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Chapter 8 Higher-Order Reified Adaptive Network Models with a Strange Loop



Abstract In this chapter, as in Chap. 7, the challenge of exploring plausible reified network models of order higher than two is addressed. This time another less usual option for application was addressed: the notion of Strange Loop which from a philosophical perspective sometimes is claimed to be at the basis of human intelligence and consciousness. This notion will be illustrated by examples from music, graphic art and paradoxes, and by Hofstadter's claims about how Strange Loops apply to the brain. A reified adaptive network model of order higher than 2 was found, that even can be considered as being of infinite order. An example simulation shows the upward and downward interactions between the different levels, together with the processes within the levels. Another example addresses adaptive decision making according to two levels that are mutually reifying each other, as in Escher's Drawing Hands lithograph.

8.1 Introduction

Like in Chap. 7, the challenge addressed in the current chapter relates to the open question left from Sect. 1.3: are there good examples of adaptive networks of third-order? Or even higher? The aim of this chapter is to get answers "yes" on both questions for a domain as described by the idea of Strange Loop in (Hofstadter 1979, 2006, 2007).

Hofstadter (1979) describes a Strange Loop as the phenomenon that going upward through a hierarchy of levels after a while you find yourself back at the level where you started; a hierarchy of levels that turns out to form of cycle. He illustrates this notion for different domains, such as music (Bach), graphic art (Escher), and paradoxes and logic (Gödel). For example, in logic the idea was exploited by Gödel by defining a coding by natural numbers of all logical statements on arithmetic, as formalised by logical formulae. Such coding is a form of reification as statements become numbers, which has some similarity with the reification used for combination functions in reified temporal-causal networks. Using this coding, he was able to prove his famous incompleteness theorems in

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logic; e.g., (Nagel and Newman 1965; Smorynski 1977). Other literature relates Strange Loops, for example, to architectural education (Gannon 2017), advertising (Hendlin 2019), self-representation in consciousness (Kriegel and Williford 2006), and psychotherapeutic understanding (Strijbos and Glas 2018). Hofstadter's claim is that the brain also makes use of Strange Loops as an essential ingredient for human intelligence and consciousness. This has been used as a source of inspiration in the current chapter.

In the current chapter, it is shown how a Strange Loop can be modeled by a reified network. Here the lowest (base) level also functions as a reification level for the highest level, so that the reification levels form a cycle. Since a cycle has no end, this can be considered an example of a reified adaptive network of order ∞ .

Hofstadter (1979)'s illustration of the notion of Strange Loop by its application in music (Bach), graphic art (Escher), and paradoxes and logic (Gödel) will be very briefly summarized in Sect. 8.2, and Hofstadter's ideas about Strange Loops in the brain are discussed. Section 8.3 presents an example 12-level reified network model for a Strange Loop, and a simplified 4-level version of it. Section 8.4 shows an example simulation of it; this adaptive network model follows Escher (1960)'s design of his Ascending and Descending lithograph (Fig. 8.2). Another example is addressed in Sect. 8.5, of a two-level reified network where each level reifies the other level, following Escher (1948)'s design of his Drawing Hands lithograph (Fig. 8.3). It shows how a Strange Loop can be used to model adaptive decision making. Section 8.6 shows an example simulation of this mutually reified network model.

8.2 The Notion of Strange Loop

In this chapter, like in Chap. 7, the inspiration for examples of reified network models may feel a bit out of the ordinary, as the aim is to use the concept of reified network to explore the idea of Strange Loop described by Douglas Hofstadter in (Hofstadter 1979, 2007).

8.2.1 Strange Loops in Music, Graphic Art and Paradoxes

Hofstadter illustrates the idea first metaphorically from a music context as follows:

There is one canon in the Musical Offering which is particularly unusual. Labeled simply "Canon per Tonos", it has three voices. The uppermost voice sings a variant of the Royal Theme, while underneath it, two voices provide a canonic harmonization based on a second theme. The lower of this pair sings its theme in C minor (which is the key of the canon as a whole), and the upper of the pair sings the same theme displaced upwards in pitch by an interval of a fifth. What makes this canon different from any other, however, is that when it concludes-or, rather, seems to conclude-it is no longer in the key of C minor, but now is in D minor. Somehow Bach has contrived to modulate (change keys) right under the listener's nose. And it is so constructed that this "ending" ties smoothly onto the beginning again; thus one can repeat the process and return in the key of E, only to join again to the beginning. These successive modulations lead the ear to increasingly remote provinces of tonality, so that after several of them, one would expect to be hopelessly far away from the starting key. And yet magically, after exactly six such modulations, the original key of C minor has been restored! All the voices are exactly one octave higher than they were at the beginning, and here the piece may be broken off in a musically agreeable way. (Hofstadter 1979), p. 18

