UNIVERSIDADE FEDERAL DE SANTA CATARINA
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CONTRIBUIÇÕES PARA A ENUMERAÇÃO E PARA A ANÁLISE DE MECANISMOS E MANIPULADORES PARALELOS

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Roberto Simoni

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Roberto Simoni

CONTRIBUIÇÕES PARA SÍNTESE DO NÚMERO E ANÁLISE DE MECANISMOS E MANIPULADORES PARALELOS

Esta Tese foi julgada adequada para a obtenção do Título de “Doutor em Engenharia Mecânica”, e aprovada em sua forma final pelo Programa de Pós-Graduação em Engenharia Mecânica.

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The fascination of this work arises from the fact that understanding how a mechanism or a parallel manipulator functions is fairly easy, but comprehending how it was originated and why it was designed in a particular form in which it exists is more difficult.

Erdman (1993)

RESUMO

A fase de projeto conceitual de mecanismos e manipuladores paralelos, i.e. estruturas cinemáticas, destina-se ao desenvolvimento da concepção da cadeia cinemática. As etapas fundamentais para o desenvolvimento da concepção da cadeia cinemática são síntese e análise. A síntese corresponde à enumeração de concepções e a análise corresponde à seleção das concepções mais promissoras considerando os requisitos de projeto. O objetivo deste trabalho é aplicar ferramentas da teoria de grupos e teoria de grafos para a enumeração e para a análise de estruturas cinemáticas. A enumeração será desenvolvida de forma sistemática em três níveis: enumeração de cadeias cinemáticas, enumeração de mecanismos e enumeração de manipuladores paralelos. A aplicação de ferramentas da teoria de grafos e grupos permite desenvolver novos métodos para enumeração e, consequentemente, obter novos resultados. A análise será simplificada considerando um novo método que avalia as simetrias das cadeias cinemáticas. Uma cadeia cinemática é representada de forma unívoca através de um grafo. A representação através do grafo permite a manipulação computacional do problema de enumeração de cadeias cinemáticas. A aplicação de ferramentas integradas da teoria de grafos e teoria de grupos permite identificar as simetrias das cadeias cinemáticas através do grupo de automorfismos do grafo e, assim, é possível identificar quais são as possíveis escolhas de base para novos mecanismos e avaliar quais são as possíveis escolhas de base e efetuador final para manipuladores paralelos. O primeiro nível da síntese corresponde à enumeração de cadeias cinemáticas com determinada mobilidade, número de elos, número de juntas que operam num determinado sistema de helicoides. O segundo nível da síntese corresponde à enumeração de mecanismos. Um mecanismo é uma cadeia cinemática com um elo escolhido para ser a base. Assim, a enumeração de mecanismos consiste em determinar todas as possíveis escolhas de bases para uma determinada cadeia cinemática. O principal conceito empregado neste nível é o de simetria de grafos não coloridos e órbitas do grupo de automorfismos. O terceiro nível da síntese corresponde à enumeração de manipuladores paralelos. Um manipulador paralelo é uma cadeia cinemática com um elo escolhido para ser a base e outro para ser o efetuador final. Em outras palavras, um manipulador paralelo é um mecanismo com um elo escolhido para ser o efetuador final. Assim, a enumeração de manipuladores paralelos consiste em determinar todas as possíveis escolhas de efetuador final para um determinado mecanismo. O principal conceito empregado neste nível é o de simetria de grafos coloridos e órbitas do grupo de automorfismos de grafos coloridos. Na etapa de análise das concepções enumeradas serão abordadas propriedades bem estabelecidas na literatura: mobilidade, variedade, conectividade, grau de controle, redundância e simetria. Mobilidade e variedade são propriedades globais das estruturas cinemáticas. Conectividade, grau de controle e redundância são propriedades locais, i.e. entre dois elos da estrutura cinemática e são dadas por matrizes $n \times n$, onde n é o número de elos da cadeia. A simetria pode ser considerada uma propriedade global e/ou local da estrutura cinemática. A aplicação de ferramentas integradas da teoria de grafos e teoria de grupos permite demonstrar que as propriedades locais são invariantes pela ação do grupo de automorfismos do grafo, i.e. elas são propriedades simétricas. Desta forma, a representação matricial é reduzida de $n \times n$ para $o \times n$, onde o é o número de órbitas do grupo de automorfismos do grafo associado à estrutura cinemática. Essa abordagem permite simplificar a análise de estruturas cinemáticas apenas considerando as simetrias das cadeia associadas.

Palavras-chave: Projeto conceitual. Enumeração. Análise. Cadeias cinemáticas. Mecanismos. Manipuladores paralelos. Teoria de grupos. Teoria de grafos. Teoria de helicoides. Grupo de automorfismos. Simetria. Ações. Órbitas.

ABSTRACT

The conceptual design of mechanisms and parallel manipulators corresponds to the enumeration of kinematic structures (synthesis) and the selection of the most promising solutions (analysis). In mechanisms and machines theory, the conceptual design is known by several expressions such as: structural synthesis, Grübler synthesis, topological synthesis, etc. This thesis deals with enumeration and analysis of kinematic structures with a number of links and joints related by the mobility equation. A kinematic structure can be uniquely represented by a graph whose vertices correspond to links and whose edges correspond to joints, this approach simplify the conceptual design problem. The enumeration will be considered into three levels: the first level corresponds to the enumeration of kinematic chains, which are a set of links connected by joints; the second level corresponds to the enumeration of mechanisms, which are kinematic chains with one link fixed on the base; and the third level corresponds to the enumeration of parallel manipulators, which are mechanisms with a link selected to be the end-effector. The analysis of the kinematic structures enumerated will be simplified exploring the symmetries of the associated graph. In this work, we apply integrated tools of the graph theory and group theory to capture the internal symmetry of kinematic chains and mechanisms leading, respectively, mechanisms and parallel manipulators. The main concept applied is the orbits of the automorphisms group of a graph, i.e. graph symmetry, which represents a kinematic chain. Using this approach, it is possible to enumerate precisely all the conceptions of mechanisms and parallel manipulators with appropriate attributes.The application of integrated tools of graph and group theory permits a simplification of kinematic analysis of kinematic structures. We prove that important properties of kinematic structures are invariants by the action of the automorphism group of the associated graph, i.e. they are symmetrical properties. Considering that kinematic chains have symmetries, it is possible to apply group theory to reduce the matrix representation of properties of the kinematic structures (connectivity, degrees-of-control and redundancy) and consequently facilitate the structural analysis.

Keywords: Conceptual design. Enumeration. Analysis. Kinematic chains. Mechanisms. Parallel manipulators. Graph theory. Group theory. Screw theory. Isomorphisms. Automorphisms. Symmetry. Actions. Orbits.

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LIST OF ABBREVIATIONS

IFToMM	International Federation for the Promotion of Mechanism and Machine Science
ASME	American Society of Mechanical Engineers
IFR	International Federation of Robotics
DoF	Degrees of Freedom

LIST OF SYMBOLS

G	Group
X_n	Symmetric group
$Aut(G)$	Automorphism group
$\mathscr{O}_x$	The orbit of a point $x \in X$
$A = (a_{ij})$	Adjacency matrix
X	Kinematic chain or graph
M	Mobility
λ	Order of screw system
n	Number of links
j	Number of joints
f_i	Degrees of relative motion permitted by joint i
ν	Number of independent loops of the kinematic chain
M'	Biconnected subchain mobility
V	Variety
C_{ij}	Connectivity
K_{ij}	Degrees-of-control
R_{ij}	Redundancy

CONTENTS

1 INTRODUCTION

In this thesis, we study two classical problems of conceptual design of mechanisms and parallel manipulators, kinematic structures in short: enumeration and analysis. Conceptual design studies play an important role in the design of mechanisms and parallel manipulators. In the conceptual design phase, many design alternatives as possible are generated and evaluated against the functional requirements and the most promising concept is selected for design detailing (TSAI, 2001). In the conceptual design, the characteristics of mechanisms are entirely determined by the interconnectivity pattern among the links and are unaffected by changes in the metric properties; the interconnectivity pattern includes the mobility, the number of links, the number of joints and the order of screw system to which all the joint screws belong (YAN, 1998; TSAI, 2001; MRUTHYUNJAYA, 2003; SUNKARI, 2006).

The focus of this thesis are two classical problems of conceptual design phase:

- **enumeration of kinematic structures**: enumeration of kinematic chains of mechanisms (selecting the base) and parallel manipulators (selecting the base and the end-effector) satisfying the mobility criterion, and

- **analysis of kinematic structures**: kinematic structures generated are analyzed to select the most promising kinematic chain for design detailing.

The enumeration and analysis developed in this thesis are related with the number synthesis which generates all the possible solutions using graph theory and combinatorial analysis (TSAI, 2001; MRUTHYUNJAYA, 2003).

This introduction contextualizes the problem of conceptual design of mechanisms and parallel manipulators in the literature, presents the motivation to work, the state of the art and the overview of the thesis.

1.1 LOCALIZATION AND DEFINITION OF THE PROBLEM

This section provides a brief review of the design process and a systematic methodology for creation of mechanisms and parallel manipulators. Design is the creation of synthesized solutions in the form of products or systems that satisfy customer's requirements (PAHL; BEITZ, 1996; TSAI, 2001). Design is a continuous process of refining customer's requirements into a final product, the process is iterative and the solutions are usually not unique.

Firstly, the problem addressed, i.e. enumeration and analysis of mechanisms and parallel manipulators, will be described according to two systematic methodologies of design: Tsai's methodology "Mechanisms Design: Systematic Design Methodology" (TSAI, 2001), and Back et al. methodology "Integrated Design of Products: Planning, Conception and Modelling" (BACK et al., 2008), developed in the Center for Integrated Development of Products (NeDIP/UFSC). These two methodologies are chosen because they are recent, Tsai's methodology is specific for mechanisms design and Back et al. methodology, developed in the UFSC, is a more detailed methodology.

According to Tsai (2001), the design process can be logically divided into three interrelated macro-phases:

- **Specification and planning**: in this phase the customer's requirements are identified and translated into engineering specifications, in terms of the functional requirements, time and money available for the development, and planning of design.

- **Conceptual design**: during this phase, as many design alternatives as possible are generated (using graph theory and combinatorial analysis) and evaluated against the functional requirements; the most promising concept is selected for design detailing. A rough idea of how the product will function and what it will look like is developed.

- **Product design**: in the last phase, a design analysis and optimization are performed, together with a simulation of the selected concept. Function, shape, material, and production methods are considered. Several prototype machines are constructed and tested to demonstrate the concept. An engineering documentation is produced and the design goes into the production phase.

Tsai's methodology for mechanisms design is summarized in the following steps (TSAI, 2001):

1 - Identify the functional requirements, based on customer's requirements, of a class of mechanisms of interest.

2 - Determine the nature of motion (i.e. planar, spherical, or spatial mechanism), DoF, type, and complexity of the mechanisms.

3 - Identify the structural characteristics associated with some of the functional requirements.

4 - Enumerate all possible kinematic chains that satisfy the structural characteristics using graph theory and combinatorial analysis.

5 - Sketch the corresponding mechanisms and evaluate each of them qualitatively in terms of its capability in satisfying the remaining functional requirements. This evaluation results in a set of feasible mechanisms.

6 - Select a most promising mechanism for dimensional synthesis, design optimization, computer simulation, prototype demonstration, and documentation.

7 - Enter the production phase.

Feedback among design phases mentioned above is usually needed to improve the design.

Back et al. (2008) present a systematic methodology for integrated design of products. Mechanisms and parallel manipulators design may be regarded as a process of product design. Back et al. methodology is decomposed into three macro-phases which are divided in eight different phases summarized below.

- **Project planning**: the first macro-phase involves the preparation of project product plan;

  1 - Project planning: it is for planning the new project by the business strategies of the company and the organization of work to be developed throughout the process. Identification of stakeholders and plan for management of communications.

- **Elaboration of the product design**: involves the development of the product design and manufacturing plan. It is divided into four phases and the main results of each one are:

  2 - Informational design: customer's requirements are identified and translated into design requirements considering different attributes: functional, ergonomic, safety, reliability, modularity, legal aspects, and so on. The design requirements are translated into design specifications in the form of geometry, material, color, size, actuation, and so on.

  3 - Conceptual design: as many design alternatives as possible are generated and evaluated against the functional requirements. Criteria are used to select the best conception.

4 - Preliminary design: technical and economic feasibility. Development of the final layout identifying requirements of form, material, safety, review of patents and legal aspects, and so on. Development of virtual prototyping and optimization.

5 - Detailed design: product documentation. Approval of the prototype, finalization of the components specifications, manufacturing plan.

- **Pilot production**: involves the execution of the manufacturing plan in the company production and the closure of the project. It is divided into three phases and the main results of each one are:

  6 - Preparation of the production: product liberation for pilot production and assembling validation.

  7 - Product marketing: product launch on the market, guidelines for final updates and release of mass production.

  8 - Product validation: evaluation of costumers/stakeholders satisfaction, monitoring performance, and so on.

Figure 1 shows the correspondence between Tsai's systematic mechanism design methodology and Back et al. integrated product design methodology. The methodology most cited for conceptual design of mechanisms and parallel manipulators is the Tsai's methodology (TSAI, 2001).

According to Tsai (2001), Back et al. (2008), and other authors, 75% of the manufacturing cost of a typical product is committed during the first two macro-phases. Decisions made after the conceptual design phase have only influence of 25% on the manufacturing cost. Therefore, it is critical that we pay sufficient attention to the product specification and conceptual design phases (TSAI, 2001). In particular, to parallel manipulators design the cost is high and the decisions must be the most corrects in the first phases of design. Thus, the conceptual design phase is very important and received attention of the academic community in recent years as indicated by Mruthyunjaya (2003) in the 41 pages long review paper "Kinematic structure of mechanisms revisited".

We note that the conceptual design in the Tsai's methodology consists of two engines: a generator and an evaluator. The focus of this thesis is in the use of the two engines (generator and evaluator) in the conceptual design phase of mechanisms and parallel manipulators which correspond to the macro-phase two of Tsai's methodology and to phase three of Back et al. methodology (see Figure 1). The generator corresponds to enumeration of kinematic structures, using graph theory and combinatorial analysis, satisfying the mobility criterion (see Equation 3.1 on page 37) and without

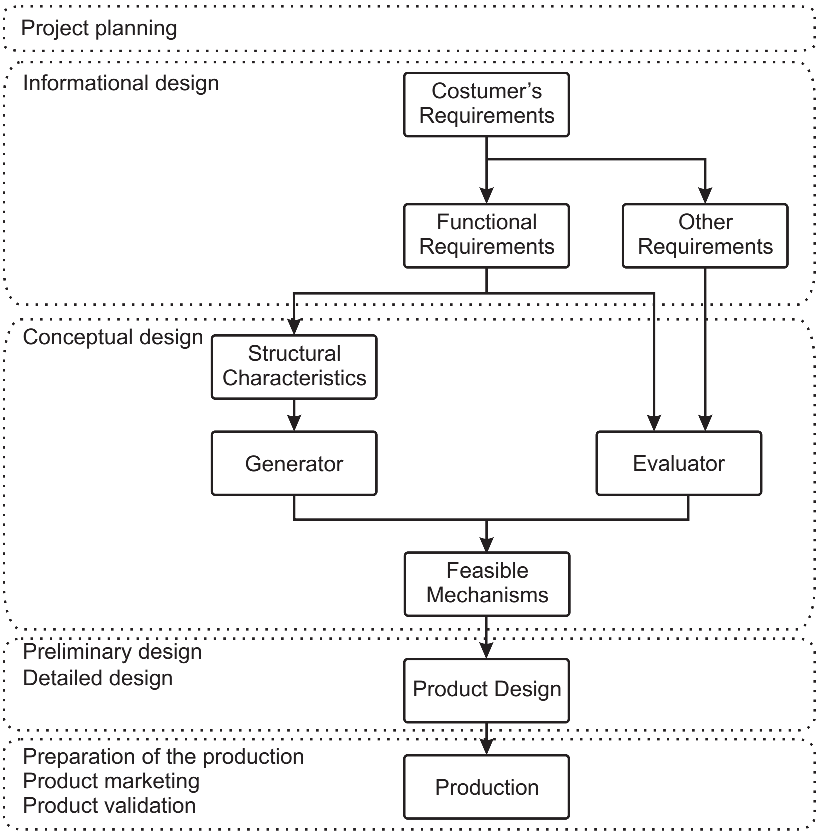


Figure 1 – Correspondence between Tsai's systematic mechanism design methodology and Back et al. integrated product design methodology.

isomorphisms. The generator engine of Tsai's methodology is also known in mechanisms and machines theory as enumeration of kinematic chains, enumeration of mechanisms, enumeration of parallel manipulators, number synthesis, structural synthesis, synthesis of mechanisms, Grübler's synthesis, topological synthesis, and so on. In fact, the generator corresponds to enumeration of kinematic structures and, therefore, between several denominations, we will use the term "enumeration" to describe this problem in this thesis. The evaluator corresponds to select the most appropriate kinematic structure, which satisfying the customer's requirements, for design detailing. Some functional requirements are translated into criteria for structural analysis (variety, connectivity, degrees-of-control, redundancy, symmetry, and so on) and then, using these criteria, it is possible to select the most appropriate kinematic structure.

1.2 THE STATE OF THE ART

This section presents the state of the art in enumeration and analysis of kinematic structures.

1.2.1 Enumeration of kinematic structures

The most important phase in the study of kinematic structure of mechanisms is the structural synthesis or classification and enumeration of kinematic chains with a given number of links and degree of freedom as related by the mobility equation (MRUTHYUNJAYA, 2003).

The approaches for enumeration are mainly founded on graph theory, group theory, Lie subgroups of displacements, screw theory and evolutionary morphology as shown in Table 1. The first two approaches are known as number synthesis and the last three approaches are known as type synthesis in mechanisms and machines literature. Number synthesis is an abstract approach and type synthesis is a geometrical approach of conceptual design.

Number synthesis is the process of finding the arrangements of a given number of links and joints which result in kinematic chains with the desired mobility (CROSSLEY, 1964). Number synthesis deals with the determination of the number of links, type and number of joints needed to achieve a given mobility of a desired mechanism and involves the enumeration of all feasible kinematic chains which satisfy these requirements. A complete list of kinematic chains, mechanisms and parallel manipulators are enumerated without isomorphisms based on the mobility criterion (see Equation 3.1 on page 37). In the enumeration, the joints are assumed to be 1-DoF, i.e. P or R joints. In terms of graphs, the number synthesis correspond to enumeration of a complete list of graphs that satisfy the mobility criteria, without isomorphic and improper graphs, where the vertices correspond to links and the edges correspond to joints. Number synthesis is considered an abstract approach and it is used to create new concepts of mechanisms and parallel manipulators.

Several methods have been developed for enumeration of kinematic chains (ALIZADE; BAYRAM, 2004; MRUTHYUNJAYA, 2003; TISCHLER et al., 1995a, 1995b; TSAI, 2001; SUNKARI; SCHMIDT, 2006; SIMONI et al., 2009, 2008). In particular, the technique presented by Tuttle et al. (1989b, 1989a), Tuttle (1996), Simoni (2008), Simoni et al. (2009) apply

Table 1 – Approaches for enumeration of kinematic structures.

Enumeration	Approach	References
Number Synthesis	Graph Theory	(TISCHLER et al., 1995a, 1995b; TSAI, 1998, 2001; ALIZADE; BAYRAM, 2004; ALIZADE et al., 2007; SUNKARI; SCHMIDT, 2006; SIMONI; MARTINS, 2007; SIMONI, 2008; SIMONI et al., 2009)
	Group theory	(TUTTLE, 1996; SIMONI et al., 2009, 2008)
Type Synthesis	Lie sub-groups of displacements	(HERVÉ, 1978, 1994, 1999; HERVÉ; SPARACINO, 1991; LI et al., 2004; ANGELES, 2004; TSAI, 1998; FANG; TSAI, 2002, 2004; FRISOLI et al., 2000; KONG; GOSSELIN, 2005, 2004a, 2007; HUANG; LI, 2002, 2003; LI; HUANG, 2003; CARRICATO, 2005; SIMONI; MARTINS, 2009)
	Evolutionary morphology	(GOGU, 2008, 2009)

group theory tools to enumeration of all mechanisms that a kinematic chain can lead. The technique presented by Simoni et al. (2008) apply group theory tools, in particular the concepts of symmetry, actions and orbits of the automorphism group of colored vertex graphs, for enumeration of parallel manipulators. The kinematic chains, mechanisms and parallel manipulators generated by these approaches are complex since your enumeration satisfy the mobility equation.

Type synthesis of parallel manipulators consists in finding all the possible types of parallel manipulators generating a specified motion pattern of the moving platform (KONG; GOSSELIN, 2007). Type synthesis is based on the selection of a particular type of mechanism (linkage, cam, gear, etc.). The selection depends to a great extent on the functional requirements of a machine and other considerations such as materials, manufacturing processes, and cost. In the type synthesis, a geometrical tool is introduced to define the type of joints, even generally the proposed methods use P and R joints, need to generate the proposed motion. These approaches consist basically of legs enumeration and assembling of these legs to form the parallel manipulators. The parallel manipulators generated by these technique are of base-platform type.

Type synthesis is generally based on Lie subgroups of displacements and screw theory and is considered a geometrical approach. Several works dealt with the problem of type synthesis (KONG; GOSSELIN, 2007, 2004a, 2005; FANG; TSAI, 2004, 2002; GOGU, 2008, 2009; LI et al., 2004; HUANG; LI, 2003; SICILIANO; KHATIB, 2008).

1.2.2 Analysis of kinematic structures

The main concepts used to analyze and classify kinematic chains, mechanisms and parallel manipulators are: mobility, variety, connectivity, degrees-of-control, redundancy and symmetry. These concepts are well established in mechanisms and machines literature.

The *mobility* (M) of a kinematic structure is the number of independent parameters required to completely specify the configuration of the kinematic chain in the space, with respect to one link chosen as the reference. The mobility may be calculated by the general mobility criterion (see Section 3.3 on page 37).

Hunt (1978) introduced the concept of connectivity. The *connectivity* (C) between two links of a kinematic structure is the relative mobility between the two links. The importance of the connectivity is emphasized by Hunt (1978), Tischler et al. (2001, 1995b), Liberati and Belfiore (2006), Belfiore and Benedetto (2000), Shoham and Roth (1997) which drives the efforts to find an algorithm for the numerical calculation of connectivity.

Tischler et al. (1995b) present the concept of *variety* (V) in kinematic structures which has application in the selection of actuated pairs. Tischler et al. (1995b) summarize the relationship between variety and connectivity by a series of conjectures and propositions proved later by Martins and Carboni (2007). If the Variety of a kinematic structure with j joints is $V = 0$, the actuated pairs may be selected at random.

Belfiore and Benedetto (2000) introduced the concept of degrees-of-control. The *degrees-of-control* (K) between two links of a kinematic structure is the minimum number of independent actuating pairs needed to determine the relative position between the two links, possibly leaving some other link-relative position undetermined.

Based on the concepts of degrees-of-control and connectivity we can introduce the concept of redundancy. The *redundancy* (R) between two links of a kinematic structure is the difference between the number of degrees-of-control and the connectivity between these links (BELFIORE; BENEDETTO, 2000; MARTINS; CARBONI, 2007).

Symmetry is another concept used for several authors to select the most promising kinematic structures (RAO, 2000; TISCHLER et al., 1998; PERNETTE et al., 1997; CAMPOS et al., 2008; HUANG; LI, 2003, 2002; HESS-COELHO, 2006). The kinematic and dynamic equations of motion of any mechanism or parallel manipulator should be as simple as possible since, generally, positions, velocities and accelerations are calculated in real time. Symmetric kinematic structures lead to considerable simplifications in kinematic and dynamic equations. Thus, it is important to identify symmetries in the kinematic chains in early stages of design and that is possible analyzing its associate graph.

Based on these concepts it is possible to classify the enumerated kinematic structures and to select the most promising for design detailing.

1.3 THESIS CONTRIBUTION

This thesis contributes to the conceptual design of mechanisms and parallel manipulators. We will address two steps of conceptual design, i.e. enumeration and analysis of mechanisms and parallel manipulators.

1.3.1 Contributions to the enumeration of mechanisms and parallel manipulators

The contribution is to develop the enumeration of kinematic chains, mechanisms and parallel manipulators, in a systematic procedure, applying integrated tools of graph theory, group theory and screw theory. The enumeration process will be consider into three levels: kinematic chains, mechanisms and parallel manipulators.

- **Level 1: Enumeration of kinematic chains:** From structural characteristics (number of links, number of joints, mobility, order of screw system) kinematic chains are enumerated. It is important to remember that a kinematic chain is an assembly of links and joints. The attributes of kinematic chains in this level are: number of links (n), number of 1-DoF joints (j), mobility (M), order of screw system (λ). The main tools considered in this level are graph theory and screw theory.

- **Level 2: Enumeration of mechanisms:** Each kinematic chain originates mechanisms selecting all different bases. It is important to remember that a mechanism is a kinematic chain with one of its components (links) taken as a frame (IONESCU, 2003). In terms of graph theory, a mechanism corresponds to a graph with one of its vertices detached (colored) to represent the fixed link (SIMONI et al., 2008). The attributes of mechanisms in this level are: number of links (n), number of 1-DoF joints (j), mobility (M), order of screw system (λ) and base of mechanism. The tools considered in this level are graph theory, group theory and screw theory; mainly the concepts of symmetry, actions and orbits of the automorphism group of non-colored vertex graphs.

- **Level 3: Enumeration of parallel manipulators:** Each mechanism originates parallel manipulators selecting different links to be the end-effectors. It is important to remember that a parallel manipulator is a kinematic chain with one of its components (links) taken as a frame and the other taken as an end-effector. In terms of graph theory, a parallel manipulator with one end-effector corresponds to a graph with two detached vertices (colored with distinct colors), one to represent the fixed link and another to represent the end-effector (SIMONI et al., 2008). The attributes of parallel manipulators in this level are: number of links (n), number of 1-DoF joints (j), mobility (M), order of screw system (λ), base and end-effector. The tools considered in this level are graph theory, group theory and screw theory; mainly the concepts of symmetry, actions and orbits of the automorphism group of colored vertex graphs.

Using this systematic procedure we will enumerate all mechanisms and parallel manipulators that a kinematic chain can originate, without isomorphisms. Applying integrated tools of graph and group theory and the concept of symmetry, we will present a new method of enumeration of parallel manipulators and several new results. Using the concept of symmetry we will present an improvement of the enumeration of mechanisms method presented by Simoni (2008).

1.3.2 Contributions to the analysis of kinematic structures

The contributions to the analysis are to apply integrated tools of graph theory and group theory to identify the symmetries of the kinematic structures and to reduce the complexity of the matricial analysis. The analysis in the context of this thesis is to describe the connectivity, the degrees-of-control and redundancy matrices in a compact form and simplify the selection of the best kinematic structure satisfying the costumer's requirements.

The main contribution to the analysis of kinematic structures is to prove the invariance of connectivity, degrees-of-control and redundancy by the action of the automorphism group of the associated graph. The connectivity, degrees-of-control and redundancy are symmetrical properties of a kinematic chain, i.e. links which are symmetric by the action of automorphism group of the graph have the same properties. Considering that symmetric links are identified by the orbits of its automorphism group of the graph we reduce the matricial representation considering one representative element of each orbit. Thus, the order of the matrices are reduced from $n \times n$ to $o \times n$ where ”n” is the number of links of the kinematic chain and ”o” is the number of orbits of the automorphism group of the graph.

Will be evident from the examples considered in Chapter 6 that this strategy simplifies the analysis considerably because most of the mechanisms and parallel manipulators have a large number of symmetries.

1.4 MOTIVATION

In the last two decades, parallel manipulators aroused attention from researchers and industry. Conceptual design phase, in particular, is a field which has been increasing in the parallel manipulators literature (MRUTHYUNJAYA, 2003). Parallel manipulators can be considered a well-established option for many different applications as shown in Section B.2.6. As opposed to serial manipulators, in which the number of kinematic arrangements (types) is somewhat limited, parallel manipulators can lead to a very large number of kinematic arrangements for a given motion pattern (KONG; GOSSELIN, 2007). However, existing architectures of parallel manipulators have been traditionally designed by the designer’s intuition, ingenuity, and experience (TSAI, 2001; KONG; GOSSELIN, 2007).

In the last years, the research in the field of conceptual design of parallel manipulators increased and some books on the subject were published addition to numerous scientific papers. The main authors and books are:

- Creative Design of Mechanical Devices (YAN, 1998).

- Mechanism Design: Enumeration of Kinematic Structures According to Function (TSAI, 2001).

- Parallel Robots (MERLET, 2006).

- Type Synthesis of Parallel Mechanisms(KONG; GOSSELIN, 2007).

- Structural Synthesis of Parallel Robots: Part 1: Methodology (GOGU, 2008)

- Structural Synthesis of Parallel Robots: Part 2: Translational Topologies with Two and Three Degrees of Freedom (GOGU, 2009).

The importance and need of conceptual design of parallel manipulators is emphasized in several other works.

Mruthyunjaya (2003) in the 41 pages long review paper "Kinematic structure of mechanisms revisited" wrote:

> The most important phase in the study of kinematic structure of mechanisms is the structural synthesis or classification and enumeration of kinematic chains with a given number of links and degree of freedom as related by the Chebyshev-Gruebler equation.

Moon and Kota (2002) wrote:

> conceptual design of mechanisms is still a mixture of art of science.

It means that conceptual design of mechanisms and parallel manipulators is still an open problem.

Torgny Brogårdh (2002) from ABB Automation Technology Products/Robotics, in his conference paper entitled "PKM Research - Important Issues, presented as seen from a Product Development Perspective at ABB Robotics" presented in the "Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators", wrote:

> This paper will address some of the most important PKM research issues as seen from a robot manufacturer's point of view. ... section 3, which is the most important part of this paper, the urgent need for a systematic topology synthesis is put forward.

Jean Pierre Merlet from INRIA Sophia Antipolis - France, author of the first book on parallel robots, in his paper entitled "Still a long way to go on the road for parallel mechanisms" presented in ASME 2002 DETC Conference (MERLET, 2002), wrote:

> Synthesis of parallel robot is an open field (there is a very limited number of papers addressing this issue) and, in my opinion, one of the main issue for the development of parallel robots in practice.

The importance of synthesis of parallel manipulators is also reiterated by Jean Pierre Merlet in the paper "Optimal design of robots" (MERLET, 2005):

> Synthesis of robots may be decomposed into two processes: structural synthesis (determine the general arrangement of the mechanical structure such as the type and number of joints and the way they will be connected) and dimensional synthesis (determine the length of the links, the axis and location of the joints, the necessary maximal joint forces/torques). The performances that may be obtained for a robot are drastically dependent on both synthesis.

Clément Gosselin and Xianwen Kong in his book "Type Synthesis of Parallel Mechanisms" (KONG; GOSSELIN, 2007), wrote:

> Parallel manipulators have been largely synthesized using intuition and ingenuity. As opposed to serial kinematic chains, in which the number of kinematic arrangements (types) is somewhat limited, parallel manipulators can lead to a very large number of kinematic arrangements for a given motion pattern. Therefore, a systematic approach is needed in order to reveal all types of parallel manipulators thereby allowing the development of the most promising designs. This fundamental issue, namely type synthesis, is the focus of this book.

Grigore Gogu, in his books "Structural Synthesis of Parallel Robots: Part 1: Methodology (GOGU, 2008)" and "Structural Synthesis of Parallel Robots: Part 2: Translational Topologies with Two and Three Degrees of Freedom (GOGU, 2009)" emphasizes the need for methodologies devoted to the systematic design of parallel manipulators:

> Structural synthesis is directly related to the conceptual phase of robot design, and represents one of the highly challenging subjects in recent robotics research. ... In general, parallel manipulators performances are highly dependent on their mechanical architecture, so that structural synthesis becomes the central problem in the conceptual design phase, but only a few works can be found in the literature on this topic. We note that this is the first book focusing on the structural synthesis of the mechanical architecture of parallel robots. The topic of this book addresses the problem of structural, also called topological, synthesis of parallel robotic manipulators in a systematic way. This is an urgent need put forward by robot manufacturers and scientists.

Jean Pierre Merlet and Clément Gosselin in the Handbook of Robotics (SICILIANO; KHATIB, 2008), wrote:

> Determining all potential mechanical architectures of parallel robots
> that can produce a given motion pattern at the moving platform
> is a challenging problem.

These comments motivate the research in this interdisciplinary field of science. The early research in this field can be considered recent and little was developed in the conceptual design of parallel manipulators. The approaches already proposed should be improved and other tools should be explored.

Another motivation is the interest of the UFSC robotics research group front of current challenges. New projects of the UFSC robotics laboratory have a current trend towards parallel manipulators.

The reader is invited to read the Appendix B (page 167) which is also part of the motivation. Appendix B presents a review of the parallel manipulators implemented by laboratories of research and industry, and presents the main applications of parallel manipulators.

1.5 THESIS ORGANIZATION

The thesis is divided into seven chapters and two appendices. The first chapter consists of this introduction to contextualizes the problem, presents the state of the art in the enumeration and analysis and, the motivation to work.

The second chapter presents the mathematical tools used in conceptual design and it is composed by four main sections: graph theory, group theory, symmetry analysis and screw theory.

The third chapter presents the fundamentals concepts and terminology of mechanisms and machines theory.

The fourth chapter presents a review of the main methods for enumeration of kinematic structures and criteria for kinematic analysis.

The fifth presents contributions to the enumeration of kinematic structures. We will present the systematic procedure considering the three levels, discussed in Section 1.3.

The sixty chapter presents an application of integrated tools of group theory and graph theory for analysis of enumerated kinematic structures in the synthesis process.

The seventh chapter presents the conclusions and perspectives to further works.

Appendix A presents an application of enumeration techniques developed in fifth chapter for enumeration of planar metamorphic robots configurations. With the development of science, technology and with space exploration, hazardous environment work, production requirements of small batch, short run and quick change-over, the traditional concept of mechanisms and robot development is facing a challenge in the 21st century to adaptability and reconfigurability. Therefore, new concepts were emerging as modular robots, metamorphic robots and variable topology mechanisms. The group theory tools are applied successfully to reduce the problem of enumeration of planar metamorphic robots configurations.

Appendix B presents a review and the applications of parallel manipulators.

2 MATHEMATICAL TOOLS

As discussed in the introduction, several mathematical tools are used in conceptual design. This chapter introduces these mathematical tools which are used in the remainder of this work. The chapter is divided in four main sections:

- *Group theory*: This section introduces the fundamental concepts of group theory. Group theory is important for enumeration and analysis of mechanisms and parallel manipulator because it captures the symmetries of the structure of the kinematic chain of parallel manipulator. A preliminary contribution of this thesis is to apply group theory tools to enumeration of mechanisms and parallel manipulators.

- *Graph theory*: This section introduces the fundamental concepts of graph theory. Kinematic chains, mechanisms and parallel manipulators can be represented by a graph. The graph representation permit us to give an abstract treatment for enumeration and analysis of mechanisms and parallel manipulators in the first phases of design.

- *Symmetry analysis*: This section presents a precise definition of symmetry in kinematic chains. Since the aim of this work is to apply symmetry to simplify the enumeration and analysis of mechanisms and parallel manipulators, which are represented in biunivocal correspondence by graphs, it is necessary to define what is meant by kinematic chain symmetry. It is an original contribution of this thesis.

- *Screw theory*: This section introduces some screw system where kinematic structures will work.

2.1 GROUP THEORY

Groups are abstract structures used in mathematics and science in general to capture the internal symmetry of a structure in the form of automorphism group. In general, groups can be thought as sets of symmetry operations. The definition of a group is the abstraction of the properties of symmetry operations. Thus, group theory methods are useful whenever there are symmetries. In the enumeration and analysis of mechanisms and parallel manipulators, the group theory can be used by identifies symmetries of kinematic chains and mechanisms (SIMONI et al., 2008, 2009). We will be interested in symmetries of kinematic structures, i.e. kinematic chains of mechanisms and parallel manipulators.

The concepts presented in this section are essential for the application proposed in this thesis. More details on group theory can be found in Alperin and Bell (1995), Burrow (1993), Rotman (1995), Scott (1964), Selig (2005).

2.1.1 Groups and subgroups

A group is a set endowed with a binary operation $\cdot : G \times G \to G$ satisfying certain axioms, detailed below. Thus, whenever a set has a group structure the whole group can be described in terms of a set of generators. This follows from the fact that the equation $a \cdot x = b$ always admits a unique solution $x = a^{-1} \cdot b$ in G.

Definition 1 (Group). *Let G be a set and $\cdot : G \times G \to G$. The pair $(G, \cdot)$ is a group if the following conditions are satisfied:*

1. *associativity: for all a, b and c in G, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.*
2. *identity element: there exists an element $e \in G$ such that for all $a \in G$, $e \cdot a = a \cdot e = a$.*
3. *inverse element: for every $a \in G$, there exists an element $a^{-1} \in G$ such that $a \cdot a^{-1} = a^{-1} \cdot a = e$.*

Definition 2 (Subgroup). *A subset $H \subset G$ is a subgroup of a group G if the operation induced by the operation on $(G, \cdot)$ satisfies the three conditions in Definition 1. This is equivalent to a requirement that $x = h^{-1} \cdot g \in H$, for all $h, g \in H$.*

Definition 3 (Group generators). *A set $\beta = \{g_1, ..., g_n\} \subset G$ is a set of generators for a group G if any element $g \in G$ can be written as the product of elements in β. In this case, we denote $G =< g_1, \ldots, g_n >$.*

Example 1 (Symmetric group). *Let $X_n = \{x_1, x_2, \ldots, x_n\}$ and $S_n = \{\sigma : X_n \to X_n \mid \sigma$ is bijective$\}$ (permutations). Consider $\cdot : S_n \times S_n \to S_n$ the operation given by the composition law $\sigma \cdot \tau = \sigma \circ \tau : X_n \to X_n$. Thus, $(S_n, \cdot)$ is the n^{th}-symmetric group. In order to describe the elements of S_n in a convenient way, let us consider a bijection $\sigma : X_n \to X_n$;*

$$\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) \end{pmatrix}.$$

For $n = 2$, we have $2! = 2$ elements

$$S_2 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}.$$

For $n = 3$, we have $3! = 6$ elements

$$S_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\}$$

The group S_n has $n!$ elements.

Definition 4 (Isomorphism group)**.** *Consider the groups $(G_1, \cdot_1)$ and $(G_2, \cdot_2)$.*

1. *A map $\phi : G_1 \to G_2$ is a homomorphism if $\phi(x \cdot_1 y) = \phi(x) \cdot_2 \phi(y)$, for all $x, y \in G_1$.*

2. *A homomorphism $\phi : G_1 \to G_2$ is an isomorphism if ϕ is bijective.*

An isomorphism is called an automorphism if $G_1 = G_2$.

Definition 5 (Automorphism group)**.** *Let G be a group. An isomorphism of G in G is called an* automorphism. *The set of all automorphisms of G form a group, which is called the automorphism group and denoted by Aut(G).*

2.1.2 Actions and orbits

The group structure is present in a model in the form of the group action, also called group representation. For the sake of simplicity, from now on let us denote the product of two group elements $g, h \in G$ by gh.

Definition 6 (Left group action)**.** *A left group action of a group G on a set X is a map $\alpha : G \times X \to X$, usually denoted by $\alpha(g, x) = g \cdot x$, satisfying the following conditions:*

1. *For all $g, h \in G$ and $x \in X$, $g \cdot (h \cdot x) = (gh) \cdot x$.*

2. *For all $x \in X$, $e \cdot x = x$.*

Analogously, a right group action can be defined. From now on, we use the term action for left action.

Example 2. *The symmetric group is a matrix group and the actions can be represented by a binary matrix operation. For instance,*

$$\sigma = \begin{pmatrix} a & b & c \\ b & a & c \end{pmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

can be represented as (left group action)

$$\sigma = \begin{bmatrix} b \\ a \\ c \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

A space X endowed with a G-action is named a G-space.

