A FINITE ELEMENT/QUATERNION/ASYMPTOTIC NUMERICAL METHOD FOR THE 3D SIMULATION OF FLEXIBLE CABLES.

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Summary A flexible cable is modeled by a geometrically exact beam model with 3D rotations described using quaternion parameters. The boundary value problem is then discretized by the finite element method. The use of an asymptotic numerical method to solve the problem, requiring quadratic equations, is well suited to the quaternion parametrization. This combination of methods leads to a fast, robust and accurate algorithm very well-adapted for the simulation of the assembly process of cables. This is proved by running many examples involving complicated solutions.

INTRODUCTION

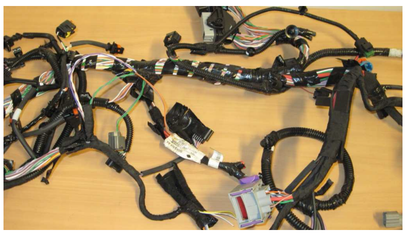

Figure 1: Cable harnesses.

The use of numerical tools for design is crucial in automotive industry. Indeed, they allow to limit the mock-up phases and thus reduce the design duration and the manufacturing costs. Most of the car pieces being rigid body solids, CAD software are nowadays widely used. However, these tools are not able to accurately predict the behavior of flexible pieces. In particular, cables simulation represents an outstanding challenge for the assembly process departments. Their structure is complex : a wire is made up of copper filaments wrapped in an elastomer duct. These wires are most of the time gathered in bundles which are themselves surrounded by various protections such as tape, PVC tube... All these features lead to very complex geometries. In addition, the large displacements undergone by cables, due to their flexibility, bring geometric nonlinearities. Furthermore, design departments require fast computation algorithms such that several design configurations can be tried. The current commercial softwares using computationally costly 3D elements are thus not adapted. In this context, we propose to build a dedicated tool in which the cables are modeled by the geometrically exact beam theory and simulated by an original combination of techniques: the finite element method for discretization, the quaternion parameters to describe rotations and an asymptotic numerical method (ANM) to compute the branches of solution.

TECHNIQUES USED

Beam formulation

The slender geometry of cables and their flexibility make a beam model with large displacements/large rotations and small strains really appropriate. These geometrically exact beam models consist in giving the position of the centerline of the beam and the rotation of the sections. They have been widely studied during the last decades but are still of interest for research, mainly because of the difficulty to describe 3D rotations. Indeed, they are not unique and among the main parametrizations existing, most of the papers have been focusing on rotational vector-like formulation, highlighting the use of only 3 parameters [1]. However, it has been proved that at least 4 parameters are necessary to avoid any singularities. Consequently, quaternions based on Euler parameters seem an optimal choice (least parameters avoiding singularities). Beyond their computational efficiency they also offer an alternative description of rotations avoiding trigonometric functions, that we found very well-adapted to be coupled with an ANM.

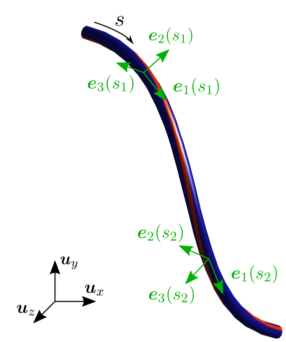

Figure 2: Beam kinematics.

Finite element analysis

Several authors have recently used the quaternions coupled with both finite element methods (FEM) [2] and finite difference methods [3], leading to promising results. For the purposes of our application, we preferred the use of the former. Indeed, the FEM is really convenient for the simulation of complicated geometries, featuring easy assembly of multibody systems (such as piecewise cables). Our FE formulation is obtained from the weak formulation of the beam. As recommended in [2], in numerical purposes, we added a Lagrangian multiplier to keep the unity of quaternions, seen as a supplementary variable. In our code, we have used isoparametric elements with a linear interpolation which seems sufficient if enough elements are employed.

