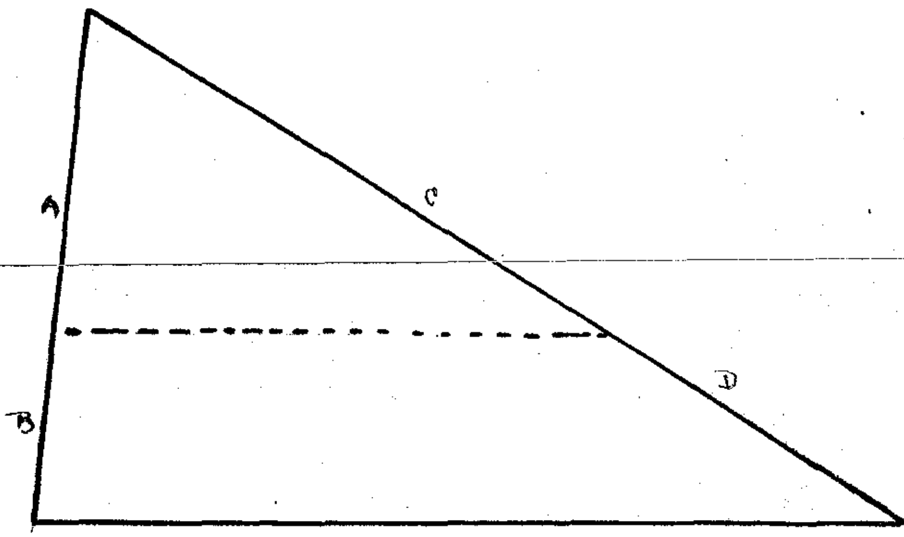
1. Draw straight lines no and no'	1. Possible
2. xn = x'n'	2. construction
3. xo = x'o'	3. construction
4. no = n'o'	4. construction
5. △ nxo ≅ △ n'o'x'	5. s.s.s = s.s.s.
6. ∠oxn = ∠o'x'n'	6. corro. ∠s of ≅ △s.

Franklyn Albright

Design



Testing Theorem I



A = 1.52 C = 2.93

B = .91 D = 1.75

$$\frac{A}{B} = \frac{C}{D}$$

$$\frac{1.52}{.91} = \frac{2.93}{1.75}$$

2.93	1.52
.91	1.73
293	456
2637	1064
	152
2.6663	2.6296

2.6663 = 2.6296

approxiniently equal

Lawrence Webdell

LABORATORY GUIDE TO

EXPERIMENTAL

AND

OBSERVATIONAL GEOMETRY

or

LABORATORY AND

FIELD GEOMETRY

RELATION OF GEOMETRY TO OTHER BRANCHES OF MATHEMATICS
CHAPTER V

ARITHMETIC

The student knows from his previous mathematical studies that arithmetic treats of positive numbers above zero, and with computations arising out of the use of these numbers.

ALGEBRA

In the study of algebra, the student is taught the use of negative as well as positive numbers, and the use of letters to represent certain values. As has been said, algebra is the science of numbers generalized. The fundamental processes of addition, subtraction, multiplication, division, factoring, fractions, etc.,are treated in this more general aspect; that is, we go over about the same ground during the first part of the year as in arithmetic, only using positive and negative values connected with the letters of the alphabet, with an extended use of the equation. The use of the equation with the orderly methods used in solving the problems will assist the student in solving his geometrical problems.

GEOMETRY

In arithmetic and algebra, number concepts are extensively used, while in geometry the form concept is mainly stressed. The student of algebra or arithmetic has used a number of theorems introduced from geometry; especially is this true in the study of mensuration in arithmetic. Also in geometry number values and principles learned in arithmetic and algebra are in constant use in the discussion of lines, planes, and solids.

Geometry is a prerequisite for much of the more advanced mathematics which follow, especially in such subjects as analytical geometry, mechanical drawing, trigonometry and calculus. It is hoped that the algebra student will be free to use the principles of algebra in his study of geometry.

The student of algebra and arithmetic has obtained some information concerning geometry when he studied in these subjects concerning angles, squares, rectangles, areas of figures, volume of solids. Geometry is the science which treats of lines, surfaces, solids, and their relationship; and as every one has observed the use of geometrical figures in the form of architectural construction in nature, in furniture and art, and has obtained some information of the subject in his previous study of elementary mathematics, it is hoped that by proper method of study, experiment and observation, he may make rapid and enjoyable progress through his study of geometry. Since geometry has an objective basis for its subject-matter, which to a certain degree has come more or less into the experience of every child, a number of teachers think it would be better to give geometry before algebra. It is believed that the laboratory and observational methods of approach will help the student to connect his previous knowledge of geometrical forms with his study in the class-room.

PLAN OF COURSE

During the middle ages most studying was of the memory type. With the advent of vocational training and laboratory work into the schools, the reading-memory method of studying has to a certain extent been replaced by the method of the learning by doing. But in a number of subjects, including geometry, students still are using methods of memory without much of the practical experiments and observations which might be given with the text.

This course is planned to furnish the student something to do, also to guide him in his methods of reasoning and observing; so that he may have real mental growth. The aim of education should be to furnish opportunity for growth. Mental growth by individual effort is what gives power. Studying through a theorem by analysis in an understanding way is worth so much more than a laborious effort to memorize proofs without a clear understanding. In one method he is not compelled to understand why certain steps were taken - the student follows the statements of the author. In the other he learns originality because he anticipates the author and thus acquires skill and power, because he is led to do his own thinking.

Geometry may be presented to the class in a variety of ways. A teacher may use only the formal theoretical proofs, with a number of originals, and give no special instructions concerning the construction of the figures, and use only the synthetical proof as given in the text. This is the usual method of presentation using oral and written proofs and examinations. To this may be added the proof by analysis, thus causing the student to understand, and not merely memorize the proof given in the text. The plan of this course is to use, in conjuction with the methods just mentioned, experimental work, and observations of geometrical forms, and a study of the applications of geometry.

These methods are given in outline form below:

A. THEORETICAL PROOF

(a) Synthetical

This is the kind of proof usually given in the text. The temptation may come to the student, especially if he lacks time to memorize the proof first, and understand its meaning later. This furnishes a great objection to using this method of teaching altogether. The use of other methods with the synthetical method will largely remove the objection.

(b) Analytical

Most oral proof should be of this kind as it is necessary for the student to devise his own proof, and he must give the plan of the proof as he gives the proof.

B. MENSURATIONAL

Arithmetical and algebraic calculations concerning geometrical figures. As this is treated usually to sufficient extent in arithmetic and geometry texts, not much space will be devoted to it here.

C. PRACTICAL

Students are encouraged to find all manner of applications of geometry in the arts, in nature, and in subjects in the school courses.

D. LABORATORY

(a) Experimental

No teacher would think of teaching the sciences without appropriate laboratory exercises. As the sciences are made more interesting and real by laboratory work, so geometry likewise can be made more interesting and to come into closer contact with the student, by presenting experiments in the laboratory dealing with geometrical materials. It is suggested in this manual to introduce most theorems by first using the laboratory method of approach, so as to give the student intimacy with the form of the figures, and of the principles involved in the theorems.

(b) Constructional

1. Drawing of Geometrical Figures. A correlation between geometry and mechanical drawing may be made, if neat and accurate figures are required to be made with appropriate instruments.

2. Material Constructions. A number of figures and geometrical constructions can be made by paper folding, moulding with clay, formation of figures with wire by using string and sticks, or by cutting of forms from wood or potatoes, etc.

E. OBSERVATIONAL GEOMETRY OR FIELD WORK

The student can learn much outside the text book if he is directed in the proper field work. Every flower, bud, leaf, tree, and building, has its geometrical forms which show applications of geometrical principles. In studying circles, students can observe arches in windows; and when studying proportion, he should learn to see proper proportions in furniture construction, in arranging furniture in room, etc.

Observational work assists the student to become more original, and helps him to connect his past knowledge of geometrical forms with the scientific treatment given in the text. The student who has thus observed the principles outside the text, cannot help but get a broader training and better appreciation of the subject.

The plan, of the course, then will be to analyze the theorem, develop the experiments which will make principles clearer, and observe these principles outside text. In undertaking this plan it is well to have a well equipped laboratory, in connection with a number of books where students may be given opportunity to make individual ingestigations and readings.

DIRECTIONS TO STUDENTS IN USING THE MANUAL

GENERAL

The student should get the distinction clearly in mind as to what constitutes a general proof for a theorem. The proofs by analysis and synthesis and by reductio ad absurdum, constitute a general method of proof, which for a general proof is superior to experimentation and observation in the following respects:

1. By the theoretical proof a number of things can be shown to be true which are not subject to direct measurement or observation.

2. Direct measurement is not always possible.

3. In these cases where measurement is possible, the measurements made cannot be absolutely exact, hence conclusions based upon measurements are subject to a possible error. Argumentational proofs are absolute and general.

4. The argument used in the theoretical proofs applies to all figures in which the given condition exists, whereas testing by measurement applies only to those cases actually measured or observed.

5. Often mistakes may occur in observation work, as is shown by a consideration of geometric illusions.

Nowhere in this laboratory guide is the student asked to accept mere measurement in place of theoretical proof, but only to exemplify and help him to understand what has been proved in the text. Experimentation nevertheless has its place and use.

Thoroughly understanding the limitations of experimentation and observations, the student should next consider the benefits and value attached to these methods.

BENEFITS

While the experimenting is not given to supplant the theoretical proofs, it has a great value in enabling the student to get first hand present information concerning the figures under discussion and of the proof under consideration. For illustration, take the theorem, "Two triangles are congruent if three sides of one are equal respectively to three sides of the other", which may be stated, three sides determine a triangle. If the student has first attempted to construct as many different triangles as possible from three given sides and finds that all the triangles are the same; and has made a triangle using sticks each respectively equal to the given length of sides or a multiple of these lengths, and then compares the stability of this combination with four sticks fastened together so as to form a quadrilateral, he will better be prepared to understand and appreciate the proof given in the text. Later in his observation he will notice in construction work all quadrilaterals are broken up into triangles by a diagonal so as to make the combination stable.

These experiments add an interest and give reality to the theorem, and help very much to connect the student's learning of geometry with practical every-day experience.

SPECIFIC DIRECTIONS TO THE STUDENT

FOR OBSERVING

Use your best effort to see a number of applications of the principles of geometry every day. Keep a list of your observations, stating:

(a) Date of the observation,

(b) What you observed,

(c) Where you observed it, and

(d) What principle in geometry was illustrated.

FOR MAKING CONSTRUCTION WORK

1. Make all drawings according to directions when such are given. When you must discover the method, use suggestions under "how to study."

2. Be neat and accurate and through.

3. Do not copy, be the originator.

4. Constructions marked thus (*) are to be constructed properly, and a record of proof made in a note-book according to the following outline:

(a) Preliminary

1. Statement of problem to be constructed;

2. What parts are given;

(a) Name these by appropriate lettering;

(b) Make drawing of these parts.

3. What is to be constructed.

(b) Construction

4. The figure showing construction lines and method of work.

5. Method of construction described. To be stated in phrase form (unit lines) with reasons.

(c) Proof

6. Proof should be given to show that the method used produces the required construction.

(d) Discussion

EXERCISES WHICH INVOLVE THE CONTENT AND DEFINITION OF GEOMETRY

After reading the definitions of geometry as given in a standard dictionary, and several geometry texts, answer the following, as a preparation for class discussion, so as to get a clear idea of the definition and content of a course in geometry:

1. What geometric plane figures do you find in the class room? What figures do you find elsewhere?

2. What geometrical solids do you observe in the class room? In the home?

3. Make a list of some of the figures you studied in arithmetic. What facts concerning them to you remember? If you fail to recall, get an arithmetic and see what is included under mensuration, etc.

4. What solids did your arithmetic treat? Make a list of the properties of the plan and solid figures studied.

5. Name some smooth serfaces which are not plane.

6. Make a list of the things you hope to learn in studying geometry.

7. In what lines of employment are geometrical principles used?

8. What advantages can you get from studying geometry besides those which are immediate practical?

ILLUSTRATIONS OF THE STRAIGHT LINE

EXPERIMENTS

Carefully fold a piece of paper. Then crease it by pressing a ruler firmly along the fold. This crease is a good illustration of a straight line (it may be used as a ruler or straight edge).

Tie a heavy weight to a fine thread, and let the weight be suspended by the thread. The position the thread takes is a straight line. If the thread has no thickness, it would be a straight line.

OBSERVATIONS

We cannot see or draw a straight line, but we can name a number of things which have the property of unchanging direction which is one of the chief characteristics of a straight line. What we draw to represent a straight line is in reality a solid, but as this may represent for us the property of unchanging direction, we take these drawings to display the position and direction and the extent of a straight line.

1. Name six other illustrations of a straight line as shown by objects in class room or elsewhere.

2. Which one do you consider the best illustration of a straight line?

3. How does a carpenter secure a long straight line (one too long to use a square or ruler conveniently)? Demonstrate this method by using a string and chalk on the board.

4. A point moving in one direction generates a straight line.

5. Can you name cases where straight lines are generated?

6. How does a plasterer make a plane surface in plastering a room?

7. Show how the definition of a plane is illustrated by the method used in making a sidewalk.

EXPERIMENT.

1. How many straight lines can you draw through a point if:

(a) All the lines are in the same plane?

(b) All the lines not in the same plane?

How many lines can you draw in each case? Endeavor to visualize the appearance in each case. Make a sketch to show the different between (a) and (b) above.

2. Mark two points on your paper, A. and B. How many straight lines can you draw through A. and B? How many lines can you draw if they need not be straight? Make a general statement of this truth. What is the shortest line connecting two points on a flat surface? What is the shortest line connecting two points on a curved surface? Like the earth's surface? Why did Lindbergh take the course he did in going to France? This question is answered more clearly in solid geometry; or you may see by observing position of places by looking on a globe.

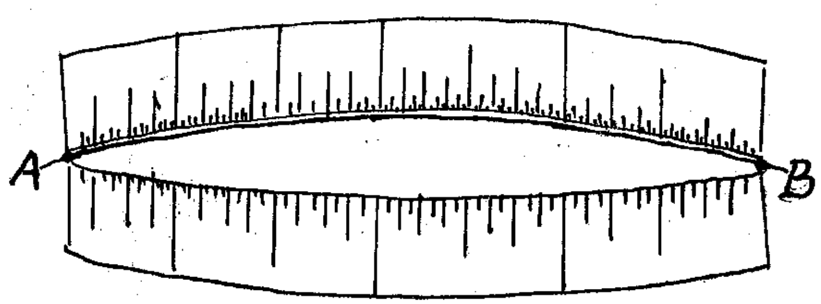
3. Tests for a straight edge, which illustrates a characteristic of a straight line.

Observing the definition for a straight line, test your ruler to see if it is a straight edge.

Test for a straight edge:

(a) Sight along its edge. Does the edge look like a point? If not, it is not straight. A line viewed from the end appears to be a point.

(b) Place the ruler on the paper and draw a line along its edge. Then mark the ends of the line A and B, as in the drawing below. Now change the ends with the ruler, and draw another line through the same points A and B. Do you have two lines through A and B, or only one? Do you see how this is using the definition for a straight line to test the ruler?



EXERCISES

1. See how many straight lines you can draw through a point.

2. See how many straight lines you can draw through two points.

3. How many curved or broken lines can you draw through two points?

4. Can you draw a straight line on a sphere (globe)?

5. Draw a curved line on a plane surface.

6. Can you draw a curved line on a sphere?

7. Can you draw a straight line on a cylinder?

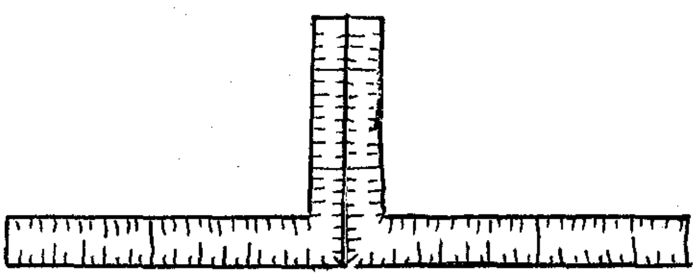
8. Mark two points on a globe. Draw a line which appears to you to be the shortest distance connecting them. Is this a straight line? See definitions. How is the shortest distance between two points on a sphere found?

9. Can ships sail in a straight line?

10. How would a ship sail in order to go the shortest distance between two ports if the two ports are in the same latitude? Would the captain sail on a parallel of latitude?

ILLUSTRATING DEFINITIONS OR THEIR APPLICATION

1. How would you test a carpenter's square? (see drawing) State definition which harmonizes with this test.



Testing a carpenters square

2. Take a small piece of paper folded upon itself, double it so that the bottom half exactly coincides with the top half, then crease. Mark by means of letters or figures the two angles the crease makes with the side of the paper. See that these angles are equal since they coincide. Now unfold the crease. The crease is perpendicular to the edge of paper. State corresponding definition.

INSTRUMENTS USED IN GEOMETRICAL CONSTRUCTION AND METHODS

INSTRUMENTS USED

Geometry text books usually recommend for the construction of geometric figures the use of the straight edge and the compasses. Plato was the first to limit the construction of the figures to these two instruments. This has no practical value, and wholly abiding by this custom now is only of imaginary interest.

In this laboratory course a number of other instruments are also required, such as, protractor, T-square, the triangles, dividers, ruling pen, etc. The student thus will not only know how to work with ruler and compasses alone, but will also become acquainted with a number of instruments which will greatly aid him in constructing certain figures with greater facility, and will give him a better preparation for mechanical drawing. The draftsman and other workers have and use a variety of instruments in their geometric constructive work.