Definition 7 (Orbits)**.** *Let X be a G-space. The orbit of a point $x \in X$, by the action of G, is the space*

$$\mathscr{O}_x = \{g \cdot x \mid g \in G\}.$$

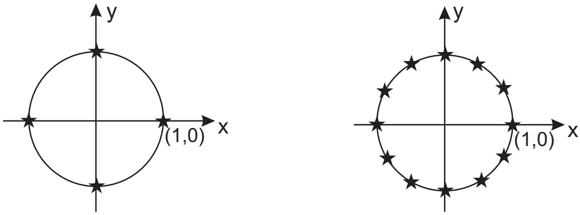
A partition of a G-space X is obtained by considering the space of G-orbits. This can be seen by defining the following equivalence relation: $x \sim y$ if and only if there exists an element $g \in G$ such that $y = g \cdot x$. The equivalence classes are exactly the orbits under the G-action. Therefore, if $x \sim y$, then $\mathscr{O}_x = \mathscr{O}_y$. It is well known that the equivalent classes define a partition.

Example 3. *Consider the $SO(2)$ group, i.e. the planar rotation group. The action is rotation of a point in the plane about the origin by an angle θ. The orbit of a point at distance r from the origin is the circle of radius r. Figure 2(a) shows the orbit of point $(1,0)$ by an angle $\theta = \frac{\pi}{2}$, i.e. $\mathscr{O}_{\frac{\pi}{2}} = \{1, e^{i\frac{\pi}{2}}, e^{i\pi}, e^{i\frac{3\pi}{2}}\}$, where $e^{i\theta} = \cos\theta + i\sin\theta$. Figure 2(b) shows the orbit of point $(1,0)$ by an angle $\theta = \frac{\pi}{6}$, i.e. $\mathscr{O}_{\frac{\pi}{6}} = \{1, e^{i\frac{\pi}{6}}, e^{i\frac{\pi}{3}}, e^{i\frac{\pi}{2}}, e^{i\frac{2\pi}{3}}, e^{i\frac{5\pi}{6}}, e^{i\pi}, e^{i\frac{7\pi}{6}}, e^{i\frac{4\pi}{3}}, e^{i\frac{3\pi}{2}}, e^{i\frac{5\pi}{3}}, e^{i\frac{11\pi}{6}}\}$.*

2.2 GRAPH THEORY

In this section, some fundamental concepts of graph theory are introduced. The definitions adopted in this work are obtained mainly from Gross and Yellen (2003), Jonsson (2007), Murota (2000), Biggs (1993a), Tsai (2001), Thomas et al. (2001).

A graph is a simple, intuitive and abstract concept used to represent



(a) Orbits of point $(1,0)$ by an angle $\theta = \frac{\pi}{2}$.

(b) Orbits of point $(1,0)$ by an angle $\theta = \frac{\pi}{6}$.

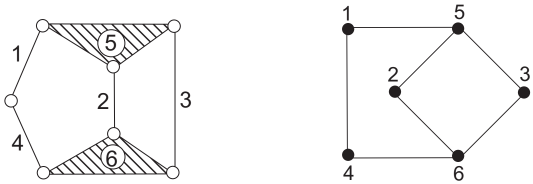
Figure 2 – Orbits of subgroups of the planar rotation group SO(2).

the idea of some kind of relationship between objects. Configurations of nodes and connections occur in a great diversity of applications. They can be present in electrical circuits, roadways, organic molecules, databases, and so on. In special, as discussed in the introduction they are essential for topological analysis and enumeration of mechanisms and parallel manipulators. It is important to remember that the topology of a kinematic chain can be uniquely identified by its graph representation, where links and joints of the kinematic chain are represented, respectively, by the vertices and edges of the graph.

2.2.1 Graphs and subgraphs

A graph $X = (V,E)$ consists of a finite set $V(X)$ of vertices and a family $E(X)$ of subsets of $V(X)$ of size two called edges. Usually, the pair $\{$x,y$\}$ denotes an edge, and the number of edges incident to a vertex v is the degree of the vertex v ($deg(v)$). A vertex of zero degree is called an isolated vertex. A vertex of degree two is called a binary vertex, a vertex of degree three a ternary vertex, and so on. A subgraph of a graph X is a graph Y such that $V(Y) \subseteq V(H)$, $E(Y) \subseteq E(H)$. A graph is dense when $|E| \ll |V|^2$ and sparse when $|E| \approx |V|^2$.

It is important to remember that a kinematic chain can be uniquely represented by the graph whose vertices correspond to links and whose edges correspond to joints of the chain (TSAI, 2001; MRUTHYUNJAYA, 2003; DOBRJANSKYJ; FREUDENSTEIN, 1967). Figure 3 shows this correspondence, Figure 3(a) shows the Stephenson kinematic chain with labeled links and Figure 3(b) shows the corresponding graph (DOBRJANSKYJ; FREUDENSTEIN, 1967).



(a) Stephenson kinematic chain.

(b) Graph representation.

Figure 3 – Correspondence between graphs and kinematic chains.

Equation 2.1 shows the adjacency matrix of the graph shown in Figure 3(b). Adjacency matrix is a means of representing which vertices of a graph are adjacent to which other vertices. The adjacency matrix of X is the $n \times n$ matrix $A(X) = (a_{ij})_{n\times n}$ such that $a_{ij} = 1$ if vertex i is adjacent to vertex j, and $a_{ij} = 0$ otherwise (including $i = j$). Other possible representations are incidence matrix, adjacency list, graph6 and sparce6 (see graph6 and sparce6 formats in McKay (2009a)). The graph shown in Figure 3(b) is represented by ECxo in the graph6 format and by :EkGChG˜ in the sparce6 format.

$$A(X) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \tag{2.1}$$

A path between two vertices v_a and v_b is a sequence $v_0, v_1, v_2, \ldots, v_k$ of vertices and edges, such that $v_0 = v_a$, $v_k = v_b$ and for all $i \in [1,k]$, $\{v_{i-1}, v_i\} \in E$. The length of a path is the number of its edges. The distance between two vertices v_a and v_b, denoted by $\delta(v_a, v_b)$, is the length of the shortest path between v_a and v_b. If each vertex appears once, except that the beginning and ending vertices are the same, the path forms a circuit or loop. In the graph shown in Figure 3(b) the sequence (1, {1,2}, 2, {2,3}, 3, {3,4}, 4) is a path and the sequence (1, {1,2}, 2, {2,3}, 3, {3,4}, 4, {4,5}, 5, {5,1}, 1) is a circuit.

A graph X is said to be connected if every vertex in X is connected to every other vertex by at least one path. The minimum degree of any vertex in a connected graph is equal to one. A connected graph is biconnected if the removal of any single vertex (and all edges incident on that vertex) can not disconnect the graph. Articulation points are vertices whose removal would increase the number of connected components in the graph. Thus, a graph without articulation points is biconnected. Figure 4 illustrates the articulation points and biconnected components of a small graph.

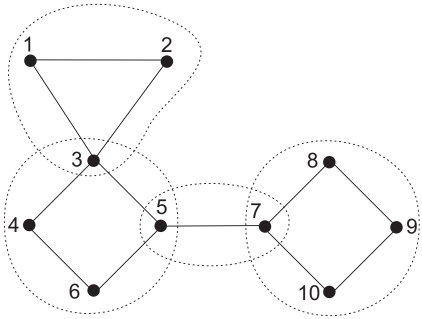


Figure 4 – A connected graph and its biconnected components with dashed boundaries. The vertices 3, 5 and 7 are cut vertices and belong to more than one biconnected component.

2.2.2 Actions

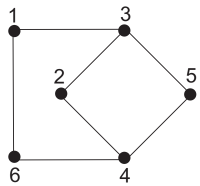
Given a graph X, a bijective map $\sigma : V(X) \rightarrow V(X)$ defines a permutation of the elements of $V(X)$. Assuming $V(X)$ has n elements, the set of permutations endowed with the operation of composition is the group S_n and we can apply the definitions presented in Section 2.1.

Example 4 (Actions)**.** *Figure 3(a) shows the Stephenson kinematic chain and Figure 3(b) its graph (X). Figures 5(a) and 5(b) show the action of $\sigma_1(X)$ and $\sigma_2(X)$, respectively, on the labels of the Stephenson graph, where*

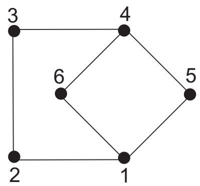
$$\begin{aligned}
\sigma_1(X) &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 1 & 6 & 2 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 & 6 \\ 5 & 6 & 2 \end{pmatrix} \\
&= (134)(256)
\end{aligned}$$

and

$$\begin{aligned}\sigma_2(X) &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 1 & 6 & 5 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 6 & 5 \end{pmatrix} \\ &= (14)(23)(56).\end{aligned}$$



(a) $\sigma_1(X)$



(b) $\sigma_2(X)$

Figure 5 – Action of σ_1 and σ_2 in the Stephenson graph X.

2.2.3 Isomorphisms and automorphisms

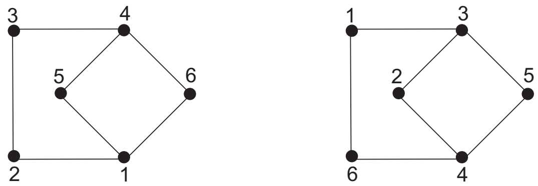
Definition 8. *Two graphs X and Y are isomorphic if there is a bijection $\sigma : V(X) \to V(Y)$ such that*

$$\{xy\} \in E(X) \Leftrightarrow \{\sigma(x)\sigma(y)\} \in E(Y).$$

If isomorphism exists between two graphs, then the graphs are called isomorphic and we write $X \simeq Y$ (GROSS; YELLEN, 2003).

Isomorphic graphs clearly have the same numbers of vertices and edges. On the other hand, equality of these parameters does not guarantee isomorphism. In general, if two graphs X and Y are isomorphic they are said to be identical and we written $X \simeq Y$. If two graphs are identical, they can clearly be represented by identical diagrams. For example, the graphs X and Y in Figure 6 can be represented by diagrams which look exactly the same, the sole difference lies in the labels of their vertices.

In order to show that two graphs are isomorphic, one must indicate an



(a) X. (b) Y.

Figure 6 – Isomorphic graphs.

isomorphism between them. The mapping (σ) defined by

$$\sigma := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 1 & 6 & 2 \end{pmatrix}$$

is an isomorphism between the graphs X and Y in Figure 6.

Most isomorphism tests are based on graph invariants which preserve the properties or parameters of graphs under isomorphism, such as degree sequence, distance matrix, vertex ordering, etc. (KING; TZENG, 1999).

Definition 9. *The automorphism of a graph is the graph's isomorphism with itself. The automorphism group of a graph X is denoted by $Aut(X)$.*

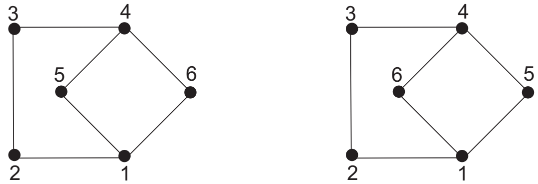
A labeled graph is mapped into another labeled graph when the labels of vertices are permuted. For some permutations, a labeled graph may map into itself. The set of those permutations which map the graph into itself form a group called automorphism group of a graph. This automorphism group is said to be a vertex-induced group (TSAI, 2001). Similarly, the edges of a graph may be labeled. We call the group of permutations that maps the graph into itself an edge-induced automorphism group.

The automorphism group of the graph is a subgroup of the symmetric group and contains all possible permutations of the vertices that preserve the adjacency. The automorphism group of a graph characterizes its symmetries, and are, therefore, quite useful for determining some of its properties. We denote the set of all automorphisms of a graph X by $Aut(X)$. It can be verified that $Aut(X)$ is a group under the operation of composition.

For example, the mapping

$$\sigma := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 1 & 6 & 5 \end{pmatrix}$$

define an automorphism between the graphs X and Y in Figure 7. Note that, in this case, the adjacency and degree are preserved.



(a) X. (b) Y.

Figure 7 – Automorphic graphs.

2.3 SYMMETRY ANALYSIS

Since the aim of this work is to apply symmetry to simplify the kinematic analysis, and, kinematic chains are represented in biunivocal correspondence by graphs, it is necessary to define what is meant by graph symmetry. This is carried out using the concept of a group defined in the previous sections. To the best of the authors' knowledge, there are no a precisely definition of symmetry in kinematic chains in the literature. Rao (2000) discusses symmetries in kinematic chains but does not presents a formal definition or a technique to obtain the symmetries of a kinematic chain. Tischler et al. (1995a) define left and right symmetry to avoid generation of isomorphic kinematic chains in the Farrell's method (more details in Chapter 4).

The symmetry of a graph corresponds to an element of the automorphism group of the graph. According to Erdős and Rényi (1963) and Petitjean (2007), a graph is considered to be symmetric when it has more than one automorphism, i.e. the automorphism group has a degree greater than 1. In the definition below we extend the concept of graph symmetry to kinematic chains.

Definition 10 (Symmetry of a kinematic chain)**.** *The symmetry of a kinematic chain is the symmetry of its corresponding graph. A kinematic chain is symmetric when it has more than one automorphism (SIMONI et al., 2010).*

In the definition below we extend the concept of symmetry order, found in Erdős and Rényi (1963) and Wright (1974), to kinematic chains.

Definition 11 (Symmetry order)**.** *We write r for the order of the automorphism group of the kinematic chain (X), i.e. $r = |Aut(X)|$, and we say that the kinematic chain is of symmetry order r.*

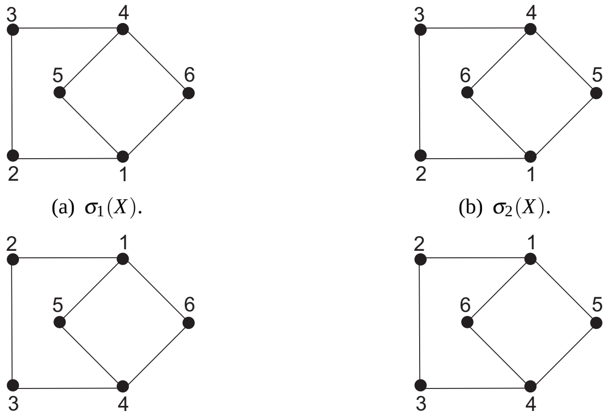
A kinematic chain which is not symmetric is called asymmetric and, for such a kinematic chain, obviously $r = 1$.

Symmetric links are identified by the orbits of the automorphism group of the graph.

Example 5. *Let X be the Stephenson graph shown in Figure 3(a) (page 22). In this case,*

$$Aut(X) = \left\{ \begin{array}{ll} \sigma_1 = (1)(2)(3)(4)(5)(6), & \sigma_2 = (1)(2)(3)(4)(56), \\ \sigma_3 = (14)(23)(5)(6), & \sigma_4 = (14)(23)(56) \end{array} \right\}.$$

Therefore, Stephenson graph is of symmetry order $r = 4$. The generator set is $Aut(X) =< \sigma_2, \sigma_3 >$. The action of the automorphism group in the Stephenson graph is shown in Figures 8(a), 8(b), 8(c) and 8(d).



(a) $\sigma_1(X)$.　(b) $\sigma_2(X)$.

(c) $\sigma_3(X)$.　(d) $\sigma_4(X)$.

Figure 8 – Action of the automorphism group in the Stephenson graph.

Following the definition of group operation, we can construct a multiplication table shown in Table 2. We conclude that every product is an element of the group; the associative law holds; σ_1 is the identity element; and every element is its own inverse. Therefore, σ_1, σ_2, σ_3, and σ_4 form a automorphism group.

Symmetric vertices (links) are identified by orbits. The orbits are:

$$\mathscr{O} = \{\{1,4\},\{2,3\},\{5,6\}\}.$$

Table 2 – Group operation table for the automorphism group of Stephenson graph.

$\circ$	σ_1	σ_2	σ_3	σ_4
σ_1	σ_1	σ_2	σ_3	σ_4
σ_2	σ_2	σ_1	σ_4	σ_3
σ_3	σ_3	σ_4	σ_1	σ_2
σ_4	σ_4	σ_3	σ_2	σ_1

2.4 SCREW THEORY

In the mobility equation (see Equation 3.1 on page 37) we have the parameter λ which represent the order of screw system to which all the joint screws belong. Therefore, this section presents the definition of a screw and some screw systems important for enumeration of kinematic chains mechanisms and parallel manipulators. More details on screw theory can be found in (HUNT, 1978; BALL, 1998; DAVIDSON; HUNT, 2004; KONG; GOSSELIN, 2007; CHIRIKJIAN et al., 2001).

2.4.1 Screw systems

In screw theory (HUNT, 1978; BALL, 1998; DAVIDSON; HUNT, 2004), a unit screw \$ is defined by a pair of vectors

$$\$ = \begin{bmatrix} \$_F \\ \$_S \end{bmatrix} = \begin{cases} \begin{bmatrix} s \\ s \times s_0 + hs \end{bmatrix} & \text{if } h \text{ is finite} \\ \\ \begin{bmatrix} 0 \\ s \end{bmatrix} & \text{if } h \to \infty \end{cases} \tag{2.2}$$

where s is a unit vector along the axis of the screw \$, s_0 is a vector directed from origin of the reference frame O-xyz to any point on the axis of the screw, and h is called the pitch. There are two vector components (F-first, S-second) or six scalar components in the above presentation of the screw.

A screw[1] is a geometric element composed by a directed line (axis) associated to a scalar parameter h denominated pitch (CAMPOS, 2004).

A screw system of order λ $(0 \leq \lambda \leq 6)$ comprises all the screws that

[1]Notation: $\$_0$, $\$_h$ and $\$_\infty$ are used to represent a screw of 0-pitch, a screw of h-pitch and a screw of ∞-pitch respectively (HUNT, 1978; DAVIDSON; HUNT, 2004; KONG; GOSSELIN, 2007).

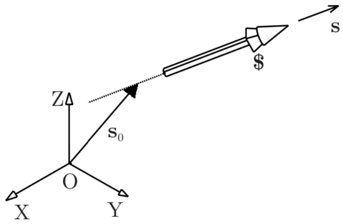


Figure 9 – The geometry of a screw.

are linearly dependent on λ given linearly independent screws. A screw system of order λ is also called an λ-system. Any set of λ linearly independent screws within an λ-system forms a basis of the λ-system. There are many types of screw systems, see for instance Hunt (1978), Davidson and Hunt (2004), Gibson and Hunt (1990a, 1990b). For example, the canonical base screw

$$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0), \tag{2.3}$$

represent a 1-system (DAVIDSON; HUNT, 2004). There are two special cases of note; one when the pitch is $h_\alpha = 0$, i.e. $\$_0 = (1\ 0\ 0; 0\ 0\ 0)$, and another when the pitch $h_\alpha = \infty$, i.e. $\$_\infty = (0\ 0\ 0; 1\ 0\ 0)$. The mechanical generators of this two 1-systems are respectively the R and P joints.

A geometric treatment on screw systems was presented by Kong and Gosselin (2007). Table 3 presents eleven most important screw systems. The description was obtained from Kong and Gosselin (2007) and the canonical base screws from Hunt (1978), Gibson and Hunt (1990b), Davidson and Hunt (2004).

2.5 CONCLUSIONS

This chapter presented group, graph and screw theory tools essential for the application proposed in this thesis. Through several examples we showed the potential of the integrated application of graph and group theory tools.

This chapter presented a precise definition of the symmetry of a kinematic chain in terms of the automorphism group of the associated graph. The definition of symmetry is an original contribution of this thesis and it will be used in the remainder of this text. Three important applications to the symmetry in this thesis are: enumeration mechanisms, enumeration of parallel manipulators and simplification to the analysis of these kinematic structures.

Table 3 – Summary of some screw systems.

λ-system	Description	Canonical base screws
1-systems		
1-$\$_\infty$-system	composed of all the $\$_\infty$ along a same direction	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $h_\alpha = \infty$
1-$\$_0$-system	composed of all the $\$_0$ along a same line	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $h_\alpha = 0$
1-$\$_h$-system	is a finite screw motion about a line	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $h_\alpha = const$
2-systems		
2-$\$_\infty$-system	composed of all the $\$_\infty$ whose directions are parallel to a same plane	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $h_\alpha = h_\beta = \infty$
1-$\$_\infty$-1-$\$_0$-system	composed of all the $\$_0$ whose axes are coplanar and parallel as well as the $\$_\infty$ whose direction is perpendicular to the axes of all the $\$_0$	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $h_\alpha = 0,\ h_\beta = \infty$
2-$\$_0$-system	composed of all the $\$_0$ whose axes intersect at a common point and are coplanar. The common point is called the center of the 2-$\$_0$-system.	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $h_\alpha = h_\beta = 0$
3-systems		
3-$\$_\infty$-system	composed of all the $\$_\infty$	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $\$_\gamma = (1\ 0\ 0; 0\ 0\ h_\gamma)$ $h_\alpha = h_\beta = h_\gamma = \infty$
2-$\$_\infty$-1-$\$_0$-system	composed of all the $\$_0$ and all the $\$_\infty$ whose directions are parallel to a plane that is not perpendicular to the axis of the $\$_0$	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $\$_\gamma = (1\ 0\ 0; 0\ 0\ h_\gamma)$ $h_\alpha = h_\beta = \infty,\ h_\gamma = 0$
1-$\$_\infty$-2-$\$_0$-system	composed of a $\$_\infty$ as well as all the $\$_0$ whose axes are located on a plane which is perpendicular to the direction of the $\$_\infty$	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $\$_\gamma = (1\ 0\ 0; 0\ 0\ h_\gamma)$ $h_\alpha = h_\beta = 0,\ h_\gamma = \infty$
3-$\$_0$-system	composed of all the $\$_0$ whose axes intersect at a common point. The common point is called the center of the 3-$\$_0$-system.	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $\$_\gamma = (1\ 0\ 0; 0\ 0\ h_\gamma)$ $h_\alpha = h_\beta = h_\gamma = 0$
4-systems		
3-$\$_\infty$-1-$\$_0$-system	composed of all the $\$_\infty$ and all the $\$_0$ whose axes are all parallel to one line	$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0)$ $\$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0)$ $\$_\gamma = (1\ 0\ 0; 0\ 0\ h_\gamma)$ $\$_\psi = (1\ 0\ 0; 0\ 0\ h_\psi)$ $h_\alpha = h_\beta = h_\gamma = \infty, h_\psi = 0$

3 MECHANISMS AND PARALLEL MANIPULATORS: A BIBLIOGRAPHY REVIEW

This chapter introduces the basic concepts of mechanisms and machines theory. The terminology used in this chapter is established in accordance with the terminology proposed by the International Federation for the Promotion of Mechanism and Machine Science (IFToMM), see Ionescu (2003).

3.1 LINKS AND JOINTS

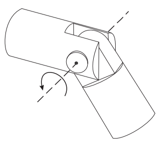
A material body is a rigid body if the distance between any two points of the body remains constant. In reality, rigid bodies do not exist, since all known materials deform under stress. However, we may consider a body as rigid if its deformation under stress is small and can be considered negligibly. The individual rigid bodies making up a mechanism, a machine or a parallel manipulator are called links (TSAI, 2001). A link is called a binary link if it is connected to only two other links, a ternary link if it is connected to three other links, a quaternary link if it is connected to four other links, and so on. A rigid body in space can move in various ways, in translation or rotation motion. These are called its degrees of freedom (DoF).

The links in a mechanism, a machine or a parallel manipulator are connected in pairs and this connection is called a joint. A joint physically adds some constraints to the relative motion between the two links. Two such paired elements form a kinematic pair (TSAI, 2001).

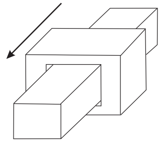
Kinematic pairs (or joints) are classified according to type of the contact between the paired elements into lowers and higher pairs (REULEAUX, 1876; TSAI, 2001; IONESCU, 2003). Lower pairs have superficial contact and higher pairs have linear or punctual contact. There are six lower pairs as shown in Figure 10 and two higher pairs as shown in Figure 11 which are frequently used in mechanisms, machines and parallel manipulators. Figure 10(g) shows the universal joint. The universal joint is sometimes referred to as the Hooke joint, ball-and-socket joint or Cardan joint.

Table 4 summarizes the DoF and the types of motion associated with each joint.

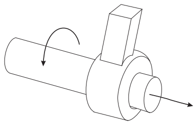
Joints with more than 1-DoF can be replaced/obtained by combinations of joints with 1-DoF, see Figure 12. The universal joint is kinematically equivalent to two intersecting revolute joints (see Figure 12(a)), therefore, it is a 2-DoF joint. The cylindric joint is kinematically equivalent to a revolute



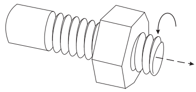
(a) Revolute joint (R joint).



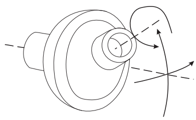
(b) Prismatic joint (P joint).



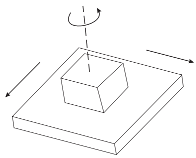
(c) Cylindric joint (C joint).



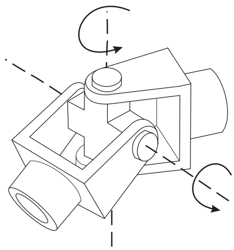
(d) Helical joint (H joint).



(e) Spherical joint (S joint).



(f) Planar joint.

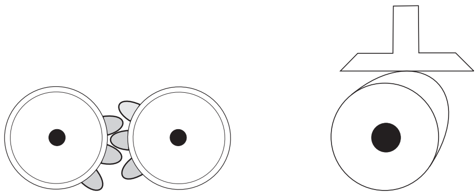


(g) Universal joint (U joint).

Figure 10 – Six lower kinematic pairs and universal joint formed by two revolute joints (SIMONI, 2008).

joint in series with a prismatic joint with their joint axes parallel to or coincident with each other (see Figure 12(b)), therefore, it is a 2-DoF joint. The spherical joint is kinematically equivalent to three intersecting revolute joints (see Figure 12(c)), therefore, it is a 3-DoF joint. The planar joint is kinematically equivalent to two prismatic joints with axis parallel to plane and a revolute joint with axis perpendicular to plane (see Figure 12(d)), therefore, it is a 3-DoF joint.

A joint is called a binary joint, if it connects only two links, and a multiple joint, if it connects more than two links.



(a) Gear. (b) Came.

Figure 11 – Two higher kinematic pairs (SIMONI, 2008).

Table 4 – Summary of kinematic pairs (joints) frequently used in mechanisms and machines (TSAI, 2001).

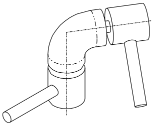
Kinematic pair	Figure	Symbol	DoF	Rotational	Translational
Lower kinematic pairs.					
Revolute	10(a)	R	1	1	0
Prismatic	10(b)	P	1	0	1
Cylindric	10(c)	C	2	1	1
Helical	10(d)	H	1	1	coupled
Spherical	10(e)	S	3	3	0
Planar	10(f)	E	3	1	2
Very used kinematic pair based on lower kinematic pairs.					
Universal	10(g)	U	2	2	0
Higher kinematic pairs.					
Gear	11(a)	G	2	1	1
Cam	11(b)	C_p	2	1	1

3.2 KINEMATIC STRUCTURES

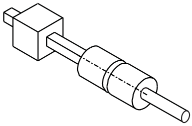
In the standard terminology, i.e. IFToMM (IONESCU, 2003), a kinematic chain is defined as an assembly of links and joints.

3.2.1 Kinematic chains

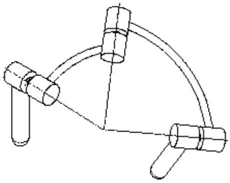
There are three types of kinematic chains: open-loop, closed-loop and hybrids. An open-loop kinematic chain has every link connected to every



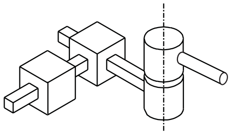
(a) Universal joint (KONG; GOSSELIN, 2007).



(b) Cylindric joint.



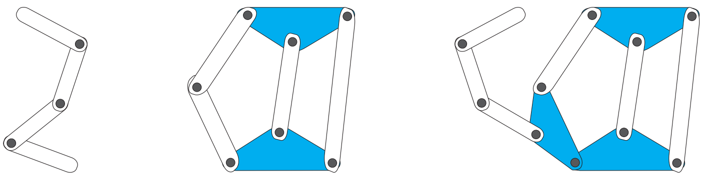
(c) Spherical joint (KONG; GOSSELIN, 2007).



(d) Planar joint.

Figure 12 – Joints obtained by combinations of 1-DoF joints.

other link by one and only one path (see Figure 13(a)). A closed-loop kinematic chain is a kinematic chain which each link is connected with at least two other links. In other words, a closed-loop kinematic chain has every link connected to every other link by at least two distinct paths (see Figure 13(b)). Clearly, it is possible for a kinematic chain to contain both closed- and open-loop kinematic chain which is called a hybrid kinematic chain (see Figure 13(c)). A kinematic chain whose joints are equivalent to lower pairs only is called a linkage.



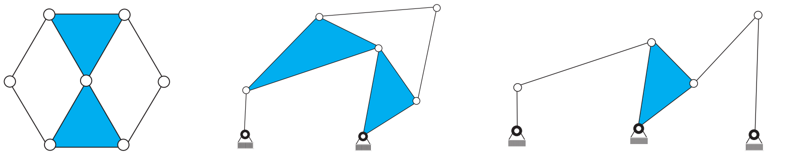
(a) (b) (c)

Figure 13 – Three types of kinematic chains: (a) Open-loop kinematic chain. (b) Closed-loop kinematic chain. (c) Hybrid kinematic chain.

3.2.2 Mechanisms

In design terms, a mechanism is kinematic chain with one of its components (links) fixed to the ground or base (or taken as a frame) (TSAI, 2001; IONESCU, 2003). The link that is fixed to the base is called the fixed link. In kinematic terms, a mechanism is a system of bodies designed to convert motions of, and forces on, one or several bodies into constrained motions of, and forces on, other bodies (TSAI, 2001; IONESCU, 2003).

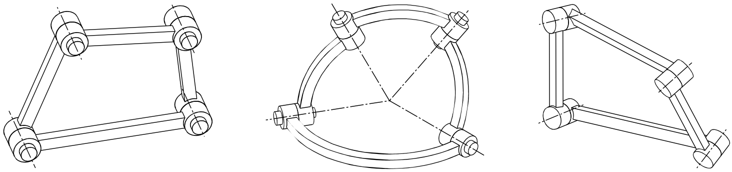
Figure 14(a) shows the Watt kinematic chain which originate two mechanisms with different characteristics of the movement relative to the base fixing one of the links of the kinematic chain. Figures 14(b) and 14(c) shows these classical mechanisms, i.e. Watt I and Watt II, originated fixing different links of the kinematic chain.



(a) Watt kinematic chain. (b) Watt I. (c) Watt II.

Figure 14 – Watt kinematic chain and the two classical mechanisms originated: Watt I and Watt II.

Mechanisms can be classified according to their nature of motion into three types: planar, spherical and spatial (TSAI, 2001). A rigid body is said to be under planar motion if the motion of all particles in the rigid body are constrained in parallel planes. A planar mechanism is a mechanism in which all points of its links describe paths located in parallel planes. A rigid body is said to be performing a spherical motion if the motions of all particles in the rigid body are confined on concentric spherical surfaces. A spherical mechanism is a mechanism in which all points of its links describe paths located on concentric spheres. A rigid body is said to be undergoing a spatial motion if its motion is not planar or spherical. A spatial mechanism is a mechanism in which some points of some of its links describe non-planar paths, or paths located in non-parallel planes. In other words, a spatial mechanism cannot be classified as planar or spherical. Figure 15 shows an example of the three types of mechanisms according to their nature of motion.



(a) Planar mechanism. (b) Spherical mechanism. (c) Spatial mechanism. Bennett mechanism.

Figure 15 – Classification of mechanisms according to their nature of motion (WANG, 2006; WANG et al., 2008).

3.2.3 Machines

By Tsai (2001), a machine is an assembly of one or more mechanisms are assembled together with other hydraulic, pneumatic, and electrical components such that mechanical forces of nature can be compelled to do work. By Reuleaux (1876), a machine is a collection of mechanisms arranged to transmit forces and do work. By IFToMM terminology (IONESCU, 2003) a machine is a mechanical system that performs a specific task, such as the forming of material, and the transference and transformation of motion and force. Figures 86 and 87 (pages 176 and 177) shows two 5-axis machines used in machine-tool.

3.2.4 Parallel manipulators

A parallel manipulator is a mechanical system under automatic control, that performs operations such as handling and locomotion, and controls the motion of its end-effector by means of at least two kinematic chains going from the end-effector towards the frame (TSAI, 2001; MERLET, 2006; IONESCU, 2003; KONG; GOSSELIN, 2007; GOGU, 2008). In other works, a parallel manipulator is a kinematic chain which one of its components (links) is fixed to the ground or base and another is chosen to be the end-effector. Several examples of parallel manipulators are presented in Appendix B.

Basically, the difference between a machine and a parallel manipulator is in function of the tasks developed. While a machine is designed to developed a “specific task” a parallel manipulator can develop “several tasks”.

3.3 MOBILITY OF KINEMATIC STRUCTURES

Mobility is the main structural parameter of a mechanism and a parallel manipulator, kinematic structures in short, and also one of the most fundamental concepts in the kinematic and the dynamic modelling of mechanisms (GOGU, 2009).

The mobility (M) or number of degrees of freedom (DoF) of a kinematic chain is the number of independent parameters required to completely specify the configuration of the kinematic chain in the space, with respect to one link chosen as the reference (IONESCU, 2003; TSAI, 2001; GOGU, 2008).

Mobility is used to verify the existence of a kinematic structure, to indicate the number of independent parameters in robot modelling and to determine the number of actuators needed to drive the kinematic structure. The mobility of a kinematic structure is given by

$$M = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i \tag{3.1}$$

where λ is the order of screw system to which all the joint screws belong (see some screw systems in Table 3 on page 30), n is the number of links, j the number of joints and f_i are the degrees of relative motion permitted by joint i (HUNT, 1978; TSAI, 2001; MRUTHYUNJAYA, 2003; MERLET, 2006). As joints with more than 1-DoF can be replaced by a combination of 1-DoF joints (see Figure 12) the mobility equation (Equation 3.1) becomes

$$M = \lambda(n - j - 1) + j \tag{3.2}$$

It is also possible to establish an equation that relates the number of independent loops (ν) to the number of links and number of joints in a kinematic chain

$$\nu = j - n + 1. \tag{3.3}$$

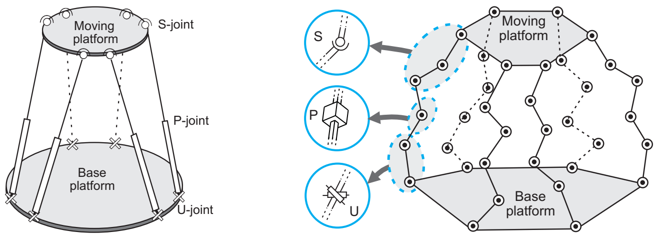
Combining Equation 3.3 with Equation 3.2 yields

$$M = j - \lambda\nu. \tag{3.4}$$

Equation 3.4 is known as the loop mobility criterion (TSAI, 2001).

Example 6. *Figure 16 shows two representations of the classical Stewart platform. Figure 16(a) shows the Stewart platform with f_i-DoF joints, i.e. 1-DoF joints (prismatic), 2-DoF joints (universal) and 3-DoF joints (spheri-*

cal). In this case, $\lambda = 6$, $n = 14$, $j = 18$, $f_1 = 6$, $f_2 = 6$ *and* $f_3 = 6$. *Applying the Equation 3.1 we have* $M = 6(14 - 18 - 1) + 36 = 6$, *as expected. Figure 16(b) shows the Stewart platform with 1-DoF joints, i.e.* f_i*-DoF joints are replaced by 1-DoF joints (see Figure 12). In this case,* $\lambda = 6$, $n = 32$ *and* $j = 36$. *Applying the Equation 3.2 we have* $M = 6(32 - 36 - 1) + 36 = 6$, *as expected.*

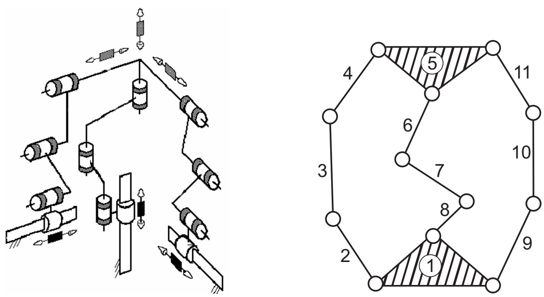


(a) f_i-DoF joints. (b) 1-DoF joints.

Figure 16 – Representations of Stewart platform.

Equations 3.1 and 3.2 are well known in mechanisms and machines theory and can be applied to several mechanisms and parallel manipulators. This equation is used for quick calculation of mobility, however, it fails in several cases. For example, let us apply the mobility criterion to 3-PRRR Cartesian Parallel Manipulator presented in Figure 17, which only contains R and P joints (KIM; TSAI, 2002, 2003). In this case, we have $n = 11$ links and $j = 12$ joints. Applying the Equation 3.1 we obtain: $M = 6$ (if we assume $\lambda = 3$), $M = 4$ (if $\lambda = 4$), $M = 2$ (if $\lambda = 5$) and $M = 0$ (if $\lambda = 6$). However, Kim and Tsai's manipulator has 3-DoF, i.e. $M = 3$ (KIM; TSAI, 2002, 2003). That equation is not applicable to many other types of recent parallel manipulators, for example the Star (HERVÉ; SPARACINO, 1992), H4 (PIERROT et al., 1999), Orthoglide (WENGER; CHABLAT, 2000), Tripteron (GOSSELIN; KONG, 2002), Isoglide family (GOGU, 2008, 2009), and others.

There are several versions of generalized equations suggested in discussions on mobility and DoF in the literature. However, mobility calculation still remains a central subject, and not solved, in the mechanisms and machines theory and should be investigated. Equation 3.1 must be used for a quick calculation of mobility in early stages of design. The more recent review of mobility calculation was presented by (GOGU, 2005, 2008). Gogu (2005) presents a critical review of several versions of generalized equations suggested in the literature and apply the theory of linear transformations to derive a new equation for mobility calculation of parallel manipulators.



(a) (b)

Figure 17 – CPM - Cartesian Parallel Manipulator (KIM; TSAI, 2002; GOGU, 2008).

3.3.1 Full mobility, partial mobility and fractionated mobility

A kinematic chain can have the following types of mobility based in the concept of mobility criteria (see Equation 3.1):

1. *Fractionated mobility*: A kinematic chain has fractionated mobility if it has a separation link or joint, when cut into two, splits the chain into separate (closed) kinematic chains. Hence, the graph of a non-fractionated kinematic chain is a biconnected graph.

2. *Partial mobility*: A kinematic chain with $M > 0$ degrees of freedom, has partial mobility if it has at least one closed subchain with M' number of degrees of freedom, such that $0 \leq M' < M$.

3. *Total mobility*: A kinematic chain with $M > 0$ degrees of freedom, has total mobility if all its closed subchains have $M' \geq M$ number of degrees of freedom.

3.3.2 Instantaneous versus full-cycle mobility

A parallel manipulator is said to be instantaneous if both its mobility and corresponding properties cannot remain unchanged after an arbitrary feasible finite motion (KONG; GOSSELIN, 2007). For example, if a parallel manipulator has 3-DoF translational at a moment, and has 2-DoF translational and 1-DoF rotational at another moment, the parallel manipulator is instantaneous.

A parallel manipulator that do not change their motion pattern after a finite motion is called full-cycle (global mobility, general mobility) parallel manipulator (MOHAMED; DUFFY, 1985).

