

ON LABOR MARKET HISTORIES

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Introduction

The purpose of this study is to identify the class of dynamic labor market models (based on job search theory) which justify the empirical procedures used in the vast majority of recent studies on labor market flows. It will be shown that this class of models is much smaller than is usually thought to be the case. A particularly interesting example is that job search theory without major (not yet accomplished) revisions, cannot be used to justify an increasing hazard rate in unemployment.¹

There is now a vast number of studies which have empirically investigated labor market flows. A large majority of these studies have confined their efforts to considering the determinants of the flow of workers from unemployment to employment [see, for example Nickell (1979a), Kiefer and Neumann (1979), Lancaster (1980)]. A much smaller number of studies have investigated the flow from employment to unemployment [see, for example Jovanovic and Mincer (1980)]. More recently, a number of studies have investigated the flows between two states (employment and unemployment) or between three states (employment, unemployment and out of the labor force) simultaneously [see, for example, Burdett et al. (1980, 1984, 1985), Flinn and Heckman (1982a,b), Tuma and Robins (1980), Toikka (1976)].

¹In the simple job search model where the unemployed individual receives unemployment insurance payments for a fixed (finite) number of consecutive periods, the reservation wage declines with duration. Assuming that the arrival rate of job offers and the distribution of potential offers are not affected by the duration of unemployment, gives an increasing hazard rate in unemployment. If, however, the arrival rate and distribution of potential wage offers are adversely affected the hazard rate could increase or decrease with the duration of a spell.

Most of these studies have used what will be termed a two-state (or three-state) "Markovian" approach.² The "Markovian" approach assumes only that the fixed characteristics of a worker (such as education, race, sex, date of birth, etc.) and possibly the worker's duration in the state currently occupied influence the probability of leaving the state in a given period of time. The individual's labor market history is assumed to play no role.

There appear to be three major reasons for this "Markovian" approach. The first is a practical one. To include a significant number of aspects of a worker's labor market history would increase the number of "right-hand-side" variables and thus reduce the power of any empirical work. Second, given some elements of a worker's labor market history are included it is difficult to know where to stop. Should the wages earned in the last two jobs held by the worker be included but not the last three? To what extent should the timing and lengths of past unemployment spells be included?³ Third, the vast majority of empirical studies in this area have cited job search theory as justification of the procedure used and most studies on job search do not include such elements as a worker's previous experience.

It should be stressed that not all empirical studies have cited job search to rationalize the estimation procedure used. Some have been agnostic about the theory; some have mentioned other theories that can be used to rationalize the empirical analysis. For example, job matching theories can be used for studying labor turnover [see, Jovanovic (1979, 1984)].

²We develop a two-state model but it can be easily generalized to a larger state space.

³In addition to "Markovian" state dependence and duration dependence, Heckman and Borjas (1980) consider occurrence dependence and lagged duration dependence. Their paper is concerned with the development of statistical techniques for testing for the presence of different types of state dependence.

Nevertheless, job search theory has provided the theoretical centerstone of much work on labor market flows. More importantly, though search theory is often utilized to justify the "Markovian" approach, very few studies have formally derived a model so that structural interpretations can be made.

It is now well-known that, within the job search framework usually considered, the optimal strategy of an unemployed worker can be characterized by a reservation wage $R(t)$, where t indicates the time since last employed. A wage offer at unemployment duration t is accepted if and only if it is at least as great as $R(t)$. Given a stationary environment, it can be shown that $R(t)$ is a constant for all t . This implies the hazard from unemployment is a constant, i.e., the distribution of completed spells of unemployment is exponential [see McCall (1970), Mortensen (1970)]. Further, if, in addition, job separation in a given time interval is independent of the duration of employment, than an individual's labor market history can be described by a two-state Markov process.

Many have argued that the reservation wage of an individual does change with the duration of a spell of unemployment. Burdett (1979), for example, assumes unemployment insurance payments decline with the duration of a spell of unemployment.⁴ If the reservation wage declines with unemployment, ceteris paribus, the hazard rate out of unemployment increases. Other models argue that the hazard rate in unemployment decreases with duration.⁵

⁴The reservation wage could decline for other reasons -- workers have finite lives [Gronau (1971)], liquidity constraints [Danforth (1979)] and learning of opportunities by a worker as he searches [Burdett and Vishwanath (1988)].

⁵For example, see Vishwanath (1986), which uses a stigma or scarring effect to theoretically justify decreasing hazard. Empirical studies by Lancaster and Nickell (1982) and Flinn and Heckman (1982) find negative duration dependence in unemployment.

Thus, it has been argued that determining the sign of the hazard is an empirical matter.

The above analysis is based on the (typically implicit) assumption that individuals only change state immediately after an event such as a job offer, or a layoff, but not necessarily after every event. We show that this assumption is restrictive. If the class of strategies is enlarged it not only increases the expected return but also

- (a) the optimal strategy cannot be characterized by a single reservation wage, and
- (b) implies the individual's labor market history cannot be described by a two-state (employment-unemployment) SMP, i.e., the individual's labor market history is important in this case.

The paper is organized as follows. In the next section semi-Markov and Markov labor market histories are defined. In Section II a general labor market choice model is specified and the key restrictions identified. Sections III and IV deal with analyzing optimal strategies under different scenarios and the last section concludes.

I. Semi-Markov and Markov Labor Market Histories

At any moment in time an individual who participates in a labor market can be envisaged to occupy one of two labor market states: employment (state 1) or unemployment (state 2). As time passes events occur (such as new job offers, layoffs, etc.) which imply the individual may choose (or be forced) to change state. The probability a given individual will leave a state in a given period of time is a number of obvious interest to those who study labor market flows. Given such probabilities are well specified it is possible to construct the distribution of completed spells.

