VU Research Portal

Temporal starvation in multi-channel CSMA networks

Zocca, A.

published in
Performance Evaluation Review
2019

DOI (link to publisher)
10.1145/3308897.3308923

document version
Publisher's PDF, also known as Version of record

document license
Article 25fa Dutch Copyright Act

Temporal starvation in multi-channel CSMA networks: an analytical framework

Alessandro Zocca
Department of Computing and Mathematical Sciences
California Institute of Technology
Pasadena, California
azocca@caltech.edu

ABSTRACT

In this paper we consider a stochastic model for a frequency-agile CSMA protocol for wireless networks where multiple orthogonal frequency channels are available. Even when the possible interference on the different channels is described by different conflict graphs, we show that the network dynamics can be equivalently described as that of a single-channel CSMA algorithm on an appropriate virtual network. Our focus is on the asymptotic regime in which the network nodes try to activate aggressively in order to achieve maximum throughput. Of particular interest is the scenario where the number of available channels is not sufficient for all nodes of the network to be simultaneously active and the well-studied temporal starvation issues of the single-channel CSMA dynamics persist. For most networks we expect that a larger number of available channels should alleviate these temporal starvation issues. However, we prove that the aggregate throughput is a non-increasing function of the number of available channels. To investigate this trade-off that emerges between aggregate throughput and temporal starvation phenomena, we propose an analytical framework to study the transient dynamics of multi-channel CSMA networks by means of first hitting times. Our analysis further reveals that the mixing time of the activity process does not always correctly characterize the temporal starvation in the multi-channel scenario and often leads to pessimistic performance estimates.

Categories and Subject Descriptors

G.3 [**Mathematics of Computing**]: Probability and Statistics—*Stochastic Networks*; G.2.2 [**Mathematics of Computing**]: Discrete Mathematics—*Graph Theory*

Keywords

CSMA algorithm, throughput analysis, Markov chain, hitting times, mixing times

1. INTRODUCTION

The Carrier-Sense Multiple-Access (CSMA) algorithm is a popular distributed medium-access control mechanism and indeed various incarnations of which are currently implemented in IEEE 802.11 WiFi networks. CSMA is a random-access algorithm, in the sense that it relies on randomness to both avoid simultaneous transmissions and share the medium in the most efficient way. Being intrisincally a randomized scheme, many stochastic models have been developed in the literature to study its performance in terms of throughput, stability, delay, and spatial fairness.

It is well-known that the delay performance of CSMA algorithms can be rather poor and much worse than for other mechanisms, such as MaxWeight. One of the root causes for these poor delay performances has been identified in the *temporal starvation phenomenon*: even in scenarios where all nodes have a good long-term throughput, they may individually experience long sequences of transmissions in rapid succession, interspersed with extended periods of starvation. These starvation effects are particularly pronounced when the nodes become more aggressive in trying to activate, which is the regime in which the network should operate to achieve maximum throughput.

Most of the literature focuses on single-channel CSMA algorithms, in which all the transmission occur on the same frequency. In this paper we consider the natural generalization in which multiple orthogonal frequency channels are available; various variations of this model have been studied in the literature. Our ultimate goal is understanding whether temporal starvation effects can be effectively mitigated by the usage of multiple channels. In order to make a fair comparison, we do not assume that additional channels can be added to the existing one, but rather that the total available wireless spectrum could be divided into C non-overlapping channels with smaller capacity on which nearby nodes can transmit without interfering with each other. In this way, at the cost of a potentially lower throughput, the starvation effects in the network can be alleviated or even eliminated and in this way obtain substantial improvements in delay performance. The aim of the present paper is to investigate this non-trivial trade-off and for this reason we analyze the aggregate throughput of multi-channel CSMA networks and develop an analytical framework for quantifying temporal starvation effects for these networks.

2. MULTI-CHANNEL CSMA NETWORK

We consider a network of transmitter-receiver pairs sharing a wireless medium according to a CSMA-type algorithm. A *node* indicates potential data transmission between a transmitter and a receiver. We assume each node can sense the interference and transmit on any of the C available channels, but at most on one at a time. For every $c = 1, \dots, C$ all possible conflicts between nodes on channel c are described by a conflict graph $G_c = (V, E_c)$. Node i has a different back-

IFIP WG 7.3 Performance 2018. Dec. 5–7, 2018, Toulouse, France

off timer for each of the C available channels and we model these timers as C independent Poissonian clocks, ticking at rates $\nu_{i,1}, \dots, \nu_{i,C}$. When the first of these C clocks rings for an inactive node, it activates on the corresponding channel, say c, if and only if the neighboring nodes of i in G_c are not active on the same channel. The transmission times of node i on channel c are independent and exponentially distributed with mean $1/\mu_{i,c}$. A network activity state is described by a vector $x \in \{0, 1, \dots, C\}^N$ where $x_i = 0$ if node i is inactive and $x_i = c$ if node i is active on channel c with $1 \leq c \leq C$. Let $\mathcal{X}_C$ be the collection of feasible network activity states, which consists of all the vectors $x \in \{0, 1, \dots, C\}^N$ such that for every $c = 1, \dots, C$ two neighboring nodes in G_c are not simultaneously active on channel c. If the vector $X_C(t)$ describes the activity state of the network at time t, then $\{X_C(t)\}_{t \geq 0}$ is a Markov process on the state space $\mathcal{X}_C$.