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Asymptotic Numerical Method

To solve the nonlinear algebraic problem obtained from the FEM $\boldsymbol{f}_{\text{int}}(\boldsymbol{u}) = \lambda \boldsymbol{f}_{\text{ext}}$ (with internal force vector $\boldsymbol{f}_{\text{int}}$, external force vector $\boldsymbol{f}_{\text{ext}}$, unknowns vector $\boldsymbol{u}$, load parameter λ), predictor-corrector algorithms are usually used. However, these algorithms require the delicate choice of a step size which has to be small enough to get convergence but not too small to keep efficiency. Besides, the convergence toward the equilibrium solution is never assured. An alternative is to use an asymptotic numerical method [4] [5]. Such a method principle is, starting from a solution point, to seek the branch of solution as an asymptotic expansion of an arc length measure, leading to a semi-analytical solution. The range of validity of the series expansion is automatically computed and allows to define a new starting point for the next branch. The full equilibrium diagram can then be computed iteratively. This method features 3 main advantages : the user does not have any parameter to tune, it requires less computation time than a classical predictor-corrector method and it is not concerned by convergence problems [4]. The main difficulty using this method is to put the system of equations under the quadratic form $\mathcal{R}(\boldsymbol{u}) = \boldsymbol{f}_{\text{int}}(\boldsymbol{u}) - \lambda \boldsymbol{f}_{\text{ext}} = \mathcal{C} + \mathcal{L}(\boldsymbol{u}) + \mathcal{Q}(\boldsymbol{u}, \boldsymbol{u}) = \mathbf{0}$ (with $\mathcal{C}$, $\mathcal{L}$ and $\mathcal{Q}$ respectively constant, linear and quadratic operators). However, the use of quaternions instead of the usual trigonometric functions leads to polynomial expressions which makes the quadratization process very straightforward. Using both quaternions and the ANM is thus a very good combination.

RESULTS AND CONCLUSION

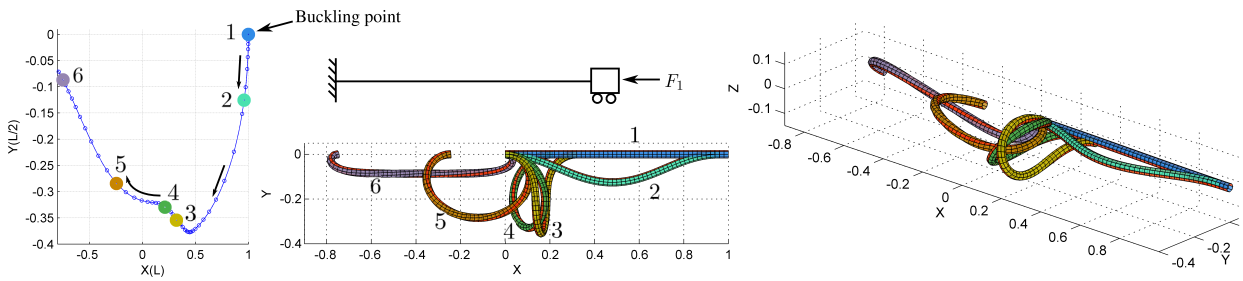

Figure 3: Post-buckling of a clamped-clamped beam. Equilibrium curve (left) and corresponding deformed shapes.

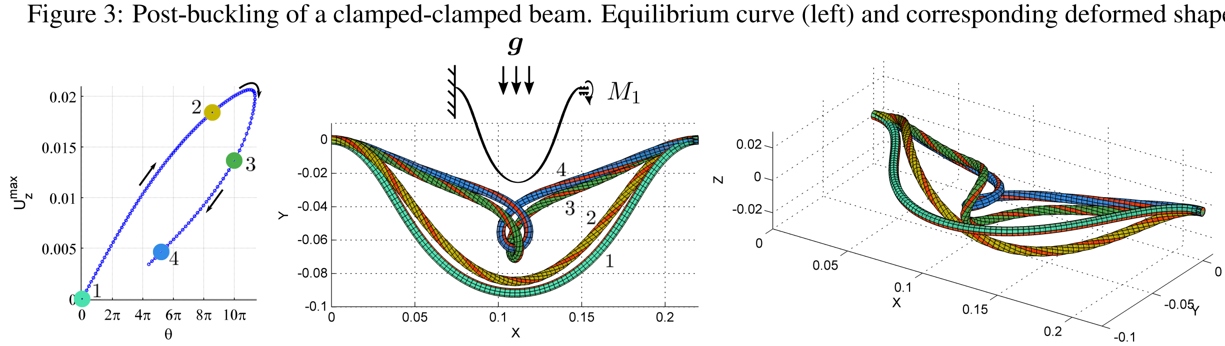

Figure 4: Formation of a plectoneme [3]. Equilibrium curve (left) and corresponding deformed shapes.

The full method has been implemented in Matlab. To solve the quadratic system $\mathcal{R}(\boldsymbol{u}) = 0$ with the ANM, use of the free online package ManLab [6] has been made. Several standard examples of the literature for which an analytical solution is known [1] [2], as well as examples featuring a more intricate bifurcation diagram have been run. On figure 3, the post-buckling of a clamped-clamped beam is illustrated. On figure 4, the experiment of [3] is reproduced numerically : it shows the appearance of a plectoneme (loop) on a cable undergoing gravity by the sole rotation of the right end-section. These results demonstrate the accuracy and the robustness of the method for a simple cable (one piece). They will lead our future developments which will consist in testing the method on a great variety of cables and on more complex geometries. In particular, we will focus on bundle-type structures whose cross-sections contain several wires. If concluding, this will allow to simulate accurately the assembly process of cables and will constitute a very useful tool for industry.

References

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