Let $H_i(t_i; y, x, z, s_0)$ denote the probability a completed spell in state i lasts no longer than time t_i ($t_i \geq 0$), given

- (a) y denotes the wage rate currently faced if employed;
- (b) x is a vector describing the individual's labor market history in that it specifies when jobs were obtained in the past, when jobs were lost, the wage rate faced at each previous instant, etc. (but not y or t_i);
- (c) z is a vector describing the individual's fixed characteristics such as date of birth, sex, race, years of education, qualifications, etc.;
- (d) s_0 denotes the historical time state i was last entered.

Note that the distribution function $H_i(\cdot)$ need not be a proper distribution in that $\lim_{t_i \rightarrow \infty} H_i(t_i; y, x, z, s_0) < 1$ if there is a positive probability the individual will not leave state i , $i = 1, 2$. The distribution functions specified above are very general and would be difficult (if not impossible) to estimate from any reasonable data set. To make these distribution functions more tractable to estimate, restrictions need to be imposed. Such practical considerations motivate the specifications to be made below.

An individual's labor market history can be described by a two-state continuous time (homogeneous) semi-Markov process (SMP) if there exists a continuous distribution function $G_i(\cdot; z)$, $i = 1, 2$, such that

$$G_i(t_i; z) = H_i(t_i; y, x, z, s) \quad (1.1)$$

for all t_i , y , x , and s . Further, letting

$$q_i(z) = \lim_{t_i \rightarrow \infty} G_i(t_i; z) \quad (1.2)$$

when (1.1) is satisfied, it follows that

$$J_i(t_i; z) = G_i(t_i; z) / q_i(z) \quad (1.3)$$

denotes the probability a completed spell in state i takes no longer than t_i , given the individual eventually leaves state i , $i = 1, 2$. Note that $J_i(\cdot; z)$ is a proper conditional distribution function in that it limits to 1 as t_i goes to infinity. Thus, given an individual's vector of fixed characteristics, z , his labor market history can be represented by a two-state SMP only if the state currently occupied, and the duration of time in the current spell, are the only factors required to determine the probability of changing state in a given period of time. Note, if the above restriction is satisfied many of the commonly used data sets on labor supply can be used to estimate labor market flows.

An individual's labor market history can be described by a two-state continuous time (homogeneous) Markov process (MP) if (1) is satisfied with the added restriction that $J_i(\cdot; z)$ is an exponential distribution function. When this restriction is satisfied, the probability the individual changes state during any particular time interval depends only on the state currently occupied given his vector of characteristics z .

For many purposes it is more useful to consider the transition rate (or hazard rate) out of state i . The following expression:

$$\hat{\pi}_i(t_i; y, x, z, s_0) = \lim_{\epsilon \rightarrow 0} \frac{[H_i(t_i + \epsilon; y, x, z, s) - H_i(t_i; y, x, z, s)]/\epsilon}{[1 - H_i(t_i; y, x, z, s)]} \quad (1.4)$$

defines the transition rate out of state i , $i = 1, 2$. Intuitively the transition rate out of state i is the "instantaneous probability" of leaving state i given the duration of the completed spell in state i is at least t_i .

Note that the transition rate out of state i is a unique transformation of the distribution of completed spells in state i . Thus,

it can be shown that an individual's labor market history can be described by a two-state continuous time SMP if there exists $\Pi_i(\cdot; z)$, $i = 1, 2$, such that

$$\Pi_i(t_i; z) = \tilde{\Pi}_i(t_i; y, x, z, s_0) \quad (1.5)$$

for all t_i , y , x , z , and s_0 . Further, a worker's labor market history can be described by a two-state MP if it can be represented by a SMP where

$$\tilde{\Pi}_i(z) = \Pi_i(t_i; z) \quad (1.6)$$

for all $t_i \geq 0$, $i = 1, 2$.

Much of the empirical work in this area has concentrated on how the transition rates out of each state change when individuals with different vectors of fixed characteristics are considered. Such problems are not addressed in the present study. Essentially, the thought experiment performed here is to consider a large number of individuals with the same vector of fixed characteristics and to see what behavioral conditions are required for a two-state SMP to describe their histories. As z is assumed to be the same for all, it is suppressed in the notation developed. Nevertheless, we shall indicate where z is expected to play an important part in the story.

II. Framework For Labor Market Choice

Below a reasonably general model of job search with layoffs is presented. The objective is to develop a framework in which it is possible to investigate the conditions required for an individual's labor market history to be described by a SMP or MP.

The same notation will be used as in the previous section with one important exception. The wage faced by an individual, y , will now be

termed the base wage rate. The actual wage rate faced depends on the base wage rate and seniority, where seniority is defined as the time since last unemployed. Specifically, assume the actual wage paid to an employed worker at time t , since last unemployed. $w(t, y_1)$ can be written as

$$w(t, y_1) = y + \phi(t_1) \quad (2.1)$$

where $\phi(\cdot)$ is a bounded differentiable function with $\phi(0) = 0$. It should be noted that seniority, as defined, is transferred during job-to-job movements but is lost as soon as the individual becomes unemployed. This is obviously restrictive as seniority is usually thought of as some function of the total time spent in employment. If, however, a worker's seniority is defined in a more general way, then a worker's previous labor market experience becomes important which rules out the possibility that labor market histories can be described by a SMP or a MP on a reasonably parsimonious state space.

The following restriction on function ϕ is imposed:

A1: $\phi(\cdot)$ is a non-decreasing bounded measurable function of t_1 .

Thus, given a base wage rate, seniority, as defined, does not decrease the wage faced. Given the caveats mentioned above, A1 is probably a close approximation of what happens in real world labor markets.

At any moment in time a worker will choose to occupy a state based on prevailing conditions. From time-to-time, however, the situation faced changes and may lead to a change in state. Below two types of events are modelled; wage events (when new job offers are received) and layoff events (when employed workers are terminated and forced to become unemployed).