In a course in mechanical drawing, a subject which uses principles of geometry in making the various drawings, a number of instruments are required.

USE OF INSTRUMENTS

A. USE OF RULER

To draw a straight line through any point by using a ruler, observe carefully the following points:

(a) Pencil should be sharpened to a chisel edge;

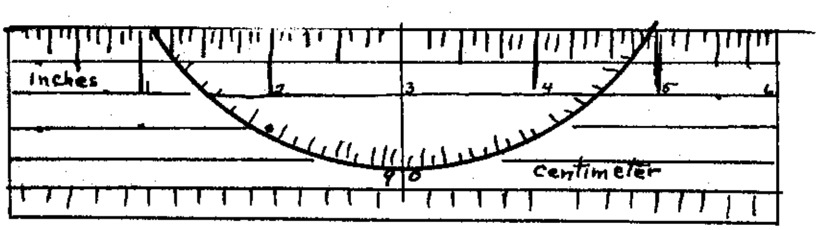
(b) Care should be taken that line passes through the point. This can be done best by observing what is called the drafts-man's rule;

(c) Drafts-man's Rule: Place the pencil on the point through which the line is to be drawn. Move the ruler up into contact with the pencil, then turn the ruler in the direction the line is to be drawn, then

(d) Holding pencil at the same angle, glide it uniformly along the edge of the ruler;

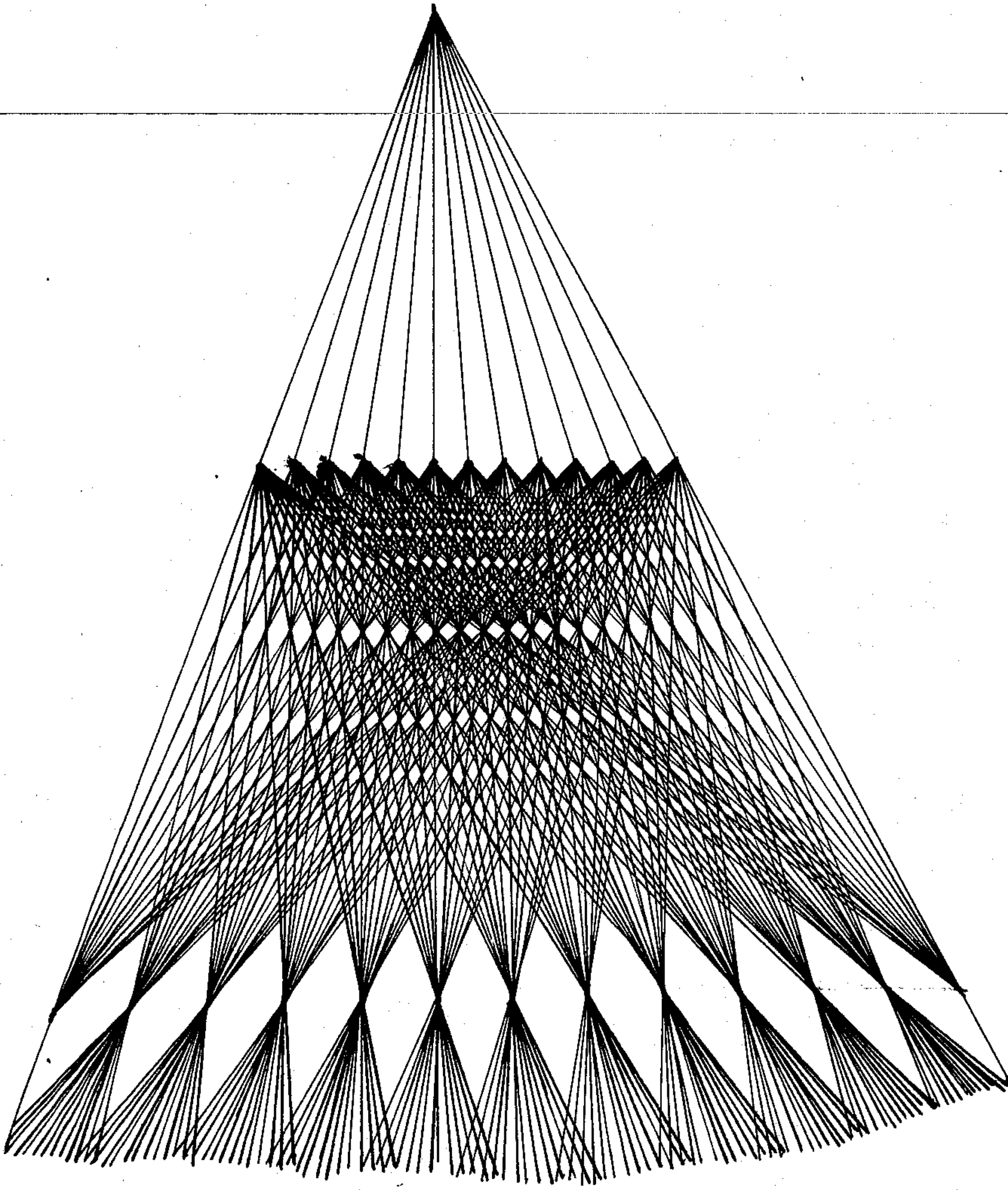
(e) As a test of your accuracy in drawing lines, construct the figure shown on the following page. Be neat and exact, and the result will show in your figure. If you are not accurate, the intersection of the lines will not be at the required points.

(f) A ruler and protractor like the one shown below is very convenient.



Ruler

RULING EXERCISE

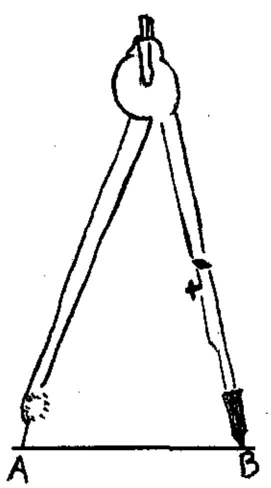


B. USE OF THE COMPASSES

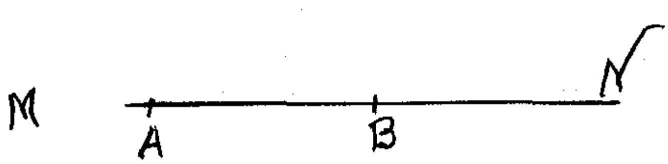
The compasses (or dividers) are used to construct circles, circular arcs, or to lay off a given segment on a line.

(a) To draw circular arcs, open the dividers to an amount equal to the length of the required radius. Place the steel point where the center of the circle (of which the arc is a part) is to be located; then strike an arc by holding the compasses near the top, and use a gliding motion to complete the curve. The student already knows how to measure a given line in inches and make a second line equal to this measured length. In formal geometry a more exact method is used as follows:

(b) To lay off a given segment on a line, open the dividers in an amount equal to the length of the required segment, as A.B.,one point of the instrument resting on each end of the segment A.B. Then using this length as a radius, strike an arc in the required place, as at A.B.



Given line AB



AB placed on MN.

USE OF PROTRACTOR

The protractor is used to find the number of degrees in a giwen angle, or it may be used to draw an angle of a required number of degrees. The protractor is a part of a circle (usually a semi-circle) divided into degrees.

(a) To measure an angle, place the center of the protractor (usually marked by a notch or line) on the vertex of the angle to be measured, and then turn the protractor so that its side rests just over one side of the angle. Next read where the other side of the angle cuts across the protractor. This gives the size of the angle in degrees.

(b) To draw an angle of a given number of degrees, draw a line where you wish one side of the angle to be located. Then place the center of the protractor where the vertex of the angle is desired. Find on the circular arc the required degree and mark a point opposite this mark. With the ruler, draw a line from this point to the vertex of the angle, or

(c) Place the center of the protractor (point A) on the place where the vertex is desired; then rotate the protractor until the right number of degrees is just over the line (point C). Then with a pencil draw the line (AB) along edge of the ruler. This is easier, and more exact than the method given in (b) above. Later, methods for constructing angles of certain size will be given in which the compasses and ruler alone will be used, and other methods in which the triangle and T-square will be needed.

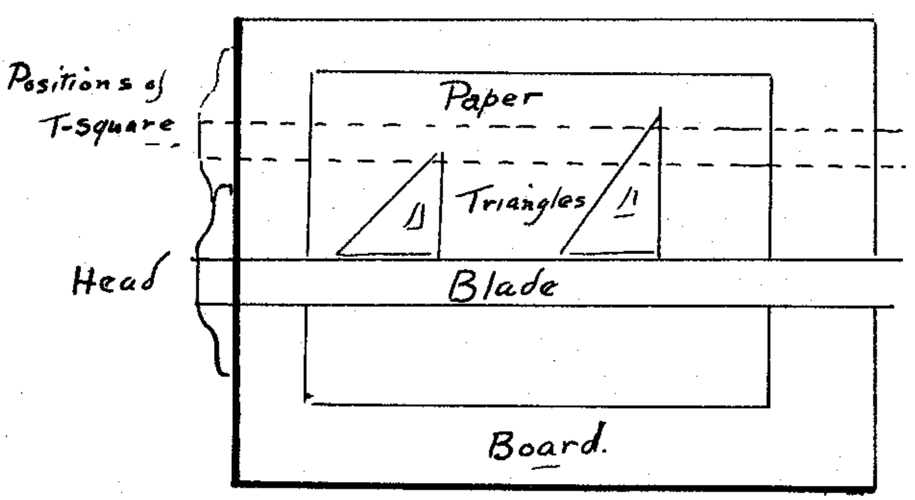
USE OF T-SQUARE

A T-Square is used by placing its head firmly in contact with edge of the drawing board, permitting the blade to extend out across the board. Lines are drawn along the top edge of the blade of the T-square. Since the head of the T-square is perpendicular to the blade, what relationship will exist between the lines drawn along the edge of the blade as the T-square is placed at various positions?

USE OF TRIANGLES

The drafts-man's triangles may be used with or without the T-square, though they are usually used with it. If a T-square is at hand, the triangles may be placed in contact with a ruler instead of the T-square. The usual triangles used are $30^{\circ} x 60^{\circ}$, and 45°. These triangles are usually made of celluloid, having

DRAWING BOARD AND T-SQUARE AND TRIANGLES
IN POSITION ON THE BOARD



one angle a right angle and acute angles as mentioned.

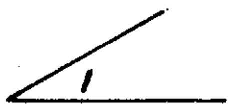
They are used to make a line perpendicular to another, also to quickly lay off 30, 60 and 45 degrees; or to make one line having an inclination of 30, 60 or 45 degrees to another.

The T-square can be used to make horizontal lines in case one desires lines perpendicular to the horizontal, or lines at an angle of 30°, 45° or 60° to the horizontal, the appropriate triangle is placed in contact with upper edge of the blade of the T-square. For making a perpendicular to the other line, place the blade of the T-square in the proper position, then use the triangles as before.

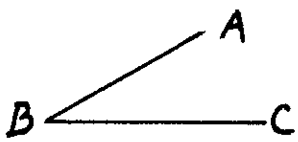
ANGLES AND THEIR MEASUREMENT

NOTATION

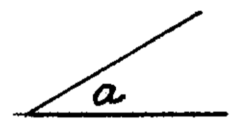
There are three ways commonly used to name an angle so as to be able to refer to it.



Named: Angle 1.



Angle ABC or CBA. Letter at vertex is read between other two.



Angle a.

CONSTRUCTIONS

Kinds of Angles:

Draw an angle to illustrate each of the following:

1. Acute angle
2. obtuse angle
3. adjacent angles
4. right angle
5. straight angle
6. reflex angle
7. complimentary angles
8. supplementary angles
9. vertical angles
10. a perigon

Letter and place the appropriate name under each drawing.

Sketch the following combination of angles:

1. Two adjacent angles;
2. Two equal angles which are adjacent;
3. Two complementary adjacent angles;
4. Two complementary non-adjacent angles;
5. Two supplementary adjacent angles;
6. Two supplementary non-adjacent angles;
7. Two equal angles and their supplements. How do they compare in size?

8. Repeat exercise No. 7, but find the complements of the equal angles instead of their supplements. Compare them as to size.

Expériment

1. Construct the following angles by paper folding:

(a) Vertical
(b) adjacent
(c) right
(d) complementary
(e) supplementary
(f) two right supplementary adjacent.

Note: the angles mentioned above may be constructed on colored stock and pasted in the notte-book.

2. Using rulers or pencils, place them in proper positions to form the angles mentioned in experiment 1. This will aid you to secure the proper relationship and meaning to the names of these angles.

3. Which of the following can you construct?

(a) One angle using 2 lines;
(b) Four angles using 2 lines;
(c) Three angles using 2 lines;
(d) Six angles using 2 lines;
(e) Eight angles using 2 lines;
(f) Twelve angles using 2 lines.

Measurement of Angles in General

The size of an angle depends upon the opening between its sides and not upon the length of the sides of the angle. It is measured in degrees. 360^{o} is equal to the sum of all the angles about one complete circle, or a perigon.

1. Reread the method given for using the protractor in measuring an angle; also the method for drawing an angle

which shall contain any number of degrees.

2. With the protractor measure the angles in each corner of your note-book paper, and see if they are right angles.

3. Draw two intersecting lines. Letter each of the four angles and measure them. Are any of them equal? What name is given to these angles?

~~4. Draw angles of various size, and measure them.~~

5. Construct the following angles: 30°, 20°, 45°, 60°, 120°, 90°, 180°, and 242°.

6. Using the protractor, construct an isosceles triangle by making the base angles equal.

7. Likewise construct an equilateral triangle.

8. Using a protractor, construct:

(a) Regular polygons of 5, 6, 7, and 9 sides. A regular polygon has all the angles between the radii of the polygon equal, as well as equal sides and vertex angles. Construct these polygons by finding the number of degrees between the radii of the polygon (number of degrees equal 360° ÷ by the number of sides of the polygon). Use this number of degrees to form the angle between the radii of the polygon. The polygon may be completed by drawing a circle of proper size, and drawing the chords.

(b) By use of the same principle, construct a five-pointed star.

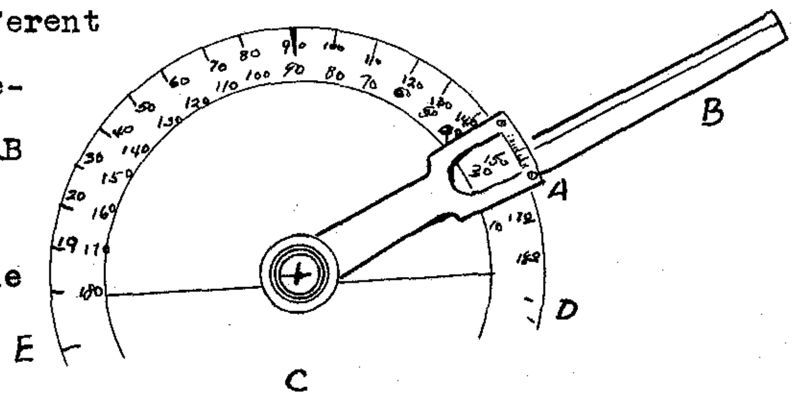
(c) Likewise construct a six-pointed or Jewish star.

ANGLES AND OBSERVATION

OBSERVATION

1..What kind of angle does the wall of your room make with the floor? What method does the carpenter use to get an angle of this size?

2. A number of protractors are provided with a sliding arm as CAB. Through how many degrees will it turn in passing from D around to E? What different angles will be formed between the movable arm CAB and the side DC? What angle is generated if the arm is rotated counter-clock wise through E



and back to D again? Through how many degrees did it turn?

3. Through what angles does a door turn on its hinges on being opened as wide as possible?

4. Beginning at 12 o'clock, what angles will be generated by the hands of a clock in one hour's time? in two hours' time? in twelve hours?

5. The angles formed may be discussed from the following view-point:

(a) The angles the minute-hand makes with its original position;

(b) The angles the hour-hand makes with its original position;

(c) The angles between each of the hands of the clock.

6. What is a perigon? Could an angle be larger than a perigon? Consider the angles formed by one of the spokes of a wheel which makes 10 revolutions. Through what angle did any part of the wheel turn in the 10 revolutions?

7. Make a record in your note-book of various kinds of angles you have observed outside of book or class room.

8. For training in judging the size of angles, draw the following angles as accurately as possible, without the aid of a protractor: (Later measure the angles and note your error).

270°, 180°, 60°, 45°, 30°, 90°, and 35°.

9. Continue this exercise on the black-board, as well as on paper, until you can do reasonably well without the use of the protractor. You should learn to draw a straight line with ease without a ruler, also to make angles near the values of 30, 45, 60, 90, and 180°. Which angles are the easiest for you to make?

