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Chapter 6
Using Multilevel Network Reification to Model Second-Order Adaptive Bonding by Homophily

Abstract The concept of multilevel network reification introduced in the previous chapters enables representation within a network not only of first-order adaptation principles, but also of second-order adaptation principles expressing change of characteristics of first-order adaptation principles. In the current chapter, this approach is illustrated for an adaptive Social Network. This involves a first-order adaptation principle for bonding by homophily represented at the first reification level, and a second-order adaptation principle describing change of characteristics of this first-order adaptation principle, and represented at the second reification level. The second-order adaptation addresses adaptive change of two of the characteristics of the first-order adaptation, specifically similarity tipping point and connection adaptation speed factor.

6.1 Introduction

In Chaps. 4 and 5, it was illustrated how second-order adaptive Mental Network models can be designed using second-order network reification. In the current chapter, it will be shown how to do this for Social Network models. This example addresses the way in which connections between two persons change over time based on the similarities or differences between the persons. This concerns the bonding-by-homophily adaptation principle as a first-order adaptation principle, that can be represented at the first reification level of a multilevel reification architecture; it is also explained as 'birds of a feather flock together'; see also (Byrne 1986; McPherson et al. 2001; Pearson et al. 2006). In this first-order adaptation principle, there is an important role for the *homophily similarity tipping point* τ. This indicates the value such that

- when the dissimilarity between two persons is less than this value, their connection will become stronger
- when the dissimilarity is more, their connection will become weaker.

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Earlier work (Sharpanskykh and Treur 2014; Holme and Newman 2006; Vazquez 2013; Vazquez et al. 2007) and (Treur 2016), Chap. 11, addressed the interaction (co-evolution) of social contagion (Levy and Nail 1993) and bonding. Such tipping points were usually considered constant, but this may not be realistic. For example, someone who already has many strong connections perhaps will be much more critical in strengthening connections than someone who has only a very few and only very weak connections. Such differences can be modeled when the tipping point value is modeled adaptively, depending on a person's circumstances, for example, on how many (and how strong) connections are already there. This makes the first-order adaptation principle of bonding based on homophily itself adaptive, where the tipping point changes over time by a second-order adaptation principle. This second-order adaptation principle can be represented at the second reification level in the multilevel reification architecture. Yet, another factor that may better be modeled adaptively is the speed of change of connections. Also, this may depend on how many (and how strong) connections someone has at some point in time. In the multilevel reified network model described here, also a second-order adaptation principle is included for the speed factor of the connection weight adaptation based on the first-order adaptation principle for bonding by homophily.

6.2 Conceptual Representation of the Second-Order Adaptive Social Network Model

In the second-order reified network model introduced here, the two main adaptation principles addressed are a first-order adaptation principle on bonding by homophily based on a tipping point τ, and one second-order adaptation principle for adaptation of this tipping point τ based on the extent of available connections and a norm $\mathbf{v}$ for this, and another second-order adaptation principle for the first-order adaptation speed. For an overview of the states and their levels, see Table 6.1.

First-order adaptation principle

- when the dissimilarity between two persons is less than tipping point τ, their connection will become stronger
- when the dissimilarity is more than tipping point τ, their connection will become weaker.

Second-order adaptation principle 1

- when for a person the number and strength of the connections is less than norm value $\mathbf{v}$, the tipping point will become higher
- when the number and strength of the total number of connections is more than norm value $\mathbf{v}$, the tipping point will become lower.

This second-order adaptation principle will have as effect that in the end, a person will have a number and strength of connections according to this norm value $\mathbf{v}$. Yet

Table 6.1 State names with their explanations

State		Explanation	Level
X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8 X_9 X_{10}		The 10 members of the example Social Network	Base level
X_{11} .. .. X_{100}	$\mathbf{W}_{X_2,X_1}$ .. .. $\mathbf{W}_{X_9,X_{10}}$	Reified representation states for the adaptive weights of the connections from member X_i to member X_j where $i \neq j$	First reification level
X_{101} .. .. X_{190}	$\mathbf{TPW}_{X_2,X_1}$ .. .. $\mathbf{TPW}_{X_9,X_{10}}$	Reified representation states for the adaptive tipping points for the reified representation states $\mathbf{W}_{X_i,X_j}$ of the adaptive connections from member X_i to member X_j	Second reification level
X_{191} .. .. X_{280}	$\mathbf{HW}_{X_2,X_1}$ .. .. $\mathbf{HW}_{X_9,X_{10}}$	Reified representation states for the adaptive speed factors for the reified representation states $\mathbf{W}_{X_i,X_j}$ of the adaptive weights of the connections from member X_i to member X_j	

another second-order adaptation principle is considered which makes the speed of change of the connections dependent on how many and how strong contacts there already are.

Second-order adaptation principle 2

- when a person has more and stronger connections than norm value $\mathbf{v}$, the speed of change for the connections will become lower
- when a person has less and weaker connections than norm value $\mathbf{v}$, the speed of change for the connections will become higher.

Note that for a model phrases such as 'more and stronger connections' still have to be made more precise. The 'more' can be related in some way to a number of connections, and the 'stronger' can be related to (average) connection weights.

Recall that a temporal-causal network model represents in a declarative manner states and connections between them that represent (causal) impacts of states on each other, as assumed to hold for the application domain addressed. The states are assumed to have (activation) levels in the interval [0, 1] that vary over time. The following three notions form the defining part of a conceptual representation of a temporal-causal network model structure; they apply both to the base network and the added reification states:

- **Connectivity**
  - Each incoming connection of a state Y, from a state X has a *connection weight value* $\boldsymbol{\omega}_{X,Y}$ representing the strength of the connection.

- **Aggregation**
  - For each state a *combination function* $\mathbf{c}_Y(..)$ is chosen to combine the causal impacts state Y receives from other states.
- **Timing**
  - For each state Y a *speed factor* $\boldsymbol{\eta}_Y$ is used to represent how fast state Y is changing upon causal impact.

The conceptual graphical representation of the multilevel reified network model is shown in Fig. 6.1. The following reification states are included.

6.2.1 Reification States at the First Reification Level

The middle (blue) plane shows how the reification states $\mathbf{W}_{X_i,Y}$ are used for the first-order reification of the connection weights $\boldsymbol{\omega}_{X_i,Y}$. The downward arrows show the network relations of these reification states $\mathbf{W}_{X_i,Y}$ to the states X_i and Y in the base network. Such network relations (including their labels, such as combination functions; see below) for reification states define the first-order adaptation principle for bonding by homophily. For this example, the speed factors and the combination functions for all base level states Y are considered constant; therefore speed factor

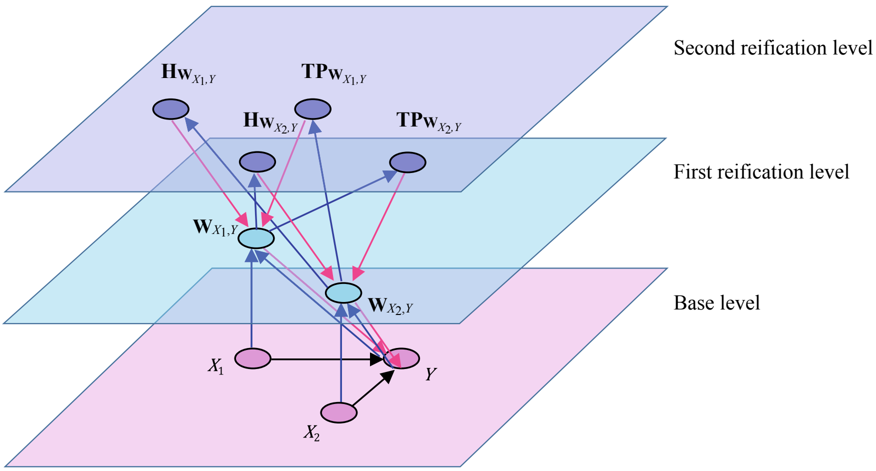

Fig. 6.1 Multilevel Reified Network model picture of a fragment of a second-order adaptive Social Network model based on Homophily. At the first reification level (middle, blue plane) the reification states $\mathbf{W}_{X_1,Y}$ and $\mathbf{W}_{X_2,Y}$ represent the adaptive connection weights for Y; they are changing based on a homophily principle. At the second reification level (upper, purple plane) the reified tipping point states $\mathbf{TP}_{\mathbf{W}_{X_1,Y}}$ and $\mathbf{TP}_{\mathbf{W}_{X_2,Y}}$ represent the adaptive tipping point values for the second-order connection adaptation based on homophily; similarly, the second-order **H**-states address the first-order adaptation speed

reification states $\mathbf{H}_Y$ and combination function reification states $\mathbf{C}_{i,Y}$ for base level states Y are not shown in the role matrices and the simulation graphs.