The arrival rate of new job offers, and hence a new wage rate, is assumed to be a function of the state currently occupied. In particular,

let λ_i denote the arrival rate of a new job offer when state i is occupied, where λ_i is the parameter of a Poisson process. Thus, $\lambda_i \cdot h$ denotes the probability a new job offer is received in a small period of time h when state i is currently occupied. No a priori restriction is placed on the relative size of λ_1 and λ_2 . In some occupations, such as lighthouse keeping, the arrival rate of new job offers when employed is likely to be small, whereas academics typically find it difficult to look for a job when unemployed.

Suppose a wage rate event occurs when an individual is currently occupying state i . The new base wage rate faced is envisaged to be the realization of a known random variable. Let $\tilde{F}_i(\cdot; \bar{y})$ denote this distribution when state i is occupied and \bar{y} indicates the previous wage rate faced. The distribution function $\tilde{F}_i(\cdot; \bar{y})$ has compact support for all \bar{y} . To simplify the analysis, assume

A2: $F_i(y) = \tilde{F}_i(y; \bar{y})$ for all y and \bar{y} where F_i has a compact support.

Thus, the wage faced before a wage event is assumed to play no role in determining the new base wage rate faced.⁶ We should note that when A2 is satisfied a worker employed at base wage \bar{y} loses the possibility of return to \bar{y} after a wage event occurs. Hence, the base wage of an employed worker can increase or decrease through time.

The arrival rate of a layoff obviously depends on the state currently occupied as it is zero when unemployed. Further, the arrival rate of layoffs may depend on seniority (in the sense defined above). Typically, it

⁶The wage distribution from which an individual samples clearly depends on his fixed characteristics z . If the vector of characteristics are captured in some index of human capital, then one can envisage that workers with higher human capital draw from some rightward translation of the distribution F .

is argued the probability of being laid off decreases with the duration of an employment spell. To allow for this, let $\mu(t_1)$ denote the arrival rate of a layoff at time t_1 since last unemployed and assume

A3: $\mu(\cdot)$ is a non-increasing function of t_1 .

This assumption appears to agree with empirical evidence. For mathematical convenience, assume a laid off worker initially faces a wage rate of zero. This harmless technical restriction guarantees a laid off worker will act as if he prefers unemployment after a layoff.]

The unemployment insurance (UI) payments paid to the unemployed worker may depend on the duration of the current spell. Let $u(t_2)$ denote the UI flow to a worker who has been unemployed for time t_2 in a spell. Throughout, $u(\cdot)$ is assumed to be bounded and piece-wise continuous and differentiable. Three alternative assumptions can be used.

A4: The UI function $u(\cdot)$ is a non-increasing (bounded measurable) function of t_2 .

A4': The UI function $u(t_2) = u$, a constant, for all t_2 .

A4'': The UI function is a strictly increasing (bounded measurable) function of t_2 .

In many countries the UI system is such that A4 is satisfied. In the United States, for example, UI payments are typically terminated if the duration of a spell of unemployment exceeds 26 weeks. The UI system in a few countries will satisfy A4'. Low paid workers in Britain, for example, get the same payment per period independent of the duration of unemployment. We know of no UI system which satisfies A4''.

An unemployed worker faces the parameters $F_2(\cdot)$ and λ_2 , whereas the employed individual faces $F_1(\cdot)$, λ_1 and u . Typically μ and λ_i , $i = 1, 2$ are assumed to be independent of z ; elements of the vector z are assumed to translate the mean of the distribution functions $F_i(\cdot)$ in a predictable way. Such complexities, although important, play no role in what follows.

As the framework described above is more general than that usually considered in the job search literature, the standard method of analysis is not immediately applicable. Later a different solution method is outlined. In the next section, however, the individuals are assumed to restrict themselves to "event strategies". An individual will be said to follow an event strategy if changes in state only take place immediately after an event, but of course, not necessarily after every event.

III. Event Strategies

Suppose an individual has been employed for a spell of length t_1 and currently faces a base wage rate y . Let $U_1(y, t_1)$ denote this individual's maximum expected discounted lifetime income given an event strategy is followed. Similarly, let $U_2(u, t_2)$ denote an individual's expected discounted lifetime income when he has been unemployed for time t_2 in the current spell and only event strategies are considered. Given the model specified in the previous section, it follows from standard techniques that⁷

$$U_1(y, t_1) = \frac{y + \mu(t_1)U_2(0) + \lambda_1\Psi_1(t_1) + U'_1(y, t_1)}{[\mu(t_1) + \lambda_1 + r]} \quad (3.1)$$

$$U_2(u, t_2) = \frac{u(t_2) + \lambda_2\Psi_2(t_2) + U'_2(u, t_2)}{[\lambda_2 + r]} \quad (3.2)$$

⁷See Sharma (1987).

where r is the discount factor, U'_i denotes the derivative with respect to t_i , and where

$$\begin{aligned}\Psi_1(t_1) &= \int \text{Max}\{U_1(e, t_1), U_2(u, 0)\} dF_1(e) \\ \Psi_2(t_2) &= \int \text{Max}\{U_1(e, 0), U_2(u, t_2)\} dF_2(e)\end{aligned}\tag{3.3}$$

The immediate implications of the above construction are stated in the following proposition:

PROPOSITION 3.1: Suppose the individual utilizes the event strategy which yields the greatest expected discounted lifetime income (henceforth referred to as the best event strategy) and A1-A3 hold. Then,

(i) if A4 or A4' hold:

- (a) U_1 is strictly increasing in y and nondecreasing in t_1
- (b) U_2 is decreasing in t_2 .

(ii) If A4" holds:

- (a) U_1 is strictly increasing in y and nondecreasing in t_1
- (b) U_2 is strictly increasing in t_2 .

Proof: It follows from (3.1) and A1-A3 that U_1 is strictly increasing in y and nondecreasing in t_1 if any versions of A4 hold. Further, from (3.2) it can be easily shown that U_2 decreases, is a constant, or increases with t_2 as A4, A4' or A4" hold respectively. ///

Proposition 3.1 implies if any version of A4 holds, the optimal strategy over all event strategies can be completely described by two functions, $R_1(t_1)$ and $R_2(t_2)$. These two functions act as generalized versions of a reservation wage, a concept used in most, if not all, studies on job search. The precise role of these two functions is summarized below.