The mobility calculated in relation to a given configuration of the parallel manipulator is an instantaneous mobility which can be different from the full-cycle mobility (HUNT, 1978). The full-cycle mobility represents the minimum value of the instantaneous mobility. For a given parallel manipulator, full-cycle mobility has a unique value. It is a global parameter characterizing the parallel manipulator in all its configurations except its singular ones.

Instantaneous parallel manipulators can be used as micro-motion parallel manipulators if necessary. As reviewed above, the classical Grübler or Kutzbach mobility criterion, which is based solely on topology, fails to provide the correct mobility in many instances. Thus, the mobility obtained by Equation 3.1 is usually instantaneous.

3.4 REPRESENTATIONS OF KINEMATIC STRUCTURES

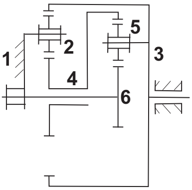
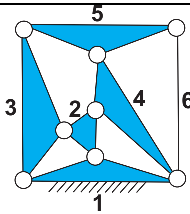
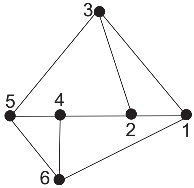
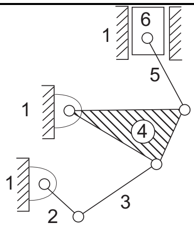
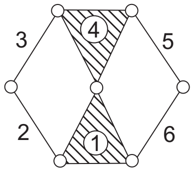
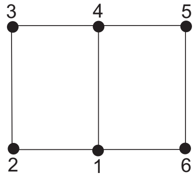
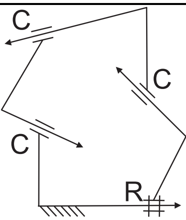
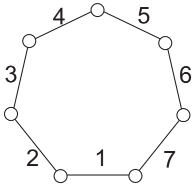
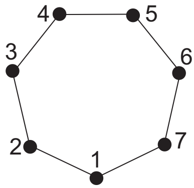
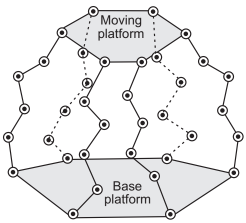
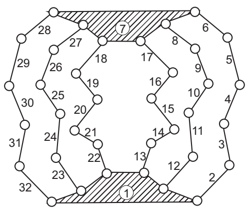
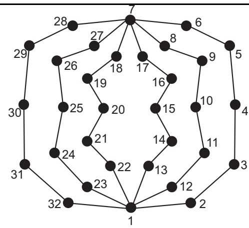
The kinematic structure of a mechanism or a parallel manipulator contains the essential information about which links are connected to which others links and type of joints. The kinematic structure can be represented in different ways. Basically, a mechanism or a parallel manipulator can be illustrated by a functional, structural and graph representation. Table 5 shows these three representations of parallel manipulators.

Functional schematic representation refers to the most familiar cross-sectional drawing of a mechanism representing physical embodiments. Shafts, gears, and other mechanical elements are drawn as such respecting the geometric relations defined by the relative positions of joint axes.

Structural representation is a more coarse representation, each link of a mechanism is denoted by a polygon whose vertices represent the joints. Specifically, a binary link is represented by a line with two end vertices, a ternary link is represented by a cross-hatched triangle, a quaternary link is represented by a cross-hatched quadrilateral, and so on.

Graph representation is an abstract representation, generally used in the initial phases of mechanisms design. A structural graph is a network of vertices or nodes connected by edges or arcs without geometric relations. Since a kinematic chain is defined by an assembly of links and joints, it can be represented in a more abstract form by a graph. In a graph representation, the vertices denote links and the edges denote joints of a mechanism.

Table 5 – Functional, structural and graph representations of mechanisms and parallel manipulators.

Functional	Structural	Graph
Epicyclic gear train: $\lambda = 2$.		
		
Watt engine: $\lambda = 3$.		
		
RCCC spatial four-bar mechanism: $\lambda = 6$.		
		
Stewart platform: $\lambda = 6$.		
		

Example 7. *Consider the mechanisms and parallel manipulators presented in Table 5. Davies (2006) showed that any gear system can be represented by $\lambda = 2$, i.e. a 2-system. Thus, applying the Equation 3.1 in the epicyclic*

gear train we have mobility $M = \lambda(n - j - 1) + j = 2(6 - 9 - 1) + 9 = 1$. *The classical Watt engine which converts a continuous rotation of link 2 to a reciprocating and oscillating motion of link 6, it works in a 3-system and it has mobility* $M = 1$, *i.e. by Equation 3.1* $M = \lambda(n - j - 1) + j = 3(6 - 1 - 1) + 7 = 1$. *The RCCC spatial four-bar mechanism which converts a continuous rotation of link 2 to a reciprocating and oscillating motion of link 4, it works in a 6-system and it has mobility* $M = 1$, *i.e. by Equation 3.2* $M = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i = 6(4 - 4 - 1) + 7 = 1$. *The classical Stewart platform works in a 6-system and it has mobility* $M = 6$ *(see Example 6).*

3.4.1 From kinematic structures to graphs

We will develop a systematic procedure for enumeration of kinematic structures based on group and graph theory tools, therefore, it is important to show how to convert kinematic structures to graphs and vice versa.

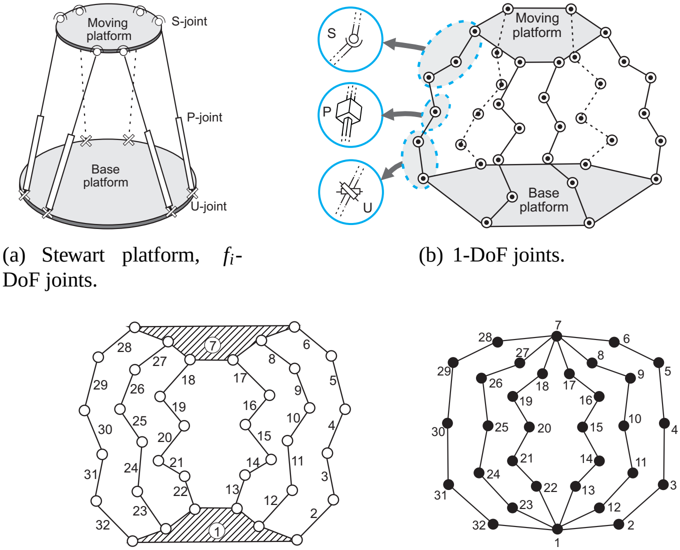
Table 5 shows the graph representation of kinematic structures. As we can see, given a kinematic structure is always possible to represent it in form of a graph where the links are represented by vertices and joints are represented by edges.

The following procedure permit us to convert a kinematic structures to a graph.

- Identify f_i-DoF joints.
- Replace f_i-DoF joints by f 1-DoF joints.
- Represent links by vertices and 1-DoF joints by edges.

In the example below, we apply these three steps to represent the classical Stewart platform to a graph.

Example 8. *Figure 18 shows the steps of representation from kinematic structure to graph. Figure 18(a) shows the Stewart platform. First, we identify the* f_i*-DoF joints as indicated in Figure 18(b): we have six prismatic joints, i.e.* $f_1 = 6$, *six universal joints, i.e.* $f_2 = 6$ *and six spherical joints, i.e.* $f_3 = 6$. *Second, we replace* f_i*-DoF joints by* f *1-DoF joints as indicated in Figure 18(c) and, third, we represent links by vertices and 1-DoF joints by edges and we have the graph representation as shown in Figure 18(d).*



(a) Stewart platform, f_i-DoF joints.

(b) 1-DoF joints.

(c) Kinematic chain.

(d) Graph Representation.

Figure 18 – From Stewart platform to graph.

3.4.2 From graphs to kinematic structures

The procedure to obtain kinematic structures from graphs is inverse of the procedure to convert kinematic structures to graphs. In the enumeration of kinematic structures via graphs, several graphs are enumerated and, we need to develop kinematic structures from these graphs.

The following procedure permit us to develop all kinematic structures from a graph.

- Identify the number of edges by leg, be k these number. Each edge correspond to 1-DoF joint. Thus, each leg correspond to a k-edge or a k-DoF.

- Make all combinations of f_i-DoF joints, $i = 1,2,3$, up to complete serial-legs with k-DoF.

- Replace the graph-legs to a k-DoF serial-legs.

- Repeat the item above for all possible k-DoF serial-legs.

In the example below we apply these four steps to develop new kinematic structures from a graph.

Example 9. *Figure 19 shows a graph of a parallel manipulator which works in a 6-system, i.e.* $\lambda = 6$*, and it has mobility* $M = 6$*. First, we built the kinematic chain identifying the number of edges by leg, replacing by joints, and the number of vertices, replacing by links. In this case, we have six legs with* $k = 6$ *edges, i.e. we have legs with 6-DoF in the kinematic chain shown in Figure 19(b). Figure 19(c) shows some of the possible combinations of* f_i*-DoF joints,* $i = 1,2,3$*, to form serial-legs with 6-DoF. Figure 19(d) shows the replacement of graph-legs to serial-legs to obtain parallel manipulators.*

We can apply this procedure to enumerate all kinematic structures from a set of graphs.

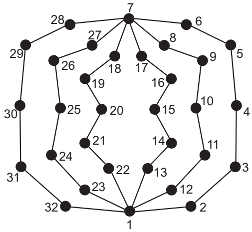
3.5 ISOMORPHIC KINEMATIC STRUCTURES

Isomorphisms avoidance is a recurrent problem in mechanisms and machines science. A major problem in the study of kinematic structures is that of detecting a possible isomorphism (structural equivalence) between two given kinematic chains, mechanisms and parallel manipulators.

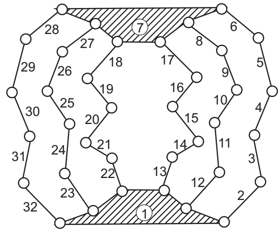
Two kinematic chains are said to be isomorphic if they share the same topological structure. In terms of graphs, there are an one-to-one correspondence between their vertices and edges that preserve the incidence. If there is not such correspondence the two kinematic chains are said to be non-isomorphic.

For example, Figure 20 shows two eight-link kinematic chains which are apparently dissimilar but are isomorphic to each other. In this example, the correspondence between the links is given by $1 \Leftrightarrow 6$, $2 \Leftrightarrow 1$, $3 \Leftrightarrow 5$, $4 \Leftrightarrow 2$, $5 \Leftrightarrow 4$, $6 \Leftrightarrow 3$, $7 \Leftrightarrow 7$, $8 \Leftrightarrow 8$.

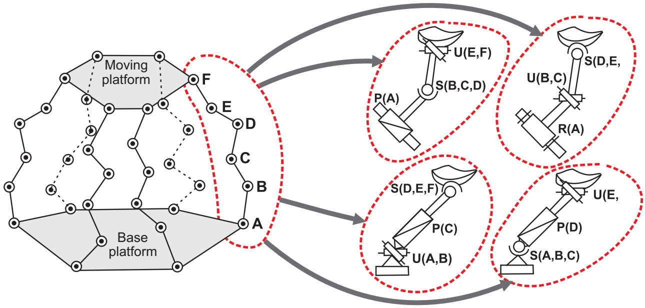
Earlier studies dealing with structural synthesis utilized visual inspection for solving this problem. Since diagrams of kinematic chains can be drawn in different ways, visual detection of isomorphisms is not always easy (see Figure 20). In view of these difficulties several attempts have been made in literature to develop reliable and computationally efficient tests for isomorphisms.



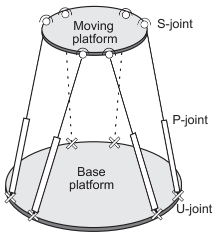
(a) Graph to $\lambda = 6$.



(b) Kinematic chain to $\lambda = 6$.



(c) Some possible combinations of legs.



(d) Classical Stewart platform.

Figure 19 – From graph to parallel manipulators.

3.5.1 Elimination of isomorphisms

Uicker and Raicu (1975) suggested that the characteristic polynomial could be used to test for isomorphisms. However, if two kinematic chains are isomorphic, it is necessary, but not sufficient, that their characteristic polynomials are identical as there are counter-examples where this method fails (TISCHLER et al., 1995a; MRUTHYUNJAYA, 2003).

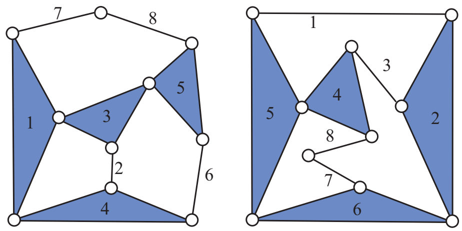


Figure 20 – Isomorphism between two kinematic chains (MRUTHYUNJAYA, 2003).

Agrawal and Rao (1987) suggested a method of identification called the optimum code. The method involves a technique for labeling the links of a kinematic chain such that a binary string obtained by concatenating the upper triangular elements of the adjacency matrix row by row, excluding the diagonal elements, is maximized. This method is called the MAX code. We can also search for a labeling of the chain that minimizes the binary string of the upper triangular elements, called the MIN code. There is a need to develop a more efficient heuristic algorithms for determination of the optimum code (TSAI, 2001; MRUTHYUNJAYA, 2003).

Rao and Raju (1991) present a method for detecting isomorphisms based on Hamming numbers of the adjacency matrix. Although no counter-examples are known, when the algorithm was applied to the detection of isomorphisms among the number of inversions of the planar, $M = 1$, ten links, some non-isomorphic inversions were omitted (TISCHLER et al., 1995a).

Siek et al. (2002) present a test of isomorphisms detection whose worst-case time complexity is $O(|V|!)$, where $|V|$ is the number of vertices.

The McKay algorithm (MCKAY, 1998, 2009b, 2007) is, to the best of the authors' knowledge, considered the fastest graph isomorphisms algorithm available today, it is exponential time $O(e^{|V|})$ (JAIN; WYSOTZKI, 2005; FOGGIA et al., 2001; MIYAZAKI, 1997).

Köbler et al. (1993) have examined the structural complexity of the graph isomorphisms problem and state that there is strong evidence to suggest that no efficient algorithms exist for this problem, i.e. the problem of isomorphisms is NP-hard[1].

[1] A problem is NP-hard if an algorithm for solving it can be translated into one for solving any NP-problem (nondeterministic polynomial time). NP-hard therefore means "at least as hard as any NP-problem", although it might, in fact, be harder (WEISSTEIN, 2009)

3.6 BARRANOV TRUSSES AND ASSUR GROUPS

The Barranov truss has a close relationship with Assur groups. Assur groups are kinematic chains in which some links contain free or unpaired elements such that when the group is connected to the frame through all its free elements it becomes a structure with zero mobility (TISCHLER et al., 1995a; MRUTHYUNJAYA, 2003). Barranov truss is a rigid structure which is formed when a link connects to all the outside pairs of an Assur group. On the other hand, removing any link of a Barranov truss, an Assur group will be obtained (HAN et al., 2000).

Figure 21 shows some basic Assur groups. Figure 22 shows the Barranov trusses originated from Assur groups in Figure 21. Three-link Barranov truss shown in Figure 22(a) was originated connecting all free links of Assur group shown in Figure 21(a) in a single link, five-link Barranov truss shown in Figure 22(b) was originated connecting all free links of Assur group shown in Figure 21(b) in a single link, and so on.

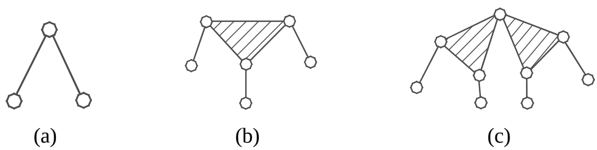


Figure 21 – Some basic Assur groups.

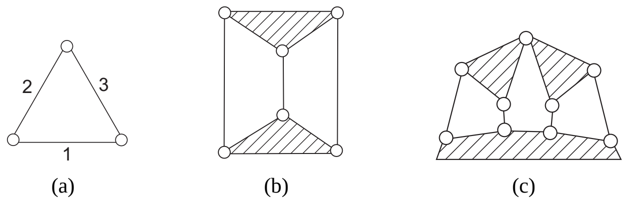
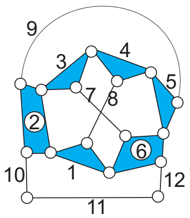


Figure 22 – Barranov trusses originated from Assur groups shown in Figure 21.

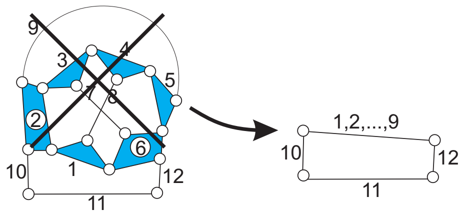
3.7 IMPROPER KINEMATIC STRUCTURES

An improper kinematic chain is a kinematic chain with $M > 0$, where at least one biconnected subchain has mobility $M' \leq 0$. If the subchain has mobility $M = 0$, them the kinematic chain has a Barranov truss as subchain.

Let the planar kinematic chains shown in Figure 23(a), the mobility is $M = 1$ but, the subchain formed by links 1-2-3-4-5-6-7-8-9, has mobility $M' = 0$ and its links act as a Barranov truss. Generally, improper chains are of no interest in pure kinematic analysis and should be discarded. Figure 23(b) shows the elimination of Barranov truss resulting in a kinematic chain more simple.



(a) Twelve-link kinematic chain.



(b) Elimination of Barranov truss.

Figure 23 – Improper planar kinematic chain with $M = 1$ because it contains a subchain (1-2-3-4-5-6-7-8-9) with $M' = 0$.

3.8 CONCLUSIONS

This chapter reviewed the main concepts used in this thesis, i.e. mobility criterion, kinematic chain, mechanisms, parallel manipulators, isomorphisms, improper chains and Barranov trusses.

In this thesis we will develop a systematic procedure for enumeration of kinematic structures based on group and graph theory tools, therefore, we will describe a procedure to convert kinematic structures into graphs and vice versa. The representation of kinematic chains by graphs is well known in the mechanisms and machines theory.

4 ENUMERATION AND ANALYSIS OF KINEMATIC STRUCTURES: A BIBLIOGRAPHY REVIEW

This chapter presents a bibliography review of enumeration of kinematic structures methods and criteria to analysis these kinematic structures. First, we review the main contributions to the enumeration of kinematic structures, i.e. kinematic chains, mechanisms and parallel manipulators. Second, we review the analysis criteria found in the literature to classify the kinematic structures enumerated.

4.1 ENUMERATION OF KINEMATIC STRUCTURES

4.1.1 Enumeration of kinematic chains

The enumeration of kinematic chains consists of the generation of a complete list of kinematic chains with a determined mobility without isomorphisms. A significant and unsolved problem in the enumeration of kinematic chains is the precise elimination of all isomorphisms and improper chains. In early stage of design, it is preferable the generation of duplicate (isomorphic) chains to the omission of a potentially useful kinematic chain (TISCHLER et al., 1995a).

It is important to remember that a kinematic chain can be uniquely represented by the graph whose vertices correspond to links and whose edges correspond to joints of the chain. Figure 24 shows this correspondence, Figure 24(a) shows the Stephenson kinematic chain with labeled links and Figure 24(b) shows the corresponding graph (DOBRJANSKYJ; FREUDENSTEIN, 1967). In graph theory terms, the enumeration of kinematic chains

(a) Stephenson kinematic chain.

(b) Graph representation.

Figure 24 – Correspondence between graphs and kinematic chains.

corresponds to the enumeration of graphs satisfying the general mobility criterion (see Equation 3.1 on page 37) and having given a number of vertices and edges. However, the problem of graphs enumeration that represent kinematic chains is a NP-Hard problem because all methods of graphs enumeration generate a great amount of isomorphisms which must be eliminated without eliminating any graph (kinematic chain) with useful potential for the accomplishment of task.

4.1.1.1 Link assortments

The first common step in enumeration of kinematic chains is the determination of the possible assortments of binary, ternary, quaternary, etc. links that can exist in a desired kinematic chain. These are given by the solutions of the following equations:

$$n = n_2 + n_3 + n_4 + \cdots \tag{4.1}$$

$$2j = 2n_2 + 3n_3 + 4n_4 + \cdots \tag{4.2}$$

where n_i is the number of links with i connections each, n is the number of links and j is the number 1-DoF joints.

The subsequent step is the formation of distinct structural patterns in which polygonal links (non-binary) can be connected together. To add the available binary links to the polygonal link patterns in all possible ways to produce closed-loop kinematic chains and finally discarding improper chains and isomorphic kinematic chains to produce the set of all distinct kinematic chains that meet the mobility criterion (see Equation 3.1 on page 37).

For the purpose of classification, each link assortment is called a *partition*. Algorithms for finding all the partitions are well documented in literature (JAMES; RIHA, 1976). Table 6 shows the partitions for constructing ten-bar kinematic chains with $\lambda = 3$ (not necessarily planar motion) and $M = 3$, where number 2 represents binary links, 3 ternary links, and so on.

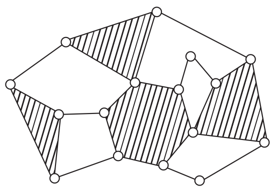
4.1.1.2 Contribution of Franke

The Franke's notation is a graphical simplification of the representation of kinematic chains (FRANKE, 1958; TISCHLER et al., 1995a). In the Franke's notation, each polygonal link is represented by one circle with a label n inside that corresponds to number of connections of the link and binary links are represented by lines. Figure 25(a) shows one 12-links kinematic

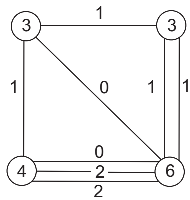
Table 6 – Partitions of the kinematic chains with ten links, with $\lambda = 3$ and $M = 3$.

Partitions	Classifications of links									
Partition 1	3	3	3	3	2	2	2	2	2	2
Partition 2	4	3	3	2	2	2	2	2	2	2
Partition 3	4	4	2	2	2	2	2	2	2	2
Partition 4	5	3	2	2	2	2	2	2	2	2
Partition 5	6	2	2	2	2	2	2	2	2	2

chain and Figure 25(b) shows the corresponding Franke's notation.



(a)



(b)

Figure 25 – Franke's notation of a 12-links kinematic chain.

In the enumeration procedure based on Franke's notation, we first consider all the possible mappings of the polygonal links (non-binary) for each possible partition. For each partition, each circle is connected by lines in all possible ways, being the incident line number in the circle equal to label of it. Each line receives a number $k \geq 0$, $k = 0$ if no binary link exists between two polygonals (DAVIES; CROSSLEY, 1966).

Care must be taken to guarantee that improper kinematic chains containing immobile sub-chains are not produced. A disadvantage of this method is that it generates a great number of isomorphisms which must be eliminated.

4.1.1.3 Contribution of Assur

Another approach for enumeration of kinematic chains is due to Assur (TISCHLER et al., 1995a; MRUTHYUNJAYA, 2003). He introduced the concept of fundamental groups, later called Assur groups. Assur groups are kinematic chains in which some links contain free or unpaired elements such that when the group is connected to the frame through all its free elements it becomes a structure with zero mobility.

Assur also proposed that kinematic chains of greater complexity (i.e. with greater number of links) could be built up by the sequential addition of these Assur groups to simpler kinematic chains (i.e. with fewer links). The basis for this idea lies in the fact that addition of an Assur's group to a link or links of an existing kinematic chain do not modify the mobility of the original kinematic chain. The method is based on visual inspection and does not require determination of partitions. Improper kinematic chains do not arise if the initial simpler kinematic chains are free from immobile sub-chains and if the free elements of an Assur's group are not all added to a single link. Figure 26 shows the addition of an Assur's group to a 4-link kinematic chain.

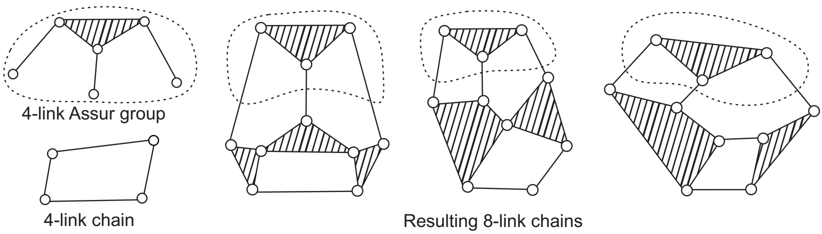


Figure 26 – Aggregation of the Assur's group to 4-link kinematic chain.

However, the method produces a large number of isomorphism. Also, it is necessary to have available atlases of chains with mobility M and number of links less that n, as well as complete atlases of all Assur groups with $(n - M - 1)$ links (MRUTHYUNJAYA, 2003).

4.1.1.4 Contribution of Farrell

Simoni and Martins (2007), Simoni (2008) implemented a modified version of the Farrell's method for enumeration of kinematic chains avoiding to enumerate the fractionated kinematic chains. Therefore, the Farrell's method will be described here with more detail and our method will be described in Section 4.1.1.5. The Farrell's method imposes a tree structure in kinematic chains generation process and is summarized in the following steps (FARRELL, 1977; TISCHLER et al., 1995a):

Step 1: Each link in the partition is assigned by a numerical label according to its degree. One of the links with the highest degree is given the number ”1”, while the link with the lowest degree is given the highest number. Two links cannot be assigned by the same number. For example, the Partition 1 in Table 6 has four ternary links, which we now labels 1, 2, 3, and 4, and six binary links we labels 5, 6, 7, 8, 9, and 10. At this stage all links are unconnected. See Figure 27.

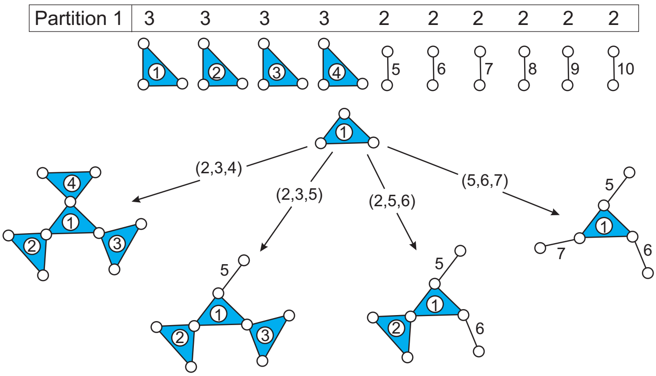


Figure 27 – Example Farrell's method: possible connections for link 1.

Step 2: The link with the lowest number (i.e. 1) is selected and the remaining links, $\{2, 3, ... , 10\}$ are grouped so that connecting link 1 to any member of the group would result in an identical, partially connected, form. Here, two distinct groups materialize, namely a group of ternary links $\{2, 3, 4\}$, and a group of binary links $\{5, 6, 7, 8, 9, 10\}$. Connecting link 1 to any member in the group $\{2, 3, 4\}$ would result in two connected ternary links, and connecting link 1 to any member of $\{5, 6, 7, 8, 9, 10\}$ would result a ternary link connected to a binary link.
Step 3: The number of connections c needed to make the link with the lowest number fully connected is determined. In this case $c = 3$, because link 1 is ternary and no connections have yet been made. All the different ways of selecting $c = 3$ links to connect to link 1 from the groups of Step 2 are found. These are; three ternary links $\{2, 3, 4\}$, two ternary links and one binary link $\{2, 3, 5\}$, one ternary link and two binary links $\{2, 5, 6\}$, and three binary links $\{5, 6, 7\}$. The partial forms which result from each of these selections are shown in Figure 27. In each case the lowest numbered link of each group are selected first. Each of the four partial forms represents a branch on the tree.
Step 4: Each of the branches in Step 3 are selected in turn and any links which are fully connected are ignored; Steps 2, 3 and 4 are repeated for the next lowest numbered link which is not fully connected. In this case the lowest numbered link will be link 2. Steps 2, 3 and 4 are repeated until all other

links are fully connected or it is impossible to connect the remaining links. When either of these two situations arises the algorithm back-tracks and continues with the next unexplored branch.
Step 5: When no unexplored branch remains the next partition is selected, and all above steps are repeated until no further partitions remain.
Step 6: Elimination of improper kinematic chains and isomorphisms and finality list the generated kinematic chains.

One disadvantage of the method is that it generates many isomorphisms which must be eliminated and the elimination requires a great computational effort. Figures 28 and 29 show the exploration of the branch 2 from the link 1.

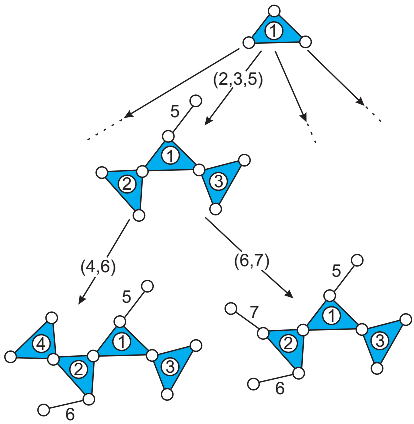


Figure 28 – Example Farrell's method: exploration of the branch 2 from the link 1.

4.1.1.5 Contribution of Simoni and Martins

Simoni and Martins (2007), Simoni (2008) presented a modification of Farrell's method in order to avoid the generation of fractionated kinematic chains. We notice that, in majority of the applications, the fractionated kinematic chains are generated without necessity. We also notice that some methods enumerate fractionated kinematic chains (LEE; YOON, 1994; SUNKARI; SCHMIDT, 2006; TUTTLE, 1996) while others do not (HWANG; HWANG, 1991; MRUTHYUNJAYA, 1984c, 1984b, 1984a; TISCHLER et al., 1995a; SIMONI et al., 2009).

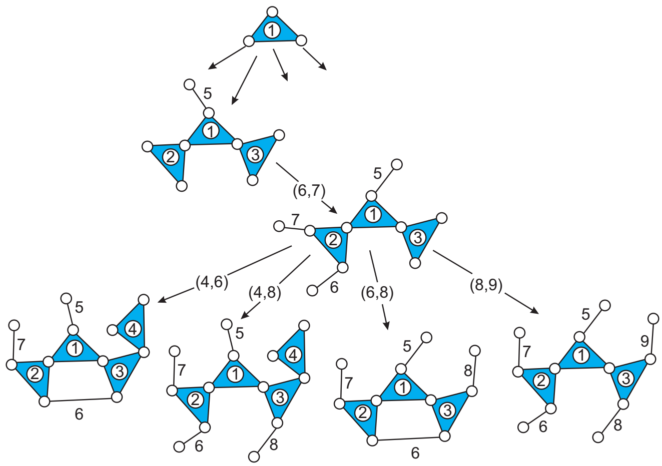
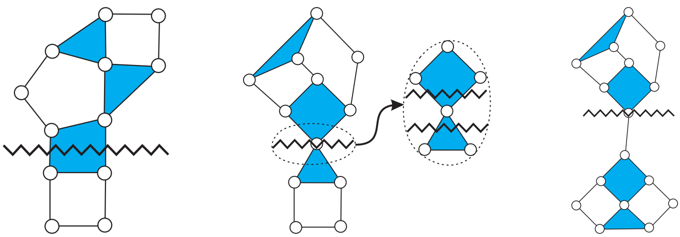


Figure 29 – Example Farrell's method: continuation of exploration of the branch 2.



(a) (b) (c)

Figure 30 – Fractionation in kinematic chains: (a) Body-fractionation. (b) Joint-fractionation. (c) Fractionation into hybrid chains.

A kinematic chain is classified as fractionated if the elimination of a single element of the chain (link or joint) divides the kinematic chain into two disconnected kinematic chains. Otherwise it is non-fractionated (TISCHLER et al., 1995a). A body-fractionated kinematic chain contains a link which divides the kinematic chain into two closed, independent, kinematic chains, see Figure 30(a). A joint-fractionated kinematic chain contains a joint whose re-motion (or disconnection) divides the kinematic chain into two closed sub-chains, see Figure 30(b). Figure 30(c) shows a kinematic chain with more complicated forms of fractionation can occur, including not only combinations of joint- and body-fractionation, but also fractionation into hybrid chains.

The method was implemented in C++ using graphs as data structure. The method imposes a tree structure in generation process similar to the Farrell's method, see Figure 31. The input data of the algorithm is the number of

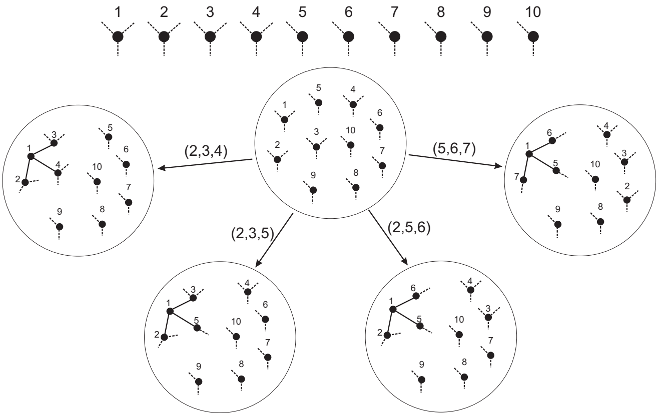


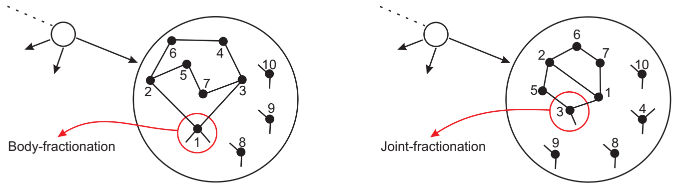
Figure 31 – Structure of proposed method in tree structure implemented using algorithms of the Boost Graph Library (SIEK et al., 2002).

vertices and the degree of each vertex. The vertices are orderly decreasing of degree and labeled with gradual number. The graph of root of tree is formed by a set of vertices labeled. Combinations of degrees of vertices are made and edges are connected in accordance with the label of each vertex. The process of adding edges is repeated to complete the degree of all the vertices.

In generation process, if a graph has a connected sub-graph with the degrees of the vertices complete except one of them, such graph do not generate more children because in this case the children will originate fractionated kinematic chains:

- if the sub-graph has only one vertex with degree 1 free, its children lead to body-fractionation as shown in Figure 32(a);
- if the sub-graph has only one vertex with degree higher than 1 free, its children lead to joint-fractionation as shown in Figure 32(b).

Some fractionated chains are generated in leaves of the tree, in this case we use the test of biconnectivity (time complexity is polynomial) of the Boost Graph Library (SIEK et al., 2002) to exclude them . Thus we avoid



(a) Avoiding generate body-fractionation. (b) Avoiding generate joint-fractionation.

Figure 32 – Eliminated graph avoiding the generation of fractionated kinematic chains.

the generation of graphs that originate fractionated kinematic chains. In the graphs of leaves of tree we run the isomorphisms test of Boost Graph Library whose worst-case time complexity is $O(|V|!)$, where $|V|$ is the number of vertices.

More details and the algorithm of the modification of Farrell's method consult Simoni and Martins (2007) and Simoni (2008).

4.1.1.6 Contribution of Sunkari and Schmidt

Recently, Sunkari and Schmidt (2006) presented a method for enumeration of kinematic chains based on the group theory techniques. They uses the McKay's method (MCKAY, 1998, 2009b) for generation of an isomorphism class representative in combination with an efficient improper testing algorithms. According to the authors of method, the algorithm is computationally efficient and it generates 318,162 planar kinematic chains whit 14 link and $M = 1$ in 37.28s on Pentium III 1.7GHz with 512MB RAM. The authors claims that the computational speed at which the kinematic chains are generated depend on McKay-type algorithm that greatly minimize the explicit isomorphism detection by using group theory techniques.

4.1.1.7 Contribution of Simoni et al.

Simoni (2008) and Simoni et al. (2009) adapt the graph generator of McKay (2009b, 1998), freely distributed together with the package *gtools*, to use the degeneracy test that Martins and Carboni (2007) use in the algorithm to calculate the connectivity and variety of kinematic chains.

The degeneracy test considered by Martins and Carboni (2007) identifies improper kinematic chains that operate in any screw system, by individualizing and calculating the mobility of all the sub-chains. Using the graph generator of McKay together with the improper kinematic chains test of Martins and Carboni (2007), we validated the kinematic chains generation method considered in Simoni and Martins (2007) and enumerated non-fractionated kinematic chains with mobility $1 \leq M \leq 6$ for several screw systems.

More details and the algorithm of the modification of Farrell's method consult Simoni (2008) and Simoni et al. (2009).

4.1.1.8 Contribution of Martins et al.

Simoni (2008) and Martins et al. (2010) present a method that generates exclusively fractionated kinematic chains, i.e. kinematic chains with body fractionation, joint fractionation, body and joint fractionation or fractionation into hybrid kinematic chains. The method is similar to the Assur method (see Section 4.1.1.3), in the sense that, kinematic chains with greater complexity (i.e. with a greater number of links) are generated by the aggregation of simpler kinematic chains (i.e. with fewer links). The advantage of the method is that degenerate kinematic chains are not enumerated if the initial simpler kinematic chains are free from immobile subchains. The number of isomorphisms is drastically reduced by applying the symmetry concept, introduced in Section 2.3, to each kinematic chain for subsequent connections.

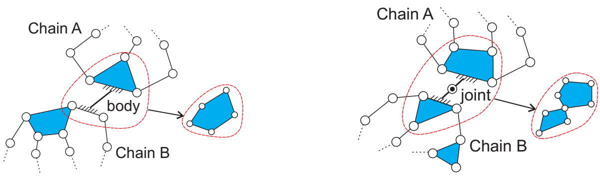
Method description

The method consists of the *aggregation* of simpler kinematic chains to form kinematic chains with greater complexity. This aggregation consists of "welding" links of two kinematic chains (one of each kinematic chain), as shown in Figure 33(a), to form kinematic chains with body fractionation or "the introduction of one joint" between two bodies to form kinematic chains with joint fractionation as shown in Figure 33(b).

A serial kinematic chain can also be introduced between the bodies of two simpler kinematic chains forming fractionation into hybrid kinematic chains as shown in Figure 34.

Isomorphism avoidance

To avoid the generation of isomorphic kinematic chains, we need to avoid "welding" or "introduction of one joint" in symmetrical links of a kinematic chain. Thus, we consider "welding" or "introduction of one joint" only



(a) Welding two links to generate a body-fractionated kinematic chain.

(b) Introduction of one joint between two links to generate joint-fractionated kinematic chain.

Figure 33 – Aggregation of kinematic chains generating kinematic chains with body- or joint fractionation.

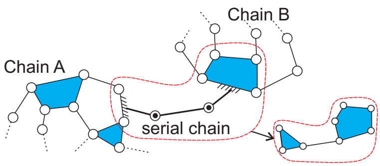


Figure 34 – Aggregation of kinematic chains generating fractionation into hybrid kinematic chains.

in links representing different inversions of the kinematic chain. For example, when we weld the serial kinematic chain shown in Figure 35 to all links of a Stephenson kinematic chain, we generate several isomorphic fractionated kinematic chains as shown in Figure 36.

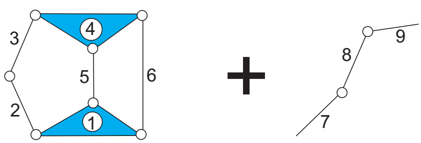


Figure 35 – Aggregation by “welding” of a serial kinematic chain and a Stephenson kinematic chain.

Considering the symmetries of the Stephenson kinematic chain, i.e. kinematic inversions or mechanisms that the Stephenson kinematic chain can assume, which are known as Stephenson I (fix link 6), Stephenson II (fix link 2) and Stephenson III (fix link 1), we avoid the generation of isomorphic kinematic chains when we weld a link of a serial kinematic chain to a link of a

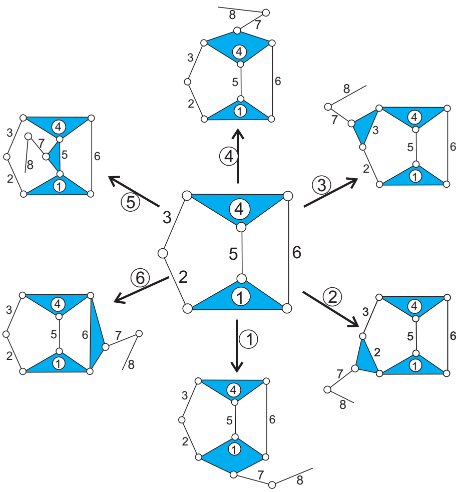


Figure 36 – List of all possible aggregations of a serial kinematic chain to a Stephenson kinematic chain. Note the generation of isomorphic fractionated kinematic chains.