Then a short description of the idea is:

In this canon, Bach has given us our first example of the notion of Strange Loops. The "Strange Loop" phenomenon occurs whenever, by moving upwards (or downwards) through the levels of some hierarchical system, we unexpectedly find ourselves right back where we started. (Hofstadter 1979), p. 18

Another metaphoric illustration from (Hofstadter 2007) considers 10 persons sitting on your lap, on top of each other on each other's laps, where it turns out you yourself sit on the lap of the 10th person sitting on top of you; magically, no collapse occurs, see Fig. 8.1, adopted from (Hofstadter 2007).

In (Hofstadter 1979, 2007) also from another context metaphorical illustrations for the Strange Loop idea were obtained: from graphic art. Maurits Cornelis Escher

Fig. 8.1 Lap loop, sitting on each other's laps. Adopted from (Hofstadter 2007), Chap 8: embarking on a strange loop safari (Douglas Hofstadter is the guy smiling at you)





Fig. 8.2 Ascending and descending (Lithograph, Escher, 1960)

(1898–1972) was a famous Dutch graphic artist who made mathematically-inspired woodcuts, lithographs, and mezzotints. In Figs 8.2, 8.3 and 8.4 three examples of his lithographs are shown that were used as an illustration in (Hofstadter 1979, 2007). For example, in Ascending and Descending (Fig. 8.2) two lines of persons are seen, one of which walks in circles upstairs all the time and one downstairs, both all the time after one cycle returning to the same level. In Drawing Hands (Fig. 8.3) the right hand is drawing the left hand which in turn is drawing the right hand. Print Gallery (Fig. 8.4) shows a more complex situation in which the art and the reality that it depicts are mixed up, not unlike having the idea that Douglas Hofstadter smiles at you in Fig. 8.1.

Hofstadter (1979, 2007) also illustrates Strange Loops by examples of paradoxes. A simple example is:

This sentence is not true

Or a similar one:

I lie

The paradox here is that when such a sentence is true, it is false, and conversely. In these cases what is referred to from within the sentence is the sentence itself, which has many similarities with some of Escher's work.



Fig. 8.3 Drawing hands (Lithograph, Escher, 1948)



Fig. 8.4 Print gallery (Lithograph, Escher, 1956)

8.2.2 Strange Loops in the Brain

A description that comes closest to the scope of the current book is the following quote, where it is analysed how different levels of rules can generate intelligent behaviour:

What sorts of "rules" could possibly capture all of what we think of as intelligent behavior, however? Certainly there must be rules on all sorts of different levels. There must be many "just plain" rules. There must be "metarules" to modify the "just plain" rules; then "metametarules" to modify the metarules, and so on. The flexibility of intelligence comes from the enormous number of different rules, and levels of rules. The reason that so many rules on so many different levels must exist is that in life, a creature is faced with millions of situations of completely different types. In some situations, there are stereotyped responses which require "just plain" rules. Some situations are mixtures of stereotyped situations - thus they require rules for deciding which of the "just plain" rules to apply. Some situations cannot be classified - thus there must exist rules for inventing new rules ... and on and on. (Hofstadter 1979), pp. 34–35

This view may be related to the approach described in (Davis and Buchanan 1977; Davis 1980); note that in that time 'rules' was an often used representation format to specify functionality in AI and knowledge-based system models. If in the above quote the word 'rule' is replaced by 'causal connection' as a different format to specify functionality, this comes even more close to the topic of the current book:

What sorts of "causal connections" could possibly capture all of what we think of as intelligent behavior, however? Certainly there must be causal connections on all sorts of different levels. There must be many "just plain" causal connections. There must be "metacausal connections" to modify the "just plain" causal connections; then "metametacausal connections" to modify the metacausal connections, and so on. The flexibility of intelligence comes from the enormous number of different causal connections, and levels of causal connections. The reason that so many causal connections on so many different levels must exist is that in life, a creature is faced with millions of situations of completely different types. In some situations, there are stereotyped responses which require "just plain" causal connections. Some situations are mixtures of stereotyped situations - thus they require causal connections for deciding which of the "just plain" causal connections to apply. Some situations cannot be classified - thus there must exist causal connections for inventing new causal connections ... and on and on. Adaptation of (Hofstadter 1979), pp. 34–35, replacing 'rule' by 'causal connection'