PROPOSITION 3.2: Suppose the individual utilizes the best event strategy and A1-A3 and any version of A4 holds, then

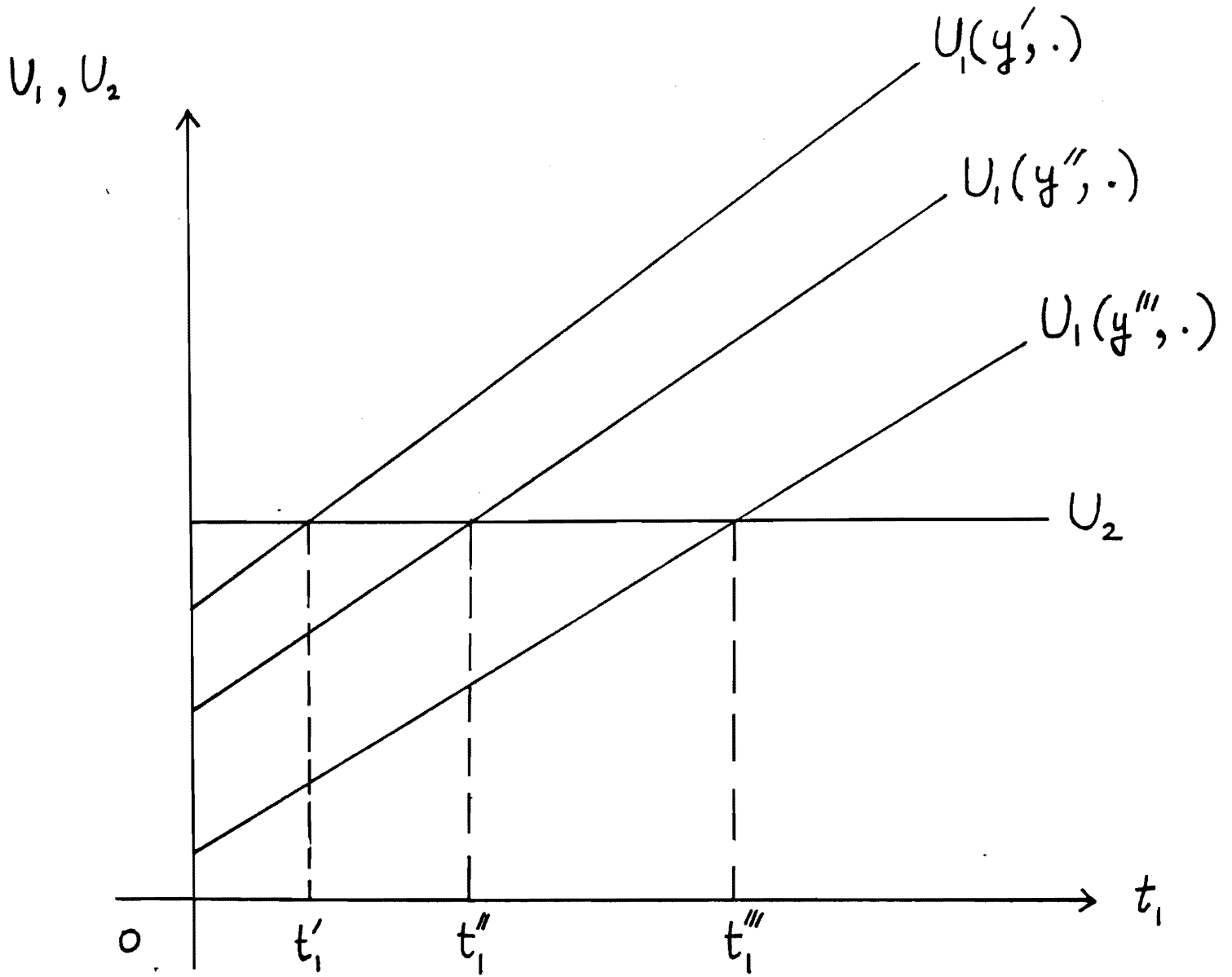
- (i) (a) $U_1(y, t_1) \geq U_2(u, 0)$ as $y \geq R_1(t_1)$
- (b) $U_1(y, 0) \geq U_2(u, t_2)$ as $y \geq R_2(t_2)$
- (ii) If A4 or A4' hold:
 - (a) R_1 is a non-increasing function of t_1 .
 - (b) R_2 is a non-increasing function of t_2 .
- (iii) If A4" holds
 - (a) R_1 is a non-increasing function of t_1 .
 - (b) R_2 is an increasing function of t_2 .

Proof: The existence of $R_1(t_1)$ follows as $U_1(y, t_1)$ increases with y , whereas $U_2(u, 0)$ is independent of y . Further, as $U_1(y, t_1)$ is a non-decreasing function of t_1 , $R_1(t_1)$ is a non-increasing function of t_1 . The existence of $R_2(t_2)$ is established in a similar fashion as $U_1(y, 0)$ increases with y and $U_2(u, t_2)$ is not influenced by y . From Proposition 3.1 it now follows that $R_2(t_2)$ increases with t_2 if A4" holds whereas $R_2(t_2)$ is non-increasing in t_2 when A4 or A4' holds. ///

The above result is illustrated by Figure 1. The expected return to employment at three different base wage rates is indicated by $U_1(y', t_1)$, $U(y'', t_1)$ and $U(y^*, t_1)$ where $y' > y'' > y^*$. As U_2 is independent of t_1 , it follows that $y' = R_1(t_1')$, $y'' = R_2(t_2'')$ and $y^* = R(t_2^*)$ where $t' < t'' < t^*$. Thus, for example, if the individual has been employed for less than time t'' in the current spell when base wage y'' is offered, then the offer is rejected and unemployment is the preferred state.