6.2.2 Reification States at the Second Reification Level

On top of the first-order reified network, a second reification level has been added (the upper, purple plane), in order to define second-order adaptation principles. The following reification states are added:

- reification states $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$ for the similarity tipping point parameter of the homophily adaptation principle for the connection weight reified by state $\mathbf{W}_{X_i,Y}$
- reification states $\mathbf{H}_{\mathbf{W}_{X_i,Y}}$ for the speed factor characteristic of the homophily adaptation principle.

Note that a double level subscript notation for second-order reification states such as $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$ should be read as tipping point for state Y, i.e., $\mathbf{TP}_Y$, for a state Y at the first reification level, in this case, $Y = \mathbf{W}_{X_i,Y}$. By substituting $\mathbf{W}_{X_i,Y}$ for Y in $\mathbf{TP}_Y$, this results in the double level subscript notation $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$; note that here for the sake of simplicity the index i of X_i is considered to be at the same subscript level as X. So, the subscript of $\mathbf{TP}$ is $\mathbf{W}_{X_i,Y}$ and this subscript itself has subscripts X_i and Y; the notation should be interpreted as $\mathbf{TP}_{(\mathbf{W}_{X_i,Y})}$. In this way, the number of reification levels is reflected in the number of subscript levels. This applies to all states at the second reification level, so, for example, also to $\mathbf{H}_{\mathbf{W}_{X_i,Y}}$. Up till now no cases of network adaptation of order higher than 2 have been explored in this book, so that more than 2 subscript levels did not show up. However, from a modeling perspective, there is nothing against adding a third reification level for the characteristics that define the second-order adaptation principles based on $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$ and $\mathbf{H}_{\mathbf{W}_{X_i,Y}}$. An example in the current context can be the addition of third-order reification states for their speed factors or their combination functions, or the parameters of these functions. In Chaps. 7 and 8 other examples of order higher than two will be shown.

Also for the above second-order reification states (upward and downward), connections have been added. These connections (together with the relevant combination functions discussed in Sect. 6.3 below) define the second-order adaptation principles based on them.

After having defined the overall conceptual representation of the reified network model, the combination functions for the new network are defined. Note that for the example simulations, speed factors and combination functions for all base states Y are considered constant; therefore speed factor reification states $\mathbf{H}_Y$ and combination function reification states $\mathbf{C}_{i,Y}$ for base states Y are not shown in the role matrices in Sect. 6.4 and in the simulation graphs in Sect. 6.5. However, for the purpose of illustration, in the general model described in Sect. 6.3, speed factor reification states $\mathbf{H}_{\mathbf{W}_{X_i,Y}}$ for the connection weight reification states $\mathbf{W}_{X_i,Y}$ are still also discussed at a general level.

6.3 Combination Functions Used at the Three Levels

In this section, the combination functions for the three levels are described. Note that all these illustrate the important role of these declarative mathematical functions:

(a) These declarative mathematical functions are powerful building blocks to define a network's structure, and this network structure determines the network's dynamics in a well-defined and unique manner
(b) By applying network reification this also covers networks that are adaptive of any order; all adaptation principles of any order are also specified just using declarative mathematical functions as building blocks.

Note that here the term declarative means that no algorithmic or procedural elements are involved in their specification, they just relate real numbers to real numbers in a time or process independent manner; they may be considered relational specifications that are functional.

A few mathematical details for the explanation below are shown in Box 6.1 (for the base level and the first reification level) and in Boxes 6.2 and 6.3 (for the second reification level). Note that the specific difference or differential equations as used in the software environment are not discussed in Chaps. 1–9. More details about them can be found in Chap. 15, Sect. 15.5, and in Chap. 10 for the universal difference and differential equation.

6.3.1 *Base Level and First Reification Level Combination Functions*

Base level combination functions (the lower, pink plane)

For this level, the combination functions for base states Y are chosen as the advanced logistic sum function $\mathbf{alogistic}_{\sigma,\tau}(\ldots)$; together with the adaptive connection weights this obtains the combination functions shown in Box 6.1.

First reification level combination functions (the middle, blue plane)

At this first reification level, the combination function for the homophily adaptation principle (at the middle plane) is needed, as shown in Box 6.1. As a measure of dissimilarity $|V_1 - V_2|$ is used. This makes that an increase or decrease in connection weight will depend on whether $\tau - |V_1 - V_2|$ is positive (less dissimilarity than τ) or negative (more dissimilarity than τ). Here τ is reified by $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$ at the second reification level. So, when the difference in states $|X_i(t) - Y(t)|$ of two persons is less than the tipping point $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}(t)$, increase of $\mathbf{W}_{X_i,Y}$ will take place, and when this difference $|X_i(t) - Y(t)|$ is more than $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}(t)$, decrease of $\mathbf{W}_{X_i,Y}$ will take place.

Box 6.1 Combination functions for the base level and first reification level
Base level (lower, pink plane)
Combination function for each of the base states X_i is the advanced logistic sum function $\mathbf{alogistic}_{\sigma,\tau}(..)$
First reification level (middle, blue plane)
See Sects. 4.3.2, and 4.2, or Chap. 3, Sect. 3.6.1, or (Treur 2016), Chap 11, Sect. 11.7, Combination function for connection weight reification state $\mathbf{W}_{X_i,Y}$ is $\mathbf{slhomo}_{\alpha}(..)$:

$$\mathbf{slhomo}_{\alpha}(V_1, V_2, W) = W + \alpha\, W(1 - W)\,(\tau - |V_1 - V_2|)$$

where for the variables used to define this function

- W refers to connection weight reification state $\mathbf{W}_{X_i,Y}(t)$
- V_1 refers to $X_i(t)$ and V_2 to $Y(t)$
- α is a homophily modulation factor for $\mathbf{W}_{X_i,Y}$
- τ is a homophily tipping point for $\mathbf{W}_{X_i,Y}$, which actually is reified by $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$ at the second reification level.

6.3.2 Second Reification Level Combination Functions

At this level, it is defined how the tipping points should be adapted according to circumstances, and similarly the first-order adaptation speed. The principle is used that the tipping point of a person will become higher if the person lacks strong connections (the person becomes less strict) and will become lower if the person already has strong connections (the person becomes more strict). This is handled using an *average norm weight* $\mathbf{v}$ for connections. This can be considered to relate to the amount of time or energy available for social contacts but also the desired extent of social contacts. So the effect is:

- if the connections of a person are (on average) stronger than $\mathbf{v}$, downward regulation takes place: the tipping point will become lower
- when the connections of this person are (on average) weaker than $\mathbf{v}$, upward regulation takes place: the tipping point will become higher.

This is expressed in the combination function for the homophily tipping point reification state $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$, a function called *simple linear tipping point function* $\mathbf{sltip}_{\mathbf{v},\alpha}(..)$, for the second reification level shown in Box 6.2. As an alternative, not the average but total cumulative weight of connections could be used. Similarly, a combination function $\mathbf{slspeed}_{\mathbf{v},\alpha}(..)$ for *simple linear speed function* is found for the speed factor reification state $\mathbf{H}_{\mathbf{W}_{X_i,Y}}$, as shown in Box 6.3. Note that the combination functions for the first- and second-order reification states all include an argument for their current value. Therefore, in role matrix **mb**, for each state at the first or second reification level, the state itself is included in the row, as can be seen in Box 6.4.