In this way, it can be seen as a rough sketch of a multilevel reified network architecture where from each level, the level beneath is adapted. That is indeed exactly what happens in a reified network model, so this can be used as inspiration for a multilevel reified temporal-causal network model. However, the text continues as follows:

Without doubt, Strange Loops involving rules that change themselves, directly or indirectly, are at the core of intelligence. (Hofstadter 1979), pp. 34–35

This suggests that such a multilevel reified temporal-causal network should get some kind of cyclic level structure as meant for Strange Loops, comparable to the metaphorical illustrations discussed above. The quote below also suggests this in some more detail: My belief is that the explanations of "emergent" phenomena in our brains - for instance, ideas, hopes, images, analogies, and finally consciousness and free will - are based on a kind of Strange Loop, an interaction between levels in which the top level reaches back down towards the bottom level and influences it, while at the same time being itself determined by the bottom level. In other words, a self-reinforcing "resonance" between different levels (...) The self comes into being at the moment it has the power to reflect itself. (Hofstadter 1979), p. 704

Moreover, he also explicitly points at the role of causality in such an architecture, in particular for the upward and downward interactions between the levels, which also brings the Strange Loop idea closer to the domain of reified temporal-causal networks:

... we will have to admit various types of "causality": ways in which an event at one level of description can "cause" events at other levels to happen. Sometimes event A will be said to "cause" event B simply for the reason that the one is a translation, on another level of description, of the other. Sometimes "cause" will have its usual meaning: physical causality. Both types of causality - and perhaps some more - will have to be admitted in any explanation of mind, for we will have to admit causes that propagate both upwards *and* downwards in the Tangled Hierarchy of mentality ... (Hofstadter 1979), p. 704

8.3 A Twelve- and Four-Level Reified Adaptive Network Model Based on a Strange Loop

So, all in all, based on these ideas, examples of reified network architectures have been designed. The Network-Oriented Modeling approach used to model this process is based on reified temporal-causal network models described in Chaps. 3 and 4; see also (Treur 2018a, b) or for a software architecture (Treur 2019b). Recall that a temporal-causal network model in the first place involves representing in a declarative manner states and connections between them that represent (causal) impacts of states on each other. The states are assumed to have (activation) levels in the interval [0, 1] that vary over time. The following three notions form the defining part of a conceptual representation of a temporal-causal network model (Treur 2016, 2019a):

- Connectivity
 - Each incoming connection of a state *Y*, from a state *X* has a *connection* weight value $\omega_{X,Y}$ representing the strength of the connection.
- Aggregation
 - For each state a *combination function* $\mathbf{c}_{Y}(...)$ is chosen to combine the causal impacts state *Y* receives from other states.
- Timing
 - For each state *Y* a *speed factor* $\mathbf{\eta}_Y$ is used to represent how fast state *Y* is changing upon causal impact.

The notion of *network reification* introduced in Chaps. 3 and 4 is a means to model adaptive networks more transparently from a Network-Oriented Modelling perspective. The concept of reification has been shown to provide substantial advantages in expressivity and transparency of models within AI; e.g., (Davis and Buchanan 1977; Davis 1980; Galton 2006; Hofstadter 1979; Sterling and Beer 1989; Weyhrauch 1980). For network models, reification can be applied by reifying network structure characteristics (such as ω_{XY} , $\mathbf{c}_{Y}(...)$, $\mathbf{\eta}_{Y}$) in the form of additional network states (called *reification states*, indicated by W_{XY} , C_Y , H_Y , respectively) within an extended network. In addition, reification states $\mathbf{P}_{i,i,Y}$ for parameters of combination functions are used. Roles W, C, H, and P are assigned to reification states according to the specific network structure characteristic they represent: connection weight reification, combination function reification, speed factor reifi*cation*, or values, respectively. Also, a role **P** for *combination function parameters* is used. A specification format based on *role matrices* is used to specify for each state which role it is playing and in relation to which other states (see Box 8.1). Role matrices are **mb** (the base connectivity role), **mcw** (connection weights, role W), ms (speed factors, role H), mcfw (combination function weights, role C), mcfp (combination function parameters, role P). Multilevel reified networks can be used to model networks which are adaptive of different orders. For more details, see Chaps. 3 and 4, or (Treur 2018a, b).