A consequence of Proposition 3.1 is that when the best event strategy is used, given any of the above sets of restrictions, an individual's labor

FIGURE 1



market history can be described by a SMP or a MP. The details are presented in the following claim which follows from the earlier propositions.

PROPOSITION 3.3: Suppose A1-A3 hold and the individual utilizes the best event strategy. Then,

- (i) if any version of A4 holds, the individual's labor market history can be described by a continuous time two-state SMP⁸ whose hazard rates can be written as

$$\Pi_1(t_1, y, X, Z, s_0) = \mu(t_1) + \lambda_1 F_1[R_1(t_1)] \quad (3.5)$$

$$\Pi_2(t_2, u, X, Z, s_0) = \lambda_2 (1 - F_2[R_2(t_2)]) \quad (3.6)$$

- (ii) If A4 or A4' hold

- (a) Π_1 is a non-increasing function of t_1
 (b) Π_2 is a non-increasing function of t_2 .

- (iii) If A4" holds

- (a) Π_1 is a non-increasing function of t_1
 (b) Π_2 is a strictly decreasing function of t_2 .

⁸Note that assumption set A1-A4(A4') gives

$$\lim_{t_1 \rightarrow \infty} F_1[R_1(t_1)] > 0 \text{ and } \lim_{t_2 \rightarrow \infty} (1 - F_2[R_2(t_2)]) > 0$$

This implies that the individual will continue to change states forever, i.e., neither of the states is an absorbing state. Hence, the distribution function describing the completed spells in state i will be a proper distribution function.

Let $v = (v_1, v_2)$ denote the stationary probability vector of the embedded Markov chain. Let η_1, η_2 be the expected duration in states 1 and 2, respectively. Then, (by Theorem in Heyman and Sobel, p. 327), the limiting or stationary probabilities of the semi-Markov process defined are:

$$\theta_1 = v_1 \eta_1 / [v_1 \eta_1 + v_2 \eta_2], \quad \theta_2 = v_2 \eta_2 / [v_1 \eta_1 + v_2 \eta_2].$$

The interpretation of θ_1 and θ_2 is that if the individual is observed over a fairly long time, one would expect him to be unemployed θ_2 proportion of the time and employed for θ_1 proportion of the time. Alternatively, θ_1 can be thought of as the expected proportion of a large number of homogeneous workers who will be in state 1, in the long run. Under this interpretation θ_2 can be regarded as the "natural" rate of unemployment for the particular group of workers.

To complete this section, restrictions are imposed which guarantee the individual's labor market history can be described by a MP. Intuitively, we require conditions such that the expected return to being in a particular state is independent of the duration in that state.

PROPOSITION 3.4: Suppose the individual utilizes the best event strategy.

If A1-A3, A4' hold with the added restrictions

$$(i) \quad \mu(t_1) = \mu \quad (\text{a constant}) \quad \text{for all } t_1, \quad (3.7)$$

and

$$(ii) \quad \phi(t_1) = 0 \quad \text{for all } t_1, \quad (3.8)$$

then an individual's labor market history can be described by a continuous time two-state MP. Further, the transition rates are given by

$$\Pi_1(y, t_1, X, Z, s_0) = \mu + \lambda_1 F_1(R) \quad (3.9)$$

$$\Pi_2(u, t_2, X, z, s_0) = \lambda_2 [1 - F(R)] \quad (3.10)$$

where $R = R_1(t_1)$ for all t_1 and t_2 .

Proof: The assumptions made guarantee that the value functions U_i , $i = 1, 2$ do not depend on the time spent in a state during the current spell. Thus R_i are independent of the duration of a spell in a particular state. Also, since $R_1(0) = R_2(0)$ the result follows from (3.5) and (3.6) ///

The above analysis shows that the structure of the two-state "Markov" model is very simple. In each state the optimizing individual uses a reservation wage strategy, and the resulting hazard rate besides depending on the state itself may depend on the duration in that state.⁹

⁹If employed workers are allowed the option of keeping their current wage when a wage "shock" occurs, an individual's labor market history is still a SMP. The structure of the process is however different from that previously obtained. Each wage level constitutes a separate state

IV. Optimal Strategies

The results obtained in the previous section were based on the restriction that an individual only considered event strategies. The purpose of this section is to investigate when such a strategy can be guaranteed to be the one which maximizes expected discounted lifetime income. This is of interest because we will show that when an event strategy is optimal the individual's transitions between states can be represented by a SMP or a MP on a reasonably simple state space. If the best event strategy is not optimal within a broader class of strategies such a representation may not be possible. In fact we show below that for an important class of popularly used models this is in fact the case.

To achieve the above goal it is shown that two stopping time functions can describe the strategy used by an income maximizing individual. This is achieved in an Appendix. In particular, the objective is to construct $V_1(y, t_1)$, the maximum expected discounted income given base wage y and t_1 the duration in the current employment spell, and $V_2(y, t_2)$ the maximum expected discounted income given unemployment insurance system specified by u and t_2 the length of the current unemployment spell. Of course these value functions need not exist. The following proposition states the conditions under which these value functions are well-defined.

PROPOSITION 4.1: Given A1, A2, A3, A4' or A4" hold

- (i) $V_1(y, t_1)$ and $V_2(y, t_2)$ exist for all y, t_1, t_2 .
- (ii) V_i is a non-decreasing function of $t_i, i = 1, 2$.

(employment now being defined as a continuum of wage states). Also, only individuals who are laid off leave employment.

- (iii) $V_1(\cdot, t_1)$ is a strictly increasing function of y if $V_1(y, t_1) > V_2(u, 0)$.

Proof: See Appendix.

Two implications of the above claims are stated in the following proposition.

PROPOSITION 4.2: (i) Given A1, A2, A3 and A4' or A4" hold, an event strategy yields the maximum expected discounted lifetime income and thus an individual's labor market history can be described by a two-state SMP such that the hazard rate out of either state decrease with duration.