Box 6.2 Combination function for the homophily tipping point reification state at the second reification level

The following combination function called *simple linear tipping point function* **sltip**$_{\mathbf{v},\boldsymbol{\alpha}}$**(..)** can be used for the second-order reification state $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$ at the second reification level (upper, purple plane):

$$\mathbf{sltip}_{\mathbf{v},\boldsymbol{\alpha}}(W_1,\ldots,W_k,T) = T + \boldsymbol{\alpha}\, T(1-T)(\mathbf{v} - (W_1 + \ldots + W_k)/k)$$

where for the variables used to define this function

- T refers to the homophily tipping point reification value $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}(t)$ for $\mathbf{W}_{X_i,Y}$
- W_j to connection weight reification value $\mathbf{W}_{X_i,,Y}(t)$
- $\boldsymbol{\alpha}$ is a modulation factor for the tipping point $\mathbf{TP}_{\mathbf{W}_{X_i,Y}}$
- $\mathbf{v}$ is a norm for Y for average (incoming) connection weight from the $X_{i.}$

This function can be explained as follows. The norm parameter **v** indicates the preferred average level of the connection weights $\mathbf{W}_{X_i,Y}$ for person Y. The part ($\mathbf{v}$ − $(W_1 + \ldots + W_k)/k$) in the formula is positive when the current average connection weight $(W_1 + \ldots + W_k)/k$ is lower than this norm, and negative when it is higher than the norm. When T is not 0 or 1, in the first case, the combination function provides a value higher than T, which makes that the tipping point value T is increased, and therefore more connections are strengthened by the homophily adaptation. So, in this case, the average connection weight will become more close to the norm **v**. In the second case, the opposite takes place: the combination function provides a value lower than T, which makes that the tipping point value T is decreased, and as a consequence, more connections are weakened by the homophily adaptation. So also now the average connection weight will become more close to the norm **v**. Together this makes that in principle (unless in the meantime other factors change) the average connection weight will approximate the norm **v**. The factor $T(1-T)$ in the formula takes care that the tipping point values T stay within the [0, 1] interval.

As an alternative, note that as a slightly different variant the division by k can be left out. Then the norm does not concern the average but the cumulative connection weights; also a logistic sum function could be used here.

Box 6.3 Combination function for the connection weight speed factor reification state at the second reification level
For the adaptive speed factor for the connection weight adaptation, the following combination function called *simple linear speed function* **slspeed**$_{\nu,\alpha}$**(..)** can be considered; it makes use of a similar mechanism using a norm for connection weights:

$$\textbf{slspeed}_{\nu,\alpha}(W_1,\ldots,W_k,H) = H + \alpha H(1-H)(\nu - (W_1 + \ldots + W_k)/k)$$

where for the variables used to define this function

- H refers to $\mathbf{W}_{X_i,Y}$ speed factor reification value $\mathbf{H}_{\mathbf{W}_{X_i,Y}}(t)$
- W_j to connection weight reification value $\mathbf{W}_{X_i,Y}(t)$
- α is a modulation factor for $\mathbf{H}_{\mathbf{W}_{X_i,Y}}$
- ν is a norm for an average of (outgoing) connection weights for Y.

This function can be explained as follows. Also here the norm parameter ν indicates the preferred average level of the connection weights $\mathbf{W}_{X_i,Y}$ for person Y. The part ($\nu - (W_1+ \ldots + W_k)/k$) in the formula is positive when the current average connection weight $(W_1+ \ldots + W_k)/k$ is lower than this norm, and negative when it is higher than the norm. When H is not 0 or 1, in the first case, the combination function provides a value higher than H, thus the speed factor is increased, and the connection weights are changing faster by the homophily adaptation. In the second case, the combination function provides a value lower than H; this decrease in the speed factor value makes the homophily adaptation slower. The factor $H(1-H)$ in the formula takes care that the values for H stay within the [0, 1] interval.

6.4 Role Matrices for the Reified Social Network Model

The role matrices are shown in Boxes 6.4 and 6.5. The specific numbers shown here relate, in particular, to Scenario 3 described in Sect. 6.5. Note that in the simulated scenarios a relatively large number of states was taken into account:

- 10 base level states for the base Social Network
- 90 first-order reification states for the first-order adaptation principle for bonding based on homophily for all of the base level connections
- 90 second-order reification states for the second-order adaptive tipping point adaptation principle for the tipping point parameter used in the first-order adaptation principle for bonding based on homophily.

For reasons of space limitations, not all these states have been written down explicitly in the role matrices here. For the same reason the second-order **H**-states have been left out here. First, in Box 6.4 role matrices **mb** and **mcw** are shown. In Box 6.5 the other three role matrices are shown, and the initial values.

Box 6.4 Role matrices **mb** for connected base states (=states of same or lower level with outgoing connections to the given state) and **mcw** for connection weights of these connections

mb	**base connectivity**	1	2	3	4	5	6	7	8	9	10
X_1		X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
X_2		X_1	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
X_3		X_1	X_2	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
X_4		X_1	X_2	X_3	X_5	X_6	X_7	X_8	X_9	X_{10}	
X_5		X_1	X_2	X_3	X_4	X_6	X_7	X_8	X_9	X_{10}	
X_6		X_1	X_2	X_3	X_4	X_5	X_7	X_8	X_9	X_{10}	
X_7		X_1	X_2	X_3	X_4	X_5	X_6	X_8	X_9	X_{10}	
X_8		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_9	X_{10}	
X_9		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_{10}	
X_{10}		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	
X_{11}	$\mathbf{W}_{X_2,X_1}$	X_1	X_2	X_{11}							
..	..	..	..	..							
..	..	..	..	..							
X_{100}	$\mathbf{W}_{X_9,X_{10}}$	X_{10}	X_9	X_{100}							
X_{101}	$\mathbf{TPW}_{X_2,X_1}$	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{101}
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
X_{190}	$\mathbf{TPW}_{X_9,X_{10}}$	X_{92}	X_{93}	X_{94}	X_{95}	X_{96}	X_{97}	X_{98}	X_{99}	X_{100}	X_{190}
X_{191}	$\mathbf{HW}_{X_2,X_1}$	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{191}
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
X_{280}	$\mathbf{HW}_{X_9,X_{10}}$	X_{92}	X_{93}	X_{94}	X_{95}	X_{96}	X_{97}	X_{98}	X_{99}	X_{100}	X_{280}

mcw	**connection weights**	1	2	3	4	5	6	7	8	9	10
X_1		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	
X_2		X_{20}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	
X_3		X_{29}	X_{30}	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}	
X_4		X_{38}	X_{39}	X_{40}	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	
X_5		X_{47}	X_{48}	X_{49}	X_{50}	X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	
X_6		X_{56}	X_{57}	X_{58}	X_{59}	X_{60}	X_{61}	X_{62}	X_{63}	X_{64}	
X_7		X_{65}	X_{66}	X_{67}	X_{68}	X_{69}	X_{70}	X_{71}	X_{72}	X_{73}	
X_8		X_{74}	X_{75}	X_{76}	X_{77}	X_{78}	X_{79}	X_{80}	X_{81}	X_{82}	
X_9		X_{83}	X_{84}	X_{85}	X_{86}	X_{87}	X_{88}	X_{89}	X_{90}	X_{91}	
X_{10}		X_{92}	X_{93}	X_{94}	X_{95}	X_{96}	X_{97}	X_{98}	X_{99}	X_{100}	
X_{11}	$\mathbf{W}_{X_2,X_1}$	1	1	1							
..	..	..	..	..							
..	..	..	..	..							
X_{100}	$\mathbf{W}_{X_9,X_{10}}$	1	1	1							
X_{101}	$\mathbf{TPW}_{X_2,X_1}$	1	1	1	1	1	1	1	1	1	1
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
X_{190}	$\mathbf{TPW}_{X_9,X_{10}}$	1	1	1	1	1	1	1	1	1	1
X_{191}	$\mathbf{HW}_{X_2,X_1}$	1	1	1	1	1	1	1	1	1	1
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
X_{280}	$\mathbf{HW}_{X_9,X_{10}}$	1	1	1	1	1	1	1	1	1	1

