

## A Multi-term Boltzmann Equation Analysis of Charged Particle Transport Properties in Electric and Magnetic Fields in Gases

Saša Dujko<sup>(1,2,\*)</sup>, Ronald D. White<sup>(2)</sup>, Zoran Lj. Petrović<sup>(1)</sup>

<sup>(1)</sup> Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Zemun, Belgrade, Serbia

<sup>(2)</sup> ARC Centre for Antimatter-Matter Studies, School of Engineering and Physical Sciences, James Cook University, 4811 Townsville, QLD, Australia

<sup>(\*)</sup> [sasa.dujko@ipb.ac.rs](mailto:sasa.dujko@ipb.ac.rs)

Since the mid to late 1990s the theoretical analysis of charged particle transport processes in neutral gases in the presence of electric and magnetic fields has advanced considerably. For electron swarms in crossed electric and magnetic fields, the advancement was motivated by the desire to overcome the limitations of the two-term approximation for solving the Boltzmann equation and various types of equivalent/effective field approximations often employed to describe the impact of a magnetic field on the transport. The first systematic multi-term analysis for electron swarms under hydrodynamic conditions in the presence of uniform dc electric and magnetic fields was given by Ness [1] and since then a considerable number of papers has been published in a relatively short time. The situation up to 2002 was reviewed by White *et al.* [2] where a unified multi-term theory for solving the Boltzmann equation valid for both electrons and ions in the presence of time-dependent electric and magnetic fields was presented. This theory was recently employed to study the influence of an orthogonal magnetic field on the transient behavior of the diagonal diffusion tensor elements for swarms undergoing conservative collisions only [3]. Since ionization plays a vital role in plasma maintenance any transport theory must include rate coefficients, and correctly account for the effects of non-conservative collisions on drift and diffusion. With these remarks as background, we extend the previous theory [4] and in this work we present a theoretical and numerical investigation of hydrodynamic and non-hydrodynamic charged particle swarms in neutral gases under the influence of dc and ac electric and magnetic fields when non-conservative collisions are operative with applications of non-equilibrium magnetized plasma discharges to plasma processing, gas laser discharges and drift chambers for detection particles in mind.

For charged particle swarms under time-dependent hydrodynamic regime, a multi-term solution of Boltzmann's equation is developed [5]. The angular dependence of the phase space distribution function in velocity space is represented in terms of an expansion in spherical harmonics. No restrictions are placed on the number of spherical harmonics in the polynomial expansion nor on the space and time-dependence of the phase space distribution function. The speed dependence of the phase space distribution function is represented by an expansion in Sonine polynomials about a Maxwellian distribution function using a well-known two-temperature method. By doing so, the Boltzmann equation is decomposed into a hierarchy of coupled kinetic equations for tensorial expansion coefficients.

For time-dependent hydrodynamic regime, the space dependence of the phase space distribution function is represented in terms of powers of density gradient operator. A second order density gradient expansion was required to highlight the explicit modification of transport coefficients about by non-conservative collisional processes of attachment and electron impact ionization. Employing the implicit finite difference scheme for evaluation of the time-derivatives, the Boltzmann equation under conditions of time-dependent hydrodynamic regime is transformed into a hierarchy of doubly infinite coupled inhomogeneous matrix equations for the time-dependent moments. Truncation of both the Sonine polynomials and spherical harmonics results in a sparse system of coupled complex equations. This system of equations is solved using a standard sparse inversion routines.

This theory is employed to consider two situations: (i) temporal relaxation of the electrons in dc electric and magnetic fields crossed at arbitrary angle; and (ii) time-dependent behavior of electron swarms in ac electric and magnetic fields crossed at arbitrary angle and at arbitrary phase difference. Recent studies on the temporal relaxation of electrons in gases performed by Loffhagen and Winkler [6] are extended by overcoming the inherent inaccuracies of the two-term approximation for solving the Boltzmann equation and by addressing the temporal relaxation of spatial inhomogeneities through a study of the diffusion tensor. The relaxation process of various electron transport properties for a range of gases is investigated. Particular emphasis is placed upon the relaxation process of the diffusion coefficients under conditions when electron transport is strongly affected by non-conservative collisions. In the framework of ac studies, the variation of the electron transport coefficients with electric and magnetic field strengths, field frequency, phase difference between the fields and angle between the fields is addressed using physical arguments for certain model and real gases. There are no restrictions on the field amplitudes nor on the frequency of the applied electric and magnetic fields. A multitude of kinetic phenomena were observed that are generally inexplicable through the use of steady-state dc transport theory. Phenomena of significant note include the existence of transient negative diffusivity, time-resolved negative differential conductivity and anomalous anisotropic diffusion. Most notably, a proposed new mechanism for collisional heating in inductively coupled plasmas has emerged from this thesis. It is shown that the synergism of temporal non-locality and cyclotron resonance effect under conditions of time-dependent, high frequency electric and magnetic fields can be used to pump the energy into the swarm. In particular, it is demonstrated that the magnetic field amplitude, phase-difference between the fields and field frequency can be tuned to exploit/control this phenomenon.

Under non-hydrodynamic conditions, however, a density gradient expansion procedure is not valid and the space dependence of the phase space distribution function is retained explicitly throughout the entire decomposition process of the Boltzmann equation. For numerical discretization in configuration space the finite difference scheme and pseudo-spectral method are employed. Boundary conditions are specified for swarms undergoing conservative collisions only under steady-state Townsend conditions and techniques for solving the resulting large system of algebraic complex equations are discussed. The explicit effects of ionization and attachment on the spatially resolved electron transport properties under non-hydrodynamic conditions are investigated by a Monte Carlo simulation technique. In particular, we identify the relations for the conversion of hydrodynamic transport properties to those found in an idealized steady-state Townsend experiment [7]. Our Monte Carlo simulation code and sampling techniques appropriate to these experiments have provided us with a way to test these conversion formulas and their convergence.

## Reference

- [1] K. F. Ness, 1993 Phys. Rev. E 47 327
- [2] R. D. White, K. F. Ness and R. E. Robson, 2002 Appl. Surf. Sci. 192 25
- [3] R. D. White, S. Dujko, K.F. Ness, R. E. Robson, Z. M. Raspopović and Z.Lj. Petrović, 2008 J. Phys. D: Applied Physics 41 025206
- [4] R. D. White, R. E. Robson, S. Dujko, P. Nicoletopoulos and B. Li, 2009 J. Phys. D: Applied Physics 42 194001
- [5] S. Dujko, 2009 Ph.D. Thesis, James Cook University
- [6] D. Loffhagen and R. Winkler, 1999 IEEE Trans. Plasma Sci. 27 1262
- [7] S. Dujko, R. D. White and Z. Lj. Petrović J. Phys. D: Applied Physics 41 245205