10. Practice drawing perpendiculars to a line: (a) from a point in the line, and (b) from a point outside the line.

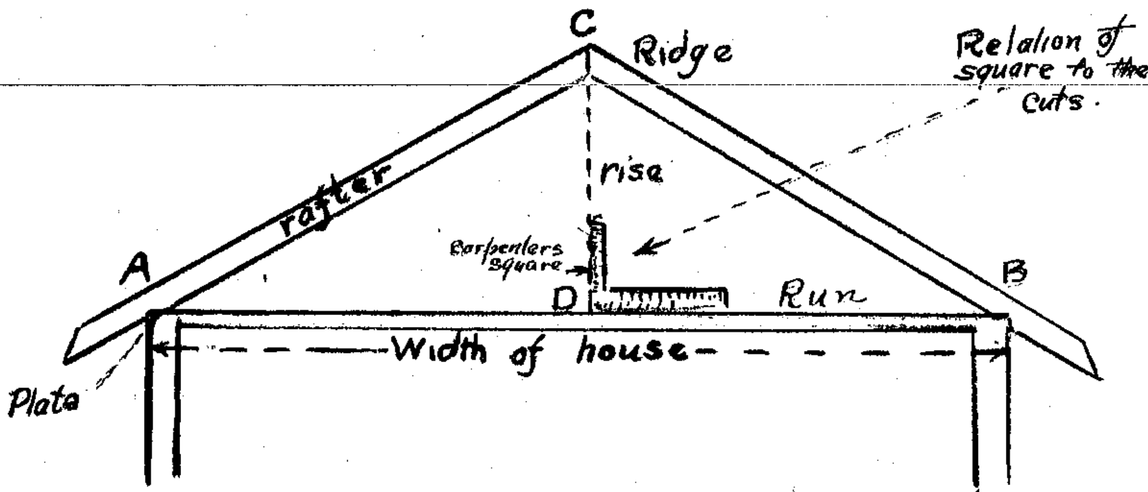
11. Notice the roof pitch (slope) of houses in your neighborhood. Make a table containing a record of your observations.

Pitch of Roof	Kind of Roof covering	Pleasing angle or non-pleasing	Serviceable in your climate?
Etc.			

Of what materials are the roofs made which have the lowest pitch? What kind use the highest? Which seems to you the most pleasing pitch?

HOW PITCH IS ESTIMATED BY BUILDERS AND CARPENTERS

Observe the method used by carpenters in designating roof pitch. The fraction obtained by dividing one-half the



distance AB between the two plates (where lower ends of rafters rest on side walls) into the distance DC of the top of the rafter to the line joining the plates is called the pitch. The pitch of roofs often used is commonly one of the following fractions: 1/5, 1/4, 1/3, 1/2, etc.

CONSTRUCTION

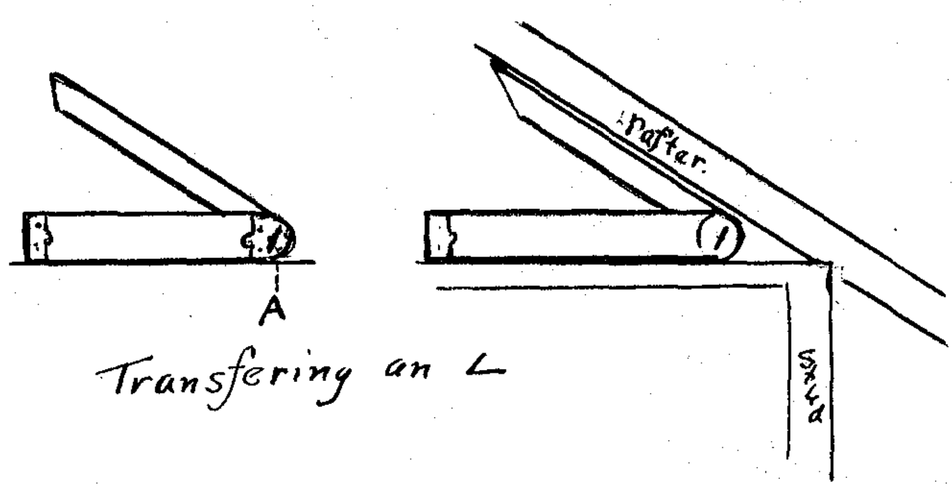
1. In your note-book make drawings to show the pitch of roof to show the values mentioned above.

2. Measure and record the base angles in each of the drawings made in exercise No. 1.

The pitch of a roof could be measured by means of a protractor, or the size of this angle can be found by a carpenter's tool called a T-bevel-square.

Later we will learn how a carpenter secures the ~~cuts on the rafters from his square.~~

A carpenter needs to know the principles in geometry to be able to do the best independent work. He uses the T-bevel-square to copy a given angle. To do this, he loosens the screw at joint A, places the square in angle to be



measured, and then tightens the screw A. The angle of the T-bevel-square is now opened to the same size as the angle to be copied, because they have been made equal by superposition. With the T-bevel-square the angle can be transferred to any desired position.

TRIANGLES

CONSTRUCTION

Draw the various types of triangles, which are listed below. Triangles are given names according to the kind of sides or kind of angles which they contain.

Named from Sides:

1. Scalene triangle
2. Isosceles triangle
3. Equilateral triangle

From the kind of angle:

4. Acute triangle
5. Obtuse triangle
6. Right triangle
7. Equiangular triangle

Kind of △	∠ A	∠B	∠c	Observations

EXPERIMENT

Measure the angles in each of the triangles you have just constructed, and record in a diagram like the one above showing the sum of the angles of each triangle, and any observations as to equality, etc.,which you may notice.

OBSERVATION

What do you find the sum of the angles of a triangle to be?

Do you find any special relationships between the angles of an isosceles triangle? Angles of a right triangle? How do the angles of an equilateral triangle compare? In the cases listed below, is it possible to construct a triangle using the parts as given below? Which are impossible to construct? Make sketches of each case:

(a) Use one acute angle with

1. Two other acute angles;
2. Another acute angle, and one right angle;
3. Another acute angle, one obtuse angle;
4. Two right angles;
5. Two obtuse angles;
6. One right angle, one obtuse angle;

(b) One obtuse angle, and

8. Two other obtuse angles;
9. One other obtuse angle, and one right angle;

(c) One right angle, and

10. Two other right angles;

(d). 11. Can an isosceles triangle have two acute angles for its base angles?

12. Can the base angles of a triangle be obtuse? What theorem, or axioms, are implied in above cases of construction?

CONGRUENCE

Two figures may be congruent, similar or equivalent. If they exactly coincide, when placed one over the other, they are said to be congruent. The parts (like angles and sides) which lie one over the other are called corresponding parts.

Two figures are similar if they have the same shape; they are equivalent, or equal, if they contain the same area; and they are congruent if they contain the same area and are similar (≅). In this connection numerous cases of the congruency of triangles will be considered, and a number of constructions and experiments pertaining to them will be made. Similarity of triangles is studied in book III. Equivalent triangles and their areas in book IV.

Congruence sometimes implies the placing of one figure upon another. Often this work of actual moving one figure and placing it on another is only imaginary, the arguments and demonstrations being carried out without actually moving any figure.

Proof by superposition is avoided as much as possible in giving a theoretical demonstration. In the experimental work in this treatise, recourse will often be had to testing and comparing two figures by the method of superposition. This is done to make the theorems more real to the student, and is not to take the place of a logical demonstration.

A. Sketch the following:

1. Two similar figures but not the same size;
2. Two equivalent figures but not the same shape;
3. Two congruent figures the same shape and size.

4. Describe briefly how you made the three constructions.

5. Find two tree leaves to illustrate the above mentioned cases of figure relationship.

6. Can two solid bodies be

(a) Congruent? (b) Equivalent? (c) Similar?

B. Congruence of triangles and figures

1. Directions to make congruent figures:

(a) Congruent figures can be easily made by drawing the design on paper. This drawing is placed over several thicknesses of paper, ply wood, etc., and then cut along the lines of the drawing.

(b) How do carpenters duplicate a pattern?

(c) What method is used in sewing, to obtain duplicates?

(d) Why do you secure duplicates in the above cases?

(e) Enumerate other applications of these methods.

2. Testing for congruence in triangles:

(a) Abbreviations used,-

S. stands for a side, and A. for an angle of a triangle;

S.A.S. $\cong$ S.A.S. signifies that two sides and the included angle of the two triangles are equal respectively.

(b) Test: S.A.S. $\cong$ S.A.S.

* (1) Construct a triangle using two inches for the base and the included angle = 25° and the other side = $1\frac{1}{2}$ inches.

(2) Construct a second triangle with the same given parts, cut with the scissors along the sides of this triangle. Test with the first triangle by superposition. Will all the triangles with these parts be the same size?

(c) Test: A.S.A. ≅ A.S.A;

* (1) Construct a triangle with a base = 2 inches ~~angle A = 25°, angle A = 45°. Complete the triangle.~~

(2) Construct another triangle with the same parts and with tracing paper placed over this triangle trace its exact size. Test with the first triangle by superposition.

(d) Test S.S.S. ≅ S.S.S.

* (1) Construct a △ with S = 3 inches, S = $1\frac{1}{2}$ inches, S = 2 inches.

(2) Devise a method for testing S.S.S. ≅ S.S.S.

3. Applications of congruence of triangles:

(a) Measurement of inaccessible distances,-

(1) By using triangles, devise a method for finding the distance across a stream. Test the method by actual trials.

(2) Likewise find the distance between two inaccessible islands located near the shore. Test this by figuring the distance between the two objects in place of the island if you are not located near the seashore.

(3) How do astronomers find the distance to the

sun or moon? Look up in some good text book on astronomy and briefly record the method used and the principles of geometry involved.

(4) How was the distance from the equator to the pole known before any one had ever gone to the north pole? Describe the method briefly. This is often discussed in astronomy under the heading of triangulation.

(5) Devise a method for finding the distance

a. Across a lake; b. Through a hill; c. Of a pipe line running under a house.

(b) How could you continue a straight line on the other side of a building? Try the method.

(c) What geometric principles are involved in the construction and use of range finders?

(d) What methods do boy scouts use in measuring and estimating distance? What kind of range finders do they sometimes use?

(e) Observe the form of a range finder used long ago. See Smith's History of Mathematics, page 362, Vol. 2.

(f) Construct a range finder for finding the distance to near objects.

(f) A person can estimate the distance across a river by the following method:

(1) With head erect pull the cap down so that sighting under its bill the opposite bank of the river is just in view.

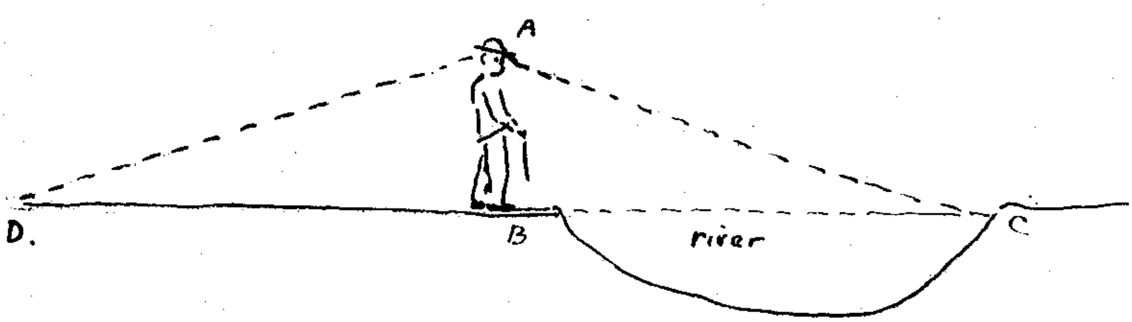
(2) Keeping the head in the same erect position, turn through 180°, or any other convenient angle.

(3) Notice the extent of the space visible under the visor of the cap. This is the distance across the river.

(4) Measure or step this distance.

(5) Give a proof for the pinciples involved.

(6) Test this method by a number of trials on level ground, then measure.

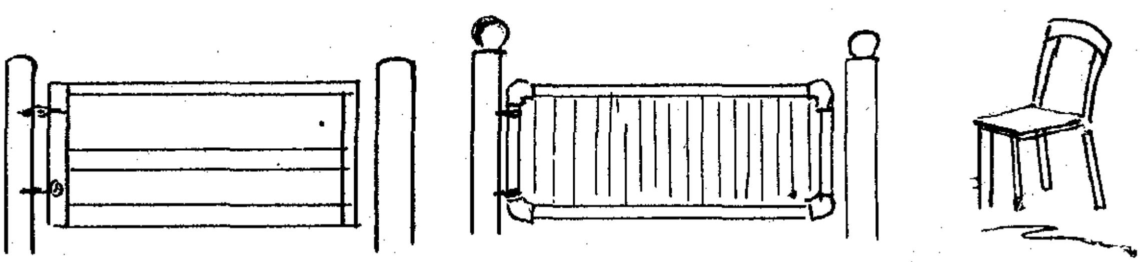


When one of Napoleon's engineers was asked to measure the distance across a river, he used this method as he did not have his instruments with him.

STABILIZING EFFECT OF THE TRIANGLE

With three given sides only one triangle can be formed, but with four sides any number of quadrilaterals can be made. Use is made of this principle to make things rigid by breaking the quadrilaterals into triangles.

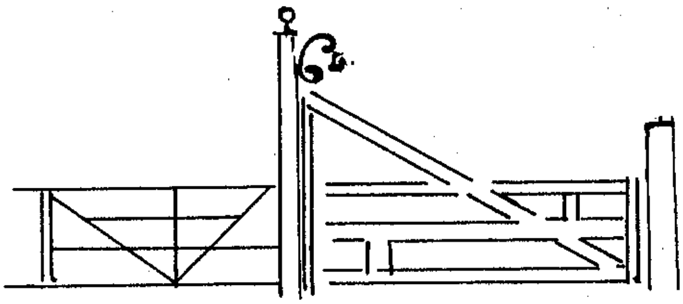
Secure four narrow strips of wood of different ~~length (like pieces of yard-stick). Drill holes near the~~ end of the pieces, and join these together with stove bolts so as to make a four-sided figure (a quadrilateral). See if it is rigid, or can it be changed into various shapes

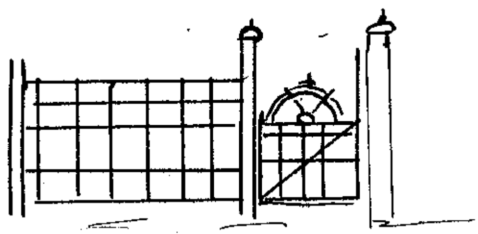


Where would lines be drawn to produce stability?

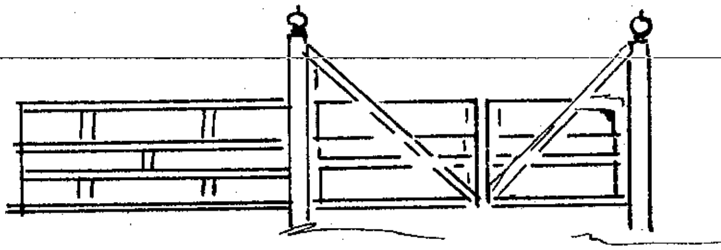
by using slight pressure on any vertex? Similarly form three pieces into a triangle. How many shapes can this have? A figure is said to be rigid if it maintains only one shape when pressure is exerted upon it. For the same amount of material what kind of windmill tower would you build, a three- or a four-post? Show by sketch how you would make the gates and chair rigid.

STABILIZING EFFECT OF THE TRIANGLE

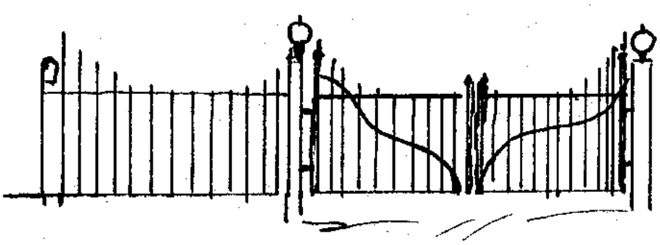




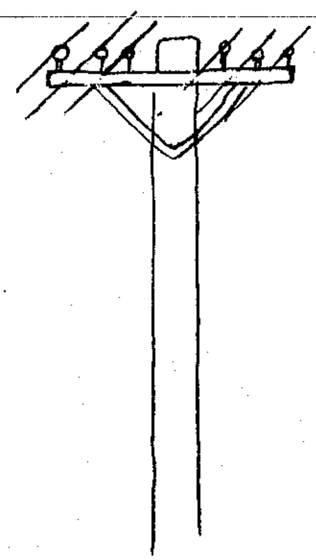
SOME GATES OF GOOD



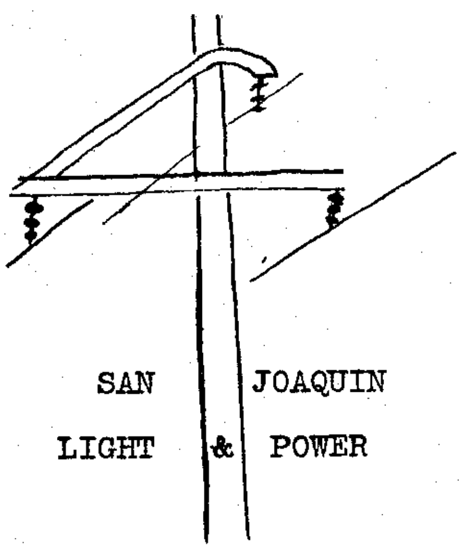
DESIGN AND



STURDY CONSTRUCTURE



BRACE ON
TELEPHONE POLES.



SAN JOAQUIN

LIGHT & POWER

POLES

STABILIZED ARMS.

THE USE OF THE TRIANGLES IN TRUSS

OBSERVATIONS

1. Look in an Encyclopoedia under the topic, Truss or Bridge Construction. Sketch and fix in mind three of the types of truss shown.

Observe in your neighborhood the forms of truss constructions found. Make sketches showing the application of the stabilizing effect of the triangle,

(a) As a roof truss (c) In bridge construction

(b) As a brace in bldg. (d) In furniture, etc.

2. Observe in your neighborhood gates of good design and sturdy construction. Make a sketch of the two best.

3. How is the triangle used in house construction?

4. How may a screen door be braced so as to prevent sagging?

5. Experiments on strength of materials, strength and stability of trusses of various types may be made by constructing any of the forms of trusses, and observe the effect of strains or forces upon their shape.