8.3.1 A Twelve-Level Reified Adaptive Network Model Forming a Cycle of Levels

A first reified network model depicted in Fig. 8.5 illustrates how the notion of Strange Loop can be modeled in a reified network of 12 levels, by shaping the levels not straight upward, but in a cyclic form. This looks like a crazy network structure, even a bit Escher-ish, in which the 12 levels of the reified network seem to be juggled around, thereby forming a circle (depicted here in the shape of a dodecagon). As a cycle never ends, the levels actually go on forever; then it can be considered to be adaptive of order ∞ . It looks similar to Escher's Ascending and Descending where after four upward (or four downward) stairs the persons find themselves at the same level again. Is that what intelligent behaviour or consciousness is about, as Hofstadter suggests?

To model this, in Fig. 8.5 starting from the horizontal plane at the left hand side in the picture, the pink arrows are downward causal connections (going anti-clockwise) and the blue arrows are upward causal connections (going clockwise). So far, so good. But on the right hand side of the picture the pink downward causal connections are displayed as upward and the blue upward causal connections are displayed as downward. So you have to keep your head upside down to see it



Fig. 8.5 Multilevel reified network of 11th-order forming a strange loop of 12 levels, which actually makes the order ∞

correctly. In fact, locally, keeping your head (or the page) in the right position, for every two adjacent levels the model is quite normal, just a first-order adaptive network. The only strange thing is that globally it is connected as a cycle; but you cannot point at a specific part where there would be something wrong locally, exactly like usually is the case in Escher's work.

This does not look like a conventional reified network as the usual examples in this book. However, is it impossible? If this is modeled and simulated at a computer, will there not be an inconsistency, like in many paradoxes where levels are mixed up? The answer to this is 'no, this is not impossible'. One main reason why no inconsistency occurs in a dynamic network modeling setting is that time keeps opposite views separate, they will not occur at the same instant.

8.3.2 A Simpler Four-Level Reified Example of an Adaptive Network Model for a Strange Loop

It seems that such a reified network model as graphically depicted in Fig. 8.5 can still be described by role matrices and then it will just run in the software environment. To test this, consider in Fig. 8.6 a simplified version of the above picture with 4 levels instead of 12, as shown in Fig. 8.5, so that the regular 12 sided polygon (called dodecagon) becomes a 4 sided regular polygon (called square). Table 8.1 shows the states and their explanations. This resembles more closely Escher (1960)'s design of Ascending and Descending shown in Fig. 8.2, with the blue arrows the upward walking line of persons, and the downward pink arrows the downward walking line of persons; see Fig. 8.9. In Box 8.1 role matrices for this example are shown.



Fig. 8.6 A third-order multilevel reified network forming a strange loop, which actually makes the order ∞

State	Explanation	Level
X_1		
X2 X3		Base level
X_4	$\mathbf{W}_{X_{15}X_{16}}$ Reified representation state for the weight of the connection from X_{15} to X_{16}	
X_5		First
X_6		reification
X7 V		level
X8	$\mathbf{w}_{X_3X_4}$ Reified representation state for the weight of the connection from X_3 to X_4	
X9 V		Second
A 10 Y.1		reification
X_{12}	$\mathbf{W}_{X_7X_8}$ Reified representation state for the weight of the connection from X_7 to X_8	level
X13		
X_{14}		I nird reification
X_{15}		level
X_{16}	$\mathbf{W}_{X_{11}X_{12}}$ Reified representation state for the weight of the connection from X_{11} to X_{12}	

Table 8.1 The states and their explanations

The summary of this example reified network model is as follows. As an example, within the level of X_5 to X_8 , the first state is an independent state X_5 for external contextual input, which can be considered a kind of sensor state for that level. It affects state X_6 which can be considered as a kind of representation for the context. That state is also affected by state X_2 at one level lower, so that this context representation X_6 covers more contextual aspects than the one via X_5 . Next, state X_7 can be considered a form of interpretation or belief about this context. Finally, state X_8 can be considered some form of (preparation for) a concluding action. It has incoming connections from both X_6 and X_7 . The connection from X_7 to X_8 is adaptive; its weight is determined by state X_{12} at the next level. This description applies to each of the four levels, where at each level a different aspect of the context may be used as an external input. Through the upward interlevel connections (the upward blue arrows) this contextual information is integrated in some way.