Stephenson kinematic chain, as shown in Figure 37. This simple example shows that the identification of symmetries of the kinematic chains reduces drastically (and sometimes eliminates) the number of isomorphisms on the output list.

Using this method Martins et al. (2010) concluded that the discrepancies reported in the literature were related to fractionation in kinematic chains. More details and examples of the generation of fractionated kinematic chains technique consult Simoni (2008) and Martins et al. (2010).

4.1.1.9 Other contributions

Tischler et al. (1995a) proposed an improvement to the Farrell method, called the Melbourne method, with the objective to reduce the number of isomorphisms in output list. The improvement consists of applying a set of four rules with the objective of reducing the number of isomorphisms on the output list. Melbourne's method was applied to enumeration of kinematic chains suitable for application as robot hands (TISCHLER et al., 1995b). Mruthyunjaya (1979) presented a method based on the transformation of binary kinematic chains for the structural synthesis of simple- and multiple-jointed kinematic chains with positive, zero or negative freedom. Davies and Crossley

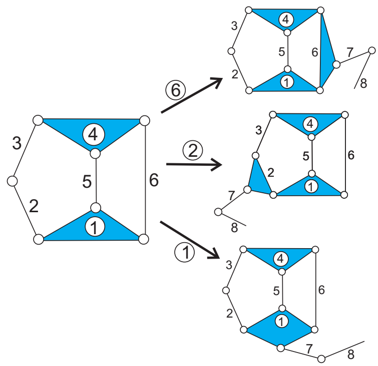


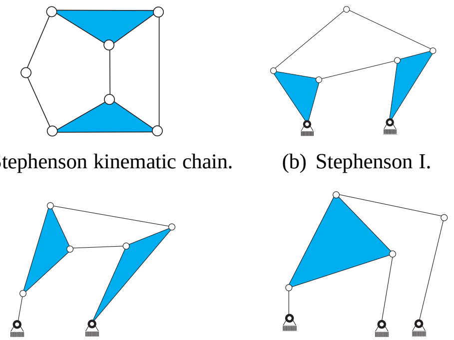
Figure 37 – List of aggregations of a serial kinematic chain in the Stephenson kinematic chain avoiding repeating symmetries. In this case, the identification of symmetries eliminates the generation of isomorphic fractionated kinematic chains.

(1966) presented a method based on Franke's notation. Tuttle and coworkers (TUTTLE, 1996; TUTTLE et al., 1989a, 1989b) enumerated the kinematic chains systematically which reduced the need for isomorphism testing. The theory of symmetry groups is used successfully by Tuttle to eliminate isomorphic entities in the generation of bases and kinematic chains.

4.1.2 Enumeration of mechanisms

Agreement with IFToMM a mechanism is a kinematic chain with one of its components (links) taken as a frame (IONESCU, 2003). The problem of define all distinct choices of bases of a kinematic chain, i.e. define the possible mechanism to a given kinematic chain is known in mechanisms and machines literature as enumeration of mechanisms, enumeration of inversions, specialization, and so on (MRUTHYUNJAYA, 2003; TUTTLE, 1996; JAMES; RIHA, 1976; WALDRON; KINZEL, 1999; YAN; HWANG, 1991; YAN, 1998).

Figure 38(a) shows the Stephenson kinematic chain which originate three mechanisms with different characteristics of the movement relative to the base fixing one of the links of the kinematic chain. Figures 38(b), 38(c) and 38(d) shows the classical mechanisms originated fixing different links of the kinematic chain, i.e. Stephenson I, Stephenson II and Stephenson III.



(a) Stephenson kinematic chain. (b) Stephenson I.

(c) Stephenson II. (d) Stephenson III.

Figure 38 – Stephenson kinematic chain and the three classical mechanisms originated: Stephenson I, Stephenson II and Stephenson III.

4.1.2.1 Contribution of Tuttle et al.

Tuttle and coworkers (TUTTLE, 1996; TUTTLE et al., 1989a, 1989b) presented a method based on the theory of finite symmetry groups to enumeration of kinematic chains minimizing the use of isomorphism testing, as part of their method they generated information on subgroup structure of symmetry groups of binary and polygonal links and utilized it for deriving distinct inversions of chains. They present a table of mechanisms with up to 14 links and possessing up to three degrees of freedom.

4.1.2.2 Contribution of Yan et al.

Yan and Hwang (1991) and Yan (1998) presented a process called specialization which consists of assigning specific types of links and joints in the available atlas of kinematic chains, subject to certain design requirements and constraints. First, they apply the permutation group to define the bases of a mechanism. Second, they apply the Polya's theory to count all specialized mechanisms with a determined number of joints (prismatic, rotative, cam, etc.) and a determined type of links (spring, rigid, ground, etc) (YAN, 1998). Simoni (2008) and Simoni et al. (2009) present a method similar to that of Yan (1998) to define the bases of a mechanism, this method will be reviewed in the next section.

4.1.2.3 Contribution of Simoni et al.

Simoni (2008) presented a method for enumeration of mechanisms based on graph and group theory techniques, however, Simoni (2008) did not presented a detailed description of the method. Simoni (2008) only used the nauty to identify the orbits of the graph represents a kinematic chain. After that work, Simoni et al. (2009) presented an improvement of the method applying the concept of symmetry presented in Section 2.3. This improvement using the concepts of symmetry, actions and orbits of the automorphism group of the associated graph will be described in Section 5.3 and it is an original contribution of this thesis.

4.1.2.4 Other contributions

Rao and Rao (1996) represent an inversion of a chain by a joint-joint symmetric distance matrix in which each entry is the shortest distance, in terms of number of links, by passing the frame link, between the corresponding joints. A numerical scheme comprising of the row-sums as numerators and the sum total of all row-sums as the common denominator is claimed to be successful in distinguishing inversions of chains with 6, 8 and 10 links (MRUTHYUNJAYA, 2003). Vijayananda (1994) successfully applied the representation set of links to distinguish mechanisms derived from a chain and carried out enumeration of mechanisms with up to 13 links and possessing up to seven degrees of freedom (MRUTHYUNJAYA, 2003).

4.1.3 Enumeration of parallel manipulators

Agreement with IFToMM a mechanism is a parallel manipulator is a kinematic chain with one of its components (links) taken as frame and the other taken as end-effector (IONESCU, 2003).

4.1.3.1 Contribution of Tsai

Tsai (2001) presents a method of enumeration of parallel manipulators with a single platform distributing the number of binary links between the number of legs of the parallel manipulator. Tsai (2001) uses four equations to characterize the structural topology of parallel manipulators:

$$M = \lambda(n-j-1)+\sum_{i=1}^{j} f_i \tag{4.3}$$

$$m = M \tag{4.4}$$

$$\sum_{k=1}^{m} C_k = \sum_{j=1}^{j} f_i \tag{4.5}$$

$$\sum_{k=1}^{m} C_k = (\lambda+1)M-\lambda \tag{4.6}$$

$$\lambda \geq C_k \geq M \tag{4.7}$$

where M is the mobility (see Equation 3.1 on page 37), m is the number of limbs made up of an open-loop kinematic chain and C_k is the connectivity of a limb and it is defined as the number of degrees of freedom associated with all the joints, including the terminal joints, in that limb. Substituting Equation 4.4 and 4.5 into Equation 4.3, we obtain the Equation 4.6. To ensure proper mobility and controllability of the moving platform, Tsai uses Equation 4.7, i.e. the connectivity of each limb should not be greater than the motion parameter or be less than the number of degrees of freedom of the moving platform.

Example 10. *For spatial manipulators,* $\lambda = 6$*. Thus, Equations 4.6 and 4.7 become* $\sum_{k=1}^{m} C_k = 7M-6$ *and* $6 \geq C_k \geq M$*. All feasible limb connectivity listings are shown in Table 7. After the classification presented in Table 7, the*

Table 7 – Classification of Spatial Parallel Manipulators according to Tsai's method.

Mobility (M)	Total Joint Degrees of Freedom ($\sum_{i=1}^{j} f_i$)	Limb Connectivity Listing ($C_1, C_2, \ldots, C_m$)
2	8	4, 4
		5, 3
		6, 2
3	15	5, 5, 5
		6, 5, 4
		6, 6, 3
4	22	6, 6, 5, 5
		6, 6, 6, 4
5	29	6, 6, 6, 6, 5
6	36	6, 6, 6, 6, 6, 6

method consider all possible combinations of revolute, prismatic, universal and spherical joints for each limb connectivity listing. For example, the (5, 5, 5) connectivity listing cam be changed by RUU, UPU, RRS, RPS, PSP, RRRU, PRRU, RRRRR, RRRRP, RPRRP, RRPRP, and so on. More details of how to convert graphs to kinematic structures consult Sections 3.4.1 and 3.4.2 (pages 42 and 43).

4.1.3.2 Contribution of Alizade and Bayram

Alizade and Bayram (2004) present a method of enumeration of parallel manipulators with single and multiple platforms, where parallel manipulators are classified according to their platform type(s) and the connections between them. The method determines simple structural groups for a given set of synthesis parameters and then a number of required actuators are added to the group to form the parallel manipulator. For certain synthesis parameters, the Alizade and Bayram's method finds one structure with the desired number and type of platforms (non-binary links) and number of binary links (ALIZADE; BAYRAM, 2004). After that, the number of binary links is distributed between the number of branches and legs originating only one parallel manipulator for the specified parameters.

The procedure can be summarized, step by step, as follows (ALIZADE; BAYRAM, 2004):

1) Select values for the number of mobile platforms, B, and the total number of joints on the platforms, j_p;

2) The number of different structural groups, G, is given by $G = 0.5j_p - B + 1$ (the structural groups correspond to partitions discussed in Section 4.1.1.1);

3) Select a value for the total number of connections (sum of number of legs and branches), c, in the interval given by $1 + 0,5j_p \leq c \leq 1 + j_p - B$;

4) Calculate the number of branches, c_b, from $j_p = c_b + c$;

5) Calculate the number of legs, c_l, from $c = c_b + c_l$;

6) Calculate the sum of mobility of all joints in the structural group f_t from $f_t = \lambda(c - B) = \sum_{i=1}^{j} f_i$

7) Place the joints on branches and legs.

8) Decide on the place to add the actuators. The DoF of the manipulator is equal to the number of actuators added. Note that one may place the actuators on legs or branches and also more than one actuator may be placed on the same leg or branch.

9) Using the principle of interchangeability of kinematic pairs, replace the single mobility kinematic pairs with other kinematic pairs as desired.

Example 11. *A spatial parallel manipulator with four degrees of freedom is required, we want to use two triangular platforms (ALIZADE; BAYRAM, 2004).*

1) $\lambda = 6$, $M = 4$, $B = 2$ $j_p = 6$.

2) $G = 0.5 * 6 - 2 + 1 = 2$, *we have only two different structural groups, i.e. partitions (4 3 3) and (3 3 3).*

3,4,5) $c = 4$ *or* $c = 5$*:*

- *For* $c = 4$*:* $c_b = 2$ *and* $c_l = 2$*;*
- *For* $c = 5$*:* $c_b = 1$ *and* $c_l = 4$.

6)
- *For* $c = 4$*:* $f_t = 6(4-2) = 12$*; since we have a total of four branches and legs, it is convenient to place* $12/4 = 3$ *joints on each leg or branch.*
- *For* $c = 5$*:* $f_t = 6(5-2) = 18$*; since we have a total of five branches and limbs, it is convenient to place three joints on each leg or branch and place the remaining three joints on some of the branches or legs as we like.*

7) The placement of joint for $c = 4$ *and* $c = 5$ *is given in Figures 39(a) and 39(c);*

8,9) In Figure 39(b), two actuators are placed on branches and two actuators are placed on legs. In Figure 39(d), all actuators are placed on the legs. Therefore the synthesis is concluded.

4.2 CRITERIA FOR ANALYSIS OF KINEMATIC STRUCTURES

Structural analysis is a field of kinematics that study of the nature of connection among the members (links and joints) of a mechanism and its mobility (TSAI, 2001). It is concerned primarily with the fundamental relationships among the mobility, the number of links, the number of joints, and

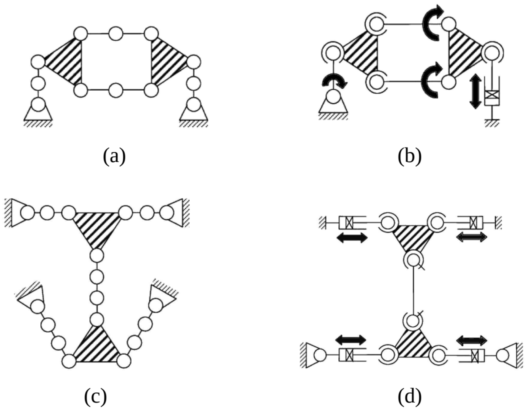


Figure 39 – Two parallel manipulators obtained by Alizade's method (ALIZADE; BAYRAM, 2004).

the type of joints used in a mechanism. The structural analysis deals only with the general functional characteristics of a mechanism and not with the physical dimensions of the links.

As we can see in Section 4.1, in general, the number of generated kinematic structures generated in the enumeration process is great and it is difficult to evaluate each chain individually. Therefore, it is necessary to develop a set of criteria to evaluate the merit of each chain without eliminating a chain with possibilities to develop the desired task. For this reason, the concepts of variety, connectivity, degrees-of-control, redundancy and symmetry can be used to classify kinematic chains according to the constraints required. They are essential for structural analysis of mechanisms and parallel manipulators.

4.2.1 Mobility

Definition 12 (Mobility)**.** *The number of* degrees of freedom*, or* mobility *(M), of a kinematic chain is the number of independent parameters required to completely specify the configuration of the kinematic chain in space, with respect to one link chosen as the reference.*

The mobility of a kinematic chain, with n links and j single degree of freedom joints, may be calculated by the general mobility criterion (HUNT, 1978; MRUTHYUNJAYA, 2003) applied to a set of n links and j single degree of freedom joints:

$$M = \lambda(n - j - 1) + j \tag{4.8}$$

where λ is the order of the screw system to which all the joint screws belong. More details on mobility calculation consult Section 3.3.

Using the graph representation of a kinematic chain (see Figure 24, on page 49), the general mobility criterion is given by

$$M = \lambda\,(|V| - |E| - 1) + |E| \tag{4.9}$$

where $|V|$ is the number of graph vertices (i.e. links) and $|E|$ is the number of graph edges (i.e. joints) (TSAI, 2001; MRUTHYUNJAYA, 2003).

4.2.2 Variety

Variety is an useful property for determining the relative connectivities within a chain and also for selecting actuated pairs. Variety may also be used to classify kinematic chains according to the constraints required (TISCHLER et al., 1995b, 2001).

A kinematic chain is of variety V if it does not contain any loop, or subset of loops, with a mobility of less than $M - V$, but does contain at least one loop, or subset of loops, which has a mobility of $M - V$ (TISCHLER et al., 1995b).

Recently, Martins and Carboni (2007) present a new definition of variety in terms of graphs.

Definition 13 (Variety)**.** *Let a kinematic chain of mobility M be represented by a graph G, the variety of the kinematic chain is:*

$$V = M - min\{M(G'_k) \forall G'_k \in B_s\} \tag{4.10}$$

where B_s is the (finite) set of all possible biconnected subgraphs G'_k of graph G and $M(G'_k)$ is the mobility of the k biconnected subchain/subgraph.

Classification of kinematic chains by variety V allows generalizations to be made about the relative connectivity of links within the kinematic chain, therefore, if a kinematic chain with variety V has a mobility M greater than the order of screw system that generally prevails λ, i.e. if $M > \lambda$, then any two links, separated by at least $\lambda - V$ joints, have relative connectivity $C \geq \lambda - V$. The variety of kinematic chains also affects the choice of the joint to be actuated. If the Variety of a kinematic chain with j joints is $V = 0$, the actuated pairs may be selected at random. Figure 40 shows three ten links planar kinematic chains with variety $V = 2$ (Figure 40a), $V = 1$ (Figure 40b), $V = 0$ (Figure 40c).

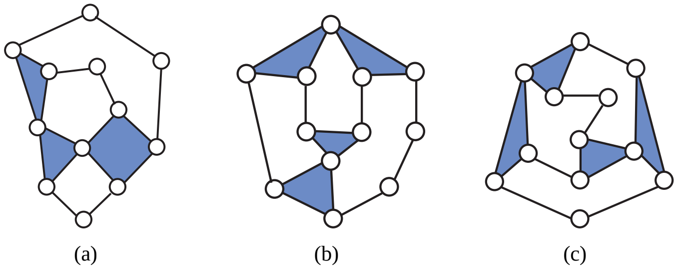


Figure 40 – Planar kinematic chains: (a) $V = 2$, (b) $V = 1$ and (c) $V = 0$.

Tischler et al. (2001) showed that the variety can be used to select those candidate mechanisms best suited to an intended function. An epicyclic transmission, designed to control the finger-tip of a dextrous robot finger, was used to demonstrate the technique. Tischler et al. (2001) identified that the most appropriate kinematic chains to the epicyclic gear train have variety $V = 1$. Among the 2271 kinematic chains with $\lambda = 2$, $M = 3$ and $\nu = 5$ enumerated in the set required, only 5 (five) have variety $V = 1$, and the other 2266 can be discarded as completely unsuitable. This example shows the potential of the variety to select the the most appropriate kinematic chains.

4.2.3 Connectivity

The connectivity C_{ij} between two links i and j of a kinematic chain is the relative mobility between links i and j. This concept was introduced by Hunt (1978). The importance of the connectivity and redundancy is emphasized by Hunt (1978), Tischler (1995), Belfiore and Benedetto (2000), Tischler et al. (2001), Liberati and Belfiore (2006).

Different algorithms for connectivity calculations were proposed by Shoham and Roth (1997), Belfiore and Benedetto (2000), Liberati and Belfiore (2006). However, all these algorithms presented some fail in connectivity calculation. An alternative definition of connectivity and a new algorithm capable of connectivity calculation for every kinematic chain can be found in Martins and Carboni (2007):

Definition 14 (Connectivity)**.** *In a kinematic chain represented by a graph X, the connectivity between two links i and j is defined in Martins and Carboni*

(2007) as

$$C_{ij} = \min\{D[i,j], M, M'_{\min}, \lambda\} \tag{4.11}$$

where $D[i,j]$ is distance between vertices i and j of X, M is the mobility of the kinematic chain considered, $M'_{\min}$ is the minimum mobility closed-loop biconnected subchain of X containing vertices i and j, and λ is the order of screw system.

Example 12. *The connectivity is an important criterion for selecting kinematic chains. For a better understanding of the importance of the connectivity consider the kinematic chain shown in Figure 41. Figure 41 represents a closed-loop kinematic chain with mobility $M = 3$, but the connectivity between any two links does not exceed 2. From this simple example, and as already outlined in previous works (SHOHAM; ROTH, 1997; BELFIORE; BENEDETTO, 2000; LIBERATI; BELFIORE, 2006; CARBONI, 2008), it is evident that connectivity, not mobility, determines the ability of an output link to perform a task relative to a frame.*

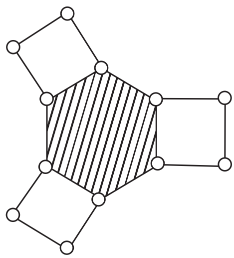


Figure 41 – Planar kinematic chain with maximum connectivity between links of 2, i.e. $C_{ij} \leq 2 \; \forall \; i, j$ (CARBONI, 2008). This kinematic chains will be eliminated for the connectivity.

4.2.4 Degrees-of-control

Belfiore and Benedetto (2000) introduced the concept of degrees-of-control. The degrees-of-control K_{ij} between two links i and j of a kinematic chain is the minimum number of independent actuating pairs needed to determine the relative position between the two links i and j, possibly leaving some other link-relative position undetermined as when K_{ij} is less than the mobility M (BELFIORE; BENEDETTO, 2000). It is an important concept to structural analysis of kinematic chains.

Recently, Martins and Carboni (2007) present a new definition of degrees-of-control in terms of graphs.

Definition 15 (Degrees-of-control)**.** *In a kinematic chain represented by a graph X, the degrees-of-control between two links i and j is*

$$K_{ij} = \min\{D[i,j], M'_{\min}\} \tag{4.12}$$

Based on the definition of degrees-of-control and connectivity, the definition of redundancy will be introduced in next section.

4.2.5 Redundancy

Redundancy is one of the most important parameters in a kinematic chain together with connectivity and variety (MARTINS; CARBONI, 2007). The redundancy can be used to prevent collisions in manipulators which operate in confined environment (SIMAS, 2008).

Definition 16 (Redundancy)**.** *In a kinematic chain represented by a graph X, the redundancy between two links i and j is the difference between K_{ij} and C_{ij}*

$$R_{ij} = K_{ij} - C_{ij}. \tag{4.13}$$

Example 13. *Consider the planar kinematic chain with 10 links and 12 joints and its graph X shown in Figure 42. By Equation 3.1 (page 37) we have*

$$M = 3(10-12-1)+12 = 3$$

the mobility is equal to three. The variety is zero, i.e. $V = 0$ (see Section 4.2.2). The adjacency matrix $A(X)$ is given by:

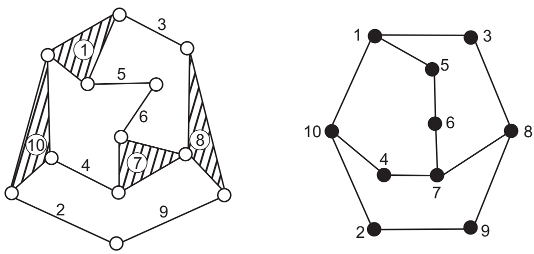


Figure 42 – Planar kinematic chain and graph representation.

$$A(X) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{4.14}$$

The connectivity matrix $C(X)$ is given by:

$$C(X) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 2 & 1 & 2 & 3 & 2 & 3 & 1 \\ 2 & 0 & 3 & 2 & 3 & 3 & 3 & 2 & 1 & 1 \\ 1 & 3 & 0 & 3 & 2 & 3 & 2 & 1 & 2 & 2 \\ 2 & 2 & 3 & 0 & 3 & 2 & 1 & 2 & 3 & 1 \\ 1 & 3 & 2 & 3 & 0 & 1 & 2 & 3 & 3 & 2 \\ 2 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 \\ 3 & 3 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 & 3 & 2 & 1 & 0 & 1 & 3 \\ 3 & 1 & 2 & 3 & 3 & 3 & 2 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & 2 & 3 & 2 & 3 & 2 & 0 \end{bmatrix} \end{array} \tag{4.15}$$

In this case, the degrees-of-control matrix $K(X)$ is equal to the connectivity matrix $C(X)$, $K(X) = C(X)$, and the redundancy matrix is null, $R(X) = 0$.

4.3 CONCLUSIONS

This chapter presented a bibliography review of the enumeration of kinematic structures methods and the criteria for analysis of kinematic structures.

First, we presented a bibliography review of the main contributions to the enumeration of kinematic structures. The main results of enumeration of kinematic chains are found in Tischler (1995), Mruthyunjaya (2003), Sunkari and Schmidt (2006), Simoni (2008), Simoni et al. (2009). The main results of enumeration of mechanisms are found in Vijayananda (1994), Tuttle (1996), Mruthyunjaya (2003), Simoni (2008), Simoni et al. (2009). Simoni (2008) presents results of enumeration of mechanisms using group and graph theories, however, the description of the method was not clear and the concepts used in his method can be improved using the definition of symmetry presented in Section 2.3. Using the concept of symmetry, it is possible to conclude that enumeration of mechanisms are related to symmetries of kinematic chains. The description of the enumeration of mechanisms method using the symmetry concept will be presented in Chapter 5. The main methods for enumeration of parallel manipulators related with the two engines (generator and evaluator) in the conceptual design phase of mechanisms and parallel manipulators of Tsai's methodology were reviewed. We noted that, for some structural parameters, Tsai's and Alizade's methods enumerated one kinematic chain of the parallel manipulator. Then, using combinatorial analysis, rotative, prismatic, universal and spherical joints are allocated in the kinematic chain to form the kinematic structure of the parallel manipulator. We also note that, Tsai's method enumerates only parallel manipulators with open-loop legs. Alizade's method also enumerates open-loop legs, however, he introduces more moving platforms. We will introduce a new method for enumeration of all parallel manipulators that a kinematic structure can originate. Using this approach, it is possible to ensure that all kinematic structure will be evaluated.

Second, we reviewed the main criteria used to classify the kinematic structures enumerated. The importance of these criteria are emphasized by several authors: Hunt (1978), Tischler (1995), Tsai (2001), Mruthyunjaya (2003), Belfiore and Benedetto (2000), Tischler et al. (2001), Liberati and Belfiore (2006), Shoham and Roth (1997), Martins and Carboni (2007), Carboni (2008), Simas (2008). Applications of these criteria will be considered in Chapter 6. For the purposes of this thesis, the criteria will be classified into global and local. In Chapter 6 we prove that local criteria are invariant by the action of the automorphism group. Therefore, it is possible to reduce the matricial representation of local criteria and to simplify the analysis.

5 CONTRIBUTIONS TO THE ENUMERATION OF KINEMATIC STRUCTURES

The contribution of this work to the enumeration of kinematic structures is to develop the enumeration of kinematic chains, mechanisms and parallel manipulators in a systematic procedure applying integrated tools of graph theory, group theory and screw theory.

First, we present the systematic procedure which considers the enumeration process into three levels: kinematic chains, mechanisms and parallel manipulators. Second, we present the current status of enumeration of kinematic chains found in the literature and we will compare and discuss the results of the methods reviewed in Section 4.1 (page 49). Third, we present an improvement of the method of enumeration of mechanisms presented by Simoni (2008) using the concept of symmetry introduce in Section 2.3 (page 26). We present the current status of enumeration of mechanisms found in the literature and we will compare and discuss these results. Fourth, we present a new method for enumeration of all parallel manipulators that a kinematic chain can originate.

This chapter provides original contributions to the enumeration of kinematic structures and it is based on the following papers:

- “Mãos Robóticas: Critérios para Síntese Estrutural e Classificação” (SIMONI et al., 2007);
- “Criteria for Structural Synthesis and Classification of Mechanisms” (SIMONI; MARTINS, 2007);
- “Enumeration of Kinematic Chains and Mechanisms” (SIMONI et al., 2009),
- “Enumeration of Parallel Manipulators” (SIMONI et al., 2008) and
- “Fractionation in planar kinematic chains: Reconciling enumeration contradictions” (MARTINS et al., 2010).

5.1 SYSTEMATIC PROCEDURE FOR ENUMERATION OF KINEMATIC STRUCTURES

The enumeration of kinematic structures will be develop into three levels: kinematic chains, mechanisms and parallel manipulators. Figure 43 shows the structure of the systematic procedure. We will briefly describe below the three levels of the systematic procedure.

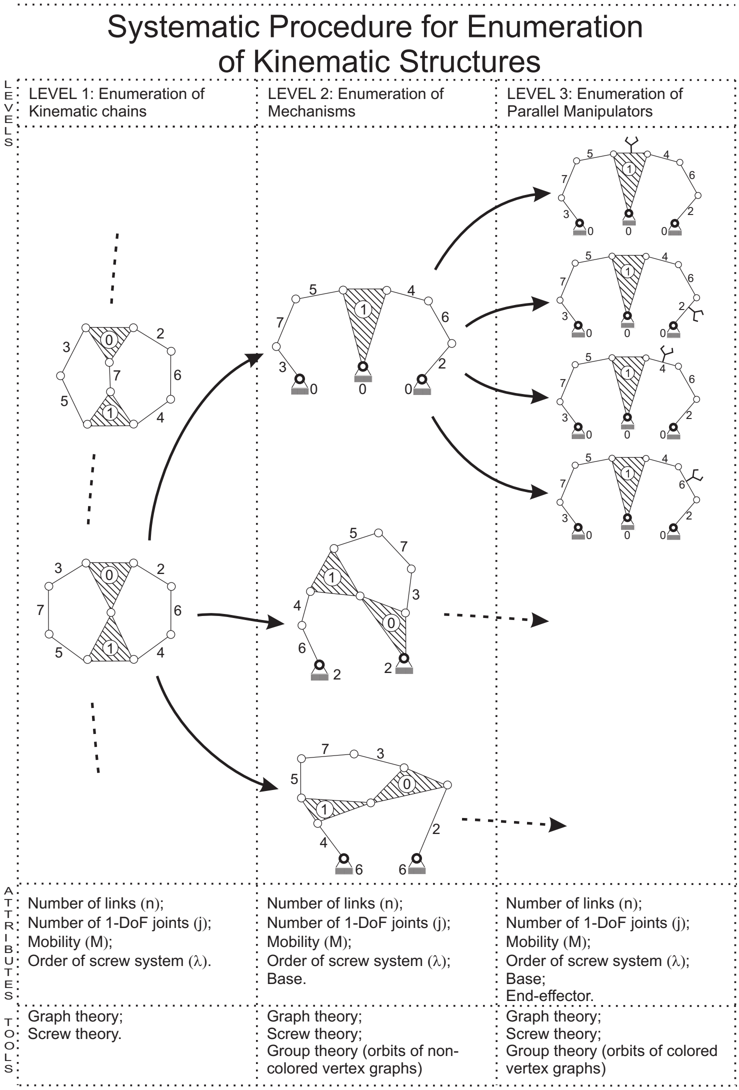


Figure 43 – Three levels of the systematic procedure for enumeration mechanisms and parallel manipulators. Each level has a description of the attributes of the kinematic structure and the mathematical tools used in the design process.

5.1.1 Level 1: Enumeration of kinematic chains

The level 1 corresponds to the enumeration of kinematic chains. Enumeration of kinematic chains satisfying a set of design specifications is still an open problem. Three difficulties are common in the enumeration process: isomorphism (NP-hard), degeneration (NP-hard) and fractionation.

From structural characteristics (number of links, number of joints, mobility, order of screw system) kinematic chains are enumerated. It is important to remember that a kinematic chain is an assembly of links and joints. The attributes of kinematic chains in this level are:

- number of links (n),
- number of 1-DoF joints (j),
- mobility (M) and
- order of screw system (λ).

The main tools considered in this level are graph theory and screw theory and the main methods for enumeration of kinematic chains were presented in Section 4.1 (page 49). There are some discrepancies in the results found in the literature, the results and these discrepancies will be commented in Section 5.2.

5.1.2 Level 2: Enumeration of mechanisms

Each kinematic chain originates mechanisms selecting all different bases. It is important to remember that a mechanism is a kinematic chain with one of its components (links) taken as a frame (IONESCU, 2003). In terms of graph theory, a mechanism corresponds to a graph with one of its vertices detached (colored) to represent the fixed link (SIMONI et al., 2008).

The attributes of mechanisms in this level are:

- number of links (n),
- number of 1-DoF joints (j),
- mobility (M),
- order of screw system (λ) and
- base of the mechanism.

The tools considered in this level are graph theory, group theory and screw theory; mainly the concepts of symmetry, actions and orbits of the automorphism group of non-colored vertex graphs.

In Section 5.3 we will present an improvement of the method of enumeration of mechanisms presented by Simoni (2008) using the concepts of symmetry, actions and orbits of the automorphism group of non-colored vertex graphs introduced in Section 2.3 (page 26). We will present the current status of enumeration of mechanisms found in the literature and we will compare and discuss these results.

5.1.3 Level 3: Enumeration of parallel manipulators

Each mechanism originates parallel manipulators selecting different links to be end-effectors. It is important to remember that a parallel manipulator is a kinematic chain with one of its components (links) taken as a frame and the other taken as an end-effector. In terms of graph theory, a parallel manipulator with one end-effector corresponds to a graph with two detached vertices (colored with distinct colors), one to represent the fixed link and another to represent the end-effector (SIMONI et al., 2008).

The attributes of parallel manipulators in this level are:

- number of links (n),
- number of 1-DoF joints (j),
- mobility (M),
- order of screw system (λ)
- base and end-effector of the parallel manipulator.

The tools considered in this level are graph theory, group theory and screw theory; mainly the concepts of symmetry, actions and orbits of the automorphism group of colored vertex graphs. We will present a new method for enumeration of parallel manipulators. The method consists of enumerating all the possible parallel manipulators with one end-effector that a single kinematic chain can originate.

Using this systematic procedure we will enumerate all mechanisms and parallel manipulators that a kinematic chain can originate, without isomorphisms. It is important to remember that the isomorphism problem is NP-hard.

5.2 ENUMERATION OF KINEMATIC CHAINS

This section corresponds to level 1 of the systematic procedure for enumeration of kinematic structures proposed in Section 5.1 and shown in Figure 43. A review of the main methods for enumeration of kinematic chains was presented in Section 4.1 (page 49). To enumerate kinematic chains we can use one of those methods. However, as indicated in Section 4.1, some methods enumerate fractionated kinematic chains while others do not. Authors in general do not confirm the presence or absence of fractionated kinematic chains in their lists. Sunkari and Schmidt (2006) present in their Tables 1-4 a series of enumeration contradictions. Tischler et al. (1995a) deal with the fractionation question in a greater detail. Mruthyunjaya (2003) comments that Tuttle and coworkers (TUTTLE, 1996; TUTTLE et al., 1989a, 1989b) generate only non-fractionated kinematic chains. However, both papers do not identify explicitly the fractionated kinematic chains generated nor present numerical results comparing non-fractionated and fractionated kinematic chains. In view of this, we will present the current status of enumeration of kinematic chains and we will discuss these results.

5.2.1 Current status of enumeration of kinematic chains

Tables 8, 9 and 10 present the current status of enumeration of kinematic chains. Note that, these tables present just the number of kinematic chains, as we can see the number is large and it is impracticable to provide all the drawings of the kinematic chains or their corresponding graphs. To the best of the authors' knowledge, the largest number of drawings of the kinematic chains is the planar case ($\lambda = 3$) with mobility $M = 3$ and number of loops $\nu = 3$ listed in Tischler et al. (1995a), Mruthyunjaya (1984a) and Martins et al. (2010).

Table 8 shows the results obtained applying the method proposed by Martins et al. (2010), Simoni (2008) described in Section 4.1.1.8 for enumeration of fractionated kinematic chains with mobility $1 \leq M \leq 6$. We notice that the table presented by Simoni (2008) has an incorrect result of 440 fractionated kinematic chains for the planar case, i.e. $\lambda = 3$, mobility equal three, i.e. $M = 3$, and four loops, i.e. $\nu = 4$, the correct result is 460 as shown in Table 8.

To the best of the authors' knowledge, the method proposed by Martins et al. (2010) and Simoni (2008) is the unique that enumerate only fractionated kinematic chains. Recently, Martins et al. (2010) conclude that the discrepancies in the results found in the literature are related with fractiona-

Table 8 – Current status of enumeration of fractionated kinematic chains obtained by Simoni (2008) andMartins et al. (2010).

λ	ν	*Mobility*					
		1	*2*	*3*	*4*	*5*	*6*
2	2	-	1	2	4	6	9
	3	-	2	11	31	74	153
	4	-	11	67	270	839	2239
3	2	-	1	2	4	6	9
	3	-	5	24	63	142	273
	4	-	86	460	1559	4222	9920
4	2	-	1	2	4	6	9
	3	-	10	41	104	222	416
5	2	-	1	2	4	6	9
	3	-	17	69	169	350	634
6	2	-	1	2	4	6	9
	3	-	27	102	246	495	882

λ is the order of screw system to which all the joint screws belong.
ν is the number of loops of the kinematic chain.

tion in kinematic chains and they indicate that most of the results are correct and the difference are the fractionated kinematic chains.

Table 9 shows the results of enumeration of non-fractionated kinematic chains. For example, with $M = 1$, $\lambda = 3$ and $\nu = 2$ we have *2* (two) kinematic chains, these are the classical Watt and Stephenson kinematic chains.

For non-planar case, i.e. $\lambda = 2,4,5,6$, the results are in agreement with Martins et al. (2010) and Simoni (2008). For planar case, i.e. $\lambda = 3$, the most results are in agreement with those of Sunkari and Schmidt (2006), Tuttle (1996), Lee and Yoon (1994). We confirm all the results (normal font style) in Table 9 up to four loops, i.e. $\nu = 4$, others results (italic font style) presented in Table 9 were obtained from Sunkari and Schmidt (2006) because they use a technique of enumeration similar to the Martins et al. (2010) and Simoni (2008) (see Sections 4.1.1.6 and 4.1.1.7).

In the following, we will indicate and comment all the discrepancies on the results of Table 9 relative to results found in the literature:

- case $M = 1$ and $\nu = 6$; the result of Tuttle (1996) is 318126 and the result of Sunkari and Schmidt (2006), presented in Table 9, is 318162. By the similarity of the numbers we believe that 318126 is a typo.

- case $M = 2$ and $\nu = 6$; the result of Tuttle (1996) is 1432608 and the result of Sunkari and Schmidt (2006) and Lee and Yoon (1994), presented

Table 9 – Current status of enumeration of non-fractionated kinematic chains.

λ	*ν*	*Mobility*					
		1	*2*	*3*	*4*	*5*	*6*
2	2	1	2	3	4	6	7
	3	3	9	20	40	70	121
	4	13	49	160	432	1033	2241
3	2	2	3	5	6	8	10
	3	16	35	74	126	212	325
	4	230	753	1962	4356	8846	16649
	5	*6856*	*27496*	*38547*	*216291*		
	6	*318162*	*1432730*	*4805764*	*13743920*		
	7	*19819281*					
4	2	3	4	6	8	10	12
	3	42	93	172	289	451	678
5	2	4	6	8	10	13	15
	3	116	225	398	621	939	1339
6	2	5	7	10	12	15	18
	3	242	454	749	1146	1661	2327

in Table 9, is 1432730. We believe that the result of Tuttle (1996) is incorrect because two authors, i.e. Sunkari and Schmidt (2006) and Lee and Yoon (1994), confirm the result of the Table 9.

- case $M = 3$ and $\nu = 4$; the result of Simoni (2008) is 1982 and the correct result is 1962.

- case $M = 3$ and $\nu = 6$; the result of Tuttle (1996) is 4805382 and the result of Sunkari and Schmidt (2006), presented in Table 9, is 4805764.

Table 10 gives the results of the enumeration of general kinematic chains, i.e. fractionated and non-fractionated kinematic chains. The entries of this table are given by summing the entries of Tables 8 and 9. For instance, the case $\lambda = 3$, $M = 3$ and $\nu = 3$ is given by 24(F) + 74(NF) = 98(G), and the case $\lambda = 6$, $M = 6$ and $\nu = 2$ is given by 882(F) + 2327(NF) = 3209(G), where (F), (NF) and (G) represent, respectively, fractionated, non-fractionated and general kinematic chains.

For non-planar case, i.e. $\lambda = 2, 4, 5, 6$, the results are in agreement with Martins et al. (2010) and Simoni (2008). For planar case, i.e. $\lambda = 3$, the most results are in agreement with those of Tischler et al. (1995a), Mruthyunjaya (1984a), Hwang and Hwang (1991), Mruthyunjaya (1984c, 1984b). We confirm all the results (normal font style) in Table 10 up to four loops, i.e. $\nu = 4$,

Table 10 – Current status of enumeration of general kinematic chains.

λ	ν	*Mobility* 1	2	3	4	5	6
2	2	1	3	5	8	12	16
	3	3	11	31	71	144	274
	4	13	60	227	702	1872	4480
3	2	2	4	7	10	14	19
	3	16	40	98	189	354	598
	4	230	839	2422	5915	13068	26569
	5	*6862*	*29704*				
4	2	3	5	8	12	16	21
	3	42	103	213	393	673	1094
5	2	4	7	10	14	19	24
	3	116	242	467	790	1289	1973
6	2	5	8	12	16	21	27
	3	242	481	851	1392	2156	3209

others planar results (italic font style) presented in Table 10 (case $M = 1,2$ and $\nu = 5$) were obtained from Hwang and Hwang (1991) and Mruthyunjaya (2003).