- (ii) Given A1, A2, A3 and A4' hold with (3.7) and (3.8) being satisfied, an event strategy yields the maximum expected discounted lifetime income and thus the individual's labor market history can be described by a two-state MP with hazard rates given by (3.9) and (3.10).

Proof: Suppose $V_1(y, t_1) \geq V_2(y, 0)$ for some y and t_1 . Proposition 4.1(b) establishes that $V_1(y, t'_1) \geq V_2(y, 0)$ for $t'_1 > t_1$. This guarantees that an individual utilizing an income maximizing strategy will only leave employment immediately after an event (but not necessarily after every event). A similar argument establishes that such an individual will only leave unemployment immediately after an event (but not necessarily after every event). The claims made now follow from Proposition 3.3. ///

For the rest of this section we assume that A1-A4 hold. Further, to focus on essentials we assume (3.7) and (3.8) hold. Not imposing (3.7) and (3.8) merely complicates the analysis (without offering any additional insights) and does not alter the general conclusions.

Suppose the individual has been unemployed for time t_2^* when a new wage offer y is received. This situation is illustrated in Figure 2. From the claims made in Proposition 3.1 it is clear that by following the best event strategy this offer should be accepted only if $t_2^* > t(y)$. As illustrated in Figure 2 a "high" wage offer y^* is always accepted because $U_1(y^*) > U_2(t_2^*)$ for all $t_2^* \geq 0$. Similar reasoning establishes that "low" wage offer y' will always be rejected.

We now enlarge the strategy space of the individual. Specifically, suppose the worker can accept wage offer y for a short period of time, ϵ and then return to unemployment. By following this plan the worker "re-initializes" the UI payment flow as $u(0)$ [greater than $u(t_2)$, for $t_2 > 0$] is received initially when returning to unemployment. For ϵ small enough, but still strictly positive, it is not difficult to imagine this option yielding a greater expected return than any event strategy.

To formalize the above intuition, suppose wage offer y is received when unemployed and the following three options are considered by the individual:

- (a) Accept the offer and remain employed at least until the next event (wage or layoff event).
- (b) Reject the offer and remain unemployed at least until the next wage offer is received.
- (c) Accept offer y but return to unemployment after time ϵ , unless an event occurs first. If a wage event occurs accept it until an event occurs or until the current employment duration is ϵ . If a layoff event occurs first, return to unemployment.

Given the above options, the strategy which yields the maximum expected discounted lifetime income will be termed the Temporary-employment strategy.

We will denote it by $TE(\epsilon)$ where ϵ indicates the minimum planned length of such employment. It should be stressed that the time interval ϵ is not a choice variable but taken as a given parameter of the decision framework.¹⁰

We can interpret the above enlargement of the strategy space as allowing the worker to engage in "temporary" employment. The individual holds the temporary job as long as it takes to become eligible for unemployment insurance. Most UI systems require that a worker be employed for a certain length of time before he becomes eligible for UI payments. In this sense, ϵ is fixed and taken to be an institutional constraint. Of course, while holding a "temporary" job an individual receives other job offers and may transit into a "permanent" job or another "temporary" job. If fired from a temporary job after an employment spell of less than ϵ , whether he receives UI payments again depends on the UI system.

Let $D_1(y, \epsilon)$ denote the maximum expected return to employment at wage y . The expected return to unemployment when the $TE(\epsilon)$ -strategy is used and t_2 is the current duration of the spell is indicated by $D_2(u, t_2, \epsilon)$. Finally, $Q(y, \epsilon, \tau)$ denotes the expected return to option (c) above, when wage y is faced and the duration in employment is $\tau (\leq \epsilon)$.

Utilizing techniques used in Section III it follows that

$$D_1(y, \epsilon) = \frac{[y + \mu D_2(u, t_2, \epsilon) + \lambda_1 \Phi_1(y, \epsilon)]}{r + \mu + \lambda_1} \quad (4.1)$$

where $\Phi_1(y, \epsilon) = \int \text{Max}\{D_1(e, \epsilon), D_2(u, 0, \epsilon)\} dF_1(e)$

$$D_2(u, t_2, \epsilon) = \frac{[u(t_2) + \lambda_2 \Phi_2(u, t_2, \epsilon) + D_2'(u, t_2, \epsilon)]}{r + \lambda_2} \quad (4.2)$$

¹⁰Note when an individual uses an $TE(\epsilon)$ -strategy he is not restricted to changing state immediately after an event. He can if he so chooses, leave a state at anytime. Also, note that an event strategy is a $TE(\epsilon)$ -strategy.

where $\Phi_2(u, t_2, \epsilon) = \int \text{Max}(D_1(e, \epsilon), D_2(u, t_2), Q(e, \epsilon, 0)) dF_2(e)$ and D_2' is the derivative with respect to t_2 . Further,¹¹

$$Q(y, \epsilon, r) = \frac{[y + \mu D_2(u, t_2, 0) + \lambda_1 \Phi_3(y, \epsilon, r) + Q'(y, \epsilon, r)]}{r + \mu + \lambda_1}$$

$$\text{where } \Phi_3(y, \epsilon, r) = \int \text{Max}[D_1(e, \epsilon), Q(e, \epsilon, r)] dF_1(e), \text{ and,} \quad (4.3)$$

$$Q(y, \epsilon, r) = D_2(u, t_2, 0) \text{ if } r = \epsilon.$$

It is clear from the enlarged strategy space used above, that the expected return to a TE(ϵ)-strategy (for any given $\epsilon > 0$) is at least as great as when any event strategy is used. The next proposition states that if the given ϵ is small enough the TE(ϵ)-strategy dominates all event strategies.

PROPOSITION 4.3: Given A1-A4 hold with (3.7) and (3.8) being satisfied, for small enough ϵ ($\epsilon > 0$), the TE(ϵ)-strategy yields a strictly greater expected discounted lifetime income than an event strategy.