Box 6.5 Role matrices **mcfw** and **mcfp** for combination function weights and combination function parameters, and for speed factors, and the initial values

mcfw	combination function weights	1 alogistic	2 slhomo	3 sltip	4 slspeed
X_1		1			
X_2		1			
X_3		1			
X_4		1			
X_5		1			
X_6		1			
X_7		1			
X_8		1			
X_9		1			
X_{10}		1			
X_{11}	$\mathbf{W}_{X_2,X_1}$		1		
..	..		..		
..	..		..		
X_{100}	$\mathbf{W}_{X_9,X_{10}}$		1		
X_{101}	$\mathbf{TPW}_{X_2,X_1}$			1	
..	..			..	
..	..			..	
X_{190}	$\mathbf{TPW}_{X_9,X_{10}}$			1	
X_{191}	$\mathbf{HW}_{X_2,X_1}$				1
..	..				..
..	..				..
X_{280}	$\mathbf{HW}_{X_9,X_{10}}$				1

ms	speed factors	1
X_1		0.5
X_2		0.5
X_3		0.5
X_4		0.5
X_5		0.5
X_6		0.5
X_7		0.5
X_8		0.5
X_9		0.5
X_{10}		0.5
X_{11}	$\mathbf{W}_{X_2,X_1}$	X_{191}
..	..	..
..	..	..
X_{100}	$\mathbf{W}_{X_9,X_{10}}$	X_{280}
X_{101}	$\mathbf{TPW}_{X_2,X_1}$	1
..	..	..
..	..	..
X_{190}	$\mathbf{TPW}_{X_9,X_{10}}$	1
X_{191}	$\mathbf{HW}_{X_2,X_1}$	1
..	..	..
..	..	..
X_{280}	$\mathbf{HW}_{X_9,X_{10}}$	1

mcfp	function	1 alogistic		2 slhomo		3 sltip		3 slspeed	
	parameter	1	2	1	2	1	2	1	2
		σ	τ_{log}	α_{homo}	τ_{homo}	α_{tip}	τ_{tip}	α_{speed}	τ_{speed}
X_1		1	1.5						
X_2		1	1.5						
X_3		1	1.5						
X_4		1	1.5						
X_5		1	1.5						
X_6		1	1.5						
X_7		1	1.5						
X_8		1	1.5						
X_9		1	1.5						
X_{10}		1	1.5						
X_{11}	$\mathbf{W}_{X_2,X_1}$			1	X_{101}				
..	..			..	..				
..	..			..	..				
X_{100}	$\mathbf{W}_{X_9,X_{10}}$			1	X_{190}				
X_{101}	$\mathbf{TPW}_{X_2,X_1}$					0.1	0.4		
..	..					..	..		
..	..					..	..		
X_{190}	$\mathbf{TPW}_{X_9,X_{10}}$					0.1	0.4		
X_{191}	$\mathbf{HW}_{X_2,X_1}$							1	0.4
..	..							..	..
..	..							..	..
X_{280}	$\mathbf{HW}_{X_9,X_{10}}$							1	0.4

iv	initial values	1
X_1		0.8
X_2		0.9
X_3		0.5
X_4		0.6
X_5		0.6
X_6		0.6
X_7		0.7
X_8		0.5
X_9		0.6
X_{10}		0.8
X_{11}	$\mathbf{W}_{X_2,X_1}$	0.5
..	..	..
..	..	..
X_{100}	$\mathbf{W}_{X_9,X_{10}}$	0.6
X_{101}	$\mathbf{TPW}_{X_2,X_1}$	0.4
..	..	..
..	..	..
X_{190}	$\mathbf{TPW}_{X_9,X_{10}}$	0.55
X_{191}	$\mathbf{HW}_{X_2,X_1}$	0.6
..	..	..
..	..	..
X_{280}	$\mathbf{HW}_{X_9,X_{10}}$	0.4

For role matrix **mcw** (see Box 6.4), note again, as in the previous chapters, that there are two types of cells here: those for constant values (in green) as also in the role matrices discussed in Chap. 2, but here it can be seen that at the base level in all cells adaptive connection weights are indicated (in peach colour), which means that all weights for all 90 connections at the base level are adaptive. In these cells, there are no values written but only names X_k of other states one reification level higher: the names X_{11} to X_{100} of the reification states $\mathbf{W}_{X_iX_j}$ for these connection weights. These refer to the first-order adaptation principle for bonding by homophily, which is further specified by the characteristics for the 90 reification states $\mathbf{W}_{X_iX_j}$ (also indicated by X_{11} to X_{100}): characteristics like their speed factors, their incoming connections, and their combination functions and the parameters thereof.

The role matrices **ms** for the speed factors and **mcfw** for the combination function weights (in Box 6.5) only contain values (in green cells) as all of them are non-adaptive. But in role matrix **mcfp** (also in Box 6.5) there are peach-coloured cells that indicate that the tipping point parameters for the first-order bonding by homophily adaptation principle for all base connections are adaptive. So, for them the names X_k of the corresponding reification states $\mathbf{TP}_{\mathrm{W}_{X_iX_j}}$ at one level higher are indicated (in red cells). This refers to the second-order adaptation principle, which is further specified by the characteristics of the reification states $\mathbf{TP}_{\mathrm{W}_{X_iX_j}}$ for these tipping point parameters (their speed factors, their incoming connections, and their combination functions and the parameters thereof). The initial values (also shown in Box 6.5) were varying from 0.1 to 0.9 for the reification states $\mathbf{W}_{X_iX_j}$ for the nonzero base connection weights and from 0.15 to 0.7 for the 90 reification states $\mathbf{TP}_{\mathrm{W}_{X_iX_j}}$ for the tipping points. For specific initial values for the scenarios, see also below, in Sect. 6.5, Tables 6.2 and 6.4.

6.5 Simulation of the Social Network Model for Adaptive Bonding by Homophily

The example simulation scenarios introduced in Sects. 6.5.1–6.5.3 concern adaptive social network scenarios with 10 persons X_1 to X_{10}. The first two scenarios address a case in which only the outgoing connections of X_1 are adaptive, the other connection weights are kept constant. For all simulations, $\Delta t = 1$ was used, and the focus in all three scenarios was on the homophily adaptation with constant connection weight speed factor $\mathbf{H}_{\mathbf{W}_{X_j,X_i}} = \boldsymbol{\eta}_{\mathbf{W}_{X_j,X_i}} = 1$. In Table 6.2 the main parameter values for Scenarios 1 and 2 can be found. In Table 6.3 the initial connection weights and tipping points for Scenario 1 and 2 are shown. Note that when connection weights are 0 or 1, they will not change due to the specific combination function for bonding by homophily chosen (see also the analysis in Sect. 6.6).

Table 6.2 Main parameter values for Scenario 1/Scenario 2

Base level		First reification level		Second reification level	
Steepness σ for **alogistic(..)** for X_i	1	Homophily modulation factor α for $\mathbf{W}_{X_1,X_i}$	1	Tipping point speed factors η for $\mathbf{TP}_{\mathbf{W}_{X_1,X_i}}$	1
Threshold τ for **alogistic(..)** for X_i	1.5	Connection weight speed factor η for $\mathbf{W}_{X_1,X_i}$	1	Tipping point modulation factors α for $\mathbf{TP}_{\mathbf{W}_{X_1,X_i}}$	0.1/ 0.9
Speed factor η for base state X_i	0.5			Tipping point connection norms ν for $\mathbf{TP}_{\mathbf{W}_{X_1,X_i}}$	0.6

Table 6.3 Scenarios 1 and 2: Initial values for connection weights and tipping points

connections	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_1		0.5	0.3	0.1	0.2	0.6	0.5	0.2	0.3	0.4
X_2	0.5		0.6	0.3	0.4	0.7	0.7	0.9	0.5	
X_3	0.3	0.6		0.7		0.4	0.4		0.6	0.8
X_4	0.6	0.4	0.6		0.4	0.6	0.7	0.8		
X_5	0.2	0.5		0.7	0.6	0.4			0.9	
X_6	0.6	0.6	0.7	0.5			0.7	0.7	0.5	0.7
X_7	0.2	0.8	0.6	0.7	0.6	0.7		0.7		
X_8	0.6	0.5				0.6	0.5		0.4	0.5
X_9	0.6		0.6	0.7	0.4		0.7			0.6
X_{10}	0.6	0.7		0.7	0.4	0.6		0.8		

	$\mathbf{TP}_{\mathbf{W}_{X_1,X_2}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_3}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_4}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_5}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_6}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_7}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_8}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_9}}$	$\mathbf{TP}_{\mathbf{W}_{X_1,X_{10}}}$
$\mathbf{TP}_{\mathbf{W}_{X_1,X_i}}(0)$	0.4	0.35	0.5	0.65	0.2	0.3	0.25	0.55	0.6

So, the connections which have weight 0 initially, will keep weight 0 forever. In Chap. 13, various other combination functions for bonding by homophily are explored, among which a class of them which do allow connections with weight 0 or 1 to change.