In the following, we will indicate and comment all the discrepancies on the results of Table 10 relative to results found in the literature:

- case $M = 3$ and $\nu = 4$; the result of Hwang and Hwang (1991) is 2442 and the result of Simoni (2008), Martins et al. (2010) and Vijayananda (1994), presented in Table 10, is 2422. By the similarity of the numbers we believe that 2442 is a typo.

- case $M = 4$ and $\nu = 4$; the result of Hwang and Hwang (1991) is 5951 and the result of Simoni (2008), Martins et al. (2010) and Vijayananda (1994), presented in Table 10, is 5915. By the similarity of the numbers we believe that 5951 is a typo.

The work initiated by Simoni (2008) and complemented by Martins et al. (2010) solves the contradictions of the results of enumeration of kinematic chains found in the literature since 1960. Enumeration lists of kinematic chains are presented in the literature (SUNKARI; SCHMIDT, 2006; SIMONI et al., 2009; TISCHLER et al., 1995a; TUTTLE, 1996; MRUTHYUNJAYA, 1984a; LEE; YOON, 1994; HWANG; HWANG, 1991; MRUTHYUNJAYA, 1984c, 1984b), but until now the contradictions of the results are not conclusive. Now, with the work of Simoni (2008) and Martins et al. (2010) it is possible to affirm without doubt that the methods (TISCHLER et al., 1995a; MRUTHYUNJAYA, 1984a; HWANG; HWANG, 1991; MRUTHYUNJAYA, 1984c, 1984b) enumerate general kinematic chains (fractionated and non-fractionated) while other methods, e.g. (SUNKARI; SCHMIDT, 2006; TUTTLE, 1996; LEE; YOON, 1994), enumerate only non-fractionated kinematic chains. In this section, we presented the current results of enumeration of kinematic chains and point out the discrepancies and incorrect results of all methods. As noted above, most results presented in the literature are correct, with minor disagreements (possibly caused by typos), most contradictions are only related to the presence or not of fractionated kinematic chains in the enumeration lists.

5.3 ENUMERATION OF MECHANISMS

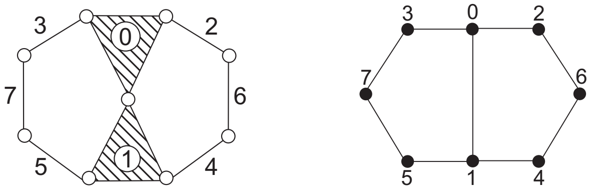
This section corresponds to level 2 of the systematic procedure for enumeration of kinematic structures proposed in Section 5.1 and shown in Figure 43. A review of the main methods for enumeration of mechanisms was presented in Section 4.1 (page 49).

In this section, first, we introduce a new representation of mechanisms in terms of graph which is an useful simplification for computational implementation. Second, we present an improvement of the method of enumeration of mechanisms proposed by Simoni et al. (2009). The improvement consists in the application of the concept of symmetry in kinematic chains presented in Section 2.3 (page 26). Using this method, it is possible to affirm that mechanisms are related to the symmetries of the kinematic chains. Third, we present the current status of enumeration of mechanisms found in the literature and, finally, we discuss these results.

5.3.1 Graph representation of mechanisms

Agreement with IFToMM (International Federation for the Promotion of Mechanism and Machine Science) a mechanism is a kinematic chain with one of its components (links) taken as a frame (IONESCU, 2003).

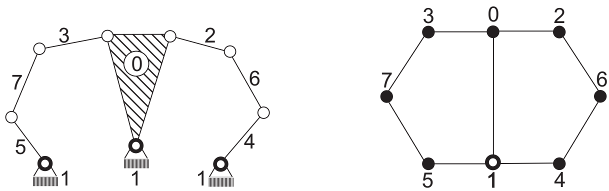
Figure 44(a) shows a kinematic chain and Figure 44(b) its graph representation. For the purposes of this thesis, kinematic chains and mechanisms are represented by graphs. This representation is a very useful simplification for analyzing the possible mechanisms which a kinematic chain can originate. A new graph representation of mechanisms was introduced by Simoni et al. (2008) to simplify the application of group theory tools for enumeration of all possible mechanisms that a kinematic chains can originate.



(a) Kinematic chain. (b) Graph representation.

Figure 44 – Graph representation of kinematic chains.

A mechanism is a kinematic chain with one of its components (links) taken as a frame (IONESCU, 2003). In terms of graph theory, a mechanism corresponds to a graph with one of its vertices detached (colored) to represent the fixed link (SIMONI et al., 2008). Figure 45(b) shows the graph of the mechanism presented in Figure 45(a) where the detached vertex represents the fixed, i.e. link 1.



(a) Mechanism. (b) Graph representation.

Figure 45 – Graph representation of mechanisms (SIMONI et al., 2008).

5.3.2 Improvement of the method proposed by Simoni (2008)

The improvement of the method proposed by Simoni (2008) consist into apply the concept of symmetry presented in Section 2.3 (page 26). Mechanisms are related to the symmetries of the kinematic chain (SIMONI et al., 2009). Ignoring dimensions, symmetrical links in the kinematic chain, when fixed, yield mechanisms with the same kinematic characteristics. For instance, any of the binary links of the Watt kinematic chain, due to the symmetry of the chain, yield the same Watt I mechanism (see Figure 14 in Section 3.2).

From the biunivocal correspondence between graphs and kinematic chains, the symmetry of a kinematic chain can be analyzed through the symmetry of its correspondent graph. As discussed in Section 2.1, using group theory, the symmetry of a graph can be identified by its automorphisms group. A kinematic chain is symmetric when it has more than one automorphism, see Definition 10 of Simoni et al. (2010). Thus, once identified symmetries in the kinematic chain we can use the graph symmetry to enumeration of mechanisms. As shown in Example of the Watt mechanisms (Figure 14, page 35), symmetric links yield the same mechanisms, i.e. Watt I and Watt II. However, mechanisms are related to the symmetries of the links of the kinematic chain and symmetric links are identified by orbits of the automorphism group of the graph associated to kinematic chain. The orbits of the automorphism group provides the sets of vertices (links) that are in the same equivalence classes, i.e. they possess the same properties of symmetry. Therefore, a mechanism may be enumerated by choosing one representative from each orbit of the automorphism group of graph that represents the kinematic chain. The number of orbits of the automorphism group induced by the graph vertices is equal to the number of mechanisms that the graph can originate. In order to determine the possible choices of the fixed link, only one representative of each orbit needs to be selected.

Figure 46 illustrates the applied techniques step by step to enumeration of mechanisms. First, given a graph, we need to identify symmetries. McKay (2009b, 1998) implemented the program *nauty* (No AUTomorphisms, Yes?) which is a set of very efficient C language procedures for determining the automorphism group of a graph with colored vertices. It provides this information in the form of a set of generators, the size of the group, and the orbits of the group. We can use nauty, without colored vertices, to determining the automorphism group of a kinematic chain using its associate graph. Second, the internal symmetry of the graph is represented in the form of an automorphism group and their orbits provide the equivalence classes under the action of the automorphism group. Symmetric links are grouped in distinct orbits because the orbits form a partition of the vertex set. Third, to enumerate all mechanisms of a kinematic chain can originate is only needed to choose one representative from each equivalence classes, i.e. from each orbit, because these equivalence classes provides links with the same characteristics of symmetry inside of the kinematic chain. Finally, since there is a biunivocal correspondence between graphs, kinematic chains and mechanisms, see Section 5.3.1, we obtain all possible choices of base in each kinematic chain originating all possible mechanisms that a kinematic chain can originate.

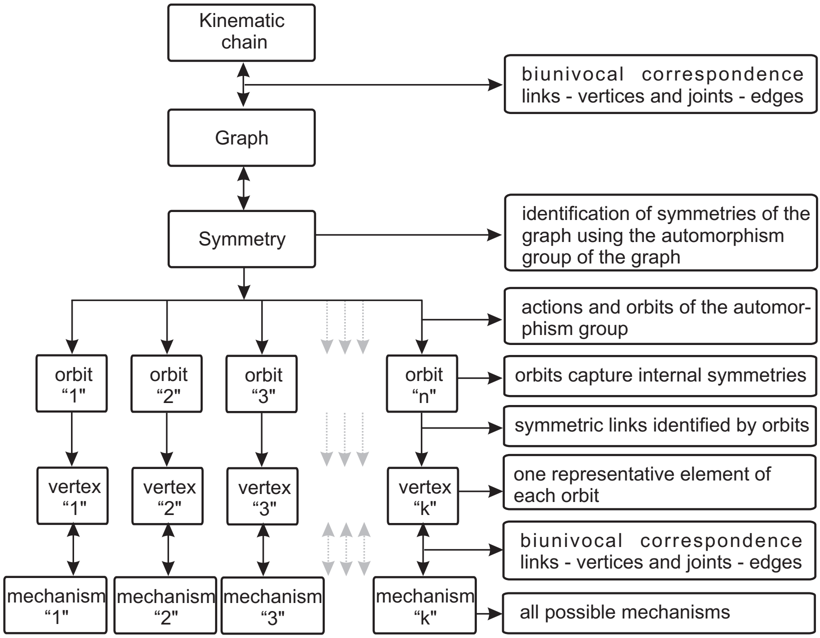


Figure 46 – Flowchart of the proposed method outlining the role of the group theory tools for enumeration of mechanisms.

Algorithm 1 shows the pseudocode of the improvement of the method of method of enumeration of mechanisms using the nauty program.

In the following, we will apply the technique to enumeration of Watt and Stephenson mechanisms, two examples well known in mechanisms and machines literature. These didactic example are chosen because the results are well established and it is easy to understand the application of the method.

Example 14. *Watt kinematic chain shown in Figure 47(a) is well known in the literature of mechanisms, we know that it originate two distinct mechanisms, i.e. Watt I and Watt II. Watt kinematic chain is represented by the labeled graph X shown in Figure 47(b). The automorphism group of the Watt graph possesses four elements:*

- $\sigma_1 = (1)(2)(3)(4)(5)(6)$,
- $\sigma_2 = (12)(34)(56)$,
- $\sigma_3 = (15)(26)(3)(4)$ *and*

Algorithm 1 Pseudocode of the improvement of the method of enumeration of mechanisms of Simoni (2008).

1 - Input:

- A non-colored vertex graph, which represents a kinematic chain.

2 - Run the nauty program:

- Determines the automorphism group of the graph.
- **nauty output:** Elements of the automorphism group.

3 - Post-processing:

Determines the number of symmetries of the non-colored vertex graph.

IF {non-colored vertex graph is symmetric, i.e. $r \neq 1$ (see Definition 11 on page 26)}

THEN

- Identify equivalence classes of vertices of the non-colored vertex graph.
- These equivalence classes are grouped into orbits of the automorphism group.
- Select one representative element of each orbit of the non-colored vertex graph, i.e. a vertex, to represent a new mechanism.
- The number of orbits of the automorphism group of the non-colored vertex graph is equal to the number of possible choices of bases, i.e. the number of mechanisms that the associate kinematic chain can originate.
- Use the graph representation of mechanisms (see Section 5.3.1) to identify the new mechanisms.

ELSE

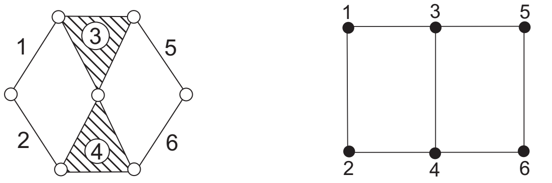
- All links, when fixed, originate distinct mechanisms.

4 - Output:

- Number mechanisms.

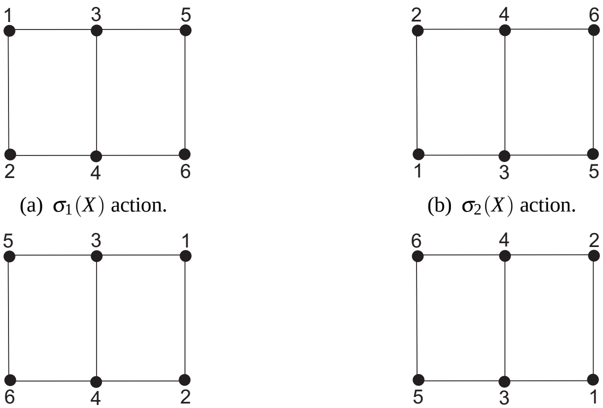
- $\sigma_4 = (16)(25)(34)$.

The generator set is $Aut(X) =< \sigma_2, \ \sigma_3 >$. *As* $Aut(X)$ *has four elements the Watt kinematic chain is symmetric, therefore, we can apply the method described above. The action of the automorphism group of the Watt graph is shown in Figures 48(a), 48(b), 48(c) and 48(d), respectively. The orbit of*



(a) (b)

Figure 47 – Watt kinematic chain and graph representation.



(a) $\sigma_1(X)$ action. (b) $\sigma_2(X)$ action.

(c) $\sigma_3(X)$ action. (d) $\sigma_4(X)$ action.

Figure 48 – Image of automorphism group action in the Watt graph.

vertex 1 is equal to the orbit of vertices 2, 5, and 6 of the graph, i.e. $\mathscr{O}_1 = \mathscr{O}_2 = \mathscr{O}_5 = \mathscr{O}_6 = \{1,2,5,6\}$, *and the orbit of vertex 3 is equal to the orbit of vertex 4 of the graph, i.e.* $\mathscr{O}_3 = \mathscr{O}_4 = \{3,4\}$. *Therefore, the orbits of the automorphism group are:*

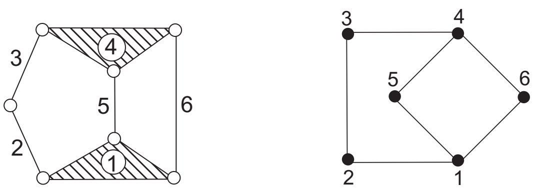
- $\mathscr{O}_{Watt \ I} = \{1,2,5,6\}$ *and*
- $\mathscr{O}_{Watt \ II} = \{3,4\}$.

Note that it is possible to identify the orbits by analyzing $Aut(X) = <\sigma_2, \sigma_3>$. *For the Watt kinematic chain, the automorphism group possesses two orbits; therefore, the number of mechanisms for the Watt chain is two and the representatives can be, for example, 1 and 3. Fixing link 1, we generate the classical Watt I mechanism. Similarly, fixing link 3 we generate the Watt II mechanism. Any choice from the orbit of link 1 (i.e. 1, 2, 5, or 6) originates Watt I mechanism, and any choice in the orbit of link 3 (i.e. 3 or 4) originates Watt II mechanism.*

Example 15. *Stephenson graph (X), shown in Figure 49, has four elements in the automorphism group:*

- $\sigma_1 = (1)(2)(3)(4)(5)(6)$,
- $\sigma_2 = (1)(2)(3)(4)(56)$,
- $\sigma_3 = (14)(23)(5)(6)$ *and*
- $\sigma_4 = (14)(23)(56)$.

As $Aut(X)$ *has four elements the Stephenson kinematic chain is symmetric, therefore, we can apply the method described above. The action of automorphism group of the Stephenson graph (Figure 49(b)) is shown in Figures 50(a), 50(b), 50(c) and 50(d),respectively. The orbit of vertex 1 is*



(a) Stephenson kinematic chain. (b) Graph representation.

Figure 49 – Correspondence between graphs and kinematic chains.

equal to the orbit of vertices 4 of the graph, i.e. $\mathscr{O}_1 = \mathscr{O}_4 = \{1,4\}$, *the orbit of vertex 2 is equal to the orbit of vertex 3 of the graph, i.e.* $\mathscr{O}_2 = \mathscr{O}_3 = \{2,3\}$, *and the orbit of vertex 5 is equal to the orbit of vertex 6 of the graph, i.e.* $\mathscr{O}_5 = \mathscr{O}_6 = \{5,6\}$. *Therefore, the orbits of the automorphism group are:*

- $\mathscr{O}_{Stephenson\ III} = \{1,4\}$,
- $\mathscr{O}_{Stephenson\ II} = \{2,3\}$ *and*
- $\mathscr{O}_{Stephenson\ I} = \{5,6\}$.

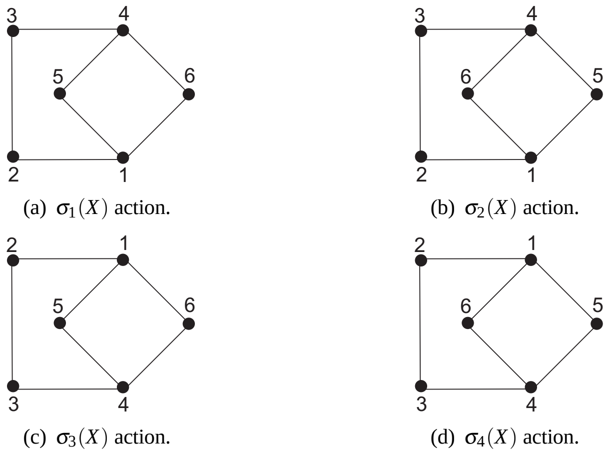


Figure 50 – Image of automorphism group action in the Stephenson graph.

For the Stephenson kinematic chain, the automorphism group possesses three orbits originating three distinct mechanisms and the representatives can be 1, 2 and 5. These choices of fixed links originate, respectively, the classical Stephenson III, II and I mechanisms. Fixing links 4, 3 and 6 would generate the same Stephenson III, II and I mechanisms because they are, respectively, in the same orbits.

5.3.3 Current status of enumeration of mechanisms

This section presents the current results of enumeration of mechanisms found in the literature. Each kinematic chain originate your own mechanisms, therefore, we can enumerate the mechanisms originated by fractionated kinematic chains, non-fractionated kinematic chains and by general kinematic chains (see Tables 8, 9 and 10). Tables 11, 12 and 13 present the current results of enumeration of mechanisms. Note that, the tables present just the number of mechanisms, as we can see the number is large and it is impracticable to provide all the drawings of the mechanisms that a kinematic chains can originate.

Following the results of enumeration of kinematic chains, we initiate with Table 11 that presents the list of fractionated mechanisms, these mechanisms are originated from kinematic chains shown in Table 8 (page 80). To the best of the authors' knowledge, the method proposed by Martins et al.

(2010) is the unique that enumerate only fractionated kinematic chains. We confirm all the results of Table 11.

Table 11 – Current status of enumeration of fractionated mechanisms obtained by Martins et al. (2010).

λ	ν	*Mobility*					
		1	*2*	*3*	*4*	*5*	*6*
2	2	-	2	6	15	27	47
	3	-	4	49	171	471	1103
	4	-	49	380	1793	6430	19323
3	2	-	3	8	19	33	56
	3	-	34	167	508	1244	2645
	4	-	742	4388	16349	48166	122411
4	2	-	3	9	21	37	62
	3	-	82	367	1043	2414	4894
5	2	-	4	11	25	43	71
	3	-	193	799	2138	4684	9068
6	2	-	4	12	27	47	77
	3	-	353	1410	3649	7757	14608

λ is the order of screw system to which all the joint screws belong.
ν is the number of loops of the kinematic chain.

Table 12 shows the current results of enumeration of non-fractionated mechanisms, i.e. mechanisms originated from kinematic chains shown in Table 9 (page 81). For example, with $M = 1$, $\lambda = 3$ and $\nu = 2$ we have *5* (five) mechanisms, these are the classical Watt I, Watt II, Stephenson I, Stephenson II and Stephenson III mechanisms originated from Watt and Stephenson kinematic chains.

For non-planar case, i.e. $\lambda = 2,4,5,6$, the results are in agreement with Simoni (2008), Simoni et al. (2009). For planar case, i.e. $\lambda = 3$, the results are in agreement with those of Tuttle (1996) and Simoni et al. (2009). We confirm all the results (normal font style) in Table 9 up to four loops, i.e. $\nu = 4$, others results (italic font style) presented in Table 9 were obtained from Tuttle (1996).

Table 13 shows the current results of enumeration of general mechanisms, i.e. mechanisms originated from kinematic chains shown in Table 10 (page 82). For non-planar case, i.e. $\lambda = 2,4,5,6$, the results are in agreement with Simoni (2008). For planar case, i.e. $\lambda = 3$, we confirm all the results in Table 13, some results are in agreement with those of Vijayananda (1994). The discrepancies occur in the following cases:

Table 12 – Current status of enumeration of non-fractionated mechanisms.

λ	ν	*Mobility*					
		1	*2*	*3*	*4*	*5*	*6*
2	2	2	5	9	15	23	33
	3	8	35	91	217	463	897
	4	45	255	1014	3248	8924	21911
3	2	5	11	18	28	39	55
	3	71	220	517	1056	1955	3387
	4	1834	7156	20737	51245	113387	231664
	5	*75397*	*335398*	*1105923*			
	6	*4274142*	*20736427*	*74387903*			
4	2	10	18	29	43	59	79
	3	324	832	1749	3245	5581	9042
5	2	17	31	45	65	86	113
	3	1196	2704	5136	8849	14256	21894
6	2	27	44	65	89	117	150
	3	3331	6813	12126	19792	30538	45118

- case $M = 6$ and $\nu = 2$; the result of Vijayananda (1994) is 110 and our result is 111.

- case $M = 3$ and $\nu = 3$; the result of Vijayananda (1994) is 648 and our result is 684. By the similarity of the numbers we believe that 648 is a typo.

- case $M = 3$ and $\nu = 4$; the result of Vijayananda (1994) is 25124 and our result is 25125.

- case $M = 4$ and $\nu = 4$; the result of Vijayananda (1994) is 67591 and our result is 67594.

As we can see, the discrepancies are very similar, the maximal discrepancy is four mechanisms in the last case. Notes that, in all cases, our method enumerate more mechanisms than Vijayananda (1994). It is an indication that the method of Vijayananda (1994) can be discarding any feasible solution.

To the best of the authors' knowledge, Tables 11, 12 and 13 shown the most complete result of enumeration of mechanisms.

Table 13 – Current status of enumeration of general mechanisms.

λ	ν	*Mobility*					
		1	*2*	*3*	*4*	*5*	*6*
2	2	2	7	15	30	50	80
	3	8	39	140	388	934	2000
	4	45	304	1394	5041	15354	41234
3	2	5	14	26	47	72	111
	3	71	254	684	1564	3199	6032
	4	1834	7898	25125	67594	161553	354075
4	2	10	21	38	64	96	141
	3	324	914	2116	4288	7995	13936
5	2	17	35	56	90	129	184
	3	1196	2897	5935	10987	18940	30962
6	2	27	48	77	116	164	227
	3	3331	7166	13536	23441	38295	59726

5.4 ENUMERATION OF PARALLEL MANIPULATORS

This section corresponds to level 3 of the systematic procedure for enumeration of kinematic structures proposed in Section 5.1 and shown in Figure 43. This section provides an original contribution to the enumeration of parallel manipulators with one end-effector and is based on the following paper:

- “Enumeration of parallel manipulators” (SIMONI et al., 2008).

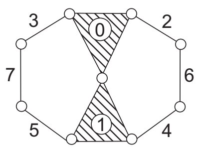
A review of the main methods for enumeration of parallel manipulators was presented in Section 4.1 (page 49). As pointed by Simoni et al. (2008), the method that we will present here is the only one that enumerates all possible parallel manipulators of a kinematic chain. The results presented in this section are new and therefore we do not have references for comparison. The advantage of this approach is that all parallel manipulators will be evaluated and the most promising will be chosen by design detailing.

In this section, first, we introduce a new representation of parallel manipulators in terms of graph which is an useful simplification for computational implementation. Second, we describe the method which uses the group theory tools (see Section 2.1), specially, we apply the concepts of symmetry, actions and orbits of the automorphism group of a colored vertex graph. The method consists of enumerating all the possible parallel manipulators with one end-effector that a single kinematic chain can originate. Third, we present some applications of the method and, finally, we discuss the results.

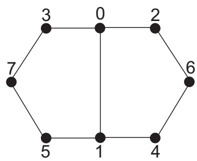
5.4.1 Graph representation of parallel manipulators

In this section we explore the number of parallel manipulators with one end-effector which a kinematic chain can originate. The exploration is carried out using graph and group theory. The representation of a parallel manipulator by a graph is a very useful simplification for analyzing all the possible parallel manipulators which the kinematic chain can originate. A new graph representation of mechanisms and parallel manipulators was introduced by Simoni et al. (2008) to simplify the application of group theory tools for enumeration of all possible parallel manipulators with one end-effector that a kinematic chains can originate.

To a better understanding of representation of a parallel manipulators by a graph, the representation of kinematic chains and mechanisms will be briefly presented below. Figure 51(a) shows a kinematic chain and Figure 51(b) its graph representation. Figure 52(a) shows a mechanism and Fig 52(b) shows their graph representation. In terms of graph theory a mechanism corresponds to a graph with one of its vertices detached (colored) to represent the fixed link (SIMONI et al., 2008).

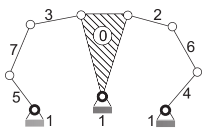


(a) Kinematic chain.

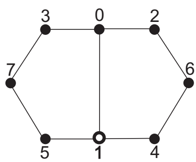


(b) Graph representation.

Figure 51 – Graph representation of kinematic chains.



(a) Mechanism.

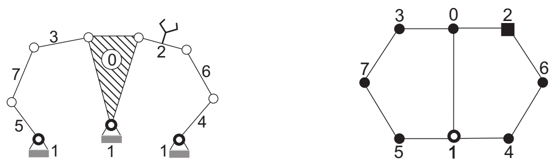


(b) Graph representation.

Figure 52 – Graph representation of mechanisms (SIMONI et al., 2008).

A parallel manipulator is a kinematic chain with one of its components (links) taken as frame and the other taken as end-effector (IONESCU, 2003). In terms of graph theory, a parallel manipulator with one end-effector corresponds to a graph with two detached vertices (colored with distinct colors), one to represent the fixed link and the other to represent the end-effector (SIMONI et al., 2008).

Figure 53(b) shows the graph of the parallel manipulator shown in Figure 53(a), where one of the detached links represents the base and the other represents the end-effector. If the parallel manipulator possess more than one end-effector, more graph vertices must be detached to represent it.



(a) Parallel Manipulator.

(b) Graph representation of parallel manipulator.

Figure 53 – Graph representation of parallel manipulators with one end-effector.

Simoni et al. (2008) used the concept of orbits of the automorphism group of non-colored vertex graphs, of group theory, to enumerate all the possible mechanisms of a single kinematic chain.

In this section, we present an extension of the enumeration of mechanism method for enumeration of parallel manipulators with one end-effector. Thus, we will represent parallel manipulators by graphs with two of their vertices colored (detached), one to represent the base and the other to represent the end-effector, and use tools from group theory for enumeration of all the possible parallel manipulators with one end-effector that a single kinematic chain can originate.

5.4.2 New method for enumeration of parallel manipulators

Our method for the enumeration of parallel manipulators consists of calculating orbits of the automorphism group of colored vertex graphs and selecting all the possible distinct label listing of vertices (one to represent the base and other to represent the end-effector) which can originate distinct parallel manipulators. Firstly, the base of the parallel manipulators are enumerated using the concepts of symmetry, action and orbits of the automorphism group. Second, the links defined as base of the parallel manipulators are colored and the automorphism group is obtained again. The symmetries are discarded and only original configurations are obtained.

5.4.2.1 Orbits of non-colored vertex graphs and corresponding bases

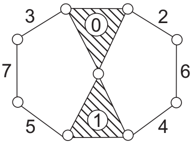
The method of enumeration of bases is the same of enumeration of mechanisms presented in Section 5.3. We use the same method because it is shown in Section 5.3.3 that the results obtained by the method of enumeration of mechanisms are effective and all possible choices of bases are enumerated. For completeness of the enumeration of parallel manipulators method we will present another example of enumeration of bases, i.e. mechanisms. Using the tools of group theory presented in Section 2.1 we can obtain all possible bases of a kinematic chain choosing a representative of each orbit of the automorphism group of a non-colored vertex graph, i.e. discarding symmetric links. The number of orbits of a non-colored vertex graph which represent the kinematic chain is equal to the number of bases that the graph (i.e. kinematic chain) can originate. To ascertain which are the possible choices for the fixed link there only needs to be chosen a representative of each orbit (SIMONI et al., 2008), this procedure guarantees that symmetric links are discarded originating only the distinct choices of base.

Example 16. *Figure 54(a) shows a planar kinematic chain with mobility three ($M = 3$) and two loops, the kinematic chain is represented by a labeled non-colored graph (without vertices detached) as shown in Figure 54(b) which will be called X. The automorphism group of graph X possesses four elements:*

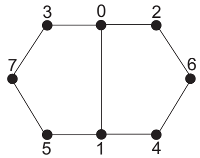
- $\sigma_1 = (0)(1)(2)(3)(4)(5)(6)(7)$,
- $\sigma_2 = (23)(45)(67)$,
- $\sigma_3 = (01)(24)(35)$ *and*
- $\sigma_4 = (01)(25)(34)(67)$.

The action of the automorphism group on the graph X is shown in Figures 55(a), 55(b), 55(c), and 55(d), respectively.

The orbit of vertex 0 is equal to the orbit of vertex 1, i.e. $\mathscr{O}_0 = \mathscr{O}_1 = \{0,1\}$, the orbit of vertex 2 is equal to the orbit of vertices 3, 4 and 5, i.e. $\mathscr{O}_2 = \mathscr{O}_3 = \mathscr{O}_4 = \mathscr{O}_5 = \{2,3,4,5\}$, and the orbit of vertex 6 is equal to

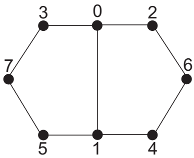


(a) Kinematic chain.

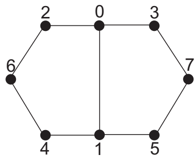


(b) H.

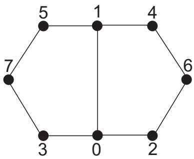
Figure 54 – Graph representation of kinematic chain.



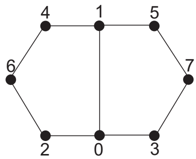
(a) $\sigma_1(X)$ action.



(b) $\sigma_2(X)$ action.



(c) $\sigma_3(X)$ action.



(d) $\sigma_4(X)$ action.

Figure 55 – Actions of automorphism group on the graph.

the orbit of vertex 7, i.e. $\mathscr{O}_6 = \mathscr{O}_7 = \{6,7\}$. *Therefore, the three orbits of the automorphism group are:*

- $\mathscr{O}_{Base_1} = \{0,1\}$,
- $\mathscr{O}_{Base_2} = \{2,3,4,5\}$ *and*
- $\mathscr{O}_{Base_3} = \{6,7\}$.

The possible choices of base for the kinematic chain shown in Figure 54(a) are obtained by choosing a representative of each orbit of the automorphism group induced by associated non-colored graph vertices, for example 0, 2 and 6.

The number of orbits of the automorphism group (i.e. 3) is equal to the number of all possible bases of the parallel manipulator that the kinematic chain can originate. The links that are in the same orbit originate identical *bases, i.e. the changing of a fixed link does not cause different characteristic in the movement of the mechanism in relation to the fixed link. The changing of a fixed link, for links that are in different orbits, leads to different characteristic in the movement of the mechanism originating distinct mechanisms for the kinematic chain.*

5.4.2.2 Orbits of colored vertex graphs and corresponding end-effectors

For enumeration of all possible parallel manipulators with one end-effector for a given kinematic chain we use colored vertex graphs.

The method of enumeration of all the possible parallel manipulators with one end-effector for a kinematic chain is similar of the method of enumeration of mechanisms and it consists of identify symmetries of the kinematic chain with one link selected as base. The symmetries are identified by orbits of the automorphism group of colored vertex graphs, the colored vertex represent the base and is obtained by method presented in Section 5.4.2.1. To enumerate all the possible parallel manipulators with one end-effector which can be originated by a single kinematic chain we only need to enumerate all the possible choices of the end-effector for each base. The simplest way of enumerate all possible choices of end-effector is to color one vertex (which originates the base) of each time and identify the symmetries calculating the orbits of the automorphism group of the colored vertex graph (with the vertex that originates the base colored). The parallel manipulator (base and end-effector) is obtained choosing a representative element of each orbit of the colored graph to represent the end-effector.

Figure 56 shows a flowchart of the method, step by step, outlining the role of the group theory tools for enumeration of parallel manipulators.

With this technique all the parallel manipulators with one end-effector that the kinematic chain can originate are enumerated. Having established the possible choices of a base, for each colored base (colored graph vertex) the automorphism group of colored vertex graph captures the internal symmetries of graph and supplies the information through the orbits of the automorphism group. In the case of colored graphs, the automorphism group captures equivalence between graph vertices in relation to the colored vertices. The vertices that are in the same orbit originate identical parallel manipulators with one end-effector. Now we present some examples of the new method.

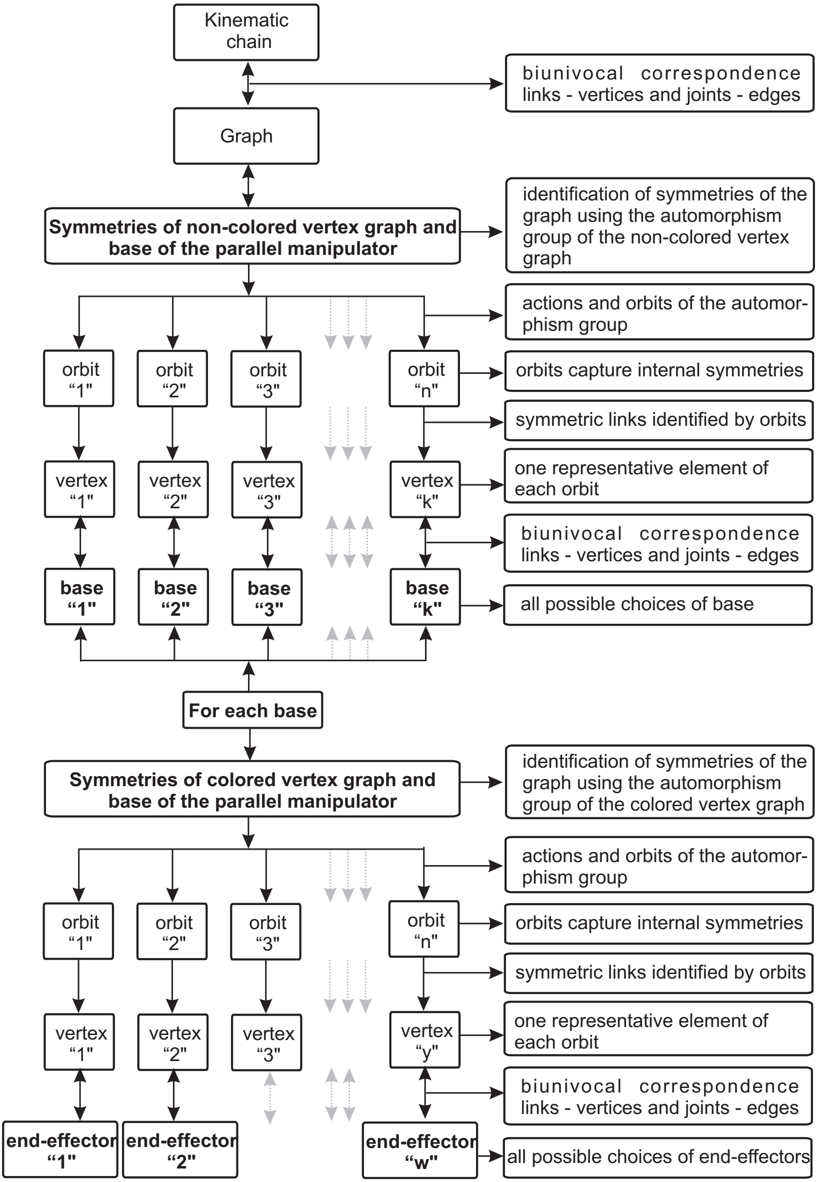


Figure 56 – Flowchart of the proposed method outlining the role of the group theory tools for enumeration of parallel manipulators.

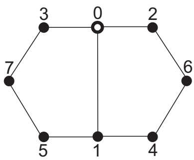
Example 17. Enumeration of planar parallel manipulators with one end-effector: *In Example 16, we enumerated all possible choices of base of the kinematic chain in Figure 54(a), i.e.* 0, 2 *and* 6. *Now, we will enumerate all the possible parallel manipulators with one end-effector for the kinematic chain in Figure 54(a).*

First, we consider the base 0. *The graph vertex of label* 0 *in Figure 54(b) is colored as shown in Figure 57(a) and the orbits of the automorphism group of the colored graph are calculated. The automorphism group of the graph with vertex* 0 *colored possesses two elements:*

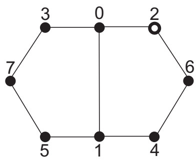
- $\sigma_1 = (0)(1)(2)(3)(4)(5)(6)(7)$ *and*
- $\sigma_2 = (23)(45)(67)$.

Therefore, the orbits of the automorphism group are:

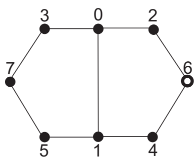
- $\mathscr{O}_0 = \{0\}$,
- $\mathscr{O}_{PM_1} = \{1\}$, *(PM - parallel manipulator)*
- $\mathscr{O}_{PM_2} = \{2,3\}$,
- $\mathscr{O}_{PM_3} = \{4,5\}$ *and*
- $\mathscr{O}_{PM_4} = \{6,7\}$.



(a) Vertex 0 colored.



(b) Vertex 2 colored.



(c) Vertex 6 colored.

Figure 57 – Graph representation of the enumeration of planar parallel manipulators with one end-effector calculating orbits of the automorphism group of colored vertex graphs which represent the mechanism.

Second, we consider the base 2. *The vertex of label* 2 *in Figure 54(b) is then colored as show in Figure 57(b). In this case the automorphism group of the graph with vertex* 2 *colored possesses only one element, i.e. the identity*

- $\sigma_1 = (0)(1)(2)(3)(4)(5)(6)(7)$.

Thus, the number of orbits is equal to the number of vertices, i.e.

- $\mathscr{O}_{PM_5} = \{0\}$,
- $\mathscr{O}_{PM_6} = \{1\}$,
- $\mathscr{O}_2 = \{2\}$,
- $\mathscr{O}_{PM_7} = \{3\}$,
- $\mathscr{O}_{PM_8} = \{4\}$,
- $\mathscr{O}_{PM_9} = \{5\}$,
- $\mathscr{O}_{PM_{10}} = \{6\}$ *and*
- $\mathscr{O}_{PM_{11}} = \{7\}$.

Third, we consider the base 6*. The vertex of label* 6 *in Figure 54(b) is colored as shown Figure 57(c). In this case the automorphism group of the graph with vertex* 6 *colored possesses two elements*

- $\sigma_1 = (0)(1)(2)(3)(4)(5)(6)(7)$ *and*
- $\sigma_2 = (01)(24)(35)$.

Orbits are;

- $\mathscr{O}_{PM_{12}} = \{0,1\}$,
- $\mathscr{O}_{PM_{13}} = \{2,4\}$,
- $\mathscr{O}_{PM_{14}} = \{3,5\}$,
- $\mathscr{O}_6 = \{6\}$ *and*
- $\mathscr{O}_{PM_{15}} = \{7\}$.

With this technique, we enumerate all the possible string listings of vertices that can originate distinct parallel manipulators selecting the colored vertex (base) and a vertex of each orbit of the automorphism group of the graph with colored vertices, where the string listings $x|y$ represent the two colored vertices of the graph, i.e. one parallel manipulator where x is the fixed link and y is the end-effector.