PARALLELS AND ANGLES

1. Draw two parallel lines and a transversal. Place numbers in the eight angles thus formed. Then measure each of these angles. Make comparisons and state the theorem which might be given to prove the angles equal in pairs.

2. Draw any acute angle. Draw two angles of different size, whose ~~sides are parallel respectively to sides of first angle. Make~~ comparisons to angle CAB, and state the wording of the theorem involved.

3. Draw an angle A.C.B, and two other angles of different size whose sides are perpendicular respectively to the sides of angle A.C.B. Make comparisons and give the theorem involved.

4. Show how to fold a piece of paper so as to have the creases in parallel lines.

5. How does a drafts-man use his triangles to make parallel lines? What principles are involved?

6. By use of the T-square and drawing board, construct a parallelogram.

7. Two railroad tracks cross at an acute angle; make a sketch to show which angles are equal.

8. Why are the sun's rays usually regarded as parallel? Draw to scale two lines to represent rays from a point on the sun and striking on the opposite sides of the earth, using a base line for the diameter of earth 1 mm. or less. Then remember the rays which come to us are not so far apart as the length of the diameter of the earth. If the rays come not from a

point on the sun but from opposite sides of the sun, then a drawing to represent the relationship of these rays would be represented by an isosceles triangle, the ratio of the base to its altitude being 860,000:93,000,000.

Draw to scale a triangle to represent this relationship. The figures used above are those generally used as the diameter of the sun, and the distance of the sun from the earth. What then must be the relationship between the rays from a distant star?

APPLICATION OF PARALLEL LINES

To Divide a Line into Any Number of Equal Parts

Methods:

A. Compasses and Ruler

* 1. Draw a line 2 inches long and divide it into three equal parts by use of the compasses and ruler.

B. Triangles and T-square.

2. Divide a 3-inch line into 10 equal parts by the use of the triangle and T-square method for making the lines parallel.

C. Mechanical

1. Measure the line and figure the length of each part, and use this part as the length for each segment. This method is seldom used, as it is not exact and is very inconvenient.

2. Paper folding. A line drawn on a piece of paper can be divided by folding, if the required number of parts is a power of two.

3. A line can be divided in a number of equal parts by using graph paper. One end of the line to be divided is placed at the number in the margin of the graph paper, which is the same as the number of parts into which the line is to be divided. Then the line is rotated until the other end strikes the margin of the paper which is perpendicular to the first margin. Next mark on the line to be divided, points as indicated by the appropriate parallels.

D. Carpenter's Method

1. To divide a board into a number of equal parts, a carpenter places his ruler diagonally across the board at such an angle so that the number of parts the board is to be divided into will be a factor of the length of the diagonal.

2. Where should the dots be placed along the edge of the ruler if the board is to be divided into 5 equal parts, and the ruler is placed so that the diagonal is 20 inches.

3. Draw a triangle and construct its three medians.

BISECTING AN ANGLE

Methods for bisecting an angle:

1. The compasses and ruler; 2. The paper folding;
3. The carpenter's.

* 1. Draw an angle and bisect it (use the compasses and ruler)

2. Construct a large triangle (general triangle with no two sides equal), bisect an angle. Do the bisectors intersect?

3. Through a point, draw any two lines. Bisect each of the four angles formed. Observe the results and make a record.

4. Do the bisectors of vertical angles form a straight line? Test. What relation exists between the bisectors of the vertical angles?

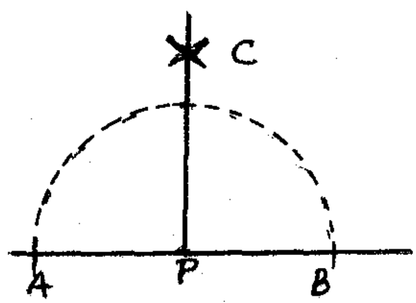
5. Show by a sketch how a carpenter divides anangle into two equal parts by means of his square (give the proof). Does a carpenter need to bisect an angle? In what type of work?

6. The bisector of vertex angle of an isosceles triangle is perpendicular to the base. Devise a wayfor testing this theorem. How did you test to see if it was perpendicular?

7. Show how to bisect an angle by paper folding. If the sides of the angle are along the sides of the paper, this becomes very easy, just a simple fold of the paper. If the angle is in the middle of the page, it becomes necessary to crease the paper along the sides of the angle, and then proceed as before. Measure equal distances from the vertex of the angle, and by folding erect a perpendicular to the sides of the angle. A line from their intersection to the vertex will be the bisector.

TO DRAW A PERPENDICULAR TO A GIVEN LINE FROM A POINT (P) IN THE LINE.

Methods:



A. The compasses and ruler

1. With point P as a center, strike arc A.B.

2. With A.and B. as centers, strike two arcs which intersect as at some point as C.

3. C.P. will be the perpendicular.

B. The paper-folding

1. Fold the paper so that:

(1) A will fall on B.,and

(2) Make the fold at point P.

(3) Crease the paper, thus forming the perpendicular

C. The mechanical drawing instruments

With the use of the T-square and triangle, or two triangles, the perpendicular can be constructed as has been previously described.

Constructions:

1. *Draw a line and locate some point P in this line. Erect a perpendicular using method A.

2. *Draw the perpendicular bisector to a line.

3. *From an external point drop a perpendicular to the line.

4. From a point near the end of the line erect a perpendicular.

5. Construct a large acute triangle. Cut the triangle out, and by paper folding draw the three altitudes, paste the triangle in your note-book.

6. Draw a large obtuse triangle and draw the three altitudes using the method given under C.

7. Draw the perpendicular bisectors to the sides of a triangle by using the compasses and the ruler.

8. Repeat number 7 by using method B.

9. Repeat number 7 by using method C.

DRAWING A LINE PARALLEL TO A GIVEN LINE

1. Through a point not in a line, to draw a line parallel to a given line (use compasses and ruler).

2. Repeat, only use the architect's method. This is done by using the T-square and the triangle, or the ruler and the triangles.

3. Construct the "Golden Oblong" (rectangle with sides in ration of approximately 2:3 or 5:8) by either of the methods suggested.

4. Likewise construct a parallelogram with the sides of the same ratio as Golden Oblong, and with an angle of 45° as one base angle.

5. Can you suggest other methods for making a line parallel to another?

6. Can a line be drawn parallel to another through a given point by the paper folding method?

7. Give a description of the method used with a drawing of the figure.

8. Make a list of the methods you can use to make a line parallel to another, through a given point.

THE BASE ANGLES OF AN ISOSCELES TRIANGLE ARE EQUAL

1. Draw an isosceles triangle using three inches for a base line. Measure and record the value of the base angles.

2. On the same base, make two other isosceles triangles, but use different length for each of the sides of the triangles constructed. Compare the angles as before.

3. Construct an isosceles triangle. Draw the perpendicular bisector to the base. Cut along this line, and the sides of triangle, and test the two angles by superposition.

4. Repeat exercise number 3, except fold along the perpendicular bisector. Do the angles coincide? Does the perpendicular bisector divide the triangle into equal parts?

5. The rafters of a certain roof are equal. How will the cuts of the rafters on the plate be? Sketch.

6. At each end of a base line 2 inches long, construct two equal angles, so as to make an isosceles triangle,measure and compare the sides.

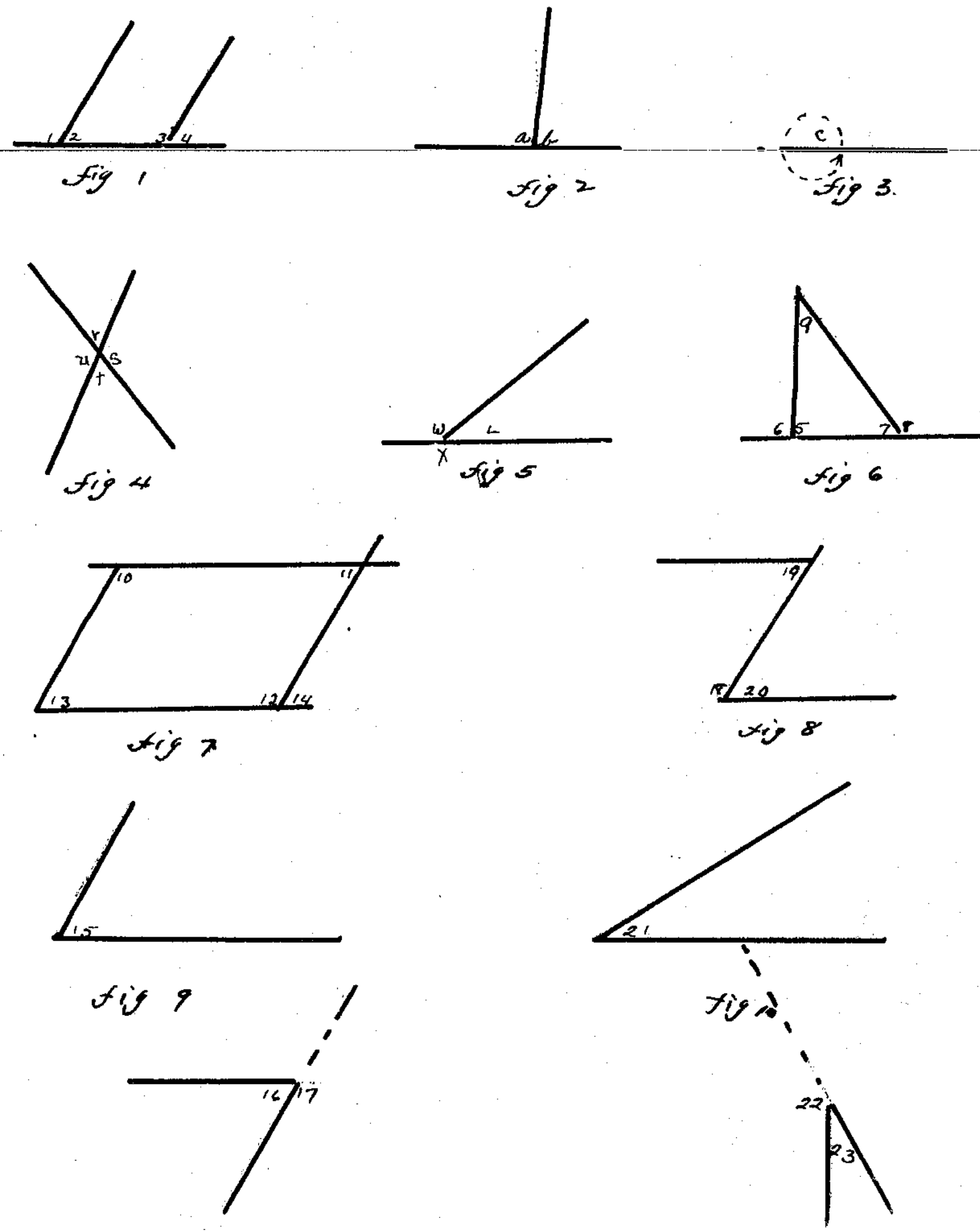
What theorems are the experiments on this page based upon?

OBSERVATIONS

Observe what applications, if any, are made of the isisceles triangle,

1. In architecture;
2. In instrument construction;
3. In other ways.

Which of the angles drawn below are equal? Which are acute, supplementary, complementary, right, or obtuse?



HOW TO COPY AN ANGLE EQUAL TO A GIVEN ANGLE

A. By using the Method Of:

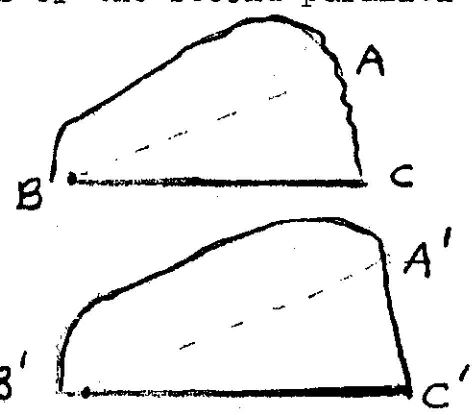
1. Ruler and compasses;

2. The Carpenters'. A carpenter can do this by placing his T-bevel-square over the angle, and setting the square exact size of the angle. This angle then may be transferred to another desired position.

3. T-square and angles. In case the angle is to be copied near the position of the first angle, this becomes very easy, if it is permitted to make the sides of the second parallel to the first angle.

4. Paper marking:

An angle can be copied equal to a given angle by using a piece of paper with one straight side,-



(a) Place the paper as in A.B.C.over the angle;

(b) Mark the points A.B.C. on the paper;

(c) Move the paper to the desired place, mark the three points A.B.C.,and draw the lines to form the angle.

B. * Construct an angle equal to a given angle.

THE SUM OF THE ANGLES OF A TRIANGLE

The sum of the angles of a triangle = 180°. Judging from its frequent use in trigonometry, this is one of the most important theorems in the geometry. Trigonometry uses three other theorems along with this one, as the main geometric basis for its work. These are:

(a) The sum of the angles of a triangle = 180°;

(b) The sum of the acute angles of a right angle = 90°;

(c) The square of the hypotenuse = the sum of the squares of the legs; and

(d) The theorem concerning the ratio of the sides of a triangle to its angles.

This theorem concerning the sum of the angbes of a triangle is so important that much space will be given to it in the form of numerous experiments and observations.

METHODS FOR PROVING THE SUM OF THE ANGLES OF A TRIANGLE = 180°:

A. By Measurements:

1. Construct a large triangle making very fine lines. By means of a protractor, measure each of the angles twice. Record the values measured in a table like the one given below.

	$\angle A$	$\angle B$	$\angle C$	Their Sum
1st Measure.				"
2nd Measurement				"
Average				"

2. Although your triangle is different in size than the triangle made by the other students, how does the sum of the angles of your triangle compare with the sum found by the other students?

3. Make a right triangle. Measure the acute angles. What do you find to be true of their sum?

4. Devise a method for finding the third angle of a triangle if two angles are given. Show the method by means of a sketch. Would your method be different in case the angles were given in degrees instead of a drawing showing the size of the angles?

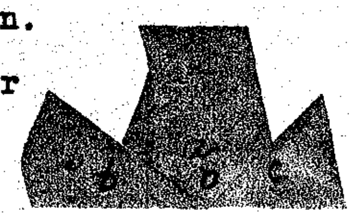
5. Without using the protractor, construct angles of 60°, 45°, and 30°.

B. By Clipping and Assembling the Angles:

The triangle shown below was constructed on colored stock; letters were placed in the angles, and the angles clipped from the triangle and reassembled about a point as shown. What does this show about the sum of the angles of a triangle?

Each student should repeat this experiment using the directions given below:

1. Construct a triangle A.B.C.
2. Place letters in the angles.
3. Cut triangle out.
4. Separate the three angles as shown.
5. Re-arrange these angles with their vertices at a common point.
6. Observe the results.



C. By Paper Folding:

1. Take any triangle piece of paper as A.B.C.

2. By folding, drop the perpendicular C.D.

3. H.E. is drawn parallel to A.B. by folding so that C. falls on D (C.D.parallel to A.B). How does this make H.E. parallel to A.B?

4. Fold so that A.F. and H.G. are perpendicular respectively to A.B.

5. When angle A. and B. and C. are each folded over to point D, you can see that they fill the straight angle A.D.B. Make this test.

D. By Paper Clipping:

1. Choose any piece of paper which only one straight edge; and

2. From any point in this line, as P, cut two straight lines forming ∠ 1, ∠ 2, and ∠ 3.

3. See if you can place the angles so as to form a triangle with the straight edges of the paper.

4. Can you get any impossible cases, by cutting the lines at various places?

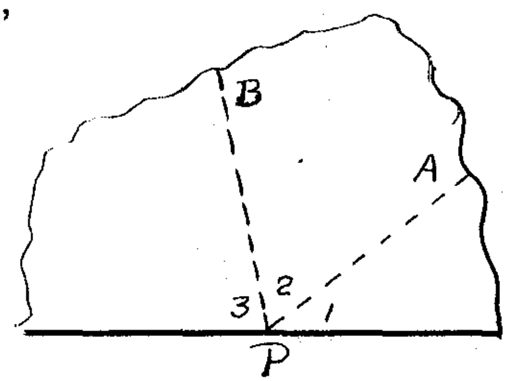


fig. 1

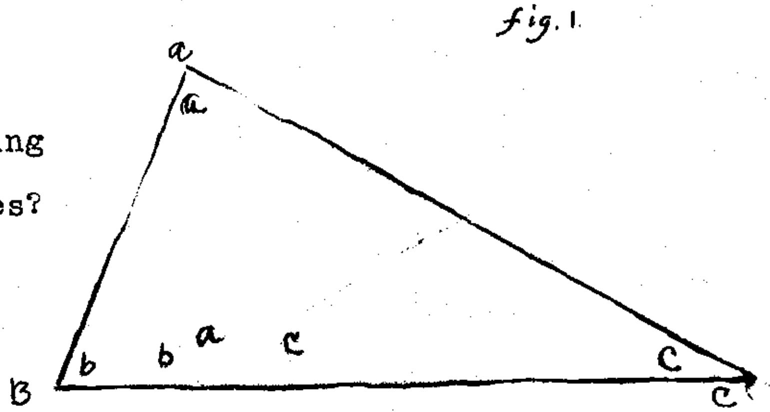


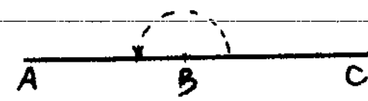
fig 2

E. By Traversing the Sides of the Triangle (indirectly).

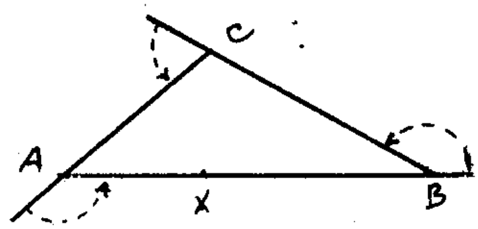
1. A straight angle = 180°.