The states X_4 , X_8 , X_{12} , or X_{16} function as reification states for the connection weights indicated below. More specifically, the downward interlevel causal connections (pink arrows) determine the weights of one of the connections at the target level in the following way:

the pink arrow from X_8	determines	the weight of the connection from X_3 to X_4
the pink arrow from X_4	determines	the weight of the connection from X_{15} to X_{16}
the pink arrow from X_{16}	determines	the weight of the connection from X_{11} to X_{12}
the pink arrow from X_{12}	determines	the weight of the connection from X_7 to X_8

Note that within each level the determined connection weight itself affects the state to which the connection is pointing to: X_4 , X_8 , X_{12} , or X_{16} . These are exactly the states that in turn determine the weights at the next level.

All this is specified in the role matrices shown in Box 8.1. Note that each level has a similar structure; this can easily be varied at any level separately as well.

Each of the role matrices **mb** (base connectivity), **mcw** (connection weights), **ms** (speed factors), **mcfw** (combination function weights), **mcfp** (combination function parameters) has a format in which in each row for the indicated state it is specified which other states (red cells) or values (green cells) causally affect it and according to which role. In role matrix **mcw**, in particular, the red cells indicate which states X_i play the role of the reification states for the weights of the connection indicated in that cell in **mb**.

mb	base connectivity	1 2		mcw	connection weights	1	2	n	ns	speed factors	1
$\begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array}$	_	$\begin{array}{c c} X_1 \\ X_1 \\ X_2 \end{array}$		$X_1 \\ X_2 \\ X_3$		1 1 1	1		$X_1 \\ X_2 \\ X_3$		0.05 0.5 0.5
X4	$W_{X_{15},X_{16}}$	X ₂ X ₃	_	X4	$W_{X_{15},X_{16}}$	1	X_8	_	X_4	$W_{X_{15},X_{16}}$	0.5
X ₅ V		X ₅ V ₅ V ₂		X5 V.		1	1		X ₅		0.03
X6 X7		X6 X6		X6 X7		1	1		А6 X7		0.5
X_8	\mathbf{W}_{X_3,X_4}	X6 X7		X_8	\mathbf{W}_{X_3,X_4}	1	X12		X_8	\mathbf{W}_{X_3,X_4}	0.5
X_9		X9		X9		1			X_9		0.02
X10		X9 X6		X_{10}		1	1		X_{10}		0.5
A11 X12	W _X ,X	A10 X10 X11		X11 X12	WY7 Ye	0.5	X16		X11 Y12	W _Y _Y	0.5
X12 X13	7**8	X13		X13		1	5110	-	X12 X12	7**8	0.015
X14		X13 X10		X_{14}		1	1		X14		0.5
X_{15}		X14		X15	W	1	V		X_{15}		0.5
X_{16}	$\mathbf{W}_{X_{11},X_{12}}$	X14 X15		X_{16}	vv x ₁₁ ,x ₁₂	1	X_4		X_{16}	$W_{X_{11},X_{12}}$	0.5
				mcfp	function		1				
			_	combir fur	ation action	alo	gistic				
mcfw	mbination function	1		norom							
COL		1		paran	ieters	1	2			1	
	weights	alogistic		раган	parameter	1 σ	2 τ	ŕ	v	initial values	1
X_1	weights	alogistic			parameter	1 σ 18	2 τ 0.2	ŕ	v X1	initial values	1