We show that the multi-channel CSMA dynamics can be represented as single-channel CSMA dynamics on an appropriate virtual network, even when the possible interference on the different channels is described by different conflict graphs. This equivalent representation allows us to immediately derive that the multi-channel CSMA network activity process $\{X_C(t)\}_{t \geq 0}$ has the following product-form stationary distribution

$$\pi_C(x) = Z^{-1} \prod_{i=1}^{N} \prod_{c=1}^{C} \left(\frac{\nu_{i,c}}{\mu_{i,c}} \right)^{\mathbb{1}_{\{x_i = c\}}}, \quad x \in \mathcal{X}_C,$$

and to prove its insensitivity to back-off and transmission time distributions.

3. MAIN CONTRIBUTIONS

Even if most of the results generalized to heterogeneous rate, for simplicity we assume homogeneous activation rates $\nu_{i,c} \equiv \nu$ and unit transmission rates $\mu_{i,c} \equiv 1$. We focus on the regime in which the activation rate ν grows large, in which the CSMA dynamics favors the activity states with a maximum number of active nodes, to which we will refer as *dominant states*. These activity states play a crucial role in our analysis, since the timescales at which transitions between them occur are intimately related to the magnitude of the temporal starvation effects.

If the number C of available channels is larger than or equal to the chromatic number of the corresponding conflict graph $\chi(G)$, then every proper C-coloring of the graph G corresponds to a dominant state for the network dynamics and the throughput analysis becomes trivial. Thus, we focus on the most relevant scenario where $C < \chi(G)$, in which there are no activity states where all nodes are simultaneously active. In this scenario all admissible activity states correspond to partial C-colorings of the graph G.

Our contributions in [1] can be summarized as follows.

- Assuming nodes are saturated, we prove various properties of the asymptotic aggregate throughput

$$\Theta(C) := \lim_{\nu \to \infty} \sum_{i \in V} \frac{1}{C} \sum_{x \in \mathcal{X}_C} \pi_C(x) \mathbb{1}_{\{x_i \neq 0\}},$$

  and, in particular, we show that that is a non-increasing function of the number C of available channels.

- We show how the temporal starvation can be evaluated in the high-activation limit $\nu \to \infty$ by studying the expected transition times between the dominant activity states. In particular, we introduce the *starvation index* $\Upsilon(C)$ of the network, that characterizes the timescale of temporal starvation phenomena as

$$\max_{i \in V} \left(\lim_{\nu \to \infty} \log_\nu (\mathbb{E}\tau_i(\nu)) \right) = \Upsilon(C) - 1,$$

  where τ_i is the first hitting time of a dominant state where node i is active starting from the worst possible starting state. The starvation index $\Upsilon(C)$ can be calculated in terms of the structure of the state space $\mathcal{X}_C$ of the Markov process describing the CSMA dynamics. More precisely, it can be obtained solving a max-min problem that involves the *communication height* $\Delta(x, y)$ between any pair of states $x, y \in \mathcal{X}_C$. This quantity is defined the minimum number of nodes (with respect to that of any dominant state) that need to become simultaneously inactive at some point along any path from x to y in $\mathcal{X}_C$, namely

$$\Delta(x, y) := \min_{\omega: x \to y} \max_{z \in \omega} (\mathcal{A}(C) - a(z)), \tag{1}$$

  where $a(x) := \sum_{i=1}^{N} \mathbb{1}_{\{x_i \neq 0\}}$ is the number $a(x)$ of active nodes in x, regardless of the channels they are active on, and $\mathcal{A}(C) := \max_{x \in \mathcal{X}_C} a(x)$ is the maximum number of active nodes in conflict graph G when C channels are available.

- We analyze the mixing time $t_{\mathrm{mix}}(\nu)$ of multi-channel CSMA dynamics using the same framework built to study the transition times between dominant states and derived the following asymptotic inequality

$$\lim_{\nu \to \infty} \log_\nu t_{\mathrm{mix}}(\nu) \geq \Gamma(C) - 1,$$

  where $\Gamma(C) \geq 1$ is the worst communication height between any pair of dominant states. We compare this new index $\Gamma(C)$ with the starvation index of the network and show that

$$\Upsilon(C) \leq \Gamma(C).$$

  These two indices can be equal, but for most conflict graphs and choices of the number C of available channels the inequality is strict, as we show with a counterexample. This fact suggests that in the high-activation regime the mixing time does not always correctly characterize the temporal starvation timescale in multi-channel CSMA networks, often leading to pessimistic performance estimates.

- By means of various counterexamples, we show that many desirable properties and performance indices are not monotone in the number C of available channels, revealing the difficulty of finding analytically the best trade-off between throughput and temporal starvation effects.

Acknowledgments

The author is supported by NWO Rubicon grant 680.50.1529 and is grateful to S.C. Borst and J.S.H. van Leeuwaarden for the precious feedback on the early stages of this work.

4. REFERENCES

[1] A. Zocca, *Temporal starvation in multi-channel CSMA networks: an analytical framework*, arXiv:1808.03602, 2018.