Table 14 shows the list of all parallel manipulators with one end-effector that the kinematic chain in Figure 54(a) can originate. Column 1 shows the orbits of the non-colored graph, column 2 shows the possible choices of base (i.e. one representative of each orbit of the non-colored graph), column 3 shows the orbits of the colored graph where the colored

Table 14 – Results of the enumeration of parallel manipulators for the kinematic chain shown in Figure 54(a).

1	2	3	4
Orbits of non-colored graph	Base	Orbits of colored graph	End-effector
0, 1	0	0	-
		1	$0\|1$
		2, 3	$0\|2$
		4, 5	$0\|4$
		6, 7	$0\|6$
2, 3, 4, 5	2	0	$2\|0$
		1	$2\|1$
		2	-
		3	$2\|3$
		4	$2\|4$
		5	$2\|5$
		6	$2\|6$
		7	$2\|7$
6, 7	6	0, 1	$6\|0$
		2, 4	$6\|2$
		3, 5	$6\|3$
		6	-
		7	$6\|7$
Total number of parallel manipulators			$\Sigma = 15$

vertex is the vertex that originates the bases shown in column 2 and the column 4 shows the possible choices of end-effector for the kinematic chain in Figure 54(a). In column 4, the parallel manipulator with one end-effector is originated from one representative of each orbit of the non-colored graph to be the base and one representative of each orbit of the colored graph to be the end-effector. Using this technique, we enumerate 15 distinct parallel manipulators with one end-effector that the kinematic chain in Figure 54(a) can originate.

Figure 58 shows some results of Table 14 in the three levels of the systematic procedure to enumeration of kinematic structures proposed in Section 5.1 and shown in Figure 43. Figure 58 shows the kinematic chain in Figure 54(a) on the first level, the mechanisms derived from this kinematic chain (i.e. , $0, 2$ *and* 6*) on the second level and the parallel manipulators with one end-effector for the first choice of base (base* 0*), i.e.* $0|1, 0|2, 0|4$ *and* $0|6$*,* *on the third level.*

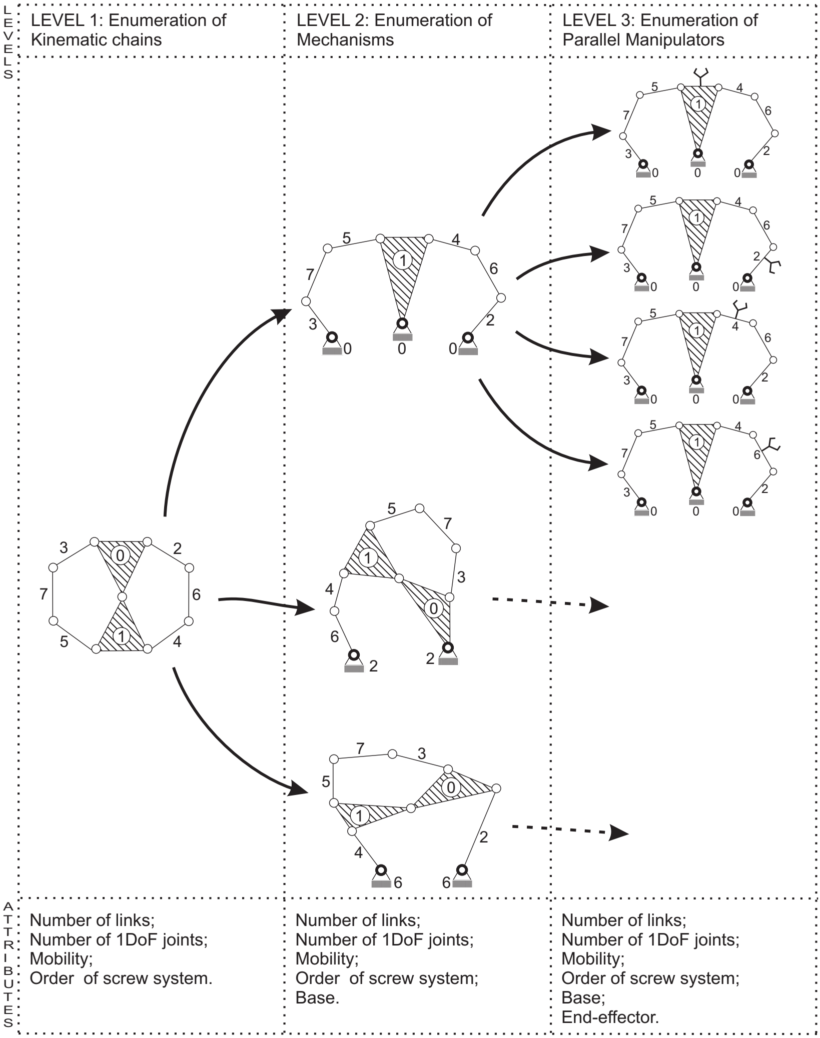
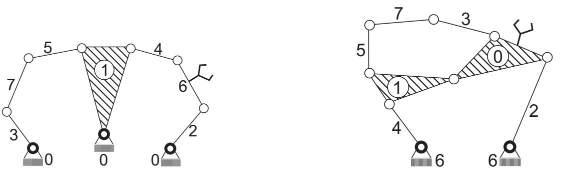


Figure 58 – Enumeration of parallel manipulators method for the results of Table 14. The systematic procedure is completed into three levels as discussed in Section 5.1 and shown in Figure 43.

We choose always the vertex of the lowest label in each orbit to represent the bases and/or the end-effectors, but the choice could be another.

Therefore, if the vertices are in the same orbit they have exactly the same kinematic and structural characteristics as mechanism or parallel manipulator. For example, in line 1 of Table 14, we choose the vertex of label 0 (see column 2) to represent the base but we could choose the vertex of label 1. The orbits of the colored graph with one of these two vertices colored (i.e. 0 or 1) will be the same as those shown in the column 3 and, consequently, the parallel manipulators indicated in the column 4 will have the same kinematic characteristics. The parallel manipulator 0|6 shown in Figure 59(a) is the same as 1|7.



(a) Base 0, end-effector 6. (b) Base 6, end-effector 0.

Figure 59 – Parallel manipulators with totally different kinematic and structural characteristics changing the base from 0 to 6 and from 6 to 0 and the end-effector from 6 to 0 and from 0 to 6, respectively.

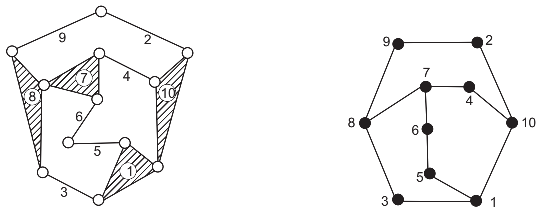
Note that the vertices that are in the same orbit on the automorphism group of the non-colored vertex graph only originate one parallel manipulator with one end-effector because the base-end-effector change does not cause alterations in the kinematic and structural characteristics of the parallel manipulator. Therefore, the parallel manipulator only appears once in Table 14, for example 0|1. The vertices that are in the same orbits on the automorphism group of different non-colored vertex graphs appear twice on the list of parallel manipulators. For example 0|6 and 6|0 (see Figures 59(a) and 59(b)), they possess totally different kinematic and structural characteristics. Often the parallel manipulators originated by the same two vertices appear camouflaged, as is the case of 2|7 and 6|3.

If the vertices are in different orbits on the automorphism group of a non-colored vertex graph then the base-end-effector change does not originate parallel manipulators with different structural characteristics and therefore they appear twice on the list of parallel manipulators.

The results presented in Table 14 are new and therefore we do not have references for comparison.

We should emphasize that, using this method, we enumerated all possible parallel manipulators that a kinematic chain can originate without isomorphisms which is a NP-hard problem. This contribution was possible through the integrated application of the graph and group theory tools presented in Chapter 2.

Example 18. Enumeration of planar parallel manipulators with one end-effector: *Figure 60 shows a planar kinematic chain with mobility three (i.e.* $M = 3$*), ten links (i.e.* $n = 10$*) and variety zero (i.e.* $V = 0$*) and its graph.*



(a) Kinematic chain. (b) Graph representation.

Figure 60 – Planar kinematic chain and graph representation.

The orbits of the automorphism group of non-colored vertex graph are:

- $\mathscr{O}_{Base_1} = \{1, 7, 8, 10\}$,
- $\mathscr{O}_{Base_2} = \{2, 5, 6, 9\}$ *and*
- $\mathscr{O}_{Base_3} = \{3, 4\}$

which originates three possible choices of base, i.e. 1, 2 and 3. Applying our method, coloring the vertex that originates distinct bases and calculating the orbits of the automorphism group of the colored vertex graph we have:

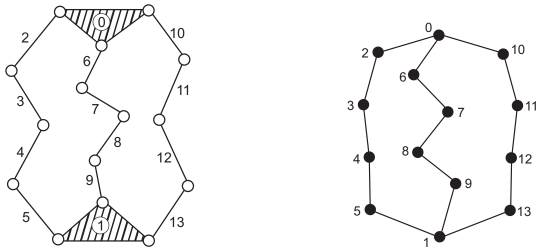
- *for vertex 1 colored, the orbits are:* $\mathscr{O}_1 = \{1\}$, $\mathscr{O}_{PM_1} = \{2\}$, $\mathscr{O}_{PM_2} = \{3\}$, $\mathscr{O}_{PM_3} = \{4\}$, $\mathscr{O}_{PM_4} = \{5\}$, $\mathscr{O}_{PM_5} = \{6\}$, $\mathscr{O}_{PM_6} = \{7\}$, $\mathscr{O}_{PM_7} = \{8\}$, $\mathscr{O}_{PM_8} = \{9\}$, $\mathscr{O}_{PM_9} = \{10\}$,
- *for vertex 2 colored, the orbits are:* $\mathscr{O}_{PM_{10}} = \{1\}$, $\mathscr{O}_2 = \{2\}$, $\mathscr{O}_{PM_{11}} = \{3\}$, $\mathscr{O}_{PM_{12}} = \{4\}$, $\mathscr{O}_{PM_{13}} = \{5\}$, $\mathscr{O}_{PM_{14}} = \{6\}$, $\mathscr{O}_{PM_{15}} = \{7\}$, $\mathscr{O}_{PM_{16}} = \{8\}$, $\mathscr{O}_{PM_{17}} = \{9\}$, $\mathscr{O}_{PM_{18}} = \{10\}$, *and*
- *for vertex 3 colored, the orbits are:* $\mathscr{O}_{PM_{19}} = \{1, 8\}$, $\mathscr{O}_{PM_{20}} = \{2, 6\}$, $\mathscr{O}_3 = \{3\}$, $\mathscr{O}_{PM_{21}} = \{4\}$, $\mathscr{O}_{PM_{22}} = \{5, 9\}$, $\mathscr{O}_{PM_{23}} = \{7, 10\}$.

Table 15 shows the possible parallel manipulators with one end-effector for the kinematic kinematic chain shown in Figure 60.

Table 15 – Results of the enumeration of planar parallel manipulators with one end-effector.

Bases	Parallel Manipulators	Total number
1	1|2; 1|3; 1|4; 1|5; 1|6; 1|7; 1|8; 1|9; 1|10;	9
2	2|1; 2|3; 2|4; 2|5; 2|6; 2|7; 2|8; 2|9; 2|10;	9
3	3|1; 3|2; 3|4; 3|5; 3|10;	5
Total number of parallel manipulators		$\Sigma = 23$

Example 19. Enumeration of spatial parallel manipulators with one end-effector: *Figure 61(a) shows a spatial kinematic chain with $M = 6$ and $n = 14$ enumerated by Tischler (1995) and Simoni et al. (2007) as one of the most promising candidates for the design of robotic fingers. The graph of the kinematic chain is shown in Figure 61(b).*



(a) Kinematic chain. (b) Graph representation.

Figure 61 – Kinematic chain and graph representation.

The orbits of the automorphism group of a non-colored vertex graph are:

- $\mathscr{O}_{Base_1} = \{0,1\}$,
- $\mathscr{O}_{Base_2} = \{2,5,6,9,10,13\}$ *and*
- $\mathscr{O}_{Base_3} = \{3,4,7,8,11,12\}$.

which originates three distinct choices of base, i.e. 0, 2 and 3. Applying our method, coloring the vertex that originates distinct bases and calculating the orbits of the automorphism group of the colored vertex graph we have:

- *for vertex 0 colored, the orbits are:* $\mathscr{O}_0 = \{0\}$, $\mathscr{O}_{PM_1} = \{1\}$, $\mathscr{O}_{PM_2} = \{2,6,10\}$, $\mathscr{O}_{PM_3} = \{3,7,11\}$, $\mathscr{O}_{PM_4} = \{4,8,12\}$, $\mathscr{O}_{PM_5} = \{5,9,13\}$,

- *for vertex 2 colored, the orbits are:* $\mathcal{O}_{PM_6} = \{0\}$, $\mathcal{O}_{PM_7} = \{1\}$, $\mathcal{O}_2 = \{2\}$, $\mathcal{O}_{PM_8} = \{3\}$, $\mathcal{O}_{PM_9} = \{4\}$, $\mathcal{O}_{PM_{10}} = \{5\}$, $\mathcal{O}_{PM_{11}} = \{6,10\}$, $\mathcal{O}_{PM_{12}} = \{7,11\}$, $\mathcal{O}_{PM_{13}} = \{8,12\}$, $\mathcal{O}_{PM_{14}} = \{9,13\}$ *and*

- *for vertex 3 colored, the orbits are:* $\mathcal{O}_{PM_{15}} = \{0\}$, $\mathcal{O}_{PM_{16}} = \{1\}$, $\mathcal{O}_{PM_{17}} = \{2\}$, $\mathcal{O}_3 = \{3\}$, $\mathcal{O}_{PM_{18}} = \{4\}$, $\mathcal{O}_{PM_{19}} = \{5\}$, $\mathcal{O}_{PM_{20}} = \{6,10\}$, $\mathcal{O}_{PM_{21}} = \{7,11\}$, $\mathcal{O}_{PM_{22}} = \{8,12\}$, $\mathcal{O}_{PM_{23}} = \{9,13\}$.

Table 16 shows the possible parallel manipulators with one end-effector for the kinematic chain in Figure 61(a).

Table 16 – Results of the enumeration of spatial parallel manipulators with one end-effector.

Bases	Parallel Manipulators	Total number
0	0|1; 0|2; 0|3; 0|4; 0|5	5
2	2|0; 2|1; 2|3; 2|4; 2|5; 2|6; 2|7; 2|8; 2|9	9
3	3|0; 3|1; 3|2; 3|4; 3|5; 3|6; 3|7; 3|8; 3|9	9
Total number of parallel manipulators		$\Sigma = 23$

5.4.3 Advantages of using symmetry

Example 18 presents the enumeration of all non-isomorphic parallel manipulators of the kinematic chain shown in Figure 60.

Using our method which analyzes the symmetries of the kinematic chain, we enumerated 23 non-isomorphic parallel manipulators shown in Table 15.

Without symmetry analysis, all possible choices of base and end-effector need to be evaluated because, in early stage of design, it is preferable the generation of duplicate (isomorphic) kinematic structures to the omission of a potentially useful solution (TISCHLER et al., 1995a). Therefore, for the kinematic chain shown in Figure 60 we have 90 possibilities of choices of base and end-effector. We have 10 links and we need to select 2 links at a time, one to be the base and another to be the end-effector. The number of arrangements that are possible when a subset of 2 items (base and end-effector) is taken from a set of 10 distinct items (links) is a “permutation of 10 objects taken 2 at a time” which can be written as P_2^{10} and is equal to $P_2^{10} = \dfrac{10!}{(10-2)!} = 90$.

For example, using symmetry analysis we identify only five (5) possibilities considering the link 3 as base, i.e. 3|1, 3|2, 3|4, 3|5, 3|10 (see Table 15), however, without symmetry analysis we have nine (9) possibilities, i.e. 3|1, 3|2, 3|4, 3|5, 3|6, 3|7, 3|8, 3|9, 3|10. As the links 1, 7, 8, 10 are symmetric, using symmetry analysis we identify only nine (9) possibilities considering ternary base, i.e. 1|2, 1|3, 1|4, 1|5, 1|6, 1|7, 1|8, 1|9, 1|10 (see Table 15), however, without symmetry analysis we have thirty-six (36) possibilities, i.e. 1|2, … 1|10, 7|1, … 7|6, 7|8, 7|9, 7|10, 8|1, … 8|7, 8|9, 8|10, 9|1, … 9|8, 9|10.

This simple example shows the potential of the proposed method. Using symmetry analysis we identify all non-isomorphic parallel manipulators and without symmetry analysis we enumerate several duplicated (isomorphic) parallel manipulators. We simplify the next step of the design, i.e. design detailing, from 90 parallel manipulators to 23 parallel manipulators. In the next chapter we will apply symmetry and use well established criteria to classify and select the most promising of these 23 parallel manipulators.

5.5 CONCLUSIONS

This chapter is based on the following papers:

- "Mãos Robóticas: Critérios para Síntese Estrutural e Classificação" (SIMONI et al., 2007);
- "Criteria for Structural Synthesis and Classification of Mechanisms" (SIMONI; MARTINS, 2007);
- "Enumeration of Kinematic Chains and Mechanisms" (SIMONI et al., 2009),
- "Enumeration of Parallel Manipulators" (SIMONI et al., 2008) and
- "Fractionation in planar kinematic chains: Reconciling enumeration contradictions" (MARTINS et al., 2010).

This chapter presented a systematic procedure for enumeration of kinematic structures, applying integrated tools of graph and group theory, into three levels: kinematic chains, mechanisms and parallel manipulators.

First, we described each level of the systematic procedure and the methods and tools used in each level.

Second, we presented and discussed the current status of enumeration of kinematic chains and indicated the discrepancies of theses results. As pointed by Martins et al. (2010) the most discrepancies are related to fractionation in kinematic chains and some incorrect results are indicated in Section 5.2.1.

Third, we presented an improvement of the method description of enumeration of mechanisms presented by Simoni (2008) using the concept of symmetry introduced in Section 2.3 (page 26). Also, we presented the current status of enumeration of mechanisms found in the literature, we compared and discussed the results.

Fourth, we presented a new method for enumeration of all parallel manipulators with one end-effector that a kinematic chain can originate. The method uses the concepts of symmetries, actions of orbits of the automorphism group of colored vertex graphs. To the best of the authors' knowledge, this is the first method for enumeration of all possible parallel manipulators which a kinematic chain can originate. The results presented in Section 5.4 are new and therefore we do not have references for comparison. We should emphasize that, using this method, we enumerated all possible parallel manipulators that a kinematic chain can originate without isomorphisms which is a NP-hard problem.

The next step is the systematization of the criteria of variety, symmetry, connectivity, degree-of-control and redundancy (MARTINS; CARBONI, 2007; BELFIORE; BENEDETTO, 2000; TISCHLER et al., 2001), that are well established concepts for kinematic analysis of the enumerated parallel manipulators. The number of parallel manipulators which each kinematic chain can originate is generally very great and it is difficult to analyze the individual merits of each parallel manipulator and we need an effective technique to the analysis of enumerated kinematic structures.

The techniques of enumeration introduced in this chapter are not only applicable for enumeration of mechanisms and parallel manipulators. Appendix A presents an application of these techniques for enumeration of planar metamorphic robots configurations. The results of Appendix A were presented in the 1^{st} ASME/IFToMM International Conference on Reconfigurable Mechanisms and Robots (ReMAR 2009) and received the best award paper on reconfigurable robots for the application of group and graph theory tools to solve the problem of enumeration of planar metamorphic robots configurations (MARTINS; SIMONI, 2009a).

6 CONTRIBUTIONS TO THE ANALYSIS OF KINEMATIC STRUCTURES

As we can see in Chapter 5, in general, the number of kinematic structures generated in the enumeration process is great and it is difficult to evaluate each kinematic structure individually. Therefore, it is necessary to use a set of criteria to evaluate the merits of each kinematic strucutre without eliminating a chain with possibilities to develop the desired task. For this reason, the concepts of variety, connectivity, degrees-of-control, redundancy and symmetry can be used to classify kinematic structures according to the constraints required (see Section 4.2). They are essential for structural analysis of mechanisms and parallel manipulators.

The contribution to the analysis is to classify the criteria to the kinematic analysis, reviewed in Section 4.2, into global and local criteria and to prove that local criteria are invariants by the action of the automorphism group of the associated graph. Global criteria are properties of the kinematic structure and local criteria are properties between members (links) of the kinematic structure.

First, we present the classification of the criteria, into global and local, and we present an example of the proposed classification of those criteria. Second, we apply integrated tools of graph and group theory to prove some lemmas and theorems about invariance by the action of the automorphism group of local criteria. The application of these lemmas and theorems results in the reduction of the matricial representation of local criteria and, consequently, in the simplification of the analysis of kinematic structures.

This chapter provides original contributions to the analysis of kinematic structures and it is based on the following papers:

- “Criteria for Structural Synthesis and Classification of Mechanisms” (SIMONI; MARTINS, 2007) and

- “Group and Graph Theories Applied to the Analysis of Mechanisms and Parallel Robots” (SIMONI et al., 2010).

6.1 CRITERIA CLASSIFICATION

Section 4.2 (page 66) presents a review of the main criteria to kinematic analysis, i.e. mobility, variety, connectivity, degrees-of-control, redundancy and symmetry. As we can see in Section 4.2, mobility and variety are properties of the kinematic structure. Already, connectivity, degrees-of-control and redundancy are properties between two links of the kinematic structure, it is evident from the Definitions 4.11, 4.12 and 4.13. Symmetry is a property of the kinematic structure, in the sense of the kinematic structure be symmetrical or asymmetrical. Also, the symmetry is a property between two links of the kinematic structure. If the kinematic structure is symmetrical, than, it is possible to explore symmetries between links or top/bottom and left/right symmetries. Thus, based on the criteria definitions reviewed in Section 4.2, we can classify the criteria into:

- *Global criteria* - properties of the kinematic structure:
  - mobility;
  - variety and
  - symmetry.
- *Local criteria* - properties between two links of the kinematic structure:
  - connectivity;
  - degrees-of-control;
  - redundancy and
  - symmetry.

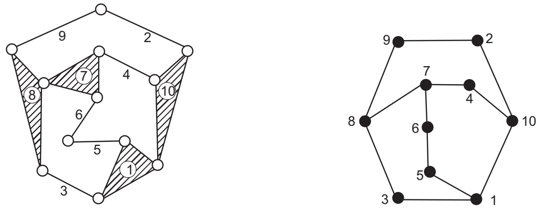
Global criteria of the kinematic structures are represented by a number, M of mobility, V of variety and r of symmetry (symmetrical if $r \neq 1$ and asymmetrical if $r = 1$, see Definition 11 on page 26). Local criteria of the kinematic structures are represented by matrices of order $n \times n$, where n is the number of links of the kinematic structure. Therefore, the symmetry of the links is represented by a string of links labels.

Example 20. *Figure 62(a) shows a planar kinematic chain with mobility three ($M = 3$), ten links ($n = 10$) and variety zero ($V = 0$). The graph of the kinematic chain is shown in Figure 62(b).*

In this case,

$$Aut(X) = \left\{ \begin{array}{l} \sigma_1 = (1)(2)(3)(4)(5)(6)(7)(8)(9)(10) \\ \sigma_2 = (1\,7)(2\,9)(3\,4)(5\,6)(8\,10) \\ \sigma_3 = (1\,8)(2\,6)(5\,9)(7\,10) \\ \sigma_4 = (1\,10)(2\,5)(3\,4)(6\,9)(7\,8) \end{array} \right\}. \tag{6.1}$$

As $Aut(X)$ has four elements the kinematic chain is of symmetry order $r = |Aut(X)| = 4$. Equation 4.15 (page 72) shows the connectivity matrix which is equal to the degrees-of-control matrix. Redundancy matrix is equal a null



(a) Kinematic chain. (b) Graph representation.

Figure 62 – Kinematic chain and graph representation.

matrix. As the kinematic chain is symmetrical we can identify the symmetries between the links of the kinematic chain. The orbits are:

$$\mathscr{O} = \{\{1,7,8,10\};\{2,5,6,9\};\{3,4\}\}.$$

Therefore, the symmetric links are: $\{1,7,8,10\}$, $\{2,5,6,9\}$ *and* $\{3,4\}$.

6.2 CRITERIA INVARIANCE BY AUTOMORPHISM GROUP

This section considers the application of integrated tools of graph and group theory to simplify the kinematic analysis. First, we will prove the invariance of degrees-of-control, connectivity and redundancy of kinematic chains by the action of its automorphism group of the associated graph. With the definition of the symmetry of kinematic chains, see Definition 10 (page 26), we will develop a method to reduce the matricial representation of the degrees-of-control, connectivity and redundancy matrices simplifying the kinematic analysis of kinematic structures.

Herein, we use some results for invariants of isomorphism and automorphism groups of graphs found in the literature (SORLIN; SOLNON, 2008; GODSIL; ROYLE, 2001; BIGGS, 1993b; LAURI; SCAPELLATO, 2003; HELL; NEŠETŘIL, 2004; GROSS; TUCKER, 2001; MCKAY, 2009b). These results, summarized below, are important to prove the Theorems 1 and 2 below.

Remark 1. *Let X be a graph, Y a subgraph of X and σ an element of $Aut(X)$.*

1. ***Degree invariance:*** $deg(\sigma(x)) = deg(x)$*, for all* $x \in V(X)$*;*
2. ***Distance invariance:*** $\delta(\sigma(x),\sigma(y)) = \delta(x,y)$*, for all* $x,y \in V(X)$*;*
3. ***Subgraph invariance:*** $\sigma(y) \simeq y$*, i.e. they are isomorphic.*

The proofs of these invariants are found in Sorlin and Solnon (2008), Godsil and Royle (2001).

To prove the Theorems 1 and 2 below we need another results which will be proved in the following lemmas.

Lemma 1 (Mobility invariance). *The mobility M of a graph (kinematic chain) is invariant by the action of the automorphism group of the graph.*

Proof: The proof follows from Definition 8. An automorphism of a graph is an isomorphism with itself and thus, the graph structure is preserved. As we can see in Example 5, the automorphism group of the graph results in the relabeling of the graph vertices and consequently the number of vertices $|V|$, the number of edges $|E|$ and the order of the screw system λ remain the same. Consequently, the mobility (Equation 4.9) is invariant. □

Lemma 2 (Subgraph mobility invariance). *The mobility M of a subgraph (subchain) is invariant by the action of the automorphism group of the graph.*

Proof: The proof follows from Remark 1 and Lemma 1. Remark 1 proves that a subgraph is invariant by the action of its automorphism group and thus, the structure of the subgraph ($|V|$, $|E|$, λ) remains the same. Lemma 1 proves that the mobility is invariant. Consequently, the subgraph mobility is invariant. □

Theorem 1 (Degrees-of-control invariance). *Let X be a graph (kinematic chain) and $Aut(X)$ its automorphism group. The degrees-of-control matrix $K(X)$ of the kinematic chain is invariant by the action of the automorphism group of the graph.*

Proof: The degrees-of-control is given by $K_{ij} = \min\{D[i,j], M'_{\min}\}$, see Equation 4.12. To prove this theorem, it is necessary to show that $D[i,j]$ matrix and $M'_{\min}$ are invariant by the action of the automorphism group. According to Remark 1 the distance of any pair of vertices is invariant by the action of the automorphism group of the graph, i.e. $D[i,j] = D[\sigma(i),\sigma(j)]$. Therefore, the $D[i,j]$ matrix is invariant by the action of the automorphism group of the graph. According to Remark 1 any subgraph is invariant by the action of the automorphism group of the graph, therefore, $M'_{\min}$ is also invariant. □

Theorem 2 (Connectivity invariance). *Let X be a graph (kinematic chain) and $Aut(X)$ its automorphism group. The connectivity matrix $C(X)$ of the kinematic chain is invariant by the action of the automorphism group of the graph.*

Proof: The proof follows from Theorem 1. The connectivity is given by $C_{ij} = \min\{K_{ij}, \lambda\}$, see Equation 4.11. K_{ij} is invariant according to Theorem 1 and λ is a property of the kinematic chain (it is not dependent on the graph) and therefore it is constant. □

Corollary 1 (Redundancy invariance)**.** *Let X be a graph (kinematic chain) and $Aut(X)$ its automorphism group. The redundancy matrix $R(X)$ of the kinematic chain is invariant by the action of the automorphism group of the graph.*

Proof: The proof follows straightforwardly from Theorems 1 and 2. The redundancy is given by $R_{ij}(x) = K_{ij}(x) - C_{ij}(x)$ (see Equation 4.13). $K_{ij}(x)$ and $C_{ij}(x)$ are invariants according to Theorems 1 and 2, consequently, $R_{ij}(x)$ is invariant. □

Theorems 1 and 2 and Corollary 1 state that the connectivity, degrees-of-control and redundancy are symmetric properties of a kinematic chain, i.e. elements which are symmetric by the action of the automorphism group of the graph have the same properties. Considering that symmetric links are identified by the orbits of the automorphism group of the graph, it is possible to reduce the matricial representation considering one representative element of each orbit.

6.3 APPLICATIONS

To show the potentialities of the results proved in Section 6.2 we will present a reduction in the matricial representation of local criteria. We have selected examples of mechanisms and parallel manipulators found in the literature where the connectivity, degrees-of-control and redundancy matrices are presented. First, we introduce the notation of the matricial representation in its reduced form.

Notation 1 (Reduced representation)**.** *The action of the automorphism group of the graph allows a reduced matricial representation. This reduced matricial representation has a subindex r as follows:*

1. *$A_r(x)$ is the reduced adjacency matrix;*
2. *$K_r(x)$ is the reduced degrees-of-control matrix;*
3. *$C_r(x)$ is the reduced connectivity matrix;*
4. *$R_r(x)$ is the reduced redundancy matrix.*

The reduced matrix corresponds to the original matrix but with rows eliminated, the elimination will be clearly shown in the following examples.

6.3.1 Example 1: Planar parallel mechanisms

This section presents the application of the method of enumeration of parallel manipulators (see Section 5.4) and the techniques to simplify the analysis (using connectivity) presented above. First, we will apply the Theorem 2 to reduce the size of the connectivity matrix. Second, using the results of the enumeration of parallel manipulators method and the reduced connectivity matrix, we will select the most promising parallel manipulators to design detailing. Third, we will present a comparison of the results using symmetry analysis and without symmetry analysis.

6.3.1.1 Reduced connectivity matrix

Let X be the kinematic chain of the parallel mechanism shown in Figure 62(a), its graph is shown in Figure 62(b). The automorphism group is

$$Aut(X) = \left\{ \begin{array}{l} \sigma_1 = (1)(2)(3)(4)(5)(6)(7)(8)(9)(10) \\ \sigma_2 = (1\ 7)(2\ 9)(3\ 4)(5\ 6)(8\ 10) \\ \sigma_3 = (1\ 8)(2\ 6)(5\ 9)(7\ 10) \\ \sigma_4 = (1\ 10)(2\ 5)(3\ 4)(6\ 9)(7\ 8) \end{array} \right\}. \tag{6.2}$$

The generator set is $Aut(X) =< \sigma_2,\ \sigma_3 >$. The orbits are:

$$\mathscr{O} = \{\{1\ 7\ 8\ 10\}; \{2\ 5\ 6\ 9\}; \{3\ 4\}\}$$

where

- $\mathscr{O}_1 = \{1\ 7\ 8\ 10\}$,
- $\mathscr{O}_2 = \{2\ 5\ 6\ 9\}$ and
- $\mathscr{O}_3 = \{3\ 4\}$.

Using a representative element of each orbit of the automorphism group (1, 2 and 3) the adjacency matrix presented in Equation 4.14 is reduced to:

$$A_r(X) = \begin{array}{c} \\ \mathscr{O}_1 \\ \mathscr{O}_2 \\ \mathscr{O}_3 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \left[\begin{array}{cccccccccc} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array} \right] \end{array}. \tag{6.3}$$

According to Theorem 2 we can reduce the connectivity matrix using a representative element of each orbit of the automorphism group. The connectivity matrix presented in Equation 4.15 is reduced to:

$$C_r(X) = \begin{array}{c} \\ \mathscr{O}_1 \\ \mathscr{O}_2 \\ \mathscr{O}_3 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \left[\begin{array}{cccccccccc} 0 & 2 & 1 & 2 & 1 & 2 & 3 & 2 & 3 & 1 \\ 2 & 0 & 3 & 2 & 3 & 3 & 3 & 2 & 1 & 1 \\ 1 & 3 & 0 & 3 & 2 & 3 & 2 & 1 & 2 & 2 \end{array}\right] \end{array} \tag{6.4}$$

where we chose as representative elements of each orbit the elements (links) 1, 2 and 3.

Note that the matrices are reduced from 10×10 to 3×10.

With the reduced adjacency and connectivity matrices shown in Equations 6.3 and 6.4 and the automorphism group shown in Equation 6.2, it is possible to rebuild the original matrices shown in Equations 4.14 and 4.15, respectively, just considering the action of the automorphism group elements on the rows of the reduced matrices. Note that it is necessary to rebuild rows 4, 5 ..., 10. Observe the action of each element of $Aut(X)$: Tables 17 and 18 show the actions which should be applied to rebuild the original matrices $A(X)$ and $C(X)$, respectively, where the first column shows the row to be rebuilt (R). To rebuild row 4 we need to choose an element of the automorphism group whose action changes a determined label x to 4. For example the action of (1 7)(2 9)**(3 4)**(5 6)(8 10) change the label $x = 3$ to 4 and, thus, it can be used to rebuild row 4 from row 3. Note that the way to rebuild the matrices is not unique, i.e. to rebuild row 10 we can use the elements **(1 10)**(2 5)(3 4)(6 9)(7 8), (1 8)(2 6)(5 9)**(7 10)** and (1 7)(2 9)(3 4)(5 6)**(8 10)**.

Table 17 – Actions of the elements of the automorphism group of the graph on the rows of the reduced adjacency matrix $A_r(X)$ for reconstruction of the original adjacency matrix $A(X)$.

R	Applied element of $Aut(X)$	Row of $A_r(X)$	Row of $A(X)$
4	(1 7)(2 9)**(3 4)**(5 6)(8 10)	1 0 0 0 0 0 0 1 0 0	0 0 0 0 0 0 0 1 0 0 1
5	(1 10)**(2 5)**(3 4)(6 9)(7 8)	0 0 0 0 0 0 0 0 1 1	1 0 0 0 0 1 0 0 0 0
6	(1 8)**(2 6)**(5 9)(7 10)	0 0 0 0 0 0 0 0 1 1	0 0 0 0 1 0 1 0 0 0
7	**(1 7)**(2 9)(3 4)(5 6)(8 10)	0 0 1 0 1 0 0 0 0 1	0 0 0 1 0 1 0 1 0 0
8	**(1 8)**(2 6)(5 9)(7 10)	0 0 1 0 1 0 0 0 0 1	0 0 1 0 0 0 1 0 1 0
9	(1 7)**(2 9)**(3 4)(5 6)(8 10)	0 0 0 0 0 0 0 0 1 1	0 1 0 0 0 0 0 1 0 0
10	**(1 10)**(2 5)(3 4)(6 9)(7 8)	0 0 1 0 1 0 0 0 0 1	1 1 0 1 0 0 0 0 0 0

Note also that the matrix representations $A_r(X)$ and $C_r(X)$ are more compact than the original $A(X)$ and $C(X)$ matrices. The more symmetric the

Table 18 – Actions of the elements of automorphism group of the graph on the rows of reduced connectivity matrix $C_r(X)$ for reconstruction of original connectivity matrix $C(X)$.

R	Applied element of $Aut(X)$	Row of $C_r(X)$	Row of $C(X)$
4	(1 7)(2 9)**(3 4)**(5 6)(8 10)	1 3 0 3 2 3 2 1 2 2	2 2 3 0 3 2 1 2 3 1
5	(1 10)**(2 5)**(3 4)(6 9)(7 8)	2 0 3 2 3 3 3 2 1 1	1 3 2 3 0 1 2 3 3 2
6	(1 8)**(2 6)**(5 9)(7 10)	2 0 3 2 3 3 3 2 1 1	2 3 3 2 1 0 1 2 3 3
7	**(1 7)**(2 9)(3 4)(5 6)(8 10)	0 2 1 2 1 2 3 2 3 1	3 3 2 1 2 1 0 1 2 2
8	**(1 8)**(2 6)(5 9)(7 10)	0 2 1 2 1 2 3 2 3 1	2 2 1 2 3 2 1 0 1 3
9	(1 7)**(2 9)**(3 4)(5 6)(8 10)	2 0 3 2 3 3 3 2 1 1	3 1 2 3 3 3 2 1 0 2
10	**(1 10)**(2 5)(3 4)(6 9)(7 8)	0 2 1 2 1 2 3 2 3 1	1 1 2 1 2 3 2 3 2 0

kinematic chain the smaller is its representation. As most parallel mechanisms found in the literature are symmetric, this representation is particularly advantageous.

6.3.1.2 Selection according to connectivity

This section presents the analysis of all possible parallel manipulators that the kinematic chain shown in Figure 62 can originate and we will classify those parallel manipulators according to connectivity. In the Example 18 (page 105) we enumerated all parallel manipulators with one end-effector, the number of parallel manipulators is shown in Table 15. We will repeat the Table 15 below, i.e. Table 19, to consider the analysis of those parallel manipulators.

Table 19 – Number of planar parallel manipulators with one end-effector.

Bases	Parallel Manipulators	Total number
1	1|2; 1|3; 1|4; 1|5; 1|6; 1|7; 1|8; 1|9; 1|10;	9
2	2|1; 2|3; 2|4; 2|5; 2|6; 2|7; 2|8; 2|9; 2|10;	9
3	3|1; 3|2; 3|4; 3|5; 3|10;	5
Total number of parallel manipulators		$\Sigma = 23$

Equation 6.4 shows the reduced connectivity matrix. If we want connectivity between base and end-effector equal to three, i.e. $C_{base,end-effector} = 3$, we have nine possible choices as indicated in boldface in Equation 6.5. Note that, the number of choices are drastically reduced when compared with

original matrix shown in Equation 4.15.

$$C_r(X) = \begin{array}{c} \\ \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 2 & 1 & 2 & \mathbf{3} & 2 & \mathbf{3} & 1 \\ 2 & 0 & \mathbf{3} & 2 & \mathbf{3} & \mathbf{3} & \mathbf{3} & 2 & 1 & 1 \\ 1 & \mathbf{3} & 0 & \mathbf{3} & 2 & \mathbf{3} & 2 & 1 & 2 & 2 \end{bmatrix} \end{array} \tag{6.5}$$

Furthermore, we can use the parallel manipulators enumerated by our method of enumeration of parallel manipulators, proposed in Section 5.4, presented in Table 19. Table 20 shows only the parallel manipulators of Table 19 with connectivity equal three.

Note that, analyzing the reduced matrix we indicate nine possible choices of parallel manipulators with connectivity equal three, however, when we analyze the number of parallel manipulators obtained by our method we discard one of them which is isomorphic to one presented in Table 20. In fact, the parallel manipulator $3|6$ is isomorphic to parallel manipulator $3|2$ and it should be discarded.

Table 20 – Number of planar parallel manipulators with one end-effector and connectivity equal three.

Bases	Parallel Manipulators	Total number
1	1|7;1|9	2
2	2|3; 2|5; 2|6; 2|7	4
3	3|2; 3|4	2
Total number of parallel manipulators		$\Sigma = 8$

Using the results of Table 20 and the kinematic chain shown in Figure 62, it is possible to incorporate “other requirements”, as indicated by Tsai’s methodology (see Figure 1, page 5), to evaluate the most adequate parallel manipulators for design detailing. For example, we have three possible choices of the base, i.e. 1, 2 or 3, if we identify that the base need to be a ternary link we have just two possible parallel manipulators as indicated by line 1 of Table 20, i.e. $1|7$ and $1|9$. Other two choices of base are binary: the base 2 is connect to one ternary link and one binary link and the base 3 is connected to two ternary links.