6.5.1 Scenario 1: Adaptive Connections for One Person; Tipping Point Modulation Factor 0.1

For this scenario (with modulation factor α = 0.1), the initial values for connection weights and tipping points can be found in Table 6.2. The average of the initial values of $\mathbf{W}_{X_1,X_i}$ is 0.344, which is below the norm $\mathbf{v}$ which is 0.6. The example simulation for this scenario shown in Figs. 6.2, 6.3, 6.4, 6.5 and 6.6 may look a bit chaotic where some connections seem to meander between high and low. However, in this scenario, it can be seen that the average connection weight, indicated by the pink line converges to 0.60145 (at time point 1750), which is close to 0.6, which was chosen as the norm $\mathbf{v}$ for the average connection weight. So at least this convergence of the average connection weight to $\mathbf{v}$ makes sense. As can be seen in Figs. 6.2 and 6.3 there is some variation of the connection weights around the average connection weight 0.60145 at time 1750. Note that the connection weights at time 1750 do not correlate to the initial values for the connections weights; they are determined by the similarity in states via the homophily principle (see Fig. 6.3). With all of these 9 persons, X_1 initially developed very strong connections (above 0.97) around time 50, but that turned out too much. Therefore 6 of the 9 were reduced between time 100 and 500, while 3 stayed high all the time: $\mathbf{W}_{X_1,X_3}$, $\mathbf{W}_{X_1,X_5}$ and $\mathbf{W}_{X_1,X_9}$. Two of these 6 stayed very low (staying at or around 0): $\mathbf{W}_{X_1,X_8}$ and $\mathbf{W}_{X_1,X_{10}}$.

As with six very low connections, this made the average of connections too low, from these six, three were increased after time 750, and the fourth one after time 1000. Eventually, two of them, $\mathbf{W}_{X_1,X_2}$ and $\mathbf{W}_{X_1,X_7}$, are around 0.6, one, $\mathbf{W}_{X_1,X_4}$, is around 0.8, and one, $\mathbf{W}_{X_1,X_6}$, is around 0.35. So what has emerged is that the person eventually has developed and kept three very good contacts with X_3, X_5, and X_9, has lost two contacts with X_8, X_{10}, and has kept the other four contacts with an intermediate type of different strengths. Figure 6.4 shows the variation in tipping point reification states over time.

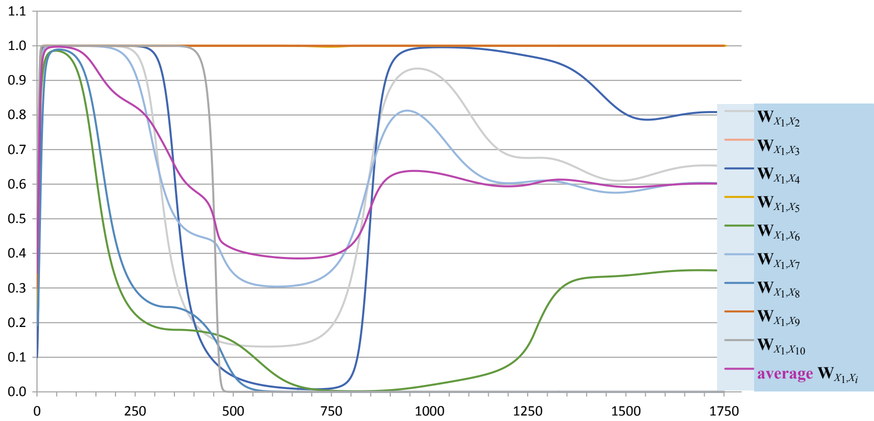

Fig. 6.2 Scenario 1: adaptive weights $\mathbf{W}_{X_1,X_i}$ of outgoing connections from X_1 over time, with the thick pink line showing the average weight of them. This average weight initially is 0.344, which is below the desired value 0.6 of the norm $\mathbf{v}$, but finally, it approximates the value 0.6 of this norm. Before that is achieved, over-controlling reactions cause strong fluctuations of having too intensive connections until time point 300, and again too low intensivity of connections between time point 400 and time point 750

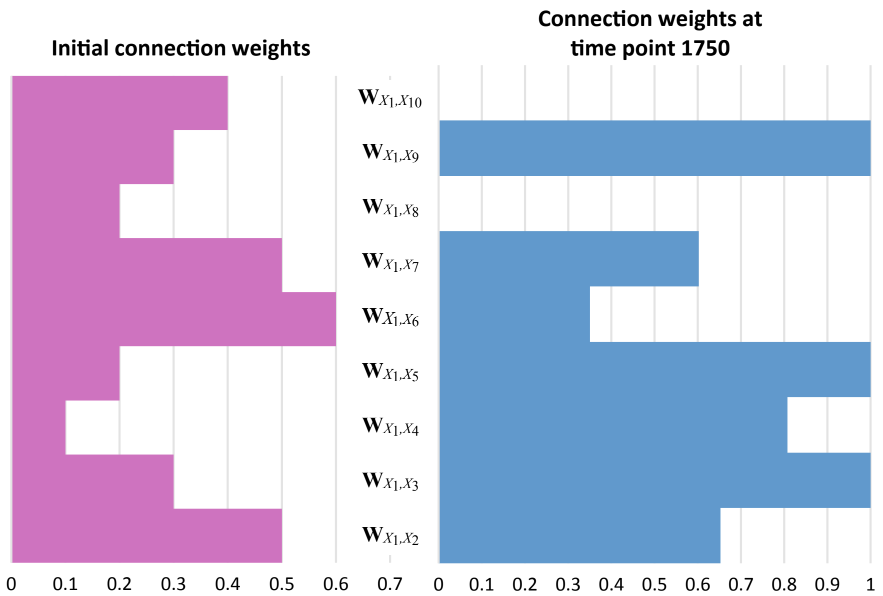

Fig. 6.3 Scenario 1 (modulation factor α = 0.1): Resulting connection weights $\mathbf{W}_{X_1,X_i}$ at time point 1750 compared to their initial values. As by the homophily principle the similarity of state values have an important effect on the dynamics of these connection weights, it can be seen that there is not much correlation between initial and final values of the weights

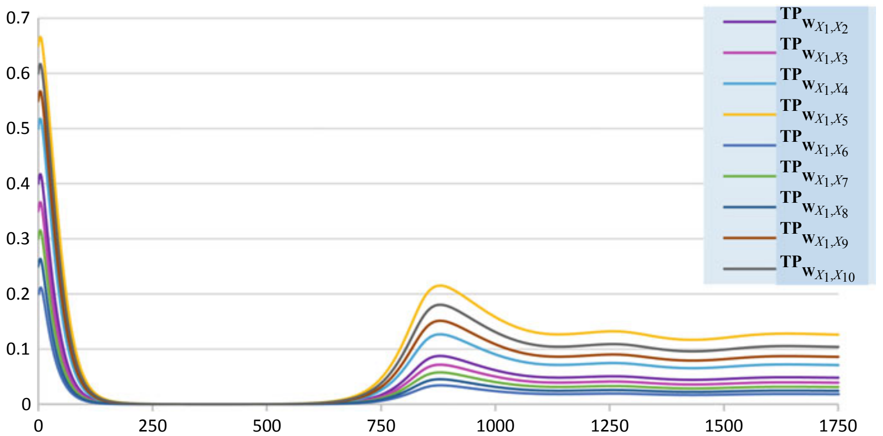

Fig. 6.4 Scenario 1 (modulation factor α = 0.1): The adaptive tipping points $\mathbf{TP}_{\mathbf{W}_{X_1,X_j}}$ over time. Initially, the tipping point values were much too high, which explains why in the first phase (until time 400) too many connections were strengthened. After the short initial phase, the tipping point values have been adapted so that they became very low so that the connections were weakened, and finally, they reached some equilibrium values

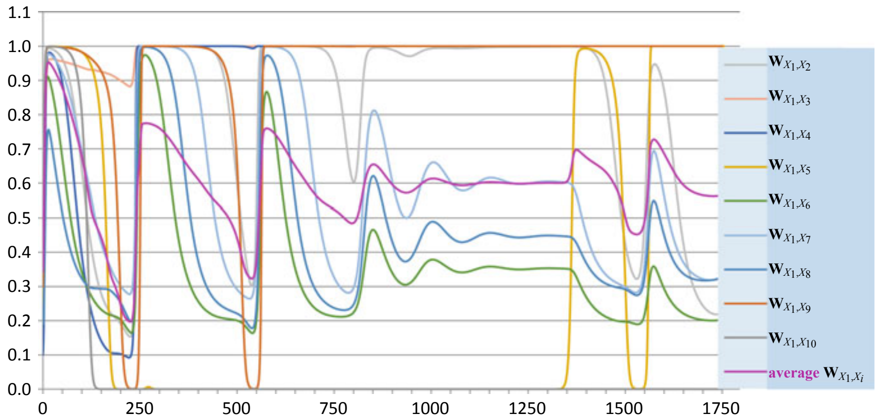

Fig. 6.5 Scenario 2 (modulation factor $\boldsymbol{\alpha}$ = 0.9): Adaptive weights of outgoing connections from X_1 over time, with the thick pink line showing the average weight for X_1. Compared to Scenario 1 this time it turns out more difficult to reach an equilibrium

