非定常磁化力対流（Pr>1）における温度境界層の考察

Scaling of unsteady magnetic convection boundary layer growth for Pr>1 fluids

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The scaling to characterize unsteady boundary layer development for thermo-magnetic convection of paramagnetic fluids with the Prandtl number greater than one is developed. Under the consideration is a square cavity with initially quiescent isothermal fluid placed in microgravity condition (g = 0) and subject to a uniform, vertical gradient magnetic field. A distinct magnetic thermal-boundary layer is produced by sudden imposing of a higher temperature on the vertical sidewall and as an effect of magnetic body force generated on paramagnetic fluid. The transient flow behavior of the resulting boundary layer is shown to be described by three stages: the start-up stage, the transitional stage and the steady state. The scaling is verified by numerical simulations with the magnetic momentum parameter m variation and the parameter γRa variation.
Key Words : Magnetic convection, Scaling analysis, Unsteady flow, Paramagnetic fluids, Thermal boundary layer

1. Introduction

Patterson & Imberger [1] made the pioneering study by using a scaling analysis to predict the transient flow behaviour in rectangular cavity when two opposite vertical sidewalls were heated and cooled simultaneously with the same amount of heating extent. Since then, extensive investigations have been made for many aspects of unsteady natural convection boundary layer flow under various flow configurations by using scaling analysis, numerical simulations and experiments, as recently reviewed by Lin, Armfield & Patterson [2]. In particularly, the scaling analysis has been used to accurately predict the Ra and A dependences of the transient natural convection flow behaviour under various flow configurations [2-3]. Nevertheless, it has also been shown that some of the scalings obtained from the scaling analysis do not perform satisfactorily with the Pr variation. This prompts us in this work to develop improved scalings by taking into account the Pr variation in the scaling analysis to predict the transient magnetic thermal boundary layer growth in microgravity environment, which extends our previous investigations on magnetic convection of paramagnetic fluids [4-5].

2. Governing equations and scaling analysis

The govering equations of motion for paramagnetic electrically non-conducting thermo-fluids subject to a magnetic field in micorgravity environment, together with the energy equation, can be written as follows:

$$\vec{\nabla}\cdot\vec{u} = 0 \tag{1}$$

$$D\vec{u}/Dt = -\rho_0^{-1}\vec{\nabla}p + \nu\nabla^2\vec{u} - (\chi_0 m\beta)/(2\mu_m)\vec{\nabla}b^2 \tag{2}$$

$$DT/Dt = \kappa\nabla^2 T \tag{3}$$

where $\vec{u}$ is the velocity vector, t is the time, p is the pressure, T is the temperature, $\vec{b}$ is the magnetic induction, β, ν and κ are the thermal expansion coefficient, the kinematic viscosity and the thermal diffusivity of the fluid at T_0, respectively, and m is the dimensionless momentum parameter for paramagnetic fluid, $m = 1+1/(\beta T_0)$ [4-5].

Under consideration is the transient flow behaviour resulting from heating of a quiescent isothermal Newtonian fluid in a 2-D open cavity of height H by imposing a fixed higher temperature, T_w, on the left-hand side vertical sidewall, as shown in Fig.1(a). The top and the bottom walls are adiabatic and the right-hand side boundary is open. All solid boundaries are non-slip. It is also assumed that the flow is laminar. Sample temperature contours at dimensionless time τ = 7.0 are shown in Fig.1(b).

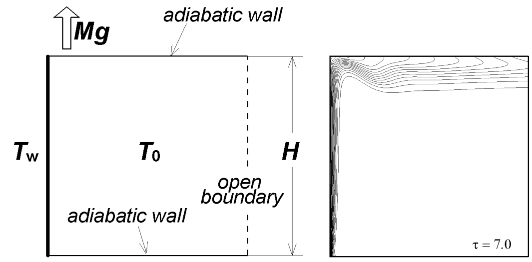

Fig.1 (a) The schematic of the physical system; (b) Simulated temperature contours at Pr = 10, $\gamma Ra = 10^7$, m = 2 and τ = 7.

The fluid is initially at rest and at a uniform temperature T_0 ($T_0 < T_w$). In the presence of a gradient magnetic field, it is the magnetic buoyancy force that acts as the driving force for the resulting natural convection. It is found that the development of the resulting boundary layer consists of three stages: the start-up stage, the transitional stage and the steady state.