2. The sum of all the angles on one side of a line equal a straight angle.

3. A person standing at B, Figure 1, and facing toward C, turns through 180° in turning so ~~as to face A.~~

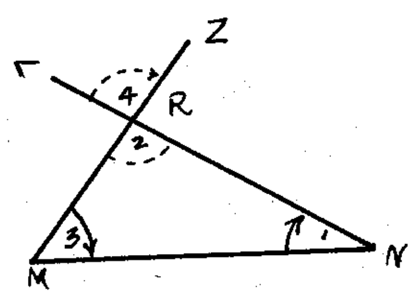


4. A person who starts from X, Figure 2, and walks along sides of triangle until he has walked along all sides back to the starting-point X, has turned through all points of the compasses. That is, the sum of the exterior angles of a triangle is equal to 360°. From this compute the value of the interior of the angles of the triangle.



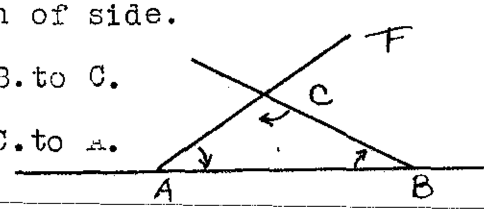
F. By Traversing the Sides (directly).

If a man stands at N facing M, and turns and walks N.R. to R, and then turns through angle 4, so as to face Z (remember angle 4 equals angle 2), and walks backward to M, and next turns through angle 3, so as to face N and walks to N; he will find he is facing in opposite direction from the position from which he started. He has therefore turned through 180°. What does this prove?



G. By Rotation of the Sides of the Triangle

1. Given triangle A.B.C. Show that the sum of the three angles is equal to 180° by the method of rotation of the sides of the triangle. Extend one side as A.B. in order to furnish sufficient length of side.



With B. as center, rotate A.B. to C.

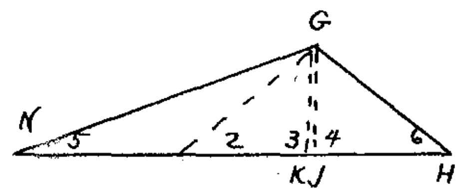
With C. as center, rotate B.C. to A.

With A. as center, rotate A.F. to B. Through how many degrees has A.B. been rotated?

H. By the method of rotating the perpendiculars to the base

Given triangle M.N.R.

Let G.K. and G.J. be two coninciding perpendiculars from N. (in the illustration they are drawn side by side for convenience).

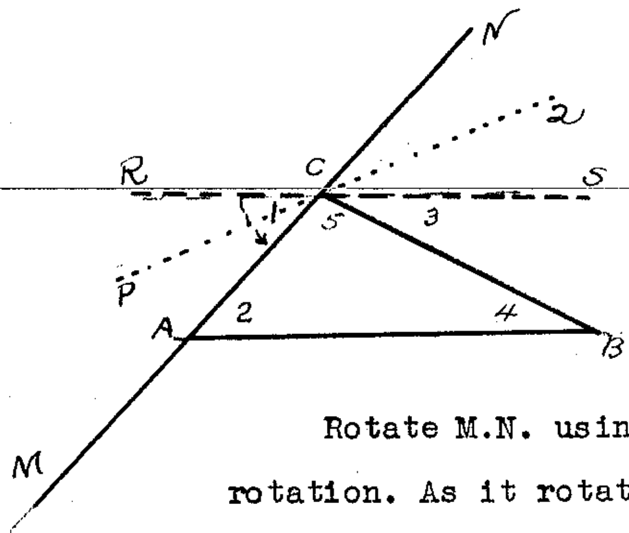


As G.K. rotates to the left about point G. over to N., what happens to angle 3? Continue the proof.

This can be seen with greater ease, if two tooth-picks or rulers are used to represent lines G.K. and G.J. Hold them in place at point G. by passing a pin through them at this point, and observe which angles change as they are rotated, one to N. and the other to R.

In any position of G.K. as angle G., what is true of ∠ 1 and ∠ 2?

I. As Shown by Rotation of Side of the Triangle.



Rotate M.N. using C as a center of rotation. As it rotates through the straight angle, M.N. from its original position it gradually increases, and evidently some time during this turning, M.N. would be in a position such as R. S. in which $\angle 1$ would be equal to $\angle 2$. This makes R.S. parallel to A.B., and therefore $\angle 1 = \angle 2$; $\angle 3 = \angle 4$ in this position; and $\angle 5 = \angle 5$ iden.

$\angle 1 + \angle 5 + \angle 3 = \angle 2 + \angle 4 + \angle 5$. Adding =

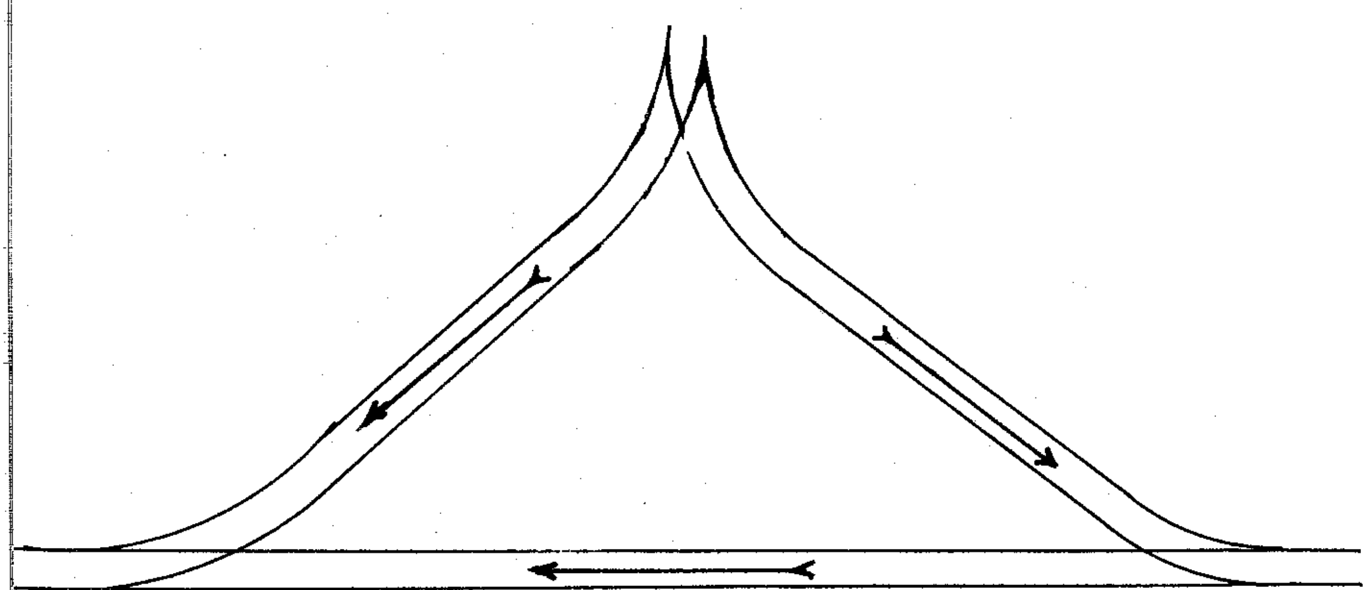
but $\angle 1 + \angle 5 + \angle 3 = 180^{\circ}$. Since R.S. is a straight line.

$\therefore$, $\angle 2 + \angle 4 + \angle 5 = 180^{\circ}$.

J. By Analogy to a Railway Y

As was shown before, to turn as to face in the opposite direction, a person turns through 180°.

In order to turn an engine in the opposite direction, in case there is no turn-table, recourse is had to the



triangle or railway Y. As the Y is in the general shape of a triangle, this proves the theorem as has been shown before.

K. By Arc Measurement

After the student has had a part of the second book in geometry, especially that portion which treats of the relation of circles to angle measurements, he can prove that the sum of the angles of a triangle equals 180°. This exercise will help the student to form a more general idea of angle measurement, since he must compute the values in various relationships. In working these exercises, use the following principles:

1. A central ∠ is measured by its intercepted arc;

2. An ∠ between two chords is measured by ½ its intercepted arc;

3. An inscribed ∠ is measured by ½ its intercepted arc;

4. An ∠ between a chord and a tangent is measured by ½ its intercepted arc;

5. An ∠ between two secants is measured by ½ the difference of the intercepted arc;

6. An ∠ between a tangent and a secant is measured by ½ the difference of the intercepted arc.

A Summary

By calling the arcs positive when an arc is on each side of the two intersecting lines which form the ∠ in question, and calling the smaller arc negative in case the two arcs are on the same side of the vertex of the angle, the six cases listed above may be incorporated into one principle, as follows:

An angle which intercepts an arc or arcs of a circle is measured by $\frac{1}{2}$ the algebraic sum of the arcs intercepted. Use this principle or the six principles given above in examination of the following figures:

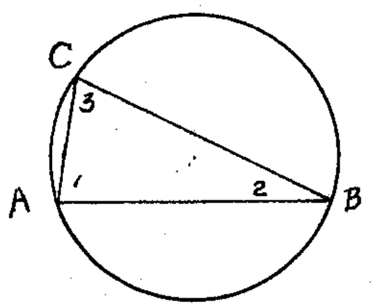


fig 1

Figure 1. With vertices of the triangle on the circumference, all angles are inscribed.

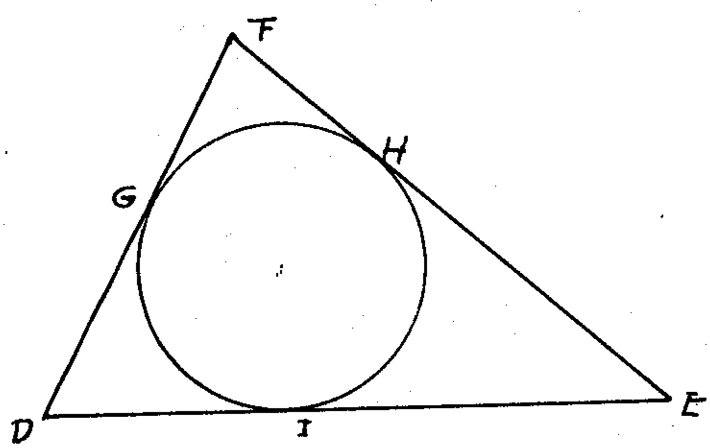


fig 2.

Figure 2. The sides of the triangle are tangents.

ARCS AND ANGLE MEASUREMENT (CON).

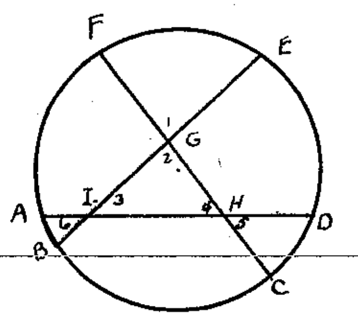


fig 3

Figure 3. The vertices of the triangle are between two intersecting chords. The angles ~~within the circle.~~

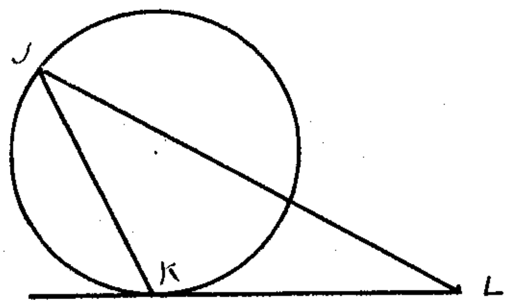


fig 4

Figure 4. An inscribed ∠, and ∠ between a tangent and chord, etc.

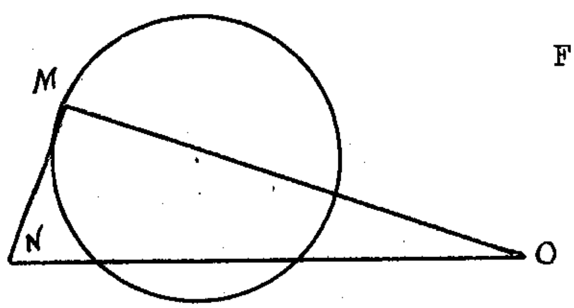


fig. 5

Figure 5. One angle on the circumference.

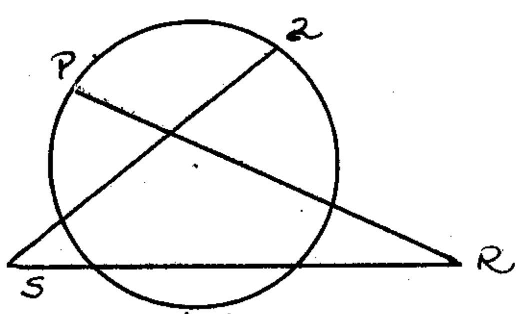


fig 6.

Figure 6. No angle on the circumference.

POLYGONS

CONSTRUCTION

Draw figures for the following polygons, and write the name of each underneath its figure:

1. Triangle
2. Quadrilateral
3. Pentagon
4. Hexagon
5. Octagon
6. Decagon
7. Duodecagon
8. Pentadecagon

EXPERIMENT

1. In constructing the polygons called for above, the sides and angles need not be made equal; in fact it would be better not to take time to make them equal. Measure each angle in the eight polygons which you have constructed and record in diagram like the one below.

Polygon	no Sides	Sum of ∠s	polygon	no. Sides	Sum of ∠s

2. Draw 4 polygons all of the same number of sides, but all different in length of sides and size of ∠s. It is best for different members of the class to choose polygons which differ in the number of sides.

3. Measure the ∠s in each polygon, and observe if the sum of the ∠s is constant for the same number of sides. Make your records in the form of a table.

4. Show by a drawing how many diagonals can be drawn from each vertex of all polygons having three to twelve sides.

Polygons	no. of sides	no. of △ formed	degrees in △s	degrees in Vertex ∠s of polygons

How many diagonals can be drawn from one vertex? What relation has this number to the number of sides of the polygon? How many diagonals for a n-gon?

5. When all the diagonals are drawn from one vertex, how many triangles are formed in each polygon of the exercise given above? In a polygon of n sides, how many triangles can be constructed by drawing diagonals from one vertex?

6. Take any point in each polygon and draw lines to each vertex in the polygon, thus dividing the polygon into triangles. Count the number of triangles, and record the values found in a table.

What relation exists between the number of triangles and ~~the number of sides of the polygon?~~

7. Measure the angles in the polygon with the protractor. Find the sum of these angles, and compare with former records.

Name of Polygon	No. of Sides	No of diagonals	No of Triangles	No of Rt. ∠s	No of degrees
Trigon	3	0	1	2	180
Quadrigon					
Pentagon					
etc.					
etc.					

THE EXTERIOR ANGLES OF POLYGONS

A. The Exterior Angles of a Triangle

Principle: The exterior angle of a triangle is greater than either of the opposite interior angles.

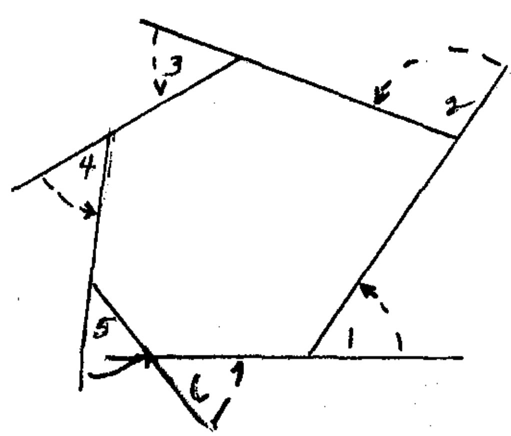
1. Draw a large scalene triangle, and by producing the sides in succession through each vertex, make three exterior angles. ~~Measure each exterior angle, and see if they are larger~~ than either of the opposite interior angles.

2. Repeat exercise number 1, but use another kind of triangle. Draw two triangles, one containing only large exterior angle, and a second triangle with a very small exterior angle. Can you make a triangle in which the exterior angle is less than either of the opposite interior angles?

B. The Exterior Angle of a Polygon

1. A person turns through 180° in turning so as to face in the opposite direction.

2. In turning a complete circuit and facing the same as before, one turns through 360°.



3. Imagine a person starting at A facing B, and walking around the polygon facing the direction shown by the arrows. At each vertex he must turn through the exterior angle of the polygon, and when he arrives at A and faces B again, he will be facing in the same direction, and has turned through 360°

Hence the sum of the exterior angles = 360°.

4. This would be true irrespective of the number of sides of the polygon. Therefore, the sum of the exterior angles of any polygon made by producing each side in succession is equal to 360°.

C. Experiment

1. Construct any polygon of seven or more sides, and produce one side at each vertex, so as to make the exterior angles of the polygon. With scissors cut the exterior angles out, and place them side by side about a point. What do you find to be true?

2. Can you make a polygon in which the sum of the exterior angles will be greater than 360°?

3. Use is made of this principle in testing a survey. A surveyor in determining the area of a plot of ground bounded by a polygon measures all the sides, and the exterior angles of the polygon, and if the sum of these angles is equal to 360°, he knows that his work is approximately correct.