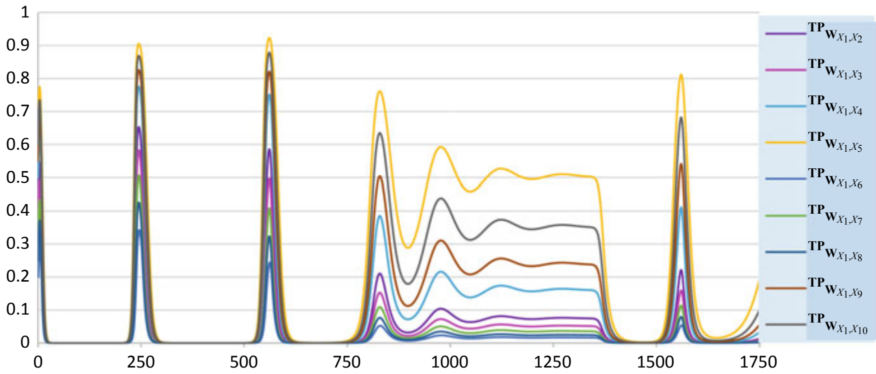

Fig. 6.6 Scenario 2 (modulation factor $\boldsymbol{\alpha}$ = 0.9): Adaptive tipping points $\mathbf{TP}_{\mathbf{W}_{X_1,X_j}}$ over time. The tipping point values react in a rather sensitive manner to the fluctuations shown in Fig. 6.5

6.5.2 Scenario 2: Adaptive Connections for One Person; Tipping Point Modulation Factor 0.9

Scenario 1 shown above is actually not one of the most chaotic scenarios; some other scenarios show a much more chaotic pattern. As an example, when for the tipping point adaptation a much higher modulation factor $\boldsymbol{\alpha}$ = 0.9 is chosen (instead of the 0.1 in Scenario 1; all other values stay the same) the pattern is still more chaotic, as shown in Figs. 6.5 and 6.6. Yet on the long term, the average connection weight moves around the set point 0.6; but notice that around time point 1250, it seemed that the process was close to an equilibrium, but that was violated by later fluctuations. Moreover, the fluctuating pattern of the tipping points in Fig. 6.6 also does not suggest that it will become stable.

6.5.3 Scenario 3: Adaptive Connections for All Persons

In Scenario 3 all connections are adaptive with main parameters shown in Table 6.4 and initial connection weight values shown in Table 6.5. The norm for average connection weight is 0.4 this time.

In Figs. 6.7, 6.8, 6.9 and 6.10, the simulation results are shown for Scenario 3. As can be seen in Fig. 6.10 eventually all connection weights converge to 0 or 1. Figure 6.7 shows, in particular, the values of the connection weights from X_1, and their average, and Fig. 6.8 shows the corresponding tipping points.

Figure 6.9 shows that in the process eventually the average connection weights per person converge in some unclear manner to a discrete set of equilibrium values: 0.111111 (connections of X_{10}), 0.222222 (connections of X_5), 0.333333 (connections of X_3, X_9), and 0.555555 (connections of X_1, X_2, X_4, X_6, X_7, X_8), all multiples of 0.111111; the overall average ends up in 0.433333 (recall that the norm $\boldsymbol{\nu}$ of $\mathbf{TP}_{\mathbf{W}_{X_i X_j}}$ for average connection weight for each person was 0.4). Also in other simulations, this discrete set of multiples of 0.111111 shows up. These specific values will be discussed as part of the analysis in Sect. 6.6.

Figure 6.10 shows that all connection weights $\mathbf{W}_{X_1,X_1}$ to $\mathbf{W}_{X_{10},X_{10}}$ (1–100) converge to 0 or 1. Note that connections from one state to itself were 0 by definition (in Fig. 6.10 the numbers 1, 12, 23, 34, 45, 56, 67, 78, 89, 100). Also this will be analysed in Sect. 6.6. The tipping points for all outgoing connections of X_1 converge to 0 (see also Fig. 6.8), and for all outgoing connections of the other persons, they converge to 1.

Table 6.4 Scenario 3: Main parameter values

Base level		First reification level		Second reification level	
Contagion alogistic steepness $\boldsymbol{\sigma}$ for X_i	0.8	Homophily modulation factor $\boldsymbol{\alpha}$ for $\mathbf{W}_{X_j,X_i}$	1	Tipping point speed factor $\boldsymbol{\eta}$ for $\mathbf{TP}_{\mathbf{W}_{X_j,X_i}}$	0.5
Contagion alogistic threshold $\boldsymbol{\tau}$ for X_i	0.15	Connection weight speed factor $\boldsymbol{\eta}$ for $\mathbf{W}_{X_j,X_i}$	1	Tipping point modulation factor $\boldsymbol{\alpha}$ for $\mathbf{TP}_{\mathbf{W}_{X_j,X_i}}$	0.4
Speed factor $\boldsymbol{\eta}$ for base state X_i	0.5			Tipping point connection norm $\boldsymbol{\nu}$ for $\mathbf{TP}_{\mathbf{W}_{X_j,X_i}}$	0.4

Table 6.5 Scenario 3: Initial connection weights

connections	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_1		0.5	0.3	0.1	0.2	0.6	0.5	0.2	0.3	0.4
X_2	0.5		0.6	0.3	0.4	0.7	0.7	0.9	0.5	
X_3	0.3	0.6		0.7	0.7	0.4	0.4		0.6	0.8
X_4	0.6	0.4	0.6		0.4	0.6	0.7	0.8		0.9
X_5	0.2	0.5		0.7		0.4		0.4	0.9	0.4
X_6	0.6	0.6	0.7	0.5			0.7	0.7	0.5	0.7
X_7	0.2	0.8	0.6	0.7	0.6	0.7		0.7		
X_8	0.6	0.5		0.4		0.6	0.5		0.4	0.5
X_9	0.6		0.6	0.7	0.4		0.7			0.6
X_{10}	0.6	0.7		0.7	0.4	0.6		0.8		

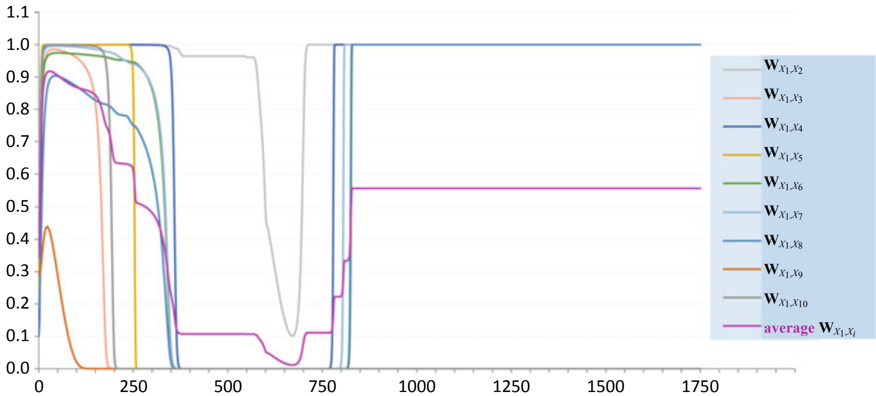

Fig. 6.7 Scenario 3: Adaptive weights of outgoing connections from X_1 over time, with the pink line showing the average weight for X_1. Here, after time 750 all connection weights become 0 or 1

6.6 Analysis of the Equilibria of the Reification States

In this section, the possible values to which certain states in the second-order reified network may converge are analysed. Recall the following definition and criterion for stationary points and equilibria.

Definition (stationary point and equilibrium)

A state Y has a *stationary point* at t if $\mathbf{d}Y(t)/\mathbf{d}t = 0$. The network is in *equilibrium* at t if every state Y of the model has a stationary point at t.