Proof: From (4.3) it follows that by making the given ϵ small enough we can make $Q(y, \epsilon, r)$ as close as we like to $D_2(u, 0, \epsilon)$. Further, $D_2(u, 0, \epsilon) \geq U_2(u, 0)$, where $U_2(u, 0)$ is defined by (3.2). From (4.1) and (4.2) it can be seen that $D_2(u, 0, \epsilon)$ is a non-increasing function of the given ϵ . The claim made now follows as $U_2(u, t_2) < U_2(u, 0)$ for $t_2 > 0$ from A.4.

///

. It should be noted that there exists a TE(ϵ)-strategy for every given ϵ and the expected return to a TE(ϵ)-strategy is greater the smaller is ϵ .

¹¹Note we assume that if an individual is fired from a "temporary" job he receives UI payments but that he will not be eligible if he voluntarily leaves a job with current employment duration of less than ϵ .

This, of course, implies that there does not exist a "best" $TE(\epsilon)$ -strategy for all $\epsilon > 0$. Nevertheless, as shown in Proposition 4.3 for ϵ small enough the expected return is greater than with all event strategies.

Suppose for a given ϵ , the $TE(\epsilon)$ -strategy dominates all event strategies. In this case an unemployed worker's strategy can be described by two functions, $R_{21}(t_2)$ and $R_{22}(t_2)$. In particular, if the worker has been unemployed for time t_2 and is then offered wage w , he will

- (a) reject the offer if $w < R_{21}(t_2)$
- (b) accept the offer for at most time ϵ , if $R_{21}(t_2) \leq w < R_{22}(t_2)$, where $R_{21}(t_2) < R_{22}(t_2)$.
- (c) accept the offer at least until the next event if $w \geq R_{22}(t_2)$.

If the $TE(\epsilon)$ -strategy does not strictly dominate all event strategies $R_{21}(t_2) = R_{22}(t_2) = R_2(t_2)$.

The next Proposition states that an individual's labor market history cannot be described by a two-state SMP if a $TE(\epsilon)$ -strategy is used.

PROPOSITION 4.4: For given ϵ , if the $TE(\epsilon)$ -strategy strictly dominates an event strategy, then an individual's labor market history cannot be described by a continuous time two-state SMP.

Proof: As shown above, given a $TE(\epsilon)$ -strategy strictly dominates an event strategy, there exists a set of wage offers which will be accepted for at most time ϵ and a set of wages which will be accepted at least until the next event. It is now straightforward to show that the transition rate out of employment depends on the wage currently faced, and thus a two-state SMP cannot describe the individual's labor market history. ///

In the job search literature several authors have presented conditions which guaranteed a declining reservation wage with the duration of a spell

of unemployment [see, for example, Burdett (1979)]. The reason for this result is that it is assumed an individual does not leave the state of employment after entering it. Once it is recognized that individuals can move between states the declining reservation wage result disappears. This also demonstrates the dangers associated with considering only the flow from one state to another and ignoring the direct or indirect feedback.

Assumptions A1-A4 guaranteed that the expected return to unemployment strictly declines with the duration of a spell when an event strategy is used. This, in turn, implied there exists a $TE(\epsilon)$ -strategy for $\epsilon > 0$ which yields a greater expected return. This argument is general in the following sense. If assumptions are made which guarantee the expected return to unemployment declines with the duration of a spell when an event strategy is used, then there exists a $TE(\epsilon)$ -strategy for some $\epsilon > 0$ which yields a strictly greater expected return.

To illustrate the above claim suppose a "scarring" model is briefly considered. Suppose A1-A4 are used when (3.7) and (3.8) are assumed to hold. As shown in Proposition (3.4), under this set of assumptions the expected return to unemployment does not change with the duration of a spell. Now assume the arrival rate of wage offers, λ_2 , is not a constant, as previously assumed, but declines with the duration of a spell of unemployment. Hence, the longer the duration of a spell of unemployment the less likely an offer is received. It is straightforward to demonstrate that such a model implies the expected return to unemployment declines with duration and also obtain a declining reservation wage result. However, by accepting a job for a reasonably short period of time and then returning to unemployment a worker can obtain an expected return greater than from any event strategy, i.e., there exists a $TE(\epsilon)$ -strategy for some $\epsilon > 0$ which

strictly dominates any event strategy.

All along we have been treating employment and unemployment as single states. Clearly, in order to examine the flows within each state we need to expand the state space. To take account of heterogeneity among the unemployed, one could identify an individual as being in a different state depending on the number of past unemployment spells. The hazard rate besides being different for each unemployment state could also depend on the duration in that state.¹² Each employment state could be a particular kind of job or set of jobs. Realistically, each job should be characterized by a base wage, expected wage growth on the job and a layoff rate. However, given these dimensions for describing a job the search strategy of the individual becomes impossible to derive. It depends in a complicated manner on the current job as well as the pattern of wage growth on alternative jobs. Moreover, with such a general description of jobs there seems to be no parsimonious state space on which an individual's labor market history can be described.

Conclusions

Labor turnover is a probabilistic phenomenon and its study requires a theoretical and empirical framework in which the role of uncertainty is explicit. The models presented in this paper were set in an intertemporal decision framework with imperfect information, and the individuals movements between labor market states viewed as the realization of a (hopefully empirically tractable) stochastic process. These simple models permit a "structural" interpretation of labor market histories and can be used for

¹²Note that UI payments, given some maximum and minimum constraints, depend on some index of past earnings. This adds to the possible heterogeneity among the unemployed. See Welch (1977).

analyzing continuous time duration data.

The paper made explicit the assumptions underlying the various models considered and specified the state space on which the individual's labor market history could be described as a Markov or semi-Markov process. It was shown that relaxing some of the assumptions could lead to defining a new (enlarged) state space or losing the Markovian nature of the stochastic process describing the individuals labor market history.