Start-up stage

Figure 2 shows a three-region structure for the boundary layers for Pr > 1. As seen, the peak velocity v_m occurs within the thermal boundary layer δ_T at a distance δ_{vm} from the wall. Also, there is a region of flow outside δ_T where there is flow which is not directly forced by buoyancy, but is the result of diffusion of momentum as the result of viscosity. This would occur at distance δ_v from the wall. Therefore, in regions I and II, the balance is between viscosity and buoyancy. However, in region III the balance is between viscosity and inertia, since there is no buoyancy there. Applying these characteristics to general scaling procedures described in [1-3], the following improved scalings are obtained for δ_T and v_m in the start-up stage of the flow development:

$$\delta_T \sim \kappa^{0.5} t^{0.5} \tag{4}$$

$$v_m = (\gamma Ra)\cdot(m\kappa^2/2H^3)\cdot(1+\mathrm{Pr}^{-0.5})^{-2}\, t \tag{5}$$

Steady stage

The boundary layer continues to grow until the time instant t_s when the convection of the heat carried away by the flow will balance the conduction of the heat transferred through the wall:

$$t_s \sim [2^{0.5} H^2 (\gamma Ra\cdot m)^{-0.5}/\kappa]\cdot(y/H)^{0.5}\cdot(1+\mathrm{Pr}^{-0.5}) \tag{6}$$

The temperature and velocity at steady state scale as:

$$\delta_{T,s} \sim [2^{0.25} H (\gamma Ra\cdot m)^{-0.25}]\cdot(y/H)^{0.25}\cdot(1+\mathrm{Pr}^{-0.5})^{0.5} \tag{7}$$

$$v_{m,s} \sim (0.5\cdot\gamma Ra\cdot m)^{0.5}\kappa H^{-1}\cdot(y/H)^{0.5}\cdot(1+\mathrm{Pr}^{-0.5})^{-1} \tag{8}$$

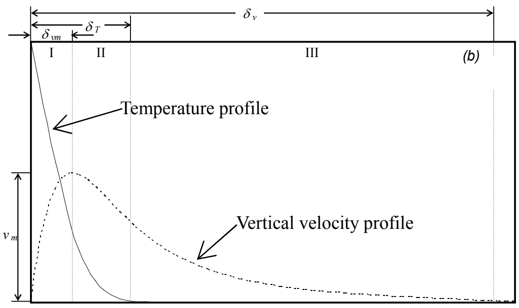

Fig.2 Numerically simulated horizontal profile of vertical velocity and temperature at height Y = 0.5 within the boundary layer at time τ = 0.25, Case 2.

Dimensionless formulation

To facilitate the numerical validation of scalings listed above, the dimensionless forms of governing equations (1)–(3) are used:

$$\vec{\nabla}\cdot\vec{U}=0 \quad (9)$$

$$D\vec{U}/D\tau=-\vec{\nabla}P+\Pr(\gamma Ra)^{-0.5}\nabla^2\vec{U}-0.5m\Pr\theta\cdot\vec{e}_y \quad (10)$$

$$D\theta/D\tau=(\gamma Ra)^{-0.5}\nabla^2\theta \quad (11)$$

For non-dimensionalization H was used as length scale, H/v_0 as time scale, $\kappa(\gamma Ra)^{0.5}/H$ as velocity scale, ρv_0^2 as pressure scale and T_w-T_0 as temperature difference scale.

Hence, during the start-up stage of the boundary layer development, the scaling relations (4) and (5) can be rewritten in dimensionless form as:

$$\Delta_T\sim\tau^{0.5}\cdot(\gamma Ra)^{-0.25} \quad (12)$$

$$V_m=0.5\cdot m\tau\cdot(1+\Pr^{-0.5})^{-2} \quad (13)$$

The dimensionless form of the scalings (6), (7) and (8) for steady states can be rewritten as follows:

$$\tau_s\sim(2/m)^{0.5}\cdot(1+\Pr^{-0.5})Y^{0.5} \quad (14)$$

$$\Delta_{T,s}\sim(\gamma Ra)^{-0.25}(1+\Pr^{-0.5})^{0.5}Y^{0.25} \quad (15)$$

$$V_{m,s}=(2/m)^{0.25}(\gamma Ra)^{-0.25}\cdot(1+\Pr^{-0.5})^{-0.5}Y^{0.25} \quad (16)$$

Cases considered

Table 1 lists values of γRa, Pr, m for 10 DNS, which are used to validate the obtained scalings. Eqs. (9)-(11) are approximated with finite difference equations and the HSMAC method is used to iterate mutually the pressure and velocity fields on staggered mesh/grid allocation system. The inertial tems in momentum equations are approximated with a third-order upwind UTOPIA scheme. The mesh size used in all simulations is 251×251 and to ensure the numerical stability, the time step is fixed to 10^{-5}.

Table 1 Values of γRa, Pr and m for 10 DNS runs.