Note that this Y also applies to the reification states. Given the differential equation for a temporal-causal network model, a more specific criterion can be found:

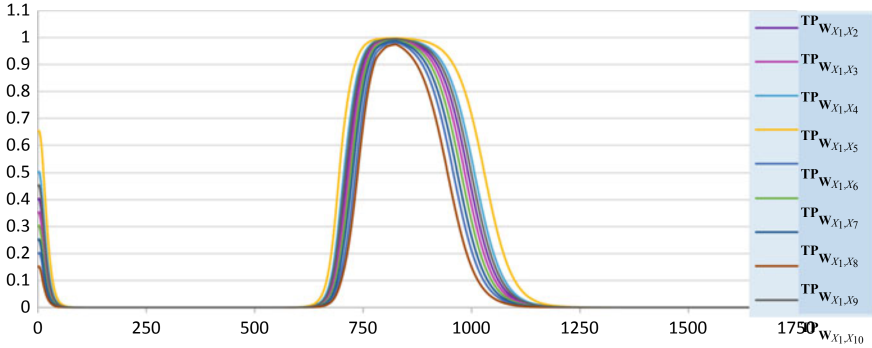

Fig. 6.8 Scenario 3: Adaptive tipping points $\mathbf{TP}_{\mathbf{W}_{X_1,X_j}}$ over time. Due to the very low connection weights between 500 and 750 (see Fig. 6.7), the tipping point values show a strong temporary increase to enable strengthening of connections

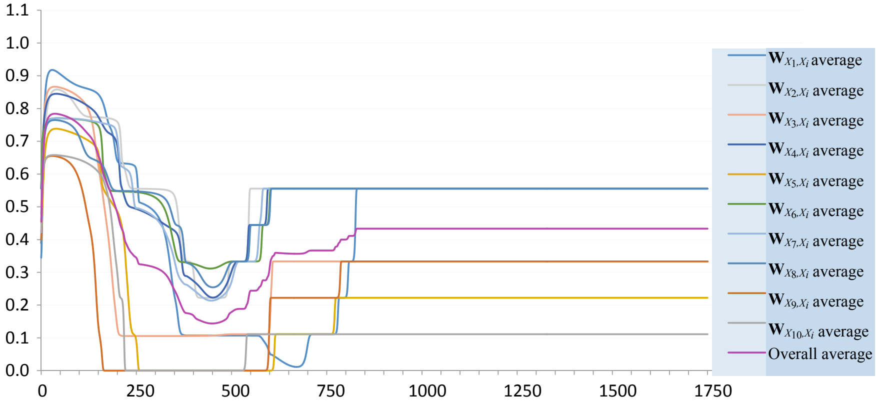

Fig. 6.9 Scenario 3: Average connection weights for each of X_1 to X_{10} and an overall average of all connections over time

Criterion for a stationary point in a temporal-causal network

Let Y be a state with speed factor $\boldsymbol{\eta}_Y$ and $X_1,\ldots, X_k$ the states from which state Y has incoming connections. Then Y has a stationary point at t iff

$$\boldsymbol{\eta}_Y = 0 \text{ or } \mathbf{c}_Y(\boldsymbol{\omega}_{X_1,Y}X_1(t), \ldots, \boldsymbol{\omega}_{X_k,Y}X_k(t)) = Y(t) \tag{6.1}$$

For an equilibrium these are called *equilibrium equations*.

This can be applied to the states at all levels in the second-order reified network, in particular to the first and second reification level. The solutions found (assuming parameters $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ and $\boldsymbol{\alpha}_3$ nonzero) are

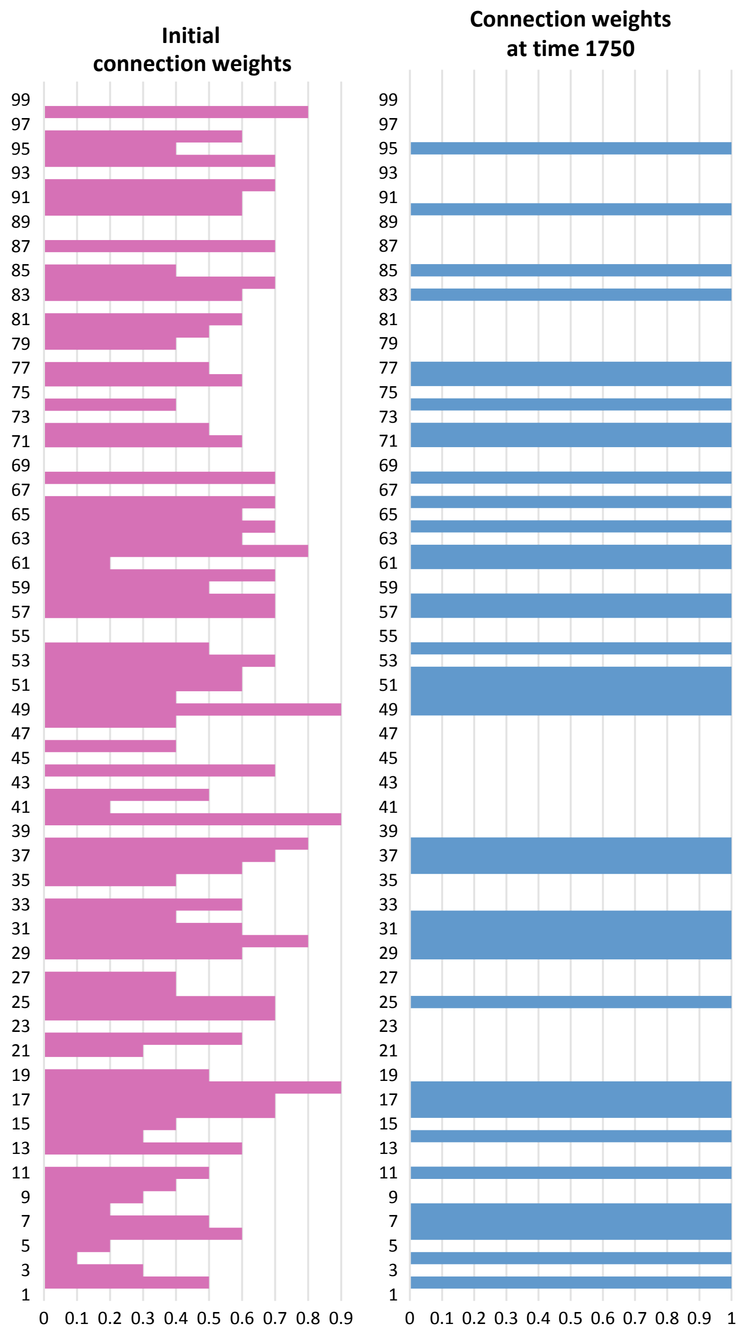

Fig. 6.10 Scenario 3: The connection weights from $\mathbf{W}_{X_1,X_1}$ to $\mathbf{W}_{X_{10},X_{10}}$ (1 to 100) initially, and finally they are 0 or 1

$$\begin{array}{llll} \underline{\mathbf{T}} = 0 & \text{or } \underline{\mathbf{T}} = 1 & \text{or } (\underline{\mathbf{W}}_1 + \ldots \underline{\mathbf{W}}_k)/k = \mathbf{v} & \\ \underline{\mathbf{H}} = 0 & \text{or } \underline{\mathbf{H}} = 1 & \text{or } (\underline{\mathbf{W}}_1 + \ldots \underline{\mathbf{W}}_k)/k = \mathbf{v} & \\ \underline{\mathbf{H}} = 0 & \text{or } \underline{\mathbf{W}} = 0 & \text{or } \underline{\mathbf{W}} = 1 & \text{or } |X_i - X_j| = \underline{\mathbf{T}} \end{array}$$

More details about this can be found in Box 6.6. In principle, this gives $4*3^2 = 36$ combined solutions, but in practice only 8. For the first two, by combining the different cases, the following five options can be found:

$$\begin{array}{l} \underline{\mathbf{T}} = 0 \quad \text{and} \quad \underline{\mathbf{H}} = 0 \\ \underline{\mathbf{T}} = 1 \quad \text{and} \quad \underline{\mathbf{H}} = 0 \\ \underline{\mathbf{T}} = 0 \quad \text{and} \quad \underline{\mathbf{H}} = 1 \\ \underline{\mathbf{T}} = 1 \quad \text{and} \quad \underline{\mathbf{H}} = 1 \\ (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k = \mathbf{v} \end{array}$$