The main point has been to illustrate that the characterization of the optimal strategy and the implied stochastic process describing an individual's movements between labor market states may not be robust to changes in assumptions, that certain institutional features of the labor market may be difficult to model in a dynamic context, and that an empirical researcher (given available data) should clearly specify the state space and the nature of the stochastic process implied by the theoretical model he has in mind before conducting an analysis of labor market flows. All models impose a special structure on labor market transitions and the estimates of parameters must be seen in this light.

APPENDIX

In this Appendix it is shown that a set of stopping rules, one for each state, can describe the income maximizing strategy of an individual in the framework considered. The method of proof is similar to that elaborated in Heyman and Sobel (1984). [See the book for references on Contraction Mappings and Markov Decision Processes.]

Suppose an individual has occupied state i for time t_1 in the current spell and faces base wage y_0 , if employed. If unemployed, y_0 can be thought of as the last wage offer received. Any particular future to be faced by the individual can be described by a sequence of doubletons

$$\omega = [(y_1, a_1), (y_2, a_2), (y_3, a_3), \dots]$$

where y_k denotes the base wage rate to be faced immediately after the k^{th} event occurs, and a_k then indicates the time from now until the k^{th} event. Let (Ω, β) be the measure space of all such sequences where β denotes the Borel sets of Ω .

The probability measure generating any particular future depends on the states chosen by the individual in the future. For the moment assume the individual under consideration plans to stay in state i (the current state) forever. Now, let $M_i(\cdot; y_0, t_1)$ denote the probability measure associated with possible futures given state i is occupied forever. Although not formally attempted here, it is reasonably straightforward to show that $M_1(\cdot; y_0, t_1)$ is constructed from $F_1(\cdot)$, λ_1 and $\mu(t_p)$ whereas $M_2(\cdot; y_0, t_2)$ is generated by $F_2(\cdot)$ and λ_2 . The immediate consequences of this construction are summarized below.

- CLAIM 1: (i) $M_2(B; y_0, t_2) = M_2(B; y_0, t'_2)$ for all t_2, t'_2 and $B \in \beta$.
- (ii) If $\mu(t_1)$ is a constant for all $t_1 \geq 0$, then $M_1(B; y_0, t_1) = M_1(B; y_0, t'_1)$ for all t_1, t'_1 and $B \in \beta$.
- (iii) If $\mu(t_1)$ is strictly decreasing for all $t_1 \geq 0$, then $M_1(\cdot; y_0, t_1)$ is such that $\Pr[w=0]$ decreases with k .

Claim 1(i) above follows because $F_2(\cdot)$ and λ_2 are independent of the duration of unemployment. Given $\mu(\cdot)$ is a constant, claim 1(ii) follows since $F_1(\cdot, \bar{y})$ and λ_1 do not depend on the length of an employment spell. Further, if $\mu(t_1)$ is decreasing in t_1 , the probability that the k^{th} event is a layoff decreases with k .

The two probability measures constructed above are based on the restriction that the individual does not change state. Nevertheless, these two measures can be used to investigate when the individual should change state. In each state an individual chooses a strategy for changing state which can be described by a stopping time function $T_i: \Omega \rightarrow \mathbb{R}$, $i = 1, 2$. Any stopping time function satisfies the following restrictions:

- (i) $T_i(\omega) \geq 0$ for all $\omega \in \Omega$, and
- (ii) if ω and ω' have the same first k elements and $T_i(\omega) \leq a_k$, then $T_i(\omega) = T_i(\omega')$ for all $\omega, \omega' \in \Omega$.

Note (ii) above implies that any strategy used depends only on what has been observed and the probability laws generating the future, but not knowledge of the future. Let \mathcal{F} denote the set of all functions $T_i(\omega)$. The individual chooses the stopping rules $T_i(\omega)$, $i = 1, 2$ to maximize expected discounted lifetime income.

Let Γ_i denote the expected discounted income given the individual is currently in state i , the stopping rule T_i is used and then optimal decisions are taken in the future. It follows that

$$\Gamma_1(y_0, t_1, T_1) = \int_{\Omega} \gamma_1(\omega, y_0, t_1, T_1) M_1(d\omega, y_0, t_1) \quad (\text{A.1})$$

where

$$\begin{aligned} \gamma_1(\omega, y_0, t_1, T_1) = & \sum_{j=0}^{k-1} \int_{a_j}^{a_{j+1}} w(y_j, t_1+s) e^{-rs} ds \\ & + \int_{a_k}^{T_1(\omega)} w(y_k, t_1+s) e^{-rs} ds + e^{-rT_1(\omega)} V_2(y_k, 0) \end{aligned} \quad (\text{A.2})$$

where $a_k \leq T_1(\omega) \leq a_{k+1}$, r is the discount rate and V_2 is defined by

$$V_2(y, t_2) = \text{MAX}_{T_2 \in \mathcal{F}} \Gamma_2(y, t_2, T_2) \quad (\text{A.3})$$

assuming the maximum exists. Similarly,

$$\Gamma_2(y_0, t_2, T_2) = \int_{\Omega} \gamma_2(\omega, y_0, t_2, T_2) M_2(d\omega, y_0, t_2)$$

where

$$\gamma_2(\omega, y_0, t_2, T_2) = \int_0^{T_2(\omega)} u(t_2+s) e^{-rs} ds + e^{-rT_2(\omega)} V_1(y_k, 0)$$

where $a_k \leq T_2(\omega) \leq a_{k+1}$ and

$$V_1(y, t_1) = \text{MAX}_{T_1 \in \mathcal{F}} \Gamma_1(y, t_1, T_1)$$

Under the assumptions of Proposition 4.1 the stopping rules T_i are well defined [see DeGroot (1970), Elfving (1967)]. Many others have studied similar questions in different frameworks, see Chow and Robbins (1961), McCall (1965), MacQueen and Miller (1960), and Mortensen (1986). Also, the value functions V_1 and V_2 are bounded measurable functions. It is straightforward now to use the method in Sharma (1987) to show that the value functions exist and are unique.

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