According to Tsai (2001), if a parallel manipulator has the number of limbs equal to the number of degrees of freedom (mobility) of the moving platform such that each limb is controlled by one actuator and all actuators are mounted on or near the fixed base, the parallel manipulator will have the advantages of low inertia, high stiffness, and large payload capacity. Thus, as the parallel manipulator has mobility equal to three, the most promising choice to the fixed link is a ternary link. As discussed in Section 4.2.3 (Example 12 on page 70) the connectivity is an important criterion for selecting the most promising parallel manipulator because the connectivity determines the ability of the moving platform to perform a determined task. Thus, for the planar kinematic chain shown in Figure 62(a) ($\lambda = 3$ and $M = 3$), the most promising choice to the connectivity between base and end-effector is $C_{base,end-effector} = 3$, i.e. the relative mobility between base and end-effector is equal to three. Therefore, we have only two possibilities of selection of the base and end-effector for the kinematic chain shown in Figure 62, i.e. $1|7$ and $1|9$. Only these two parallel manipulators have ternary base (according to Tsai (2001)) and connectivity equal to three (according to Hunt (1978), Liberati and Belfiore (2006), Martins and Carboni (2007)). Therefore, only these two parallel manipulators will be considered in the design detailing.

6.3.1.3 Comparison of the analysis of the enumerated parallel manipulators

This section presents a comparison of the results of application of the techniques presented in Chapters 5 and 6 with a general enumeration as discussed in Section 5.4.3.

Table 21 shows the comparison of the results using symmetry analysis (applying our methods) and without symmetry analysis (see Section 5.4.3). Column four of Table 21 shows the number of isomorphisms avoided applying our techniques.

Table 21 – Comparison of the analysis of kinematic structures.

Total number of parallel manipulators	Using symmetry analysis	Without symmetry analysis	Isomorphisms avoided
$C_{i,j} = 1,2,3$	23	90	67
Ternary base	9	36	27
$C_{i,j} = 3$	8	30	22
Ternary base and $C_{i,j} = 3$	2	8	6

$C_{i,j} = C_{base,end-effector}$

The first line of Table 21 shows a comparison between the total number of parallel manipulators. Using symmetry analysis, as presented in Table 19, we have 23 parallel manipulators. Without symmetry analysis, as indicated in Section 5.4.3, we have 90 parallel manipulators. The number of isomorphic parallel manipulators enumerated unnecessarily when the symmetry is not used is shown in the fourth column of Table 21.

The second line of Table 21 shows a comparison between the total number of parallel manipulators with ternary base. Using symmetry analysis, and the results of the first line of Table 20 we have 9 parallel manipulators and, without symmetry analysis, we have 36 parallel manipulators (see Section 5.4.3).

The third line of Table 21 shows a comparison between the total number of parallel manipulators with $C_{i,j} = 3$. Using symmetry analysis and the results of Table 20 we have only 8 parallel manipulators. Without symmetry analysis, all possible choices of base and end-effector with connectivity $C_{i,j} = 3$ need to be evaluated. Using the connectivity matrix presented in Equation 4.15 (page 72) we identify 30 possibilities of choices with $C_{i,j} = 3$. The number of isomorphic parallel manipulators enumerated unnecessarily when the symmetry is not used is shown in the fourth column of Table 21.

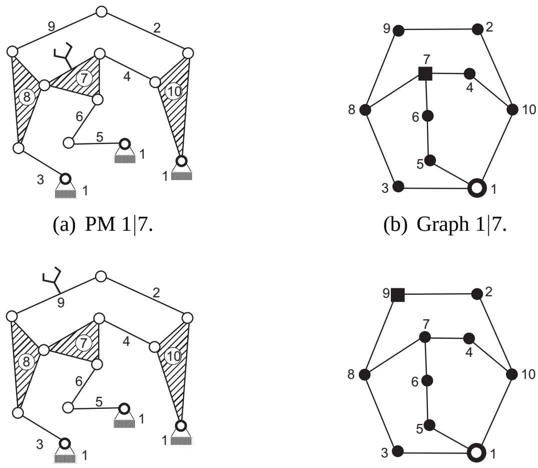
The fourth line of Table 21 shows a comparison between the total number of parallel manipulators with $C_{i,j} = 3$ and ternary base. Using symmetry analysis, and the results of the first line of Table 20 we have only 2 parallel manipulators. Without symmetry analysis, all possible choices of base and end-effector with connectivity $C_{i,j} = 3$ and ternary base need to be evaluated. Using the connectivity matrix presented in Equation 4.15 (page 72) and the kinematic chain shown in Figure 62 (page 113), we identify 8 possibilities of choices with $C_{i,j} = 3$ and ternary base because we have four ternary links, i.e. links 1, 7, 8 and 10.

If the functional requirements of the parallel manipulator are ternary base and connectivity equal to three (as discussed in Section 6.3.1.2), then only two parallel manipulators originated of kinematic chain shown in Figure 62 will be considered in the design detailing, i.e. $1|7$ and $1|9$ (see Section 6.3.1.2). Figure 63 shows these two parallel manipulators. As we can see, the techniques developed in this thesis permit us to select the most promising parallel manipulators to design detailing, the number of possible parallel manipulators is reduced from 90 to 2 (see Table 21).

This simple example shows the potential of the techniques introduced in this thesis. As we can see, the methods presented in this thesis enumerate all possible parallel manipulators and avoids isomorphisms which is a NP-hard[1] problem.

Below, we present two other examples just to show the potential of the theorems proved in this chapter. From the reduced matrix representation it is clear that the analysis will be simplified.

[1] A problem is NP-hard if an algorithm for solving it can be translated into one for solving any NP-problem (nondeterministic polynomial time). NP-hard therefore means “at least as hard as any NP-problem”, although it might, in fact, be harder (WEISSTEIN, 2009)



(a) PM 1|7. (b) Graph 1|7.

(c) PM 1|9. (d) Graph 1|7.

Figure 63 – The only two possible parallel manipulators, originated from kinematic chain shown in Figure 62, selected to design detailing.

6.3.2 Example 2: Hybrid 6-DoF Mechanisms

Let X be the kinematic chain of the hybrid 6-DoF manipulator presented in Figure 11 of Belfiore and Benedetto (2000) and shown in Figure 64.

In this case, $Aut(X)$ in terms of the generator set is given by:

$$Aut(X) = \left\langle \begin{array}{l} \sigma_1 = (7\,11)(8\,12)(9\,13)(10\,14) \\ \sigma_2 = (20\,24)(21\,25)(22\,26)(23\,27) \\ \sigma_3 = (15\,20)(16\,21)(17\,22)(18\,23) \\ \sigma_4 = (2\,7)(3\,8)(4\,9)(5\,10) \\ \sigma_5 = (1\,19)(2\,18)(3\,17)(4\,16)(5\,15)(7\,23)(8\,22) \\ \qquad (9\,21)(10\,20)(11\,27)(12\,26)(13\,25)(14\,24) \end{array} \right\rangle$$

The orbits are:

$$\mathscr{O} = \{\{1\,19\};\{2\,7\,11\,18\,23\,27\};\{3\,8\,12\,17\,22\,26\}; \{4\,9\,13\,16\,21\,25\};\{5\,10\,14\,15\,20\,24\};\{6\}\}$$

Using the Theorem 2 we can reduce the connectivity matrix using a representative element of each orbit of the automorphism group. Following the same procedure applied in the example above, the connectivity matrix $C(X)$ presented in Appendix B of Belfiore and Benedetto (2000), which is

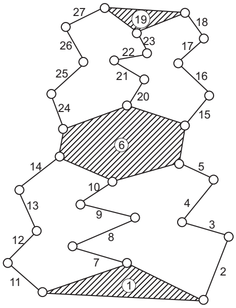


Figure 64 – Hybrid 6-DoF manipulator (BELFIORE; BENEDETTO, 2000; LIBERATI; BELFIORE, 2006).

27×27, is reduced to:

$$
C_r(X) = \begin{array}{c} \\ \mathscr{O}_1 \\ \mathscr{O}_2 \\ \mathscr{O}_3 \\ \mathscr{O}_4 \\ \mathscr{O}_5 \\ \mathscr{O}_6 \end{array}
\begin{array}{ccccccccccccccccccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 \\
0 & 1 & 2 & 3 & 3 & 3 & 1 & 2 & 3 & 3 & 1 & 2 & 3 & 3 & 4 & 5 & 6 & 6 & 6 & 4 & 5 & 6 & 6 & 4 & 5 & 6 & 6 \\
1 & 0 & 1 & 2 & 3 & 3 & 2 & 3 & 3 & 3 & 2 & 3 & 3 & 3 & 4 & 5 & 6 & 6 & 6 & 4 & 5 & 6 & 6 & 4 & 5 & 6 & 6 \\
2 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 5 & 6 & 6 & 6 & 4 & 5 & 6 & 6 & 4 & 5 & 6 & 6 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 5 & 5 & 5 & 3 & 4 & 5 & 5 & 3 & 4 & 5 & 5 \\
3 & 3 & 2 & 1 & 0 & 1 & 3 & 3 & 3 & 2 & 3 & 3 & 3 & 2 & 2 & 3 & 4 & 4 & 4 & 2 & 3 & 4 & 4 & 2 & 3 & 4 & 4 \\
3 & 3 & 3 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 3 & 3 & 2 & 1 & 1 & 2 & 3 & 3 & 3 & 1 & 2 & 3 & 3 & 1 & 2 & 3 & 3
\end{array}
$$

where we chose as representative elements of each orbit the elements (links) 1, 2, 3, 4, 5, and 6.

In this case the connectivity matrix is reduced from 27×27 to 6×27. Other properties represented by matrices, such as degrees-of-control, redundancy and adjacency, also are reduced from 27×27 to 6×27.

6.3.3 Example 3: Redundant Mechanism Employed in Space Missions

Let X be the kinematic chain of a multiple-arm robot employed in space missions presented by Belfiore and Benedetto (2000) and shown in Figure 65.

In this case, $Aut(X)$ in terms of the generator set is given by:

$$
Aut(X) = \left\langle \begin{array}{l} \sigma_1 = (1\,30)(2\,29)(3\,28)(4\,27)(5\,26)(6\,25)(7\,24) \\ \sigma_2 = (8\,16)(9\,17)(10\,18)(11\,19)(12\,20)(13\,21)(14\,22)(15\,23) \end{array} \right\rangle
$$

The orbits are:

$$
\begin{array}{l} \mathscr{O} = \{\{1\,30\};\{2\,29\};\{3\,28\};\{4\,27\};\{5\,26\};\{6\,25\};\{7\,24\};\{8\,16\}; \\ \quad \{9\,17\},\{10\,18\};\{11\,19\};\{12\,20\};\{13\,21\};\{14\,22\};\{15\,23\};\{31\}\} \end{array}
$$

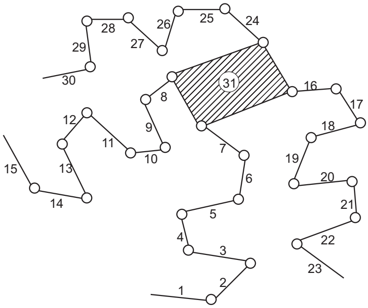


Figure 65 – Redundant Mechanism Employed in Space Missions (BELFIORE; BENEDETTO, 2000; LIBERATI; BELFIORE, 2006).

Using the Corollary 1 we can reduce the connectivity matrix using a representative element of each orbit of the automorphism group. Following the same procedure applied in the examples above, the redundancy matrix $R(X)$ presented in Appendix B of Belfiore and Benedetto (2000) is reduced to:

$Rr(X) =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
$\mathcal{O}_1$	0	0	0	0	0	0	0	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	1
$\mathcal{O}_2$	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	0
$\mathcal{O}_3$	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	0
$\mathcal{O}_4$	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	0	0	1	2	3	4	5	6	0	0	1	2	3	4	5	0
$\mathcal{O}_5$	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	0	0	0	1	2	3	4	5	0	0	0	1	2	3	4	0
$\mathcal{O}_6$	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	0	0	0	1	2	3	4	0	0	0	0	1	2	3	0
$\mathcal{O}_7$	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	0	0	0	0	0	1	2	3	0	0	0	0	0	1	2	0
$\mathcal{O}_8$	2	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	2	3	0	0	0	0	0	1	2	0
$\mathcal{O}_9$	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	0	0	0	1	2	3	0
$\mathcal{O}_{10}$	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	0	0	0	1	2	3	4	0
$\mathcal{O}_{11}$	5	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	0	0	1	2	3	4	5	0
$\mathcal{O}_{12}$	6	5	4	3	2	1	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	0
$\mathcal{O}_{13}$	7	6	5	4	3	2	1	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	0
$\mathcal{O}_{14}$	8	7	6	5	4	3	2	0	0	0	0	0	0	0	0	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	1
$\mathcal{O}_{15}$	9	8	7	6	5	4	3	1	0	0	0	0	0	0	0	3	4	5	6	7	8	9	10	3	4	5	6	7	8	9	2
$\mathcal{O}_{16}$	2	1	0	0	0	0	0	0	0	0	0	0	1	2	3	0	0	0	0	0	0	1	2	0	0	0	0	0	0	1	0

where we chose as representative elements of each orbit the elements (links) 1, 2, 3, ..., 15 and 16.

In this case the redundancy matrix is reduced from 31×31 to 16×31.

6.4 CONCLUSIONS

This chapter is based on the following papers:

- "Criteria for Structural Synthesis and Classification of Mechanisms" (SIMONI; MARTINS, 2007) and
- "Group and Graph Theories Applied to the Analysis of Mechanisms and Parallel Robots" (SIMONI et al., 2010).

In this chapter we classified the criteria to kinematic analysis, i.e. mobility, variety, connectivity, degrees-of-control, redundancy and symmetry, into global and local criteria. Global criteria are properties of the kinematic structure and local criteria are properties between members (links) of the kinematic structure.

The main contribution of this chapter is to prove the invariance of connectivity, degrees-of-control and redundancy, i.e. local criteria, by the action of the automorphism group of the graph. The connectivity, degrees-of-control and redundancy are symmetric properties of a kinematic chain, i.e. links which are symmetric by the action of the automorphism group of the graph have the same properties. Considering that symmetric links are identified by the orbits of the automorphism group of the graph, we reduce the matricial representation considering one representative element of each orbit. Thus, the order of the matrices is reduced from $n \times n$ to $o \times n$ where n is the number of links of the kinematic chain and o is the number of orbits of the automorphism group of the graph. As shown in Section 6.3 the reduced representation simplify the analysis of kinematic structures. The reduced representation presented is a minimal representation of the properties of kinematic chains in terms of symmetry.

Considering that the majority of parallel manipulators in the literature have symmetric kinematic chains (FANG; TSAI, 2002; LI et al., 2004; HUANG; LI, 2003; KONG; GOSSELIN, 2007, 2005, 2004b; GOGU, 2008, 2009), the reduced representation offers considerable advantages. As shown in the examples, if a kinematic chain has symmetry, it is possible to obtain a gain in terms of the storage of matrices, and in the simplicity of the kinematic analysis. These techniques can also be applied to kinematic chains of serial and hybrid manipulators. The only cases for which the theory presented herein is not advantageous is when the graph is fully asymmetric, i.e. in rare practical cases.

7 CONCLUSIONS AND FURTHER WORKS

This is the first thesis that considers the problems of enumeration and analysis of kinematic structures developed in the Mechanical Engineering Postgraduate Program of The Federal University of Santa Catarina. It is a broad work involving the phases of synthesis and analysis, i.e. generator and evaluator of Tsai's methodology shown in Figure 1 (page 5), and it opens a large field of research. The main focus of this work is to apply integrated tools of group and graph theory for enumeration and analysis of kinematic structures.

This work is the interest of the UFSC robotics research group front of current challenges. New projects of the UFSC robotics laboratory have a current trend towards mechanisms and parallel manipulators.

7.1 CONCLUSIONS

We improved graph and group theory tools for the application considered in this thesis and we presented bibliography reviews of mechanisms and machines concepts, enumeration of kinematic structures and criteria of analysis. Using integrated tools of graph and group theory, we presented a precise definition of symmetry in kinematic structures using the concepts of symmetry, actions and orbits of the automorphism group of graphs (see Section 2.3 on page 26). Symmetry is successfully applied in the enumeration and analysis of kinematic structures.

We presented a systematic procedure for enumeration of kinematic structures into three levels (see Figure 43 on page 76):

- **Level 1 - Enumeration of kinematic chains (Section 5.2):** From structural characteristics (number of links, number of joints, mobility, order of screw system) kinematic chains are enumerated. The attributes of kinematic chains are: number of links (n), number of 1-DoF joints (j), mobility (M) and order of screw system (λ). The main tools used in this level are: graph theory and screw theory.

- **Level 2 - Enumeration of mechanisms (Section 5.3):** Each kinematic chain originates mechanisms selecting all different bases. The attributes of mechanisms are: number of links (n), number of 1-DoF joints (j), mobility (M) and order of screw system (λ) and base of mechanism. The tools used in this level are: graph theory, group theory and screw theory; mainly the concepts of symmetry, actions and orbits of the automorphism group of non-colored vertex graphs.

- **Level 3 - Enumeration of parallel manipulators (Section 5.4):** Each mechanism originates parallel manipulators selecting different links to be end-effectors. The attributes of parallel manipulators are: number of links (n), number of 1-DoF joints (j), mobility (M) and order of screw system (λ) and base and end-effector of parallel manipulator. The tools used in this level are: graph theory, group theory and screw theory; mainly the concepts of symmetry, actions and orbits of the automorphism group of colored vertex graphs.

The main contributions of each level are summarized below:

- Level 1 - Enumeration of kinematic chains:
  - We presented the current status of enumeration of kinematic chains (see Tables 8, 9 and 10 on pages 80, 81 and 82, respectively).
  - Based on previous work of the authors, we solved the discrepancies of the results found in the literature, see Martins et al. (2010).
  - We discussed the results that are still not in compliance and we concluded that are strong evidence of typos in view of the similarities between the numbers.
- Level 2 - Enumeration of mechanisms:
  - We introduced a new notation of mechanisms in terms of graphs (see Section 5.3.1 on page 83).
  - We presented an improvement of the method of enumeration of mechanisms proposed by Simoni (2008) using the concepts of symmetry, actions and orbits of automorphism group of the graph associated to mechanism (see Section 5.3 on page 83).
  - We presented the current status of enumeration of mechanisms and we confirmed the results found in the literature (see Tables 11, 12 and 13 on pages 91, 92 and 93, respectively).
  - We indicated and discussed all the discrepancies of the results found in the literature and we indicated incorrect results by Simoni (2008).
- Level 3 - Enumeration of parallel manipulators:
  - We introduced a new notation of parallel manipulators in terms of graphs (see Section 5.4.1 on page 94).

- We presented a new method for enumeration of parallel manipulators using the concepts of symmetry, actions and orbits of automorphism group of the colored graph associated to the parallel manipulator (see Section 5.4.2 on page 95).
- We presented several new results of parallel manipulators (see Examples 17, 18 and 19 on pages 100, 105 and 106, respectively).

The graph representation of mechanisms and parallel manipulators introduced, respectively, in Sections 5.3.1 and 5.4.1 is effective and some properties of the representation are:

- Completeness: A graph represents all relationship in the kinematic structures;
- Uniqueness: Each graph represents an unique kinematic structure unless relabeling of vertices/links (automorphisms);
- Non-redundancy: The graph provides the essential information of a kinematic structures;
- Comprehensiveness: The representation is easy to understand.

The techniques of enumeration introduced in the systematic procedure described above are not only applicable for enumeration of mechanisms and parallel manipulators. Appendix A presented an application of these techniques to enumeration of planar metamorphic robots configurations. The results of Appendix A were presented in the 1^{st} ASME/IFToMM International Conference on Reconfigurable Mechanisms and Robots (ReMAR 2009) and received the best award paper on reconfigurable robots for the application of group and graph theories tools to solve the problem of enumeration of planar metamorphic robots configurations (MARTINS; SIMONI, 2009a).

In Chapter 6, we presented a new approach for structural analysis of kinematic structures using integrated tools of group and graph theory. First, we reviewed the main criteria used to classify the kinematic structure enumerated and, then, these criteria are classified into global and local criteria. Global criteria are properties of the kinematic structure and local criteria are properties between members (links) of the kinematic structure. Second, we proved the invariance of local criteria by the action of the automorphism group associated with the graph of the kinematic structure: invariance of mobility, invariance of connectivity, invariance of degrees-of-control and invariance of redundancy. By exploring the symmetries of the kinematic structure, we developed a technique to reduce the size of matricial representation of connectivity, degrees-of-control and redundancy from $n \times n$ to $o \times n$, where o

is the number of orbits by the action of the automorphism group of the associated graph. The reduced representation simplify the analysis of kinematic structures. An example is presented to show the potential of the technique.

Combining the techniques presented in this thesis, we avoid the isomorphism problem which is a NP-hard problem.

7.2 RELATED PAPERS

This work yielded several papers for journals and conferences:

- Mãos robóticas: Critérios para síntese estrutural e classificação (SIMONI et al., 2007);

- Criteria for structural synthesis and classification of mechanism (SIMONI et al., 2007);

- Enumeration of kinematic chains and mechanisms (SIMONI et al., 2009);

- Enumeration of parallel manipulators (SIMONI et al., 2008);

- Group and Graph Theories Applied to the Analysis of Mechanisms and Parallel Robots (SIMONI et al., 2010);

- Fractionation in planar kinematic chains: Reconciling enumeration contradictions (MARTINS et al., 2010);

- Enumeration of planar metamorphic robots configurations (MARTINS; SIMONI, 2009a);

- Metamorphic robots: Enumeration of configurations and motion planning (MARTINS; SIMONI, 2009b);

- Type synthesis of low-DoF parallel robots based on screw theory (MARTINS; SIMONI, 2009a);

- Progressive dynamic analysis of serial robots based on screw theory (LAUS et al., 2009);

- Progressive dynamic analysis of serial robots based on screw theory: An extension to the theory (LAUS et al., 2010).

7.3 FURTHER WORKS

It is a broad work involving the problems of enumeration and analysis of kinematic structures and it opens a larger field of research. Some interesting fields are presented below:

- Enumeration of fractionated kinematic chains to design hybrid manipulators.
- The specialization of mechanisms in the sense of to enumerate specialized mechanisms with a determined number and type of joints, i.e. rotative, prismatic, spherical, and so on (see Sections 3.4.1 and 3.4.2).
- To extend the method of enumeration of parallel manipulators from one end-effector to n end-effectors. A work in the line of research of Alizade and Bayram (2004).
- Type synthesis of kinematic structures obtained in the number synthesis process.
- Classification of symmetries of kinematic chains using automorphism group of the associated graph, girth and distance.
- Exploring symmetries in the growing field of reconfigurable robots (modular and metamorphic robots) (MARTINS; SIMONI, 2009a, 2009b).
- Synthesis of protein using group theory and graph symmetries.
- To analyze the optimal number of legs (or loops) of a mechanism and a parallel manipulator for each screw system. To analyze the influence of each leg on the complexity of kinematic and dynamic equations, to investigate the influence of each leg in the workspace, to investigate the influence of each leg to load capacity. The results of this analysis conduce to another important criteria, *loops or number of legs*, to select the best mechanism for each screw system.

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APPENDIX A -- Enumeration of Planar Metamorphic Robots Configurations

This appendix presents an original application of the techniques of enumeration presented in Chapter 5. This appendix presents the application of group and graph theory tools presented in Chapter 2 is in the field of metamorphic robotic systems. A metamorphic robotic system is a collection of mechatronic modules that can dynamically self-reconfigure in a variety of configurations, i.e. kinematic chains, to meet different or changing task requirements (CHIRIKJIAN, 1994). The contribution of this appendix to enumeration of planar metamorphic robots configurations is based on the following paper

- "Enumeration of planar metamorphic robots configurations" (MARTINS; SIMONI, 2009a).

This paper was presented in the 1^{st} ASME/IFToMM International Conference on Reconfigurable Mechanisms and Robots (ReMAR 2009) and received the best award on reconfigurable robots for the application of group and graph theory tools to solve the problem of enumeration of metamorphic robots configurations

This appendix shows how to enumerate all the non-isomorphic configurations of a planar metamorphic robotic system. Due to typical symmetries in module design, different assemblies may generate isomorphic robotic structures. A very useful simplification for metamorphic robotic systems is their representation through graphs. In this way, it is possible to apply the group theory tools discussed in Section 2.1 for the identification of symmetries of these metamorphic robotic systems. In particular, we define the concept of binary orbits of the automorphism group of graphs associated with the metamorphic robot configurations.

A.1 INTRODUCTION

A metamorphic robotic system is a collection of mechatronic modules that can dynamically self-reconfigure (CHIRIKJIAN, 1994). A change in the macroscopic morphology results from the locomotion of each module over its neighbors. Potential applications of metamorphic systems composed of a large number of modules include (CHIRIKJIAN, 1994; CHIRIKJIAN; PAMECHA, 1996; CHIANG; CHIRIKJIAN, 2001):

- obstacle avoidance in highly constrained and unstructured environments;
- "growing" structures composed of modules to form bridges, buttresses, and other civil structures in times of emergency;
- envelopment of objects, such as recovering satellites from space.

One application in particular, civil structures in times of emergency, evince the importance of previously knowing all the possible configurations that a predetermined finite number of modules can assume.

There some enticing questions in the literature of modular and metamorphic which are sometimes implicit in the context:

1. How to enumerate all possible configurations that a metamorphic robotic system can assume (CHEN; BURDICK, 1998);

2. How to find the optimal configuration for a predetermined task (CHEN; BURDICK, 1995; BI et al., 2003);

3. How to plan the movement of a metamorphic robot system, i.e. how to determine a sequence of module movements required to go from a given initial position to a desired goal configuration (PAMECHA et al., 1997; CHIANG; CHIRIKJIAN, 2001).

Questions 2 and 3 are relatively frequent in the metamorphic robot literature. Chen and Burdick (1995) consider the problem of finding an optimal module assembly configuration for a specific task. Their solution was formulated as a discrete optimization procedure. Bi et al. (2003) define the configuration space as the set of all feasible configuration variations of the robotic system and evaluate system adaptability for reconfigurable robotic systems with large variations in configurations. They also described how to achieve task-oriented configuration design of reconfigurable robotic systems.

Chirikjian and Pamecha (1996) proposed lower and upper bounds to the number of moves needed to change such systems from any initial to any final specified configuration. Pamecha et al. (1997) introduced the concept of distance between metamorphic robot configurations and demonstrate that this distance satisfies the formal properties of a metric. These metrics are applied to the automatic self-reconfiguration of metamorphic systems for computing the optimal sequence of movements required to reconfiguration. Dumitrescu et al. (2004) present a number of fast formations for both rectangular and hexagonal systems, and presented lower and upper bounds on the speed of locomotion. Kamimura et al. (2003) propose an offline method to generate a locomotion pattern automatically for a modular robot in an arbitrary module configuration.

Question 1, the problem of enumerating the set of kinematically distinct modular robot assembly configurations from a given set of modules, was addressed by Chen and Burdick (1998). They introduced a representation of a modular robot assembly configuration as an assembly incidence matrix and defined equivalence relations based on symmetries in module geometry and graph isomorphisms on the assembly incidence matrix. They also presented an algorithm to identify the kinematically equivalent robots. Chitta and Ostrowski (2006) also focused on enumeration of distinct configurations of a modular robot.

Common planar module designs are square (PAMECHA et al., 1996; DUMITRESCU et al., 2002; CHIANG; CHIRIKJIAN, 2001) and hexagonal (PAMECHA et al., 1996; ABRAMS; GHRIST, 2004; WALTER et al., 2004; DUMITRESCU et al., 2002; WALTER et al., 2002). For spatial metamorphic systems there are cubic (RUS; VONA, 2001; YOSHIDA et al., 1998) and dodecahedral (YIM et al., 1997, 2001) modules. Due to the inherent symmetries of these modules design, different assemblies of these modules may lead to several kinematically isomorphic robotic structures. To identify these symmetries, hence eliminating isomorphisms, in metamorphic robotic systems we use group theory, in particular the concept of orbits of automorphism group. This concept was previously applied to identify all mechanisms and parallel manipulators of a kinematic chains by Simoni et al. (2009, 2008). This tool helps avoiding isomorphisms in enumeration of planar metamorphic robots configurations; therefore, all non-isomorphic configurations are enumerated.

A.2 MODELLING OF METAMORPHIC ROBOTS

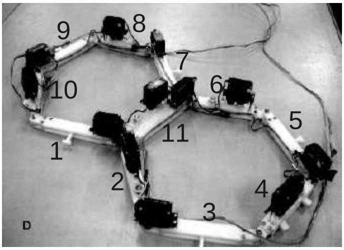
A metamorphic robot system can be modeled by a graph and the group theory tools presented in Section 2.1 can be applied to identify the symmetries of modules configurations and so it is possible enumerate all configurations that a set o modules can assume.

A.2.1 Graph representation

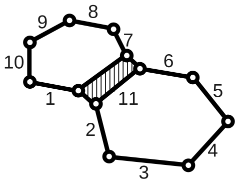
Figure 66(a) shows the metamorphic robot with two hexagonal modules presented by Pamecha et al. (1996). Figure 66(b) shows the kinematic chain of this metamorphic robot configuration and Figure 66(c) its graph representation (X).

A.2.2 Actions of automorphism group and orbits

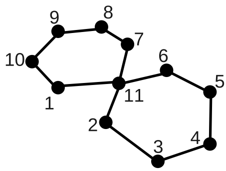
The automorphisms group of the metamorphic robot presented in Fig. 66(a) is composed by eight elements. The generators of the automorphism group are: $\sigma_1 = (1\ 7)(8\ 10)$, $\sigma_2 = (2\ 6)(3\ 5)$ and $\sigma_3 = (1\ 2\ 6\ 7)(3\ 10\ 5\ 8)(4\ 9)$,



(a)



(b)



(c)

Figure 66 – (a) metamorphic robot with two hexagonal modules (Figure 4 of Pamecha et al. (1996)); (b) kinematic chain and (c) Graph representation.

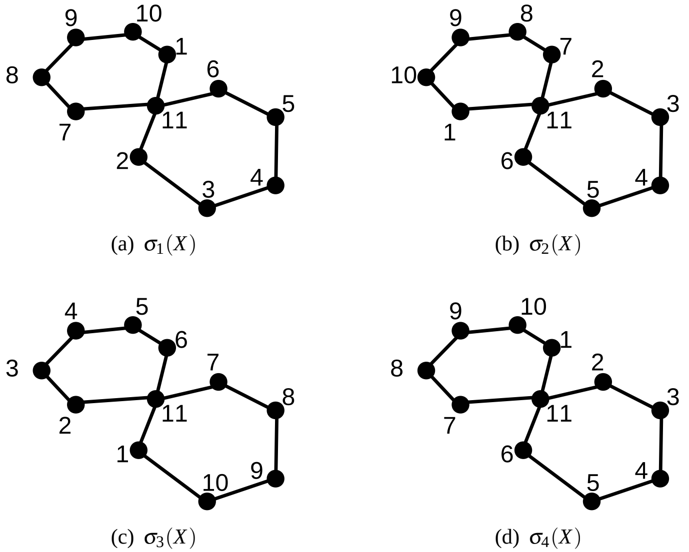
and the group is composed by $\sigma_1, \sigma_2, \sigma_3, \sigma_4 = \sigma_1 \circ \sigma_2, \sigma_5 = \sigma_1 \circ \sigma_3, \sigma_6 = \sigma_2 \circ \sigma_3, \sigma_7 = \sigma_1 \circ \sigma_2 \circ \sigma_3, \sigma_8 = e$, where e is the identity element. Figures 67(a), 67(b), 67(c) and 67(d) shows the actions of σ_1, σ_2, σ_3 and , σ_4 in G, respectively, on the labels of the metamorphic robot configuration.

For the metamorphic robot or graph shown in Figure 66 the orbits are:

- $\mathscr{O}_1 = \{1,2,6,7\}$;
- $\mathscr{O}_2 = \{3,5,8,10\}$;
- $\mathscr{O}_3 = \{4,9\}$ and
- $\mathscr{O}_4 = \{11\}$.

A.3 STANDARD MODULES AND BINARY ORBITS

In this section, we present the standard modules of metamorphic robots and discuss the symmetries of these modules. We also introduce the fundamental concepts of our technique of enumeration of planar metamorphic robots configurations: binary inversions and binary orbits.



(a) $\sigma_1(X)$ (b) $\sigma_2(X)$ (c) $\sigma_3(X)$ (d) $\sigma_4(X)$

Figure 67 – Actions of σ_1, σ_2, σ_3 and , σ_4 in the graph of metamorphic robot with two hexagonal modules shown in Figure 66(c).

As discussed in Section A.1, two standard modules applied to planar metamorphic robots are:

- square modules (PAMECHA et al., 1996; DUMITRESCU et al., 2002; CHIANG; CHIRIKJIAN, 2001), see Figure 68(a), and
- hexagonal modules (PAMECHA et al., 1996; ABRAMS; GHRIST, 2004; WALTER et al., 2004; DUMITRESCU et al., 2002; WALTER et al., 2002), see Figure 68(b).

A metamorphic robot system with square modules are represented by a four-bar kinematic chain as shown in Figure 68(c). Similarly, the hexagonal module is represent by a six-bar kinematic chain as shown in Figure 68(d). Other issues of the metamorphic robot design, such as the polarity (PAMECHA et al., 1996), were not considered during the enumeration of metamorphic robot configurations.

Figures 68(c) and 68(d) shows the internal symmetries of these modules. These symmetries may be identified by the orbits of automorphisms group. In these modules, all links (edges) have the same properties; therefore, there is a single orbit for each module:

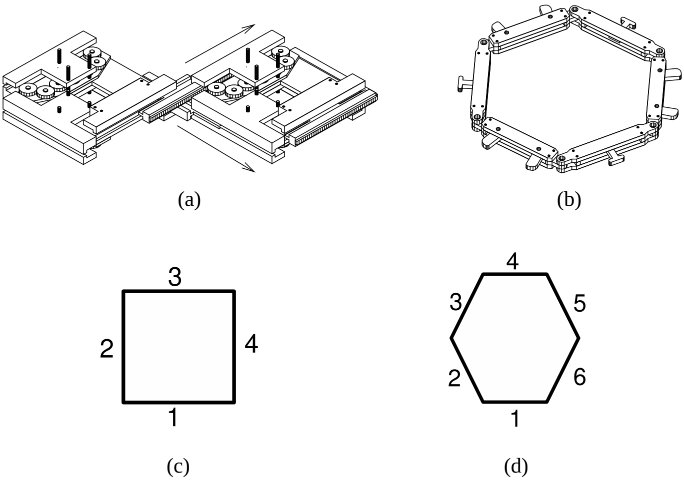


Figure 68 – Standard modules of metamorphic robots: (a) Square module (CHIANG; CHIRIKJIAN, 2001). (b) Hexagonal module (PAMECHA et al., 1996). (c) Graph representation of square module. (d) Graph representation of hexagonal module.

- square module: $\mathscr{O}_1 = \{1,2,3,4\}$ and
- hexagonal module: $\mathscr{O}_1 = \{1,2,3,4,5,6\}$.

A general metamorphic robot have multiple orbits. For example, the metamorphic robot shown in Figure 69 has the following symmetries identified by the orbits of automorphism group:

- $\mathscr{O}_1 = \{1,6,10,15\}$;
- $\mathscr{O}_2 = \{2,5,11,14\}$;
- $\mathscr{O}_3 = \{3,4,12,13\}$;
- $\mathscr{O}_4 = \{22,24\}$;
- $\mathscr{O}_5 = \{7,9,16,18\}$;
- $\mathscr{O}_6 = \{8,17\}$ and
- $\mathscr{O}_7 = \{19,30,21,23\}$.

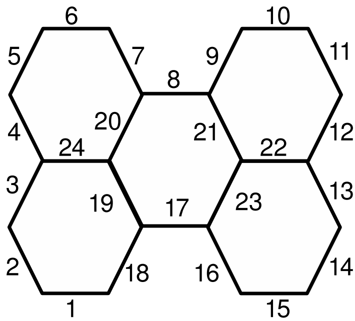


Figure 69 – Configuration of hexagonal metamorphic robot.

In kinematic terms, there are two types of links in the metamorphic robot system shown in Figure 69: binary and quaternary. Binary links 1-18 are connected to two other links while the quaternary links 19-24 are connected to four other links. Thus, the orbits $\mathscr{O}_1$, $\mathscr{O}_2$, $\mathscr{O}_3$ and $\mathscr{O}_5$ are composed by binary links and $\mathscr{O}_4$, $\mathscr{O}_6$ and $\mathscr{O}_7$ are composed by quaternary links.

Planar metamorphic robots may have other types of links, but they must have a subset of binary links since all "external" links are binary. These binary provide means for the movement of the metamorphic robot. Hence, all links of a metamorphic robot may be divided into two sets: binary and non-binary links. Another detail is that the joints are all equals in the modules, or they are R joints as shown in Figure 68(b) or they are P joints as shown in Figure 68(a). So, fixing one link we have an inversion[1] and we define:

Definition 17 (Binary inversions)**.** *Binary inversions are inversions composed only by binary links.*

Definition 18 (Binary orbits)**.** *Binary orbits are orbits composed only by binary inversions.*

A property derived from the concept of binary orbits and directly derived from the Definition 7 page 20 is:

Lemma 3 (Element of binary orbits)**.** *Every binary link is an element of a binary orbit.*

Therefore, binary links can be classified into binary orbits. Links in the same binary orbit have identical symmetry properties in the metamorphic robot configuration. Whenever a new module is connected to two or more of

[1] Inversions are related with enumeration of mechanisms (see Section 4.1.2)

the binary links from a same orbit, the resulting kinematic chains are isomorphic. For planar metamorphic robots, a new module can only be connected to links that belong to binary orbits. The binary orbits for the configuration shown in Figure 69 are $\mathscr{O}_1$, $\mathscr{O}_2$, $\mathscr{O}_3$ and $\mathscr{O}_5$ and they will be called, respectively, $\mathscr{O}_{C_1}$, $\mathscr{O}_{C_2}$, $\mathscr{O}_{C_3}$ and $\mathscr{O}_{C_4}$ where “C” means connection. Each binary orbit results in a new connection and an element of each binary orbit should be chosen to represent this connection.

In Section A.4, the configurations of metamorphic robot with “$n+1$” modules generated by configurations of metamorphic robot with “n” modules are explored.

A.4 ENUMERATION OF PLANAR METAMORPHIC ROBOTS CONFIGURATIONS

The enumeration process follows a tree structure. In root of the tree, a first module is placed. The following modules are added, one at a time, selecting just one representative for each binary orbit. See Definition 18 in Section A.3.

As orbits are equivalence classes and capture the internal symmetry of a structure (metamorphic robot); modules elements (links) in the same orbit when connected to other module elements result in isomorphic configurations. For example, Figure 70 shows a metamorphic robot with two square modules and another square module will be connected.

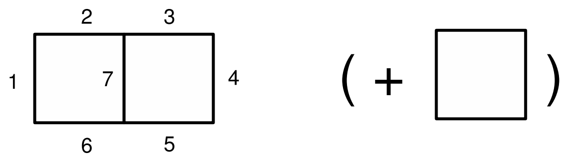


Figure 70 – Metamorphic robot with two square modules and another module for connection.

The orbits of automorphism group of metamorphic robot with two square modules are:

- $\mathscr{O}_1 = \{1,4\}$;
- $\mathscr{O}_2 = \{2,3,5,6\}$ and
- $\mathscr{O}_3 = \{7\}$.

Since link 7 is quaternary, there are just two binary orbits

- $\mathscr{O}_{C_1} = \{1,4\}$
- $\mathscr{O}_{C_2} = \{2,3,5,6\}$.