In cases that $\underline{\mathbf{H}}$ is nonzero and $|X_i(t) - X_j(t)| \neq \underline{\mathbf{T}}$ (which was the case for the simulations displayed in Sect. 6.5), all $\underline{\mathbf{W}}_j$ are 0 or 1, so with $k = 9$ the average $(\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k$ is a multiple of $\frac{1}{9}$. In the simulation case the norm $\mathbf{v} = 0.4$ is not a multiple of $\frac{1}{9}$. Therefore the cases $(\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k = \mathbf{v}$ cannot actually occur, so then $\underline{\mathbf{T}} = 0$ or $\underline{\mathbf{T}} = 1$ and $\underline{\mathbf{W}} = 0$ or $\underline{\mathbf{W}} = 1$ are the only solutions for the first two; then all solutions are:

$$\begin{array}{llll} \underline{\mathbf{T}} = 0 & \text{and } \underline{\mathbf{H}} = 0 & & \\ \underline{\mathbf{T}} = 1 & \text{and } \underline{\mathbf{H}} = 0 & & \\ & & & \\ \underline{\mathbf{T}} = 0 & \text{and } \underline{\mathbf{H}} = 1 & \text{and } \underline{\mathbf{W}} = 0 \\ \underline{\mathbf{T}} = 0 & \text{and } \underline{\mathbf{H}} = 1 & \text{and } \underline{\mathbf{W}} = 1 \\ \underline{\mathbf{T}} = 0 & \text{and } \underline{\mathbf{H}} = 1 & \text{and } |X_i - X_j| = \underline{\mathbf{T}} \\ & & & \\ \underline{\mathbf{T}} = 1 & \text{and } \underline{\mathbf{H}} = 1 & \text{and } \underline{\mathbf{W}} = 0 \\ \underline{\mathbf{T}} = 1 & \text{and } \underline{\mathbf{H}} = 1 & \text{and } \underline{\mathbf{W}} = 1 \\ \underline{\mathbf{T}} = 1 & \text{and } \underline{\mathbf{H}} = 1 & \text{and } |X_i - X_j| = \underline{\mathbf{T}} \end{array}$$

This is also shown by the simulations (e.g., see Fig. 6.10); indeed all averages are multiples of $\frac{1}{9} = 0.111111$, as found above (see Fig. 6.9). This explains the discrete set of numbers 0.111111, 0.222222, 0.333333, … observed in the simulations; as shown here, this strongly depends on the number of states. Note that although in general the reified speed factor $\mathbf{H}_{\mathbf{W}_{X_iX_j}}(t)$ may be assumed nonzero, there may also be specific processes in which it converges to 0, for example, like the temperature in simulated annealing.

Box 6.6 Solving the equilibrium equations for the second-order reification states $\mathbf{TP}_{\mathbf{W}_{X_i X_j}}$ and $\mathbf{H}_{\mathbf{W}_{X_i X_j}}$ and the first-order reification state $\mathbf{W}_{X_i X_j}$

As a first step, the above criterion applied to tipping point reification states at the second reification level is as follows where $\underline{\mathbf{T}}$ is the considered equilibrium value for $\mathbf{TP}_{\mathbf{W}_{X_i X_j}}$, and $\underline{\mathbf{W}}_j$ for $\mathbf{W}_{X_i X_j}$.

$$\mathbf{sltip}_{\mathbf{v},\boldsymbol{\alpha}_1}(\underline{\mathbf{W}}_1, \ldots, \underline{\mathbf{W}}_k, \underline{\mathbf{T}}) = \underline{\mathbf{T}}$$

This provides the following equation

$$\underline{\mathbf{T}} + \boldsymbol{\alpha}_1 \underline{\mathbf{T}}(1 - \underline{\mathbf{T}})(\mathbf{v} - (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k) = \underline{\mathbf{T}}$$

which by subtracting $\underline{\mathbf{T}}$ can be rewritten as follows

$$\boldsymbol{\alpha}_1\, \underline{\mathbf{T}}(1 - \underline{\mathbf{T}})(\mathbf{v} - (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k) = 0$$

Assuming $\boldsymbol{\alpha}_1$ nonzero, this equation has three solutions:

$$\underline{\mathbf{T}} = 0 \text{ or } \underline{\mathbf{T}} = 1 \text{ or } (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k = \mathbf{v}$$

The same can be done for the combination function for the speed:

$$\mathbf{slspeedopp}_{\mathbf{v},\boldsymbol{\alpha}_2}(\underline{\mathbf{W}}_1, \ldots, \underline{\mathbf{W}}_k, \underline{\mathbf{H}}) = \underline{\mathbf{H}}$$
$$\underline{\mathbf{H}} + \boldsymbol{\alpha}_2 \underline{\mathbf{H}}(1 - \underline{\mathbf{H}})(\mathbf{v} - (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k) = \underline{\mathbf{H}}$$
$$\boldsymbol{\alpha}_2 \underline{\mathbf{H}}(1 - \underline{\mathbf{H}})(\mathbf{v} - (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k) = 0$$

Assuming $\boldsymbol{\alpha}_2$ is nonzero this has the following solutions:

$$\underline{\mathbf{H}} = 0 \text{ or } \underline{\mathbf{H}} = 1 \text{ or } (\underline{\mathbf{W}}_1 + \ldots + \underline{\mathbf{W}}_k)/k = \mathbf{v}$$

Similarly, the criterion can be applied to connection weights at the first reification level:

$$\mathbf{adapslhomo}_{\boldsymbol{\alpha}_3}(\underline{\mathbf{H}}, X_i, X_j, \underline{\mathbf{T}}, \underline{\mathbf{W}}) = \underline{\mathbf{W}}$$

This provides the following equation, where $\underline{\mathbf{H}}$ is the considered equilibrium value for $\mathbf{H}_{\mathbf{w}_{X_i X_j}}(t)$ and $\underline{\mathbf{W}}$ the considered equilibrium value for $\mathbf{W}_{Xi,\ Xj}$:

$$\underline{\mathbf{H}}\big(\underline{\mathbf{W}} + \boldsymbol{\alpha}_3 \underline{\mathbf{W}}(1 - \underline{\mathbf{W}})\big(\underline{\mathbf{T}} - |X_i - X_j|\big)\big) + (1 - \underline{\mathbf{H}})\underline{\mathbf{W}} = \underline{\mathbf{W}}$$

which by subtracting $\underline{\mathbf{W}}$ can be rewritten into

$$\underline{\mathbf{H}}\,\boldsymbol{\alpha}\,\underline{\mathbf{W}}(1-\underline{\mathbf{W}})\left(\underline{\mathbf{T}}-\left|X_i-X_j\right|\right)=0$$

Assuming $\boldsymbol{\alpha}_3$ nonzero, this equation has four solutions:

$$\underline{\mathbf{H}}=0 \text{ or } \underline{\mathbf{W}}=0 \text{ or } \underline{\mathbf{W}}=1 \text{ or} \left|X_i-X_j\right|=\underline{\mathbf{T}}$$

6.7 Discussion

In this chapter, a second-order reified adaptive Social Network model was presented. The reified network model is based on a first-order adaptation principle for bonding-by-homophily from Social Science (Byrne 1986; McPherson et al. 2001; Pearson et al. 2006; Sharpanskykh and Treur 2014) represented at the first reification level, and in addition for a second-order adaptation principle describing change of the characteristics 'similarity tipping point' and 'speed factor' of this first-order adaptation principle.

First-order adaptive network models for bonding by homophily, that can be considered precursors of the current second-order network model, have been compared to empirical data sets in (van Beukel et al. 2019; Blankendaal et al. 2016; Boomgaard et al. 2018). It is an interesting challenge to compare the new second-order network model described here to empirical data. This is left for a future enterprise. An extension to a first-order adaptive network for bonding by multicriteria homophily was addressed in (Kozyreva et al. 2018). This also could be developed further to a second-order adaptive network model.

Also in further Social Science literature, cases are reported where network adaptation is itself adaptive. For example, in (Carley et al. 2001; Carley 2002, 2006) the second-order adaptation concept called 'inhibiting adaptation' for network organisations is described. For further work, it would be interesting to explore the applicability of the introduced modeling environment for such domains further.

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