Run	1	2	3	4	5	6	7	8	9	10
γRa	10^6	10^7	10^8	10^9	10^7	10^7	10^7	10^7	10^7	10^7
Pr	10	10	10	10	5	20	50	100	10	10
m	2	2	2	2	2	2	2	2	1	5

3. Results

To validate the scalings (12), (14), and (15), at first the time series of Δ_T for varying γRa, Pr, m and Y are obtained from the numerical simulations. Δ_T at a specific height is determined as distance from the vertical sidewall to the location where θ, the dimensionless temperature of fluid, becomes 0.01. Fig. 3(a) presents these time series with Δ_T and τ scaled respectively by $(1+Pr^{-0.5})^{0.5}Y^{0.25}/[(\gamma Ra)^{0.25}m^{0.25}]$ and $(1+Pr^{-0.5})Y^{0.5}/m^{0.5}$, which are the scales for $\Delta_{T,s}$ and τ_s at steady states, as shown by the scalings (14) and (15). At the start-up stage (before each series attains its individual peak), it is seen that all ten scaled series with varying γRa, Pr, m and Y fall onto the same straight line, confirming that $\Delta_T\sim\tau^{0.5}/(\gamma Ra)^{0.25}$ is the correct scaling for Δ_T at the start-up stage and Δ_T does not depend on Y. At steady state, these scaled series fall approximately onto the same horizontal straight line, which clearly confirms that $\Delta_{T,s}\sim(1+Pr^{-0.5})^{0.5}Y^{0.25}/[(\gamma Ra)^{0.25}m^{0.25}]$ is the correct scaling for $\Delta_{T,s}$ at steady state. Additionally, Fig. 3(a) shows that all ten scaled series attain their respective peaks almost at the same scaled time with acceptable deviations, which also validates the scaling (14).

Figure 3(b) presents further numerical results to validate the scalings (13), and (16), where the time series of V_m for varying γRa, Pr, m and Y are presented. Fig. 3(b) presents ten time series with V_m and τ scaled respectively by $m^{0.5}Y^{0.5}/(1+Pr^{-0.5})$ and $(1+Pr^{-0.5})Y^{0.5}/m^{0.5}$, which are the scales for $V_{m,s}$ and τ_s at steady state. It is found that all ten scaled time series fall approximately onto the same straight line at the start-up stage, which confirms that $V_m\sim m\tau/(1+\Pr^{-0.5})^2$ is the correct scaling for V_m at the start-up stage. At steady state, it is seen that all scaled time series fall essentially onto the same horizontal straight line, which clearly confirms that $V_{m,s}\sim m^{0.5}Y^{0.5}/(1+Pr^{-0.5})$ is the correct scaling for $V_{m,s}$ at steady state. Additionally, Fig. 3(b) shows that all ten scaled time series attain their respective peaks almost at the same scaled time, which also validates the scaling (14).

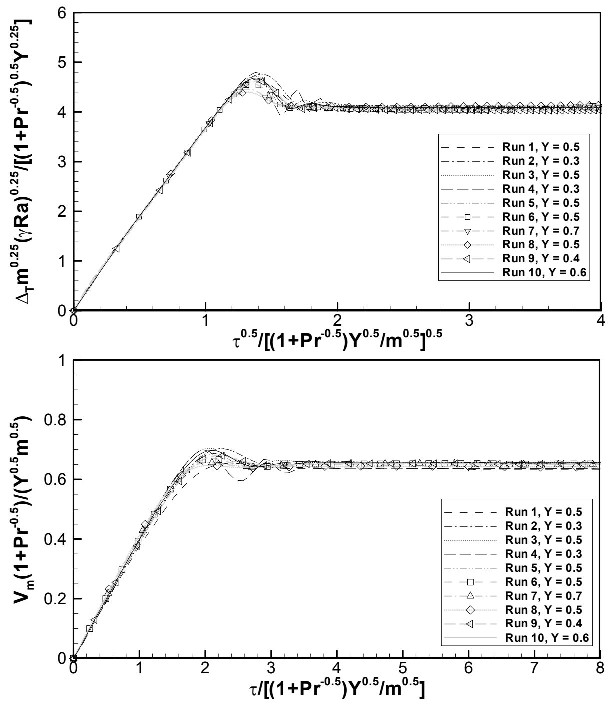

Fig.3 Scaled time series of Δ_T and V_m.

4. Conclusions

Thermo magnetic boundary layer development in 2-D enclosure filled with a paramagnetic fluid of Pr > 1 and subject to a gradient magnetic field is predicted using a scaling analysis. Numerical results demonstrate that the obtained scalings with the three-region structure represent accurately the physical behaviour of the whole stage of flow development with the Pr variation.

References

(1) J.C. Patterson, J. Imberger, J. Fluid Mech. 100 (1980), 65-86.
(2) W. Lin, S.W. Armfield, J.C. Patterson, J. Fluid Mech. 574 (2007), 85-108.
(3) S.W. Armfield, J.C. Patterson, W. Lin, Int. J. Heat and Mass Transfer 50 (2007), 1592-1602.
(4) T. Bednarz, E. Fornalik, H. Ozoe, J.S. Szmyd, J.C. Patterson, C. Lei, Int. J. Thermal Sciences 47 (2008), 668-679.
(5) T. Bednarz, C. Lei, J.C. Patterson, H. Ozoe, Int. J. Thermal Sciences 48 (2009), 26-33.