4. Measure the length of the sides of some irregular plot outside the school building. Use a surveyor's instrument or a protractor to measure the size of the angles. Test the sum of the exterior angles as a check upon your work.

PARALLELOGRAMS

1. With a ruler and compasses make a neat and accurate drawing of a parallelogram. Likewise, construct a rectangle using the ratio of width to length as 2:3.

2. Make a quadrilateral with two of its sides equal and parallel, and test to see if the figure is a parallelogram. How can you tell if it is a parallelogram?

3. Make a quadrilateral so that the opposite sides are equal, but do not try to make them parallel. Make the base 3½ inches, and other sides 2 inches. Test to see if this is a parallelogram. (What is the definition of parallelogram?)

4. Draw two lines through any point O, so as to make any convenient angle. Using O as a center and a radius of 1 inch strike off equal arcs on one of the lines on each side of O, locating points A and C. Likewise on the other line, locate points B and D, using a radius of 2 inches. Connect A to points B and D, and also connect C with B and D. Test to determine if the figure is a parallelogram.

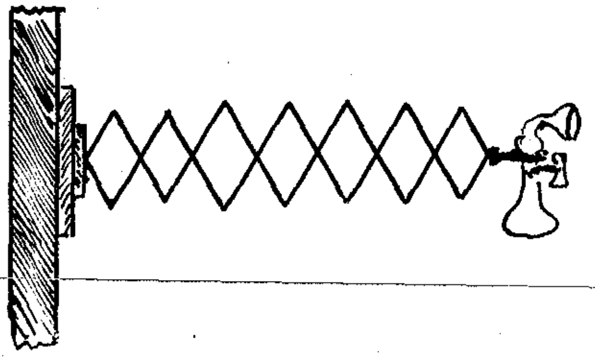
5. State the three propositions which the above exercises illustrate.

Experiment

Cut four pieces of wood, card-board, or paper, making two of them 3 inches long and two of them 2 inches long. Place these pieces with the equal lengths opposite each other, and in the form of a quadrilateral. Can you by changing the angles between the pieces place them in a position that will not form a parallelogram? Can this principle be applied in the construction of a parallel ruler?

PARALLELOGRAMS USED IN APPLIANCES

A. The Telephone Extension Bracket.



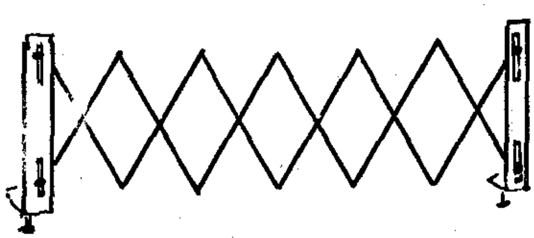
As the telephone is pulled out from the wall in a wall bracket telephone, why does the telephone always remain at the same height from the floor? Explain.

B. Folding Running-Board Luggage Carrier

Compare the type of running-board luggage carrier, which may be folded even with the running-board when not in use, with the type mentioned in C.

C. The Extension Luggage Carrier

Some bundle carriers used on automobile running-boards are constructed so that they may be lengthened to various positions, yet the end pieces always remain parallel to each other. Explain.



D. The Extension Table of the Dentist

Why does the little wall extension tool table near the dentist's operating chair always remain level, although extensible at different angles and distances?

CENTER OF GRAVITY

1. Look up the definition of center of gravity.

2. The center of gravity for flat or semi-flat objects may be found by taking the object and placing it so that it just balances on the edge of the table. Repeat with the object turned in a different position.

Draw a line as before: The center of gravity is just above the point where these lines intersect.

3. Construct the following figures on thin card-board. Cut out, and find the center of gravity by the method described in exercise number 2:

(a) Triangle

(b) Parallelogram

(c) Circle

(d) Any irregular figure

(Note: Place the figures so that one vertex will be along edge of the table).

4. Along what lines is the center of gravity in a triangle?

5. Along what lines is the center of gravity in a parallelogram?

6. Conditions of stability depend upon the location of the center of gravity. Find out what the conditions of stability are. List them.

THE RELATION BETWEEN ANGLES AND SIDES OF A TRIANGLE

Draw carefully the following figures in a convenient size:

1. A triangle A.B.S. so as to make angle A equal to angle B. Measure and compare the sides opposite these angles.

2. A triangle A.B.C. making A.B. equal to B.C. Measure ~~and compare angle A and angle B.~~

3. A triangle A.B.C. so that angle A is greater than angle B. Compare the sides opposite these angles.

4. A figure to illustrate the converse of No. 3. Make a test by measurement and comparison of parts.

5. A triangle with two of its sides very much unequal. Compare and measure angles opposite these sides.

6. A triangle A.B.C, using 1 inch for side c, $1\frac{1}{2}$ inches for side b, and make angle A equal to 30°.

Construct another triangle M.N.S. using 1 inch for sides, $1\frac{1}{2}$ inches for side n, but make angle m equal to 60°. Compare length of the third side of the first triangle with the third side m of the second triangle. What theorem is illustrated?

7. Two triangles A.B.C and H.C.F. making two sides of one equal to two sides of the other, but the third side of first greater than third side of the second. Measure and compare the angles opposite the third sides of the two triangles. What truth does this illustrate?

DIRECTIONS FOR PROBLEMS OF CONSTRUCTION

The solution of a problem of construction consists of the following steps:

1. A statement of the problem to be constructed.

2. A presentation of the given parts.

   (a) If lines of a certain length or angle of certain size, they should be drawn to scale;

   (b) Otherwise any convenient length may be drawn for what is given.

3. A statement of what is required to be done.

4. The construction. This consists of

   (a) A figure showing all construction lines by broken line;

   (b) The lines of the figure in heavy continuous lines.

5. A statement of the method of construction:

   (a) Given in phrase form; and

   (b) With reasons if needed.

6. Proof showing that following the method of the construction advocated the required figure will result.

7. A discussion of any special limitations. The student is advised to compare the sample construction given in the next few pages with the above outline.

Procedure to Obtain Solution of the More Difficult Construction Problems:

It is not always possible to construct a given geometric figure. In order for the construction to be possible, sufficient parts must be given to determine the figure. Even when sufficient parts are given, it may not always be possible to see at once just how the construction is to be made, or the parts may not be of proper length to make the construction of the figure possible.

If you cannot determine the mèthod of construction at once, proceed as follows:

1. Suppose the construction has already been made, make a sketch showing how the figure will appear when completed. You will then have a figure before you as it will finally appear, except that the construction lines are absent, and your figure will not contain the required parts, in their given length or size.

2. Indicate in this figure by heavy lines the parts which are given; and

3. From these given parts, and using the principles of geometry, find a way to proceed to the construction of the whole figure. This may be done by observing the method used in step number 4.

4. Find a part of the figure which is determined. This will usually be a triangle. Sometimes additional lines need to be drawn to form this triangle.

5. When this is found, the figure may be completed using this unit part as a foundation.

6. You are now ready to construct the entire figure from the first, using the required parts.

7. Follow general outline given for organization of the proof.

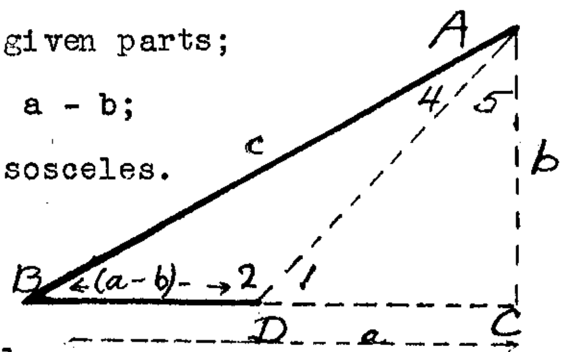
THE SOLUTION OF A CONSTRUCTION PROBLEM

To construct a right triangle given the hypotenuse and the difference between the other two sides.

Deriving the Solution:

Unless the solution is known, draw a figure and make a study of it.

Study of the Figure:



1. Heavy lines indicate the given parts;
2. Locate D, so that A.B. = a - b;
3. Then triangle A.C.D. is isosceles.
4. ∠ 1 = ∠ 5 (= 45°)

(An isosceles right triangle)

5. ∠ 2 is a supplement of ∠ 1.

Triangle A.D.B. is determined having two sides and an angle opposite one of them as ∠ 2 could only be obtuse since C is a right.

Solution:

For method of proof refer to sample of student's work as given on page 36.

Be sure to give a complete proof including also a discussion of possible cases, etc.

CONSTRUCTIONS

The following constructions are to be made and the constructions and proof written according to the plan shown previously.

Construct a right triangle, given respectively,-

1. One side and the hypotenuse;
2. The two arms A and B;
3. The hypotenuse and one acute angle;
4. The hypotenuse and difference between the other two sides;
5. The hypotenuse and one leg.

Construct an isosceles triangle, given respectively,-

6. An arm and the median to it;
7. The base and altitude drawn to the base;
8. The base and the vertex angle;
9. The base and one side;
10. The vertex angle and the altitude.

Construct an equilateral triangle, given respectively,-

11. One side;
12. One altitude;
13. The bisector of one of the angles.

Construct a triangle given, respectively,-

14. Two sides and an altitude to one of them;
15. A side, an adjacent angle, and its bisector;
16. Two sides, and the median drawn to one of them;
17. Two sides, and the altitude to one of them;

18. Two sides, and altitude to third side;

19. The mid points of its sides;

20. Two medians and the angle between them;

21. One side, the angle opposite this side, and another angle;

22. The base and the median and altitude to this side;

Construct a rectangle, given respectively,-

23. The base and the altitude;

24. A side and one diagonal;

Construct a parallelogram, given respectively,-

25. Base, an adjacent angle, and the altitude;

26. The diagonals, and the angle between them;

27. The base, one base angle, and the altitude;

28. Two adjacent sides, and angle between them;

Construct a Rhombus, given respectively, -

29. A side and the angle it forms with a diagonal;

30. The two diagonals;

31. An angle and the diagonal from its vertex;

32. One side, one diagonal;

33. A diagonal and an angle opposite it;

34. One side equal to 3 inches, and an angle of 45°.

Construct a quadrilateral, given respectively,-

35. The four sides, and an angle between two of them;

Construct a square, given respectively,-

36. Diagonal equal to 2 inches;

37. One side.

Construct a trapezoid, given:

38. The four sides.

39. Draw an isosceles triangle, and make a perpendicular bisector of the base. Does it pass through the vertex? Prove this to be true.

40. In what other triangles is exercise 38 true?

41. Draw a triangle and construct the three medians. Do they meet in a point?

42. Construct an angle of 45°.

43. Use T-square and triangles to construct perpendiculars mentioned below:

(a) Line from point in a line;

(b) A Line from point outside the line;

(c) Three altitudes for an

(1) Acute angle triangle;

(2) Obtuse angle triangle;

(3) Right angle triangle.

Do these altitudes meet? Where? How many perpendiculars can be drawn from a point in a line? Not confining your perpendiculars to same plane, how many can be drawn? Make a drawing to show this.

EXPERIMENT

1. Take a piece of colored paper, fold a crease so as to form a triangle. By paper folding, make a perpendicular from vertex to the base. (A right angle is formed by paper folding by folding an edge of paper over on to itself, then creasing the folds perpendicular to edge).

2. Construct a square by paper folding.

3. Inscribe a square within a square by paper folding. Do this in such a way that the vertices of the inscribed square are at the mid points of the sides of the first.square.

4. Repeat with another paper, and then inscribe a square in the last square you have just completed, until you have four squares each within the last square.

5. Repeat with another piece of paper, but have vertices at some other place other than the mid points of the first. Compare the sizes of the two squares. How would you construct a square double a given square?

Demonstrate by means of construction:

6. Bisectors of angles of a triangle meet within a point in the triangle. State the theorem.

Perpendiculars bisectors of the sides of a triangle meet in a point.

Make drawings to show that perpendicular bisectors

7. Of an acute angle triangle meet in a point;

8. Of an obtuse angle triangle meet in a point;

State theorem.

9. Can you draw a triangle in which the perpendicular bisectors of its sides do not meet in point?

The medians of a triangle meet in a point which is two-thirds of the distance from vertex to opposite side. Construct a triangle and measure distances from vertices to the intersections of the medians. State the theorem.

REVIEW FOR BOOK I

RECTILINEAR FIGURES

Write out this exercise in brief form, but be sure you are able to quote complet theorems if they are called for.

SUMMATION

1. Two lines are equal if (fill in blanks)

a.	f.
b.	g.
e.	h.
d.	i.
e.	j.

2. Triangles are congruent if

a.	c.
b.	d.

3. Right triangles are congruent if

a.	c.
b.	d.

4. Angles are equal if

a.	e.
b.	f.
c.	g.
d.	h.

5. List facts known concerning a parallelogram

a.	d.
b.	e.
c.	f.

6. Two lines are parallel if

a. e.

b. f.

c. g.

d. h.

7. A quadrilateral is a parallelogram if

a. d.

b. e.

c. f.

8. What is true in a polygon of n sides?

9. Give the six names which may be given to a square

a. d.

b. e.

c. f.

10. Give the four names which may be given to a parallelogram

a. c.

b. d.

GOOD PROPORTIONS IN A RECTANGLE

Whether or not a rectangle has the most pleasing appearance depends upon the ratio of its width to its length, and not so much upon the size of the rectangle.

1. With a base line of 3 inches construct a rectangle using a length for the width of the rectangle which will make what appears for you to be the most pleasing rectangle. Measure this width.

2. Choose from among a group of Christmas, Easter, and calling cards, the card which has the most beautiful outline shape. Measure and record the ratio of width to length.

3. Go to a drug store and find which is the most popular film size. What is the ratio of its dimension?

4. What is the ratio of the width to the length of the U.S. postal-card?

5. Secure opinions of other people concerning the most pleasing proportions for a rectangle.

6. Record results in a table (using nearest whole number) of objects you have measured:

Object	Width	Length	Ratio
"	"	"	"

Principles

The Greek parthenon proportions are based upon the idea that the ratio of width to length to be the most pleasing should be 2:3, or 5:8. They adhered to this principle that equal sizes were uninteresting. The Greeks, with a love for the beautiful, and a desire to use the best of space relations, chose the ratio of 2:3 for the ratio of width to the length of the rectangle. This was known as the "Golden Oblong", and is recognized as a standard for fine space relationships. Some people seem to have ability to make good space organization. Those who find it difficult to get good results in such arrangements, should use the proportions of the "Golden Oblong."

1. A rectangular mass placed within a larger rectangular frame should not be placed in the geometric center of the frame, but should have margins in which the ratio of the top, sides and bottom margins are to each other as 5:7:11.

Exercises

Mount a number of pictures in your note-book to illustrate the above given principles.

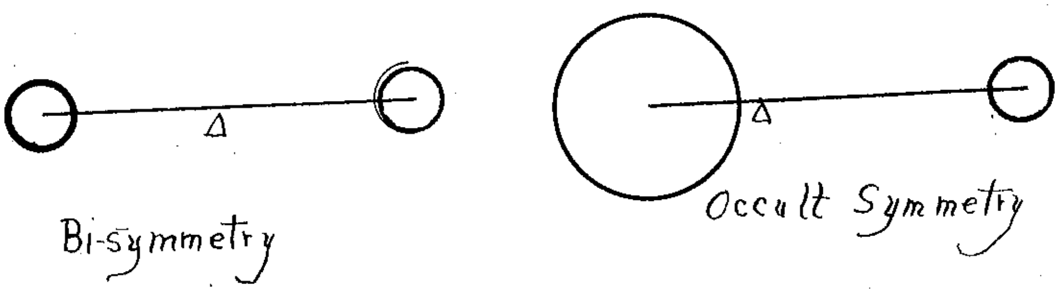
2. Observe on your home, in school, and in the city, for agreements and violations of the principles of proper relationship.

3. Do houses with square dimensions for foundation appear to be beautiful?

4. How would a house appear built on the dimensions of a cube?

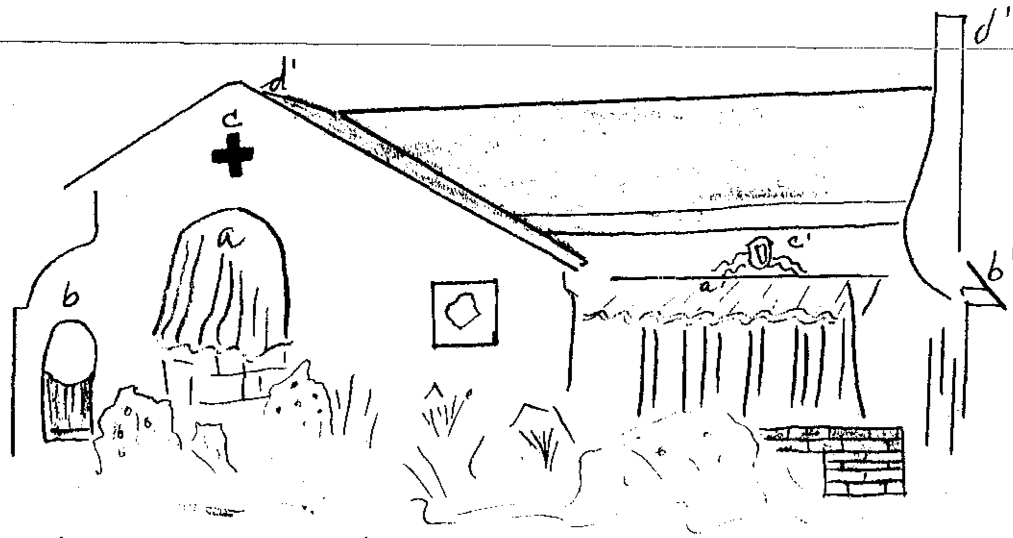
SYMMETRY

Symmetry is a form of balance. Bi-lateral symmetry (or bi-symmetry) is that type of symmetry in which the balance is very apparent to the observation. It consists of like points, line, or objects, placed at equal distances on opposite sides of a point or line as a reference. On the other hand, occult symmetry is hidden balance. In this type of balance the heavier or more important object is placed nearer to the center of the group of objects or center of the picture, while the lighter or less important is placed nearer the outside of the group of objects or picture.