The connection of a new module with links from a same orbit results in kinematically isomorphic configurations as shown in Figures 71 and 72. Figure 71 shows that the connection of a new module to the configuration of metamorphic robot on elements from the orbit $\mathscr{O}_{C_1} = \{1,4\}$ results in isomorphic configurations. Similarly, Figure 72 shows that the connection of a new module with elements from the orbit $\mathscr{O}_{C_2} = \{2,3,5,6\}$ also results in isomorphic configurations.

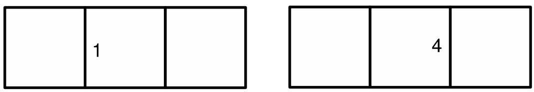


Figure 71 – Kinematically isomorphic configurations, obtained from Figure 70, by connecting another module in orbit $\mathscr{O}_{C_1} = \{1,4\}$.

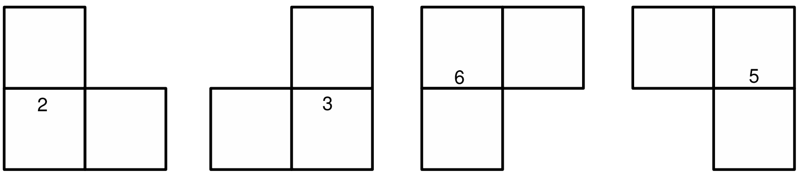


Figure 72 – Kinematically isomorphic configurations, obtained from Figure 70, by connecting another module in orbit $\mathscr{O}_{C_2} = \{2,3,5,6\}$.

Summing up, there are only two ways of connecting the new module to the current configuration, as shown in Figure 73.

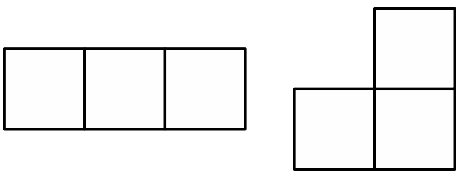


Figure 73 – Kinematically distinct (non-isomorphic) configurations of a metamorphic robot with three square modules identified by the orbits of automorphism group.

A.4.1 Metamorphic robot configurations with square modules

The technique will be presented by an example using square modules to facilitate the understanding of how the tools are applied. In Section A.4.2, we present metamorphic robot configurations with hexagonal modules.

Without loss of generality, for identification of symmetries of metamorphic robot system with square modules by group theory, we represent this module by a four-bar kinematic chain as shown in Figure 68(c).

Consider an example with a set of five square modules as shown in Figure 74. We start with a module in root of the tree (level 1) and identify all the ways to connect another module, for this we enumerate the binary orbits through the group theory tools. In example there are only one binary orbit. Figure 74 marks one representative from each binary orbit with small inclined parallel lines.

The next step is to enumerate configurations of metamorphic robots with three square modules adding another module from the second level of the tree. For this, we enumerate the binary orbits of configuration metamorphic robot of the root. In this case are two as was illustrated in Figures 71, 72 and 73. The configurations metamorphic robot with three square modules are obtained in the third level of the tree (see Figure 74).

The configurations metamorphic robot with four square modules are obtained in the fourth level of the tree. In this level, there are two isomorphic configurations to be eliminated. This isomorphisms elimination is applied in every level of the tree (see Figure 74).

Finally, to enumerate the configurations of metamorphic robot with five square modules, all non-isomorphic configurations of metamorphic robot with four square modules generated in the fourth level of the tree become roots for the fifth level. The process repeats: identification of the binary orbits, connection of a new module to single representative from each binary orbit, and elimination of the isomorphic configurations. At the end, of the process, all non-isomorphic metamorphic robot configurations with five square modules are obtained in the fifth level of the tree.

The numbers of all non-isomorphic planar metamorphic robot configurations with up to five square modules are (see Figure 74):

- 1: with a single module (level 1);
- 1: with two modules (level 2);
- 2: with three modules (level 3);
- 5: with four modules (level 4);

•12: with five modules (level 5).

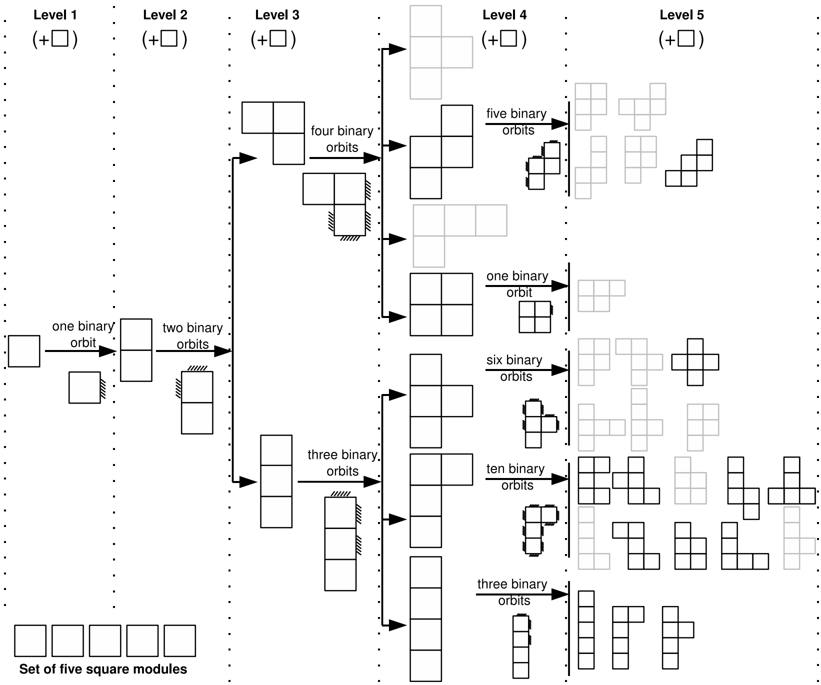


Figure 74 – Enumeration of all non-isomorphic metamorphic robot configurations (bold lines) with up to five square modules. Configurations with thin lines are those discarded due to isomorphism with previously generated kinematic chains.

A.4.1.1 Procedure in algorithmic form

In algorithm form, the procedure is summarized as:

Step 1 Calculate the binary orbits of the metamorphic robot configuration of the root.

Step 2 Assemble a new module with one element from each binary orbit, identified in the previous step, of the current metamorphic robot configuration.

Step 3 Run an (efficient) isomorphism test to eliminate the possible isomorphic configurations in each level of the tree.

Figure 75 illustrates the algorithm.

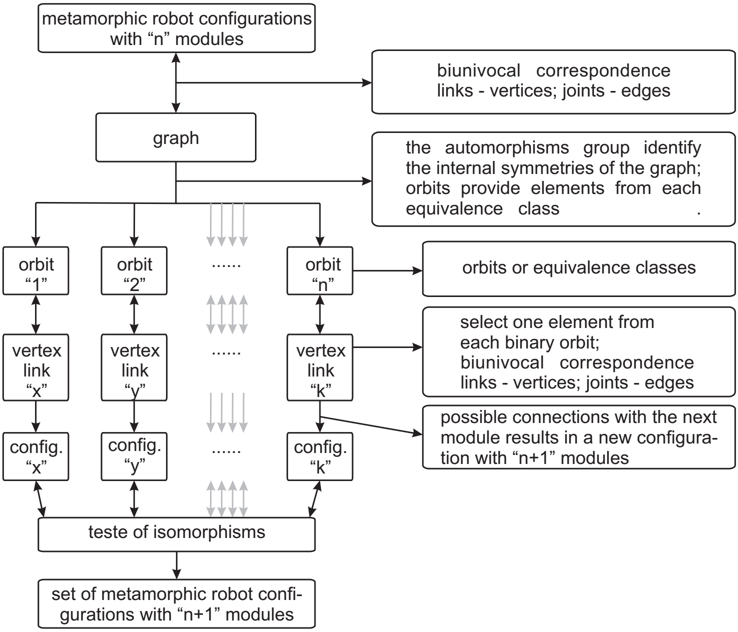


Figure 75 – Flowchart of the proposed technique outlining the role of the group theory tools for enumeration planar metamorphic robot configurations.

Group theory allows reducing the number of isomorphisms drastically by preventing symmetries during the assembling procedure. However, as the number of modules increases, the number of isomorphisms increases almost combinatorially and the process becomes computationally expensive. Hence, there is still a need of a more efficient isomorphism detection.

A.4.2 Metamorphic robot configurations with hexagonal modules

Let the enumeration of all non-isomorphic planar metamorphic robot configurations with up to four hexagonal modules. The procedure is presented in Figure 76. Besides each arrow is located the number of binary orbits. The module of the first level of the tree has only one binary orbit. The metamorphic robot in second level has three binary orbits. The third level, from left to right, has 2, 7 and 4 binary orbits, respectively.

The numbers of all non-isomorphic planar metamorphic robot configurations with up to four hexagonal modules are (see Figure 76):

- •1: with a single module (level 1);
- •1: with two modules (level 2);
- •3: with three modules (level 3);
- •8: with four modules (level 4);

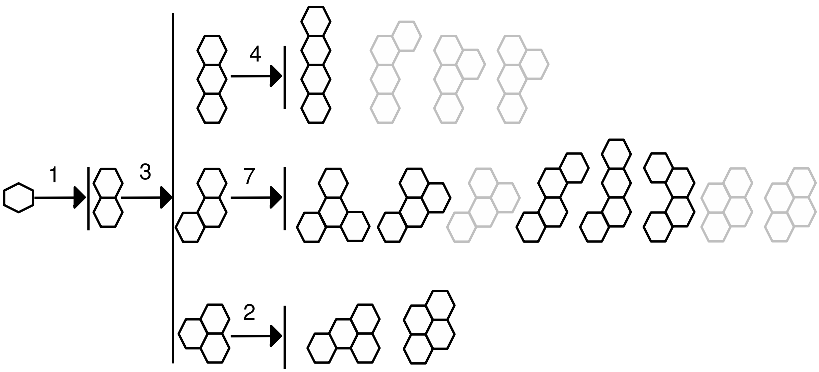


Figure 76 – Enumeration of all non-isomorphic metamorphic robot configurations (bold lines) with up to four hexagonal modules. Configurations with thin lines are those discarded due to isomorphism with previously generated kinematic chains.

A.5 CONCLUSIONS

This appendix introduced a technique for enumeration of all non-isomorphic planar metamorphic robot configurations. This technique was applied to the most common planar metamorphic robots, namely square and hexagonal modules. However, the technique may be easily extended to enumerate non-planar metamorphic robot configurations based on other types modules with only minor changes.

This appendix shows the effectivenees of the enumeration techniques presented in Chaper 5.

The enumeration consider in this appendix provide a first answer to Question 1 in Section A.1: *how many possible configurations a finite set of metamorphic robotic system can assume?*

Another related research topic is on planning the movement of a metamorphic robot system, i.e. how to determine a sequence of module movements required to go from a given initial configuration to a desired goal configuration.

Future work will be carried out to extend the method to enumeration spatial metamorphic robots configurations whit cube, dodecaedral and so on, in view of that the modules are symmetric.

Another future work is automatic configuration recognition. Automatic configuration recognition is the process by which a modular system can determine its own configuration without having it explicitly programmed.

APPENDIX B -- Parallel Manipulators

A robot manipulator is a mechanical system under automatic control that performs operations such as handling and locomotion (IONESCU, 2003). The mechanical system of a robot manipulator consists of a sequence of rigid bodies (links) interconnected by articulations (joints). From a topological viewpoint, this mechanical system formed by links and joints is known as kinematic chain. There are two fundamental structures of robots manipulators from mechanical point of view: the serial and parallel manipulators. A serial manipulator is formed by an open-loop kinematic chain: a kinematic chain is termed open-loop if every link in a kinematic chain is connected to every other link by one and only one path. A parallel manipulator is formed by a closed-loop kinematic chain; a kinematic chain is termed closed-loop if every link is connected to every other link by at least two distinct paths.

An intrinsic property of a robot kinematic chain is the mobility, i.e. the degrees of freedom (DoF). The DoF are distributed in the kinematic chain in order to have a sufficient number to execute a given task. For example, to develop a task in the plane 3-DoF are required (two for positioning a point on the object and one for orienting the object with respect to a reference coordinate frame) and to develop a task in three-dimensional space 6-DoF are required (three for positioning and three for orienting). If more DoF than task variables are available, the manipulator is said to be redundant from a kinematic viewpoint (SICILIANO et al., 2009).

The space in which a robot can operate is its work envelope, which encloses its workspace (SICILIANO; KHATIB, 2008). The workspace represents the portion of the environment that manipulator's end-effector can access. Its shape and volume depend on the manipulator structure as well as on the presence of mechanical joint limits (SICILIANO et al., 2009).

The last few years have witnessed an important development in the use of robots on the industrial world mainly due to their flexibility. Serial manipulators are by far the most common industrial robots (MERLET, 2006). According to the International Federation of Robotics (IFR), up to 2005, 59% of installed robot manipulators worldwide has anthropomorphic geometry, 20% has Cartesian geometry, 12% has cylindrical geometry, and 8% has SCARA geometry (LITZENBERGER, 2009; SICILIANO et al., 2009). Their main advantage is its large workspace with respect to its own volume and occupied floor space. However, several disadvantages are surrounding this type of robots:

- the low stiffness inherent to an open-loop kinematic structure;
- the low load/weight ratio;
- the errors are accumulated and amplified from link to link;

- they have to carry the weight of the actuators.

Thus, we see that serial manipulators are inappropriate for tasks requiring either the handling of heavy loads, an adequate level of positioning accuracy, or the ability to move fast (DOMBRE; KHALIL, 2007; MERLET, 2006).

On the other hand, the parallel manipulators are attracting interest from research and industry (MERLET; DANEY, 2008), several interesting parallel manipulators are appearing in university laboratories and some are already on the market. A recent report of the International Federation of Robotics already present statistics on parallel manipulators (LITZENBERGER, 2009). The interest of research on parallel manipulators is because current applications require high stiffness, high accuracy positioning, high speed and ability to manipulate large loads. These are the main advantages of parallel manipulators and their disadvantages are:

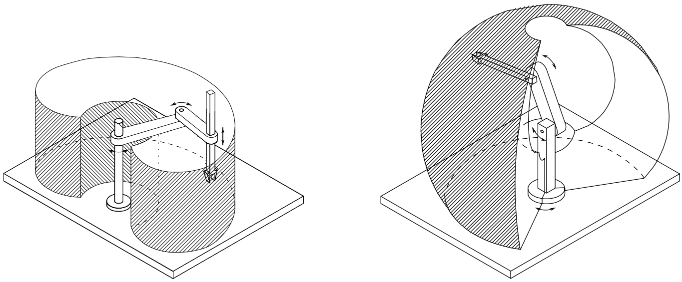
- limited workspace;
- singularities inside the workspace, where the manipulator becomes uncontrollable;
- complex design and control.

As opposed to serial manipulators, in which the number of kinematic arrangements (types) is somewhat limited, parallel manipulators can lead to a very large number of kinematic arrangements for a given DoF or motion pattern. Therefore, a systematic approach is needed in order to determine all types of parallel manipulators thereby allowing the development of the most promising designs. This fundamental issue, conceptual design of parallel manipulators, is the main focus of this work.

Parallel manipulators can be found today in the manufacturing industry, agricultural, military and domestic applications, space exploration, medicine, education, information and communication technologies, entertainment, etc.

B.1 SERIAL MANIPULATORS

Currently, the most common robot architecture is undoubtedly serial. Serial manipulators are constituted of a succession of rigid bodies linked to its predecessor and its successor by a 1- joint, from base to end-effector. As example we have the Scara which has 4-DoF as shown in Figure 77(a), the scara motion is also known in literature as Schoenflies motion. Another example of a serial manipulator with 6-DoF is shown in Figure 77(b). This design offers numerous advantages, the Figure 77 illustrate well the main advantage of this type of manipulator; its large workspace with respect to its own volume and occupied floor space. The popularity of this architecture in the industry is a clear indication of its ability to fulfill a broad variety of needs.



(a) Scara robot. (b) Anthropomorphic robot.

Figure 77 – Serial manipulators and their workspace (SICILIANO et al., 2009).

In agreement with Merlet (2006) and Dombre and Khalil (2007), the low transportable payload and poor accuracy are both inherent in the serial manipulators; the links are submitted to high forces and moments requiring them to be very rigid, and consequently very heavy (which is detrimental to fast motion), the errors of the internal sensors on the manipulator travel in an amplified manner to the end-effector. Merlet (2006, Table 1.1 and 1.2) shows that Scara type manipulators, that have a good ratio load/mass because they are direct-drive robots (without a reduction gear between the motors and the joints), the ratio load/weight is always less than 0.25 for heavy loads. For example, for a load capacity of 500 kg the robots mass of the Scara type will be at least 2000 kg. In another structures, such spherical and anthropomorphic, this ratio is worse (see Merlet (2006, Table 2)). For the positioning accuracy there are two concepts to analyze: absolute accuracy, defined as the distance between the desired and the actual position of the end-effector, and repeatability, which is the maximum distance between two positions of the end-effector reached for the same desired pose from different starting positions. Merlet (2006, Table 1.1 and 1.2) shows that the repeatability may be insufficient for certain tasks and in most cases the absolute accuracy of a serial manipulator is poor.

In summary, the serial manipulators are inappropriate for tasks requiring either the manipulation of heavy loads, or a good positioning accuracy, or to work at different scales, or the ability to move fast (DOMBRE; KHALIL, 2007; MERLET, 2006).

One alternative to avoid these problems is to use the so-called parallel manipulators, also known as parallel robots or parallel mechanisms.

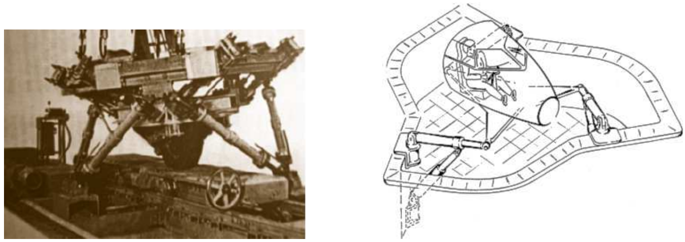
B.2 PARALLEL MANIPULATORS AND THEIR APPLICATIONS

A generalized parallel manipulator is a closed-loop kinematic chain mechanism whose end-effector (mobile platform) is linked to the base (fixed platform) by at least two independent kinematic chains called limbs (MERLET, 2006; GOGU, 2008). The mobile platform can achieve between one to three independent translations and one to three independent rotations (DoF).

Parallel manipulators are usually faster than traditional serial manipulators, since the motors can be mounted on the base, consequently saving weight. They are also stronger than serial manipulators because of the closed-loop kinematic chain and the load/weight ratio is considered very good. Another benefit is that the errors of the end-effector is less than the errors of serial manipulators since the errors are divided between all legs (as opposed to being additive as in serial manipulators). However, parallel manipulators have usually a more limited workspace; for instance, they generally cannot reach around obstacles because the legs can collide and, in addition, each leg has passive joints and each one has its own mechanical limits (BONEV, 2002). Another drawback of parallel manipulators is that they lose stiffness in singular positions. In these positions, the parallel manipulator gains finite or infinite degrees of freedom which are uncontrollable; it becomes shaky or mobile. Also, the freedom of motion on the end-effector are usually coupled together due to the multi-loop kinematic structure of the parallel manipulator (GOGU, 2008). Hence, motion planning and control usually become complicated. The advantages and current requirements in complex applications (e.g. medical applications) continue to motivate the development of parallel manipulators.

The development of parallel manipulators (PMs) can be dated back to the early 1960s when Gough and Whitehall first devised a six-linear jack system for use as a universal tire testing machine (TSAI, 1999; KONG; GOSSELIN, 2007) shown in Figure 78(a). Later, Stewart developed a platform manipulator for use as a flight simulator, (see Figure 78(b)) (TSAI, 1999).

Since 1980, there has been an increasing interest in the development of parallel manipulators. Early research on parallel manipulators have concentrated primarily on 6-DoF Gough-Stewart-type parallel manipulators. In the last decade, parallel manipulators with fewer than 6-DoF attracted industry's and researcher's attention. For some industrial applications, a parallel



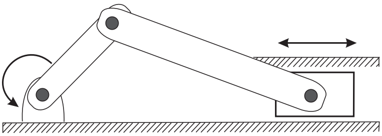
(a) (b)

Figure 78 – Gough-Stewart platforms: (a) Gough's original tire testing machine (SICILIANO; KHATIB, 2008). (b) Schematic of the Stewart platform (BONEV, 2001).

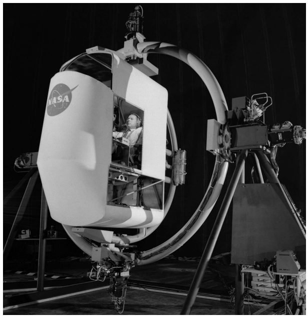
manipulator with fewer than 6-DoF, called a low-DoF parallel manipulator, is sufficient. Indeed, the study of this type of parallel manipulators is very important. Kong and Gosselin (2007) question the need of low-DoF parallel manipulators because a 6-DoF parallel manipulator could be used in all applications. However, a low-DoF parallel manipulator exhibit interesting features if compared to 6-DoF parallel manipulators; it has the advantages of simpler mechanical design, lower manufacturing and operating cost, larger workspace (reducing the legs interference and increase the maximum motion range of the remaining DoF), and simpler control (FANG; TSAI, 2002; TSAI, 2001; MERLET, 2006). Therefore, the study of low-DoF parallel manipulators recently become a main focus among the robotics research community.

B.2.1 1-DoF and 2-DoF parallel manipulators

Devices originated from closed-loop kinematic chains with 1-DoF and 2-DoF normally are not called parallel manipulators and yes mechanisms due to their low mobility. These mechanisms are used to convert motions of, and forces on, one or several bodies into constrained motions of, and forces on, other bodies and not to manipulate objects, this is the main reason for not calling them parallel manipulators. Figure 79(a) shows a slider-crank mechanism which has 1-DoF and Figure 79(b) shows a 2-DoF mechanism used by NASA as a flight simulator for the Apollo missions (NASA, 2009).



(a)



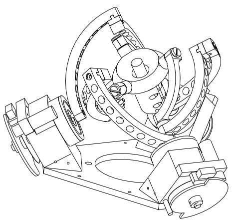
(b)

Figure 79 – 1-DoF and 2-DoF mechanisms: (a) Schematic of slider-crank mechanism. (b) 2-DoF mechanisms used by NASA as a flight simulator for the Apollo missions (NASA, 2009).

B.2.2 3-DoF parallel manipulators

Parallel manipulators with 3-DoF in translation prove extremely interesting for pick-and-place and machining operations.

Professor Clément Gosselin from Laval University Robotics Laboratory, proposed parallel manipulators with 3-DoF; 3-RRR spherical and 3-DoF 3-PRRR translational (GOSSELIN, 2009; GOSSELIN; KONG, 2002). Figure 80(a) shows the Laval University Agile Eye with three spherical DoF and the Figure 80(b) shows the Tripteron with three translational DoF (GOSSELIN; KONG, 2002).

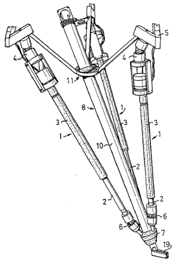


(a)

(b)

Figure 80 – 3-DoF parallel manipulators: (a) Laval University Agile Eye. (b) Tripteron (GOSSELIN, 2009; GOSSELIN; KONG, 2002).

Figure 81 shows the Tricept manipulator proposed by Neumann which has 3-DoF translational (NEUMANN, 1988). The Tricept was implemented as a 6-DoF parallel manipulator by ABB Robotics and PKMtricept SL; 3-DoF translational original from Tricept and 3-DoF spherical from a serial chain on the moving platform (see Figure 81(b)).



(a)

(b)

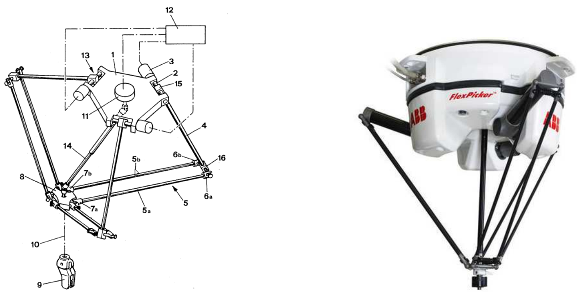
Figure 81 – Tricept parallel manipulator: (a) Schematic of the Tricept manipulator (NEUMANN, 1988). (b) Tricept T606 from PKMtricept SL (PKM Tricept SL, 2009).

B.2.3 4-DoF parallel manipulators

Parallel manipulators with 4-DoF are of great interest in industrial applications (pick-and-place, electronic industry, food industry, and so on) and they shows great advantage in medical applications and as flight simulators.

Clavel (1990), professor at École Polytechnique Fédérale de Lausanne, proposed the Delta manipulator, a parallel manipulator with 3-DoF translational and 1-DoF rotational. Figure 82(a) shows the original schematic of the Delta parallel manipulator (CLAVEL, 1990). The Delta manipulator has several applications since food industry until medical surgery. The Delta manipulator is extremely light and is said to be the fastest parallel manipulator yet made; its workspace is favorable too (DAVIDSON; HUNT, 2004). Figures 82(b) and 83(b) shows the ABB industrial Delta under the name IRB 340 FlexPicker and a medical application of a Delta, respectively. The ISIS/SurgiScope system from ISIS Robotics (PRIQUEL, 2009) using a Delta as microscope stand. Dr. Tim Lueth from MIMED (LUETH, 2009) realized the world's first head surgery (see Figure 83(b)).

Figure 83(a) shows the Adept Quattro s650H produced by ADEPT



(a) (b)

Figure 82 – Delta: (a) Schematic of the Delta (CLAVEL, 1990). (b) ABB industrial Delta under the name IRB 340 FlexPicker (ABB ROBOTICS, 2009).

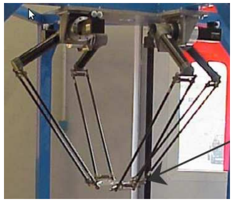
ROBOTICS (2009). The Adept Quattro s650H have Delta-type architecture with four legs.

(a) (b)

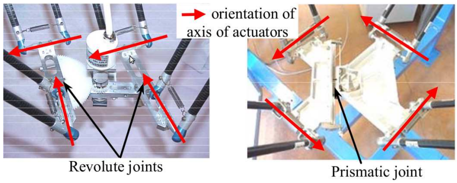
Figure 83 – Delta-type robot: (a) Adept Quattro s650H (ADEPT ROBOTICS, 2009). (b) World's first Craniomaxillofacial surgery using a Delta (LUETH, 2009).

François Pierrot and co-workers proposed the H4 and I4 family of parallel manipulators, in partnership with Toyota (PIERROT, 2009). These parallel manipulators uses various clever configurations of the moving platform to get 4-DoF, three translations and one rotation, with a design that allows for large rotation ability (PIERROT et al., 1999; KRUT et al., 2003; COMPANY et al., 2003; MERLET, 2006). I4 is an evolution of H4 architecture (see Figure 84).

(a)



(b)



(c)

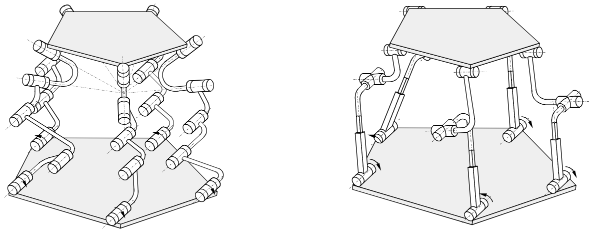
Figure 84 – H4 and I4 4-DoF parallel manipulators: (a) H4 parallel manipulator. (b) I4 parallel manipulator. (c) Clever configurations of the platforms. On the left detail of H4 platform and on the right detail of I4 platform (PIERROT et al., 2006).

B.2.4 5-DoF parallel manipulators

Parallel manipulators with 5-DoF are of great interest in the machine-tool domain, so called five-axis machining (REFAAT et al., 2006; MERLET; DANEY, 2008; MERLET, 2006). For example, in milling operation on the machine tool domain, the rotation of the platform around its normal is not needed, as the spindle will manage this DoF, hence only 5-DoF are needed. Some 5-DoF parallel manipulators have been proposed, but few parallel manipulators were implemented (KONG; GOSSELIN, 2005; LI et al., 2004; GAO et al., 2002; FANG; TSAI, 2002).

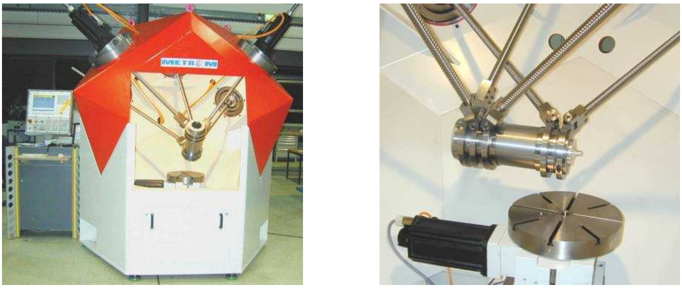
Figure 85 shows two 5-DoF parallel manipulator with identical limb structures proposed by Fang and Tsai (2002).

Figure 86 shows the 5-axis machine P800/P2000 of Metrom; this machine has a clever head mechanism that allows it to use only 5 legs (MERLET, 2006). Figure 86(b) shows details of his platform (head). Figure 87 shows the Okuma Cosmo Center PM-600 5-axis milling machine and detail of its platform (head).



(a) (b)

Figure 85 – 5-DoF parallel manipulator proposed by Fang and Tsai (FANG; TSAI, 2002): (a) 5-RRRRR. (b) 5-RPUR.



(a) (b)

Figure 86 – 5-axis machine: (a) The 5-axis P800 machine-tool from Metrom (MERLET, 2006; BONEV, 2009a). (b) The head mechanism with 3 legs attached to revolute joints sharing the same axis allows it to use only 5 legs (BONEV, 2009a).

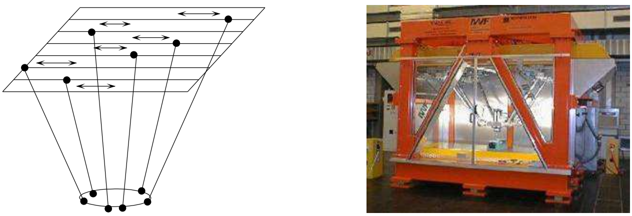
B.2.5 6-DoF parallel manipulators

Figure 88 shows the Hexaglide parallel manipulator from École Polytechnique Fédérale of Zürich. The Hexaglide is actuated by six linear actuators as shown in Figure 88(a). Figure 89(a) shows the Hexa 710-6 from Servos & Simulation (SERVOS AND SIMULATIONS, 2009), which is actuated by six rotational actuators as shown in Figure 89(a). Figure 89(b) shows an application of the Hexa for an entertainment simulator motion.

Another interesting parallel manipulator is the Eclipse, which has been conceived and designed at the Robotics Laboratory of National Seoul University, Korea (ROBOTICS LABORATORY SEUL UNIVERSITY, 1995; RYU

(a) (b)

Figure 87 – 5-axis machine: (a) Okuma Cosmo Center PM-600 (GMBH, 2009; BONEV, 2009a). (b) detail of its platform (BONEV, 2009a).



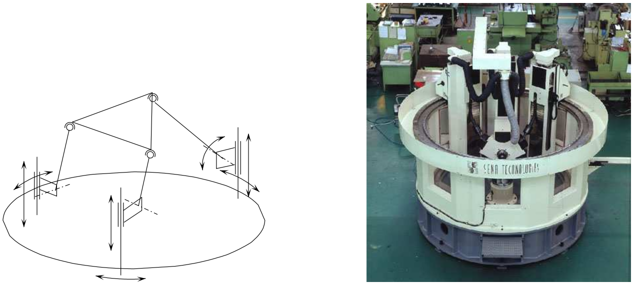
(a) (b)

Figure 88 – Hexaglide manipulator from École Polytechnique Fédérale of Zürich: (a) Schematic of the Hexaglide (MERLET, 2006). (b) Its implementation as a machine-tool (MERLET, 2006).

(a) (b)

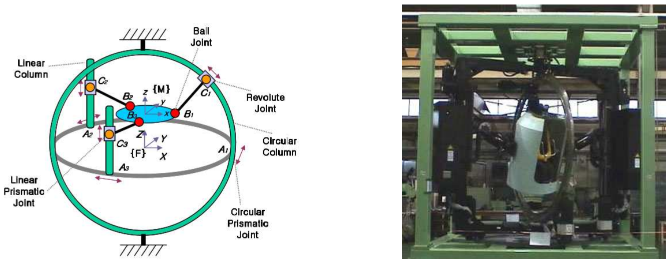
Figure 89 – Hexa 710-6 from Servos & Simulation (SERVOS AND SIMULATIONS, 2009): (a) Detail of actuation. (b) An application of the Hexa as an entertainment simulator motion base.

et al., 1998). Eclipse has seven actuators as shown in Figure 90, three carriages supporting legs on a circular rail, and on the legs are three linear actuators supporting three revolute joints connected to fixed length links, one of which is actuated (MERLET, 2006). The other ends of the links are connected to the moving platform through ball-and-socket joints. An evolution of Eclipse was proposed by Kim et al. (2002), named Eclipse II (see Figure 91(b)). The Eclipse II is not redundant (see Schematic in Figure 91(a)) and use circular railways to allow 360 degrees of rotation of the platform.



(a) (b)

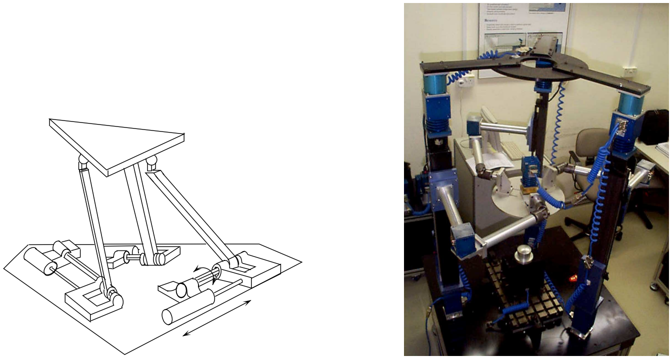
Figure 90 – Eclipse: (a) Schematic of the Eclipse (MERLET, 2006). (b) First prototype of milling machine based on the Eclipse concept (ROBOTICS LABORATORY SEUL UNIVERSITY, 1995).



(a) (b)

Figure 91 – Eclipse II: (a) Schematic of the Eclipse II (KIM et al., 2002). (b) First prototype of Eclipse II motion simulator (ROBOTICS LABORATORY SEUL UNIVERSITY, 1995).

Kohli et al. (1988) suggested a parallel manipulator with three legs and uses double actuators which are either linear and rotary, see schematic of the parallel manipulator in Figure 92(a). Figure 92(b) shows a variant of Kohli's parallel manipulator proposed by Yang et al. (2003), Chen (2009) using three RPRS legs.



(a) (b)

Figure 92 – Three leg 6-DoF manipulator: (a) Schematic of the Kohli's manipulator (MERLET, 2006). (b) A variant of Kohli's manipulators using RPRS legs (CHEN, 2009).

A more detailed review of parallel manipulators can be found in Merlet (2006) and Bonev (2009b). While several parallel manipulators have been proposed there is still a large field of research in conceptual design of parallel manipulators.

B.2.6 Applications of parallel manipulators

Some applications of parallel manipulators already have been discussed in the text above. Parallel manipulators have been successfully used in many applications and the variety of applications in which parallel manipulators are used is constantly expanding (MERLET, 2006; BONEV, 2009b; KONG; GOSSELIN, 2007; GOGU, 2008).

The applications of parallel manipulators are the most variates:

- Motion simulators and test systems; all flight simulators (MERLET, 2006; SICILIANO; KHATIB, 2008), vibrations, and so on.

- Industrial manipulators; food, electronic, ultra-accurate positioning devices, pick-and-place, fast packaging, and so on.

- Spatial applications; pointing device for telescopes (all recent land-based telescopes use parallel manipulators, either as a secondary mirror alignment system or as a primary mirror pointing device (MERLET, 2006)), simulator for the study of robotized assembly in the space, satellite instrumentation, and so on.

- Medical research; the accuracy of parallel manipulators and the fact that they are more easily “miniaturizable” than serial manipulators has led to certain research in the medical domain. Parallel manipulators are used in the medical domain for:

  - endoscopy heads (WENDLANDT; SASTRY, 1994);
  - brain surgery to position a microscope at the Necker Hospital (DOMBRE; KHALIL, 2007);
  - orthopedic surgery (BRANDT et al., 1999);
  - ophthalmic surgery (GRACE et al., 1993);
  - neurosurgery, such as the manipulator developed by the Fraunhofer Institute in Stuttgart (MCBETH et al., 2004);
  - Hepatic devices (DAVIES, 2000);
  - precise positioning, either as permanent devices such as the Delta manipulator (DAVIES, 2000).
  - ISIS/SurgiScope system from ISIS Robotics (PRIQUEL, 2009) using a Delta as microscope stand.
  - Dr. Tim Lueth from MIMED (LUETH, 2009) realized the world’s first head surgery (see Figure 83(b)).

- Miscellaneous; entertainment as tour simulator, elevator of Hydro-Gerätbau which is used for the installation of the main landing gear of the Airbus A380 (HYDRO SYSTEMS, 2007; MERLET, 2006).

- Machine-tool; in industry, numerous machine tools based on parallel structures have been designed, see for example two 5-axis machines in Figures 86 and 87.

- nanotechnology and micro-electromechanical systems; nano-manipulators and micro-manipulators were recently developed based on parallel manipulators (KONG; GOSSELIN, 2007).

For an updated comprehensive list of parallel manipulators and applications, see (MERLET, 2006; BONEV, 2009b; KONG; GOSSELIN, 2007).

Torgny Brogårdh (2002) from ABB Automation Technology Products/Robotics, in his paper entitled “PKM Research - Important Issues, presented as seen from a Product Development Perspective at ABB Robotics” presented in the “Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators”, shows a diagram exemplifying the relations between potential performance features of a parallel manipulators and the industrial applications (see Figure 93).

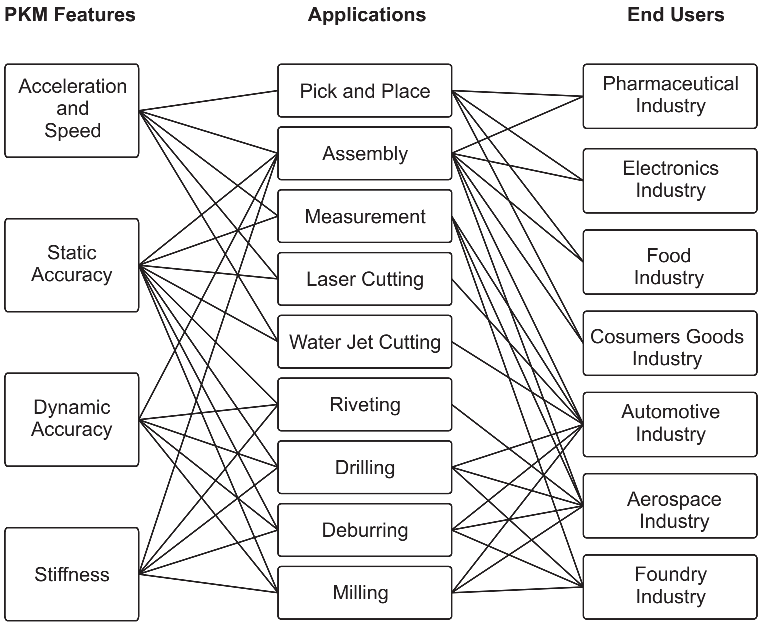


Figure 93 – Diagram exemplifying the relations between potential performance features of a parallel manipulators and the applications and industries needing this performance for improved flexible automation (BROGARDH, 2002).

The potential applications of parallel manipulators continues to motivate their design. However, the difficulties of design must still be overcome and this subject is the main objective of this thesis, to contribute to the conceptual design of mechanisms and parallel manipulators.