The teeter-board is a good illustration of the principles of symmetry. Two boys of equal weight must be placed at equal distances from the fulcrum in order to secure a balance. This is bi-lateral symmetry. If a large man were on one end of the teeter-board, and a child on the other end, it would be necessary to place them at unequal distances from the fulcrum, in order to secure a balance. It requires more care and thought to obtain occult symmetry, but it is more beautiful and not so monotonous as the bi-lateral symmetry.

A house possessing bi-symmetry has like forms of structure as windows, gables, etc. on either side of a central line. The human body has numerous elements of this type of symmetry as may be observed in ears, eyes, nose, arms, legs, etc. One arm is symmetrical to the other, but the right arm could not be placed in position of the left; neither could the right glove be used to fit the left hand.



Observe the balancing forms a, with a' etc

It gives one an aesthetic shock to observe anything which lacks symmetry. For an ear, leg or arm to be missing throws the face or body out of apparent balance. The noted statue, however, is not lacking in symmetry although it is lacking a head.

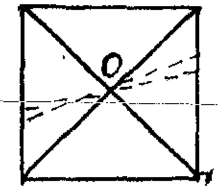
KINDS OF SYMMETRY

1. The occult symmetry in which a balance is maintained by placing the larger of two objects near the center.

2. Bi-symmetry is obtained by making both sides alike.

(a) Axial Bi-symmetry. Two points are symmetrical with respect to a line as an axis if this line bisects at right angles every line connecting the points. A figure is symmetrical with respect to a line as an axis if every point on one side has a symmetrical point on the other side.

(b) Center Symmetry. A figure is symmetrical with respect to a point as the center of symmetry if this point bisects every line drawn through it terminated by the figure.



Center Symmetry

(c) Tri-symmetry has three units of symmetry.

Tri-Symmetry

(d) Multi-symmetry has many units of symmetry.

EXERCISES

1. Name and draw two figures having by-symmetry.

2. How many axes of symmetry has each of the following:

(a) Square	(d) Regular hexagon
(b) Rectangle	(e) Equilateral triangle
(c) Parallelogram	(f) Isosceles triangle
	(g) Octagon.

3. Make sketches to show the axes for the above figures.

4. In what positions should the hands be placed in order to be symmetrical?

5. In what positions should the hands be placed in order to be unsymmetrical?

5. What parts of the human body are arranged symmetrically externally? What parts are arranged symmetrically internally?

TEST FOR CENTER OF SYMMETRY

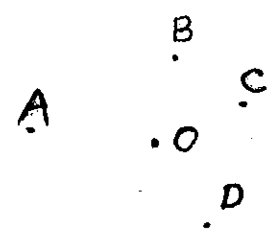
Which figures listed below have a center of symmetry?

(a) Square	(d) Regular pentagon	(g) Regular hexagon
(b) Rectangle	(e) Equilateral triangle	(h) Octagon
(c) Rhombus	(f) Circle	(i) Ellipse

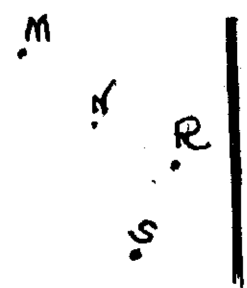
EXPERIMENT

Locating Symmetrical Points

1. Locate on a sheet of paper any point O, and four other points A.B.C.D. Then find four other points, W.X.Y.Z. symmetrical to A.B.C.D.



2. Draw any line A.B. and on one side locate at random any four points M.N.R.S. Then locate the points E.F.G.H. symmetrical with respect to A.B. as an axis of symmetry.



DRAWING SYMMETRICAL FIGURES

A. Methods.

1. Free hand.

It is usually very easy to construct a single curve free hand; but when it is required to draw another beside the first curve drawn, it will be found to be more difficult. Try drawing a vase or candle-stick free hand.

2. Geometrical construction.

(a) After one side of the desired curve has been constructed by free hand drawing, or by use of the French curve, the other side can be drawn by locating points along the path of the other curve by construction of the perpendicular bisectors.

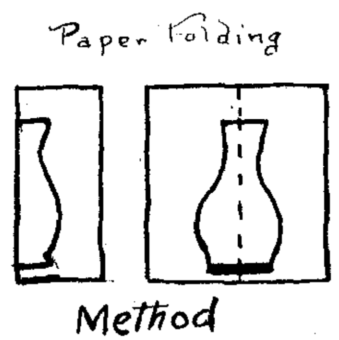
(b) Draw one part of the curve on graph paper, and then locate points for the other curve by use of the small squares on the graph paper.

(c) By making equal angles with the axis symmetrical, points can be located.

3. Mechanical.

(a) Paper folding.

(1) Construct one side of the curve as desired. Crease and fold paper along the axis, then by the use of carbon paper or sufficient pressure on tracing pencil, force outline on the other side of the fold. Or after folding, cut along the curve with the scissors.



4. Blotting.

A paper pressed on a fresh ink blot produces a figure symmetrical with the first figure.

5. Mirror

The image in the mirror is symmetrical to the object producing the image.

Use any three of the above methods to produce a symmetrical drawing of a vase or a candle-stick.

B. Observations.

1. Read various works of art concerning symmetry and its use as a principle of artistic arrangement.

2. Learn the distinction between occult (hidden balance) and bi-symmetrical balance.

3. Locate a house, the front view of which is bi-symmetrical and make a drawing of this view to scale.

4. Find a house which best illustrates your idea of good occult balance. Make a drawing of this house to scale.

5. Make a study of show window displays, and sketch the best illustrations of the two kinds of balance.

6. Make clippings from magazines or papers to show how the principles of balance may be used in arrangement of,-

(a) Furniture in the home;

(b) Flowers (see Japanese flower arrangements);

(c) Objects in a picture;

(d) Pictures on a page (as arranging photos in an album).

Follow the law of the margin as given below in arranging the pictures in a frame, or on a card:

C. Laws of the margin.-

1. Rectangular masses placed within a rectangular frame should have margins according to ratio: top, sides, bottom, as 5:7:11.

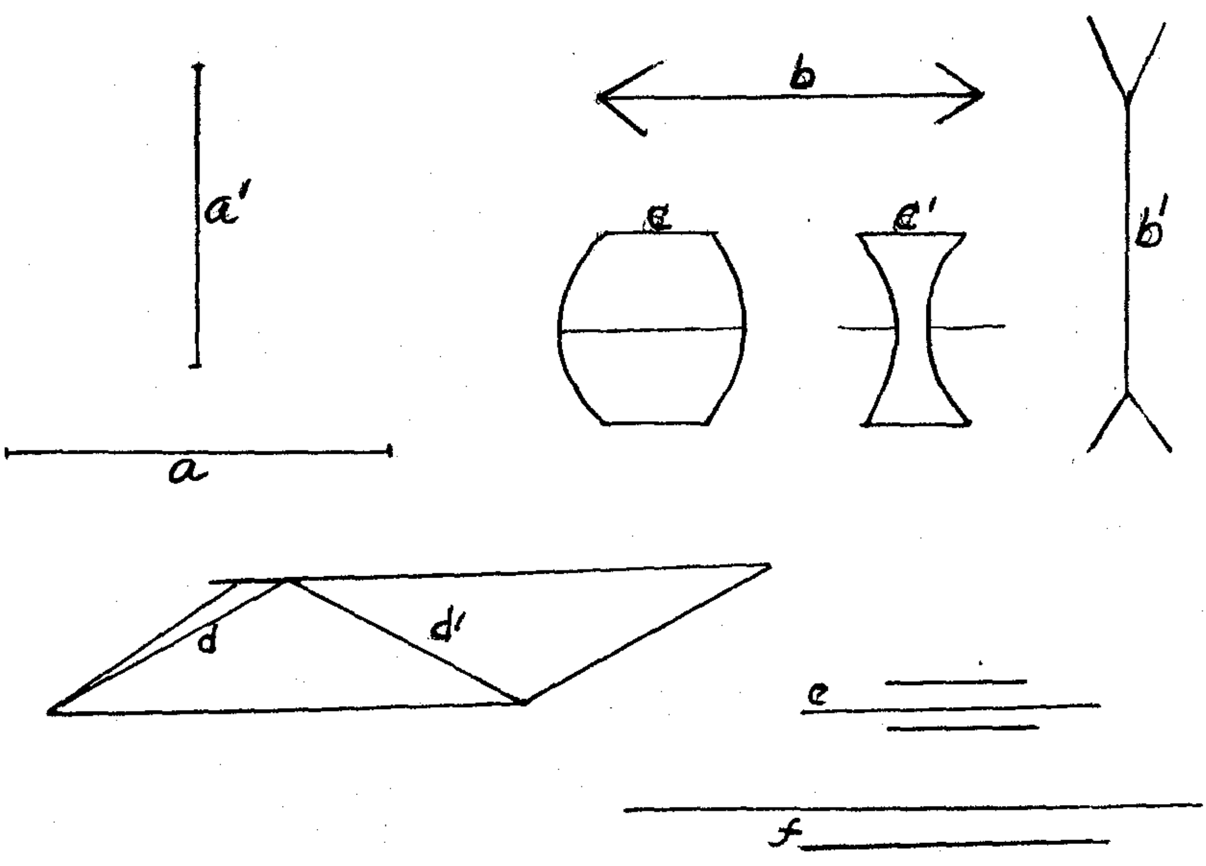
2. The important object of a photograph or picture should have a ratio of 5:8 (2:3; 3:5; 8:13).

APPEARANCES OFTEN MISLEADING

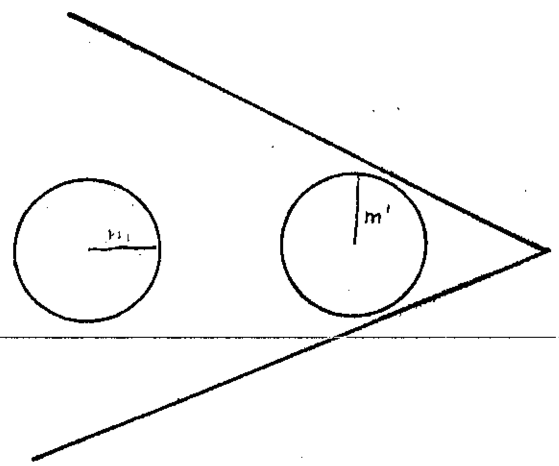
A. Geometrical Illusions

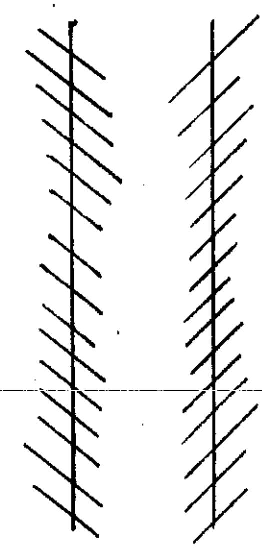
1. Often the eye is led away from the line or figure actually under observation to certain auxiliary lines, and thus we misjudge the length or size of a given figure because our eyes have slipped over to other lines, and we estimate the line either longer or shorter, depending on the position and inclination of the other lines.

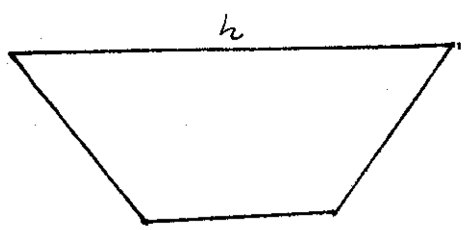
2. In the drawings given below, first estimate which is the longer line in making your comparisons, before taking any measurements.

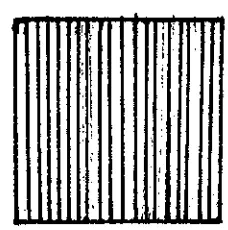


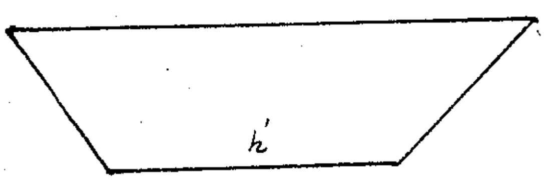
GEOMETRICAL ILLUSIONS (CON)

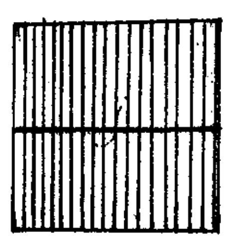












Observe the effect of line J.

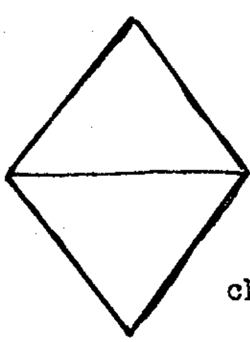
3. These illusions are given to show that we can not depend upon appearances for geometrical proofs, but must resort to the logical or general proof. Only a few of the many geometrical illusions have been given above.

B. Application to Designing.-

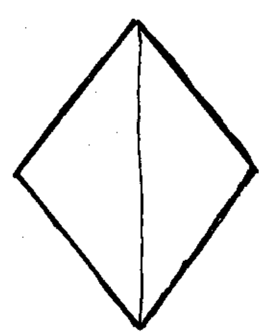
The intelligent dressmaker takes advantage of the principle that lines influence the appearance of other lines near them, in designing of dresses for various customers. Some who are stout desire to appear slender. The designer secures these results by proper position of lines as shown in the stripe of dress material, or the location and direction of trimming, or pleats, ruffles, etc.

A suggestion of the influence of lines upon the appearance refer to the figures on the following pages.

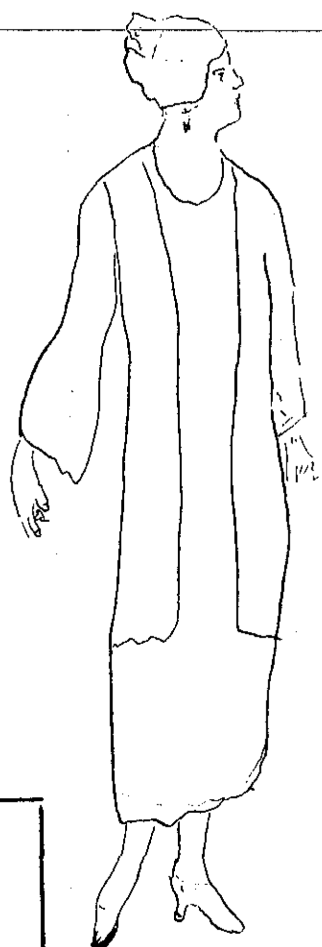
Observe the influence of the direction and position of lines upon the figures below. At the left width is emphasized, while the height is given to the figure to the right by long vertical lines.

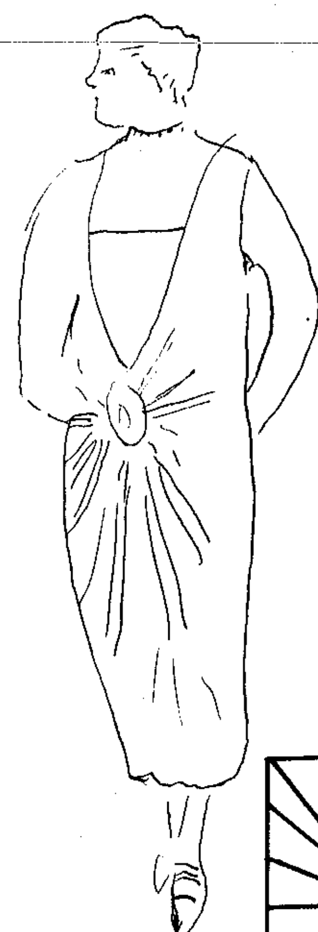


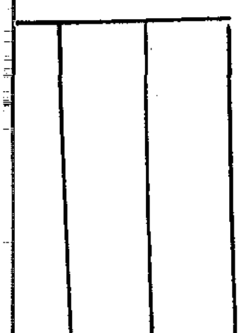
The diamonds are the same size. The direction of lines changes their appearance.

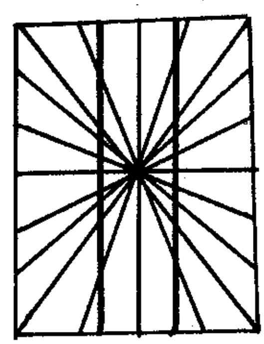


The influence of radiating lines upon parallel lines is shown in the figures below. The designer of the dress shown at the right has given emphasis to width by the lines radiating from a point. Whereas the appearance of height is given by the parallel lines in the other figure.







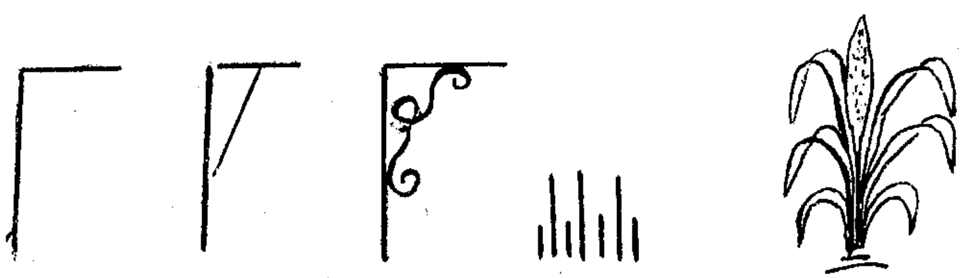


The vertical lines in figure 4 are parallel. The eye in observing the radiating lines judges the vertical lines as non-parallel.

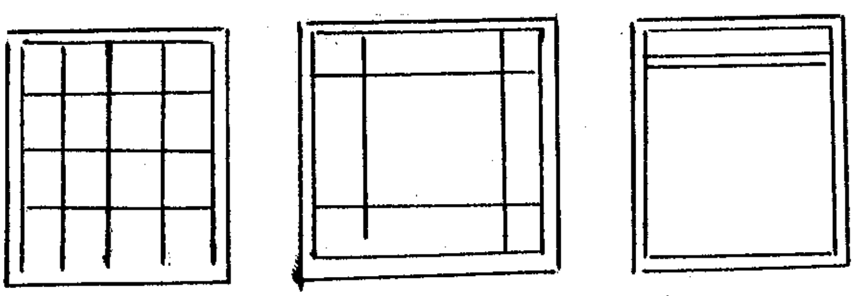
C. The influence of one line upon another

1. When two or more lines, or figures, are in close proximity, the eye does not examine each independently of the other, but one line may soften, modify or aid the eye in its examination of the other. This was shown to be true in the case of certain optical illusions. Lines may be in opposition, contradiction, transition, repetition, or in radiation in respect to another line, or lines.

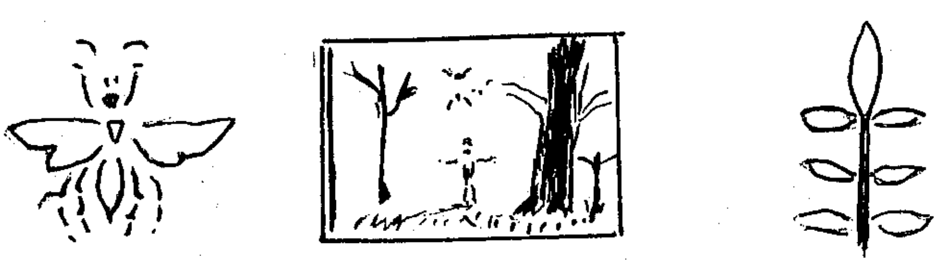
2. Beauty through opposition may be attained through proper organization of the lines.



Opposition Contradiction Transition Repetition Radiation



Regular and Varying opposition



Opposition in nature

Lines of Transition and Contradiction

Lines of contradiction modified by Transition.

Lines of Transition:

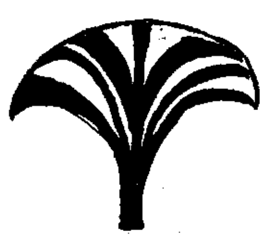
Lines of transition have a tendency to aid the eye in its turn of direction, and hence have a blending effect.

It may often be seen as an element of support in connecting two contradictory members, like in table legs, etc.

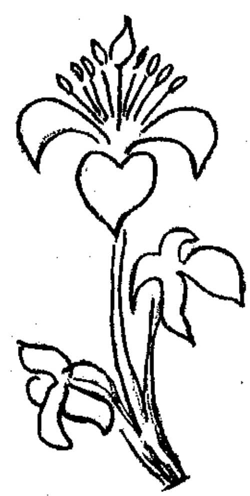
Radiating Lines

Shell

Envolute



Trellis



CHAPTER VI

(Book II)

THE PLAN OF THE OTHER BOOKS

The experiments, observations, and exercises given in Book II are provided to meet the following aims:

1. To develop the ability to use circles in designs and furnish training in constructions involving circles.

2. To give sufficient material to clarify the theorems of the text.

3. To show some of the applications.

4. To furnish sufficient material to give latitude for different interests and capacities.

5. To give an indication of the content of more advanced mathematics.

OUTLINE OF CONTENT:

A. Experiments and constructions to familiarize the student with the theorems.

1. Circles and points

Three experiments concerning points and circles.

2. Finding the center of a circle, etc.

Four experiments based upon the method for finding the center of a circle.

3. Measurements dealing with

(a) Arcs and central angles

(b) Chords, central angles, and arcs.

(c) The chord and its perpendicular bisector.

(d) Chords and their distance to center of circle.

(e) Inscribed and central angles.

4. Division of circumference.

(a) How to divide a circle into 4, 6, 8, or 12 equal parts.

(b) Divide a circle into any number of equal parts by using a scale of chords.

B. Applications:

1. Sailor's horizontal danger angle.

2. Relation of circles to construction of arches
Relation of circles to construction of windows.

3. Applications of circles to astronomy.

C. The Use of Circles in Designs:

1. The construction of monograms.

2. The curve of beauty and its uses.

3. The curve of force and its uses.

4. The relationship of curves to beauty of design.

5. Good design as shown in the Devilibus perfumers.

6. Curves used in furniture design.

D. Application to advanced mathematics.

1. Geometry should be a stepping-stone to the more advanced mathematics. If the student is given a little taste of higher mathematics, he will then know whether he would later like to spend time on the next course. If later he takes any of the higher mathematics, he has already had an introduction.

2. Construction and applications of

(a) The ellipse

(b) Hyperbola

(c) Parabola

(d) The oval

(e) Other curves.

3. Seven pages dealing with loci problems.

E. Summary:

1. The families of theorems in Book II.

2. Give 6 statements concerning equality of lines in circles.

3. Give 5 statements concerning angles and their measure.

4. Give 5 statements concerning arcs.

RATIO AND PROPORTION

(Book III)

A. Experiments, Constructions given to clarify the Theorems.

1. Testing segments made by a line parallel to the base of a triangle.

2. To divide a line into any number of proportional parts.

3. To divide a line A.B. proportionally to a given divided line C.D.

4. To find the fourth proportion.

5. To find the third proportion.

6. To find the mean proportion.

7. To divide a line into extreme and mean ratio.

8. The bisector of an angle and proportional segments.

9. Three experiments dealing with similar triangles.

10. The proportional segments of the tangent and the secant.

11. The proportional segments of two secants.

12. The proportional segments of two intersecting chords.

13. How to construct a right triangle by using a rope, as was done by the Egyptians.

14. Modern methods for constructing a right angle.

B. Trigonometric Applications.

1. Trigonometric ratios are functions of the angle only.

2. Six practical exercises treating of trigonometrical ratios.

3. How to make a table of the trigonometric function as sin., cos.,etc.

4. Sides of two similar right triangles have the same ratio.

Applications:

1. A number of experiments showing how distances to inaccessible objects may be found.

2. How to figure the height of a tree or building.

3. How distances to the planets and stars are found.

4. The law of the margin.

C. Applications:

1. After images and ratio.

2. Construction of scales.

Four experiments concerning scale drawing.

3. Drawing house plans to scale.

4. The relation of the size of the object to its image.

D. Proportion in the arts and design:

1. The vertical division of the horizontal mass.

2. The horizontal division of the vertical mass.

3. Proportion and ratio in costume construction.

4. Proportion and ratio in furniture construction.

5. Good proportions in a rectangle.

AREA OF FIGURES

(Book IV)

~~A. Derivation of the formula for the area of~~

1. Rectangle 3. Triangle

2. Parallelogram 4. Trapezoid.

The trapezoid rule for irregular figures.

The area of a triangle in terms of its side.

How to find area of a leaf, or irregular shapes.

B. Practical work in finding areas (outside laboratory):

1. Plane table work,-

(a) How to lay out on paper a scale drawing of a plot of ground.

(b) Area determinations.

2. How to find true north.

3. How to continue a line around an obstruction.

4. Checking the measurement of angles of an inclosed figure by the exterior angles.

5. Triangulation.

C. Transformation Exercises.

D. Summary of Book IV.

REGULAR FIGURES

(Book V)

A. Construction of regular figures by central angle method using a protractor:

1. Square
2. Hexagon.
3. Septagon
4. Octagon, etc.

B. Construction of regular figures by use of the ruler and compasses:

1. Square
2. Octagon
3. Hexagon
4. Duodecagon
5. Pentagon
6. Pentadecagon

C. How to make a star having five points

D. How to make a star having six points.

E. How to construct the regular polygons using a carpenter's square.

F. Make designs using regular ploygons.

G. Applications of:

Regular polygons in the following:

1. Snow crystals
2. Seed pods
3. Flowers
4. Wall paper designs
5. Linoleum
6. Designs and arts
7. Spider web
8. Honey cell
9. Architecture.

Applications of geometry to:

1. Astronomy
2. Physics
3. Carpentry
4. Mechanical drawing
5. Industrial arts
6. Furniture design.
7. Geometry in Indian rugs.

CHAPTER VII

B I B L I O G R A P H Y

A. WORKS INTENSIVELY STUDIED

1. Art and Geometrical Design

* Batchelder, E.A., Design in Theory and Practice.
The Macmillan Co., Chicago, 1922
A recognized authority on design and art, good treatment of application to geometry.

Pearson, F.A., Ticket and Show Card Designing.
Sir Isaac Pitman & Son, New York, 1924.
Furnishes a good application of geometry.

* Smith, Nettie, Designing With Flowers.
The Bruce Publishing Co., Milwaukee, 1927.
The geometric design is clearly shown.

Sprague, Elizabeth, How to Design Greeting Cards.
Bridgman Publishing Co., Pelham, New York, 1927.
Numerous suggestions on proper designing, etc.

Sykes, Mabel, Problems for Geometry.
Allyn & Bacon, Chicago, 1912.
A source book of Industrial Designs and Ornaments.

* Varnum, William H., Industrial Arts Design.
Scott Foresman & Co., Chicago, 1916.
A leader in the field of industrial Arts Design. Every geometry student and wood and metal worker should be acquainted with this work.

* Books referred to on page 143.

2. Art

* Goldstein, Harriett,)
Goldstein, Villa,) Art in Every Day Life. 2nd ed.

The Macmillan Co., Chicago, 1926.

Fine as an application of geometry of balance, symmetry, unity, proportion, emphasis, etc.

* Lemos, Pedro J., Applied Art.

~~Pacific Press Publishing Association,~~

Mountain View, California, 1920.

3. Architecture

Hodgson, F.T., Building Architectural Drawing.

Frederick J. Drake & Co., Chicago, 1904.

West, George Herbert, Gothic Architecture.

Bell & Son, London, 1927.

4. Decoration

* Holloway, E.S., The Practical Book of Finishing The Small House and Apartment.

J.B. Lippincott Co., Chicago, 1922.

A practical and useful book on decorations.

Jakaway, Bernard C., The Principles of Interior Decoration.

The Macmillan Co., Chicago, 1922.

A good treatise.

* Parsons, F.A., Interior Decoration. Its Principles and Practice.

Doubleday, Page & Co., New York, 1921.

Parsons is an authority on the subject.

* Parsons, F.A., The Art of Home Furnishing and Decoration.

Armstrong, Cork Co., Lancaster, Pa.,

Every student of geometry should read this book.

5. Geometry

Austin, W.A., Laboratory Geometry.

Scott Foresman, New York, 1926.

A few hints on drawing, but the text does not provide for laboratory work in general.

Clark, J.R., Modern Geometry.

World Book Co., Chicago, 1927.

Carroll, John, Practical Plane and Solid Geometry. 13th ed.

Browns & Oates, London, 1888.

A good book on the construction of figures.

Davies, Chas., Plane and Solid Geometry. Revision of Brewster's translation of Legendres French Text.

Wiley & Son, New York, 1837.

Palmer, Claude I.,)
Taylor, Dan P.,) Plane Geometry.

Scott Foresman, Chicago, 1915.

* Row, Sundara, Geometrical Exercises in Paper Folding. 3rd ed.

The Open Court, Chicago, 1917.

Smith, Rolland R., Beginner's Geometry.

The Macmillan Co., Chicago, 1925.

6. History of Mathematics

Cajori, Florian, A History of Mathematics.
The Macmillan Co.,Chicago, 1919.

* Smith, D.E.,History of Mathematics. 2nd, Vol. 1 + 2
Ginn Co.,Boston, 1925.

" The Teaching of Geometry.
Ginn Co.,Chicago, 1911.
Good chapter on history, etc.

7. Psychology

* Judd, C.H.,Psychology of High School Subjects.
Ginn & Co.,Boston, 1915.
An excellent treatment of Psychological analysis of Geometry and generalized experience.

* McMurry, F.M.,How to Study.
Houghton Mifflin Co.,Cambridge, Mass.,1909.
Fine on general study methods, judging the value of statements organization supplementing the thought and memorizing.

Sanford, Edmond C.,Experimental Psychology. Vol. 2.
D.C.Heath, Boston, Mass.,1898.
Excellent chapters on perception of space (150 pages)

Starch, Daniel, Educational Psychology. 15th.ed.
Macmillan Co.,New York, 1927.
Topics related to this thesis: rate of learning, how to study, transfer of training, etc.

* Witmer, Lightener, Analytical Psychology.
Ginn & Co.,Boston, Mass.,1902.
113 pages given to perception of space, etc.

8. Magazine Articles

Austin, W.A.,"A Flu Dream in Geometry".
School Science & Mathematics, 701: Nov.,1919.

Austin, W.A.,"A Dream Comes True."
School Science & Mathematics, 621: Oct.,1921.

Austin, W.A.,"Geometry a Laboratory Science."
School Science & Mathematics, 58: Jan.,1924

Austin, W.A.,"Geometry a Laboratory Science."
School Science & Mathematics, 948: Dec.,1924.

Cowley, E.B.,"Teaching of Plane Geometry".
Mathematics Teacher, n.p: Nov.,1927.

Cowley, E.B.,"Group Work in Geometry."
School Science & Mathematics, 103: 1918.

Halton, Caroline, "Falling in Love with Geometry".
(a comedy play, 40¢), 537 W. 121st.,New York.

Malsah, Fritz, "Mathematics in Germany."
Mathematics Teacher, 363: Nov.,1927.

Nyberg, J.A.,"Geometry a Laboratory Science".
School Science & Mathematics, 948: 1924

Nyberg, J A.,"Construction Exercises in Geometry".
School Science & Mathematics, 407: Dec.,1919.

Werest, Alma M, "Analysis Versus Synthesis."
Mathematics Teacher, 46: Jan.,1927.

B. OTHER AUTHORITIES CITED

1. Art

Smith, Lckaines, Looking at Pictures.
G.H. Doran & Co.,New York.,n.d.

Speer, W.W. Lessons in Form.

Donahue Hensbury,

2. Architecture

West, G.H., Gothic Architecture.

G. Bell & Son, London, 1927.

Kimball, Fiske, American Architecture.

Bobbs Merrill Co., New York, 1928.

Todd, David, New Astronomy.

American Book Co., New York, 1906.

3. Decorative Design

Branch, Z, How to Decorate Tiles.

Dodd Meed & Co., New York, 1927.

Lemos, P.J., Indian Decorative Design.

The Davis Press, Worcester, Mass., n.d.

Lemos, P.J., Oriental Decorative Design.

The Davis Press, Worcester, Mass., n.d.

Grow, J and others, Short History of Mathematics.

G.E. Stechert, New York, n.d.

Heath, Sir Thomas, History of Greek Geometry, VI

Oxford Press, New York, 1921

Heath, Sir Thomas, History of Greek Geometry, VII

Oxford Press, New York, 1921.

4. Psychology

Angel, J.R., Psychology,

Henry Holt & Co., New York, 1908.

Biggs, Thomas H, The Junior High School.

Houghton Miffin Co., Prairie Ave., Chicago, 1920.

McLellan, James,)
Dewey, John,) Psychology of Number.

Appleton, New York, 1907.

Parker, S.C.,Methods of Teaching High School Subjects.

Ginn & Co.,New York, 1920.

Titchener, E.B.,Text Book of Psychology. 17th ed.

The Macmillan Co.,New York, 1913.

Thorndyke, E.L.,"The Effect of Changed Data on Reasoning",

Educational Psychology. Vol. 5.,1920.

5. Geometry

Estill, J.G.,Numerical Problems in Plane Geometry.

Longmans-Green, New York, 1898.

Nichols, E.V.,Elementary and Constructive Geometry.

Longmans-Green, New York, n.d.

Spencer, W.G. Inventional Geometry.

American Book Co.,New York, n.d.

Touton, F.C.,Solving Geometric Originals.

Teacher's College.

Cohtribution to Education, 146: 1924.

C. BOOKS FOR ASSIGNED READING

The books which are listed in the other bibliographies that are suitable for general reading for the students of geometry class are marked with an asterick (*)

Ball, W.W.R.,Mathematical Recreations and Essays.

The Macmillan Co.,New York, n.d.

Burton, Myron G.,Shop Projects Based on Community Problems.

Ginn & Co.,New York, 1915.

Dudeny, H.E., *Amusements in Mathematics.*

G.E. Stechert, New York, n.d.

Jakaway, Bernard, *The Principles of Interior Decoration.*

The Macmillan Co., New York, 1922.

Manning, H.P., *Non-Euclidean Geometry.*

Ginn & Co., New York, n.d.

Rupert, W.W., *Famous Mathematical Problems and Theorems.*

D.C. Heath & Co., Boston, n.d.

Smith, D.E., and others, *Number Games and Rhymes.*

Teacher's College Record.

Well, Jane Warren, *Dress and Look Slender.*

David McKay, Philadelphia, Pa., 1925.