X_1 X_2 Y	weights	alogistic		X_1 X_2 Y	parameter	1 σ 18 3	2 τ 0.2 1.6	ŕ	X_1 X_2 X_2	initial values	1 0.13 0
X1 X2 X3 X4	weights W _{X-x}	alogistic		$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array}$	parameter Wx x	1 σ 18 3 3	2 τ 0.2 1.6 0.3 0.6	ŕ	X_1 X_2 X_3 X_4	initial values	1 0.13 0 0
X_1 X_2 X_3 X_4 X_5	$weights$ $W_{X_{15}X_{16}}$	alogistic		$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array}$	parameter $\mathbf{W}_{X_{15},X_{16}}$	1 σ 18 3 3 3	2 τ 0.2 1.6 0.3 0.6	i	X X_1 X_2 X_3 X_4 X_5	initial values W _{X15} X ₁₆	1 0.13 0 0 0 0
$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ \hline X_5 \\ X_6 \end{array}$	weights W _{x₁₅} x ₁₆	alogistic		X1 X2 X3 X4 X5 X6	eters parameter $\mathbf{W}_{X_{15}X_{16}}$	1 5 18 3 3 18 3	2 τ 0.2 1.6 0.3 0.6 0.2 1.3	i	$ \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{array} $	initial values Wx ₁₅ x ₁₆	1 0.13 0 0 0 0 0.12 0
X ₁ X ₂ X ₃ X ₄ X ₅ X ₆ X ₇	weights $W_{x_{15}x_{16}}$	alogistic		X1 X2 X3 X4 X5 X6 X7	eters parameter $\mathbf{W}_{x_{15}x_{16}}$	1 σ 18 3 3 3 18 3 3	2 r 0.2 1.6 0.3 0.6 0.2 1.3 0.3	i 	X X1 X2 X3 X4 X4 X5 X6 X7	initial values W _{X15} x ₁₆	1 0.13 0 0 0 0 0.12 0 0
$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{array}$	weights $W_{x_{15}}x_{16}$ $W_{x_3}x_4$	alogistic		X1 X2 X3 X4 X5 X6 X7 X8	w $x_{15}x_{16}$ W $x_{23}x_{4}$	1 5 18 3 3 18 3 3 3 18 3 3 3 18 3 3 3 3 3 3 3 3 3 3 3 3 3	2 τ 0.2 1.6 0.3 0.6 0.2 1.3 0.3 1 0.3	i	V X1 X2 X3 X4 X5 X6 X7 X8	initial values W _{x15} x ₁₆ W _{x3x4}	1 0.13 0 0 0 0 0 0 0 0 0 0 0
X1 X2 X3 X4 X5 X6 X7 X8 X9 X9	$weights$ $W_{x_{13}}x_{16}$ $W_{x_3}x_4$	1 alogistic 1 1 1 1 1 1 1 1 1 1		X1 X2 X3 X4 X5 X6 X7 X8 X9 Y	$\mathbf{W}_{X_{15},X_{16}}$ $\mathbf{W}_{X_{3},X_{4}}$	1 5 18 3 3 18 3 3 18 3 3 18 3 3 18 3 3 18 3 3 3 18 3 3 3 3 3 3 3 3 3 3 3 3 3	2 r 0.2 1.6 0.3 0.6 0.2 1.3 0.3 1 0.2 1.3	-	X X ₁ X ₂ X ₃ X ₄ X ₅ X ₆ X ₇ X ₈ X ₉ Y	$w_{x_{15}x_{16}}$ $w_{x_{15}x_{16}}$	1 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0.11
X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11	weights Wx ₁₃ x ₁₆ Wx ₃ x ₄	1 alogistic 1 1 1 1 1 1 1 1 1 1 1 1 1		X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X11	$w_{x_{15}x_{16}}$ $w_{x_{3}x_{4}}$	1 5 18 3 3 3 18 3 3 18 3 3 3 18 3 3 3 18 3 3 3 3 3 3 3 3 3 3 3 3 3	2 τ 0.2 1.6 0.3 0.6 0.2 1.3 0.3 1 0.2 1.3 0.3	-	X X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11	initial values $Wx_{15}x_{16}$ $Wx_{3}x_{4}$	1 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12	$weights$ $Wx_{15}x_{16}$ $Wx_{3}x_{4}$ $Wx_{7}x_{8}$	1 alogistic 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12	$w_{x_{15}x_{16}}$ $w_{x_{3}x_{4}}$ $w_{x_{7}x_{8}}$	1 σ 18 3 3 18 3 3 18 3 3 3 3 3	$\begin{array}{c} 2\\ \tau\\ 0.2\\ 1.6\\ 0.3\\ 0.6\\ \hline 0.2\\ 1.3\\ 0.3\\ 1\\ \hline 0.2\\ 1.3\\ 0.3\\ 1\\ \end{array}$	ř	X X1 X2 X3 X4 X5 X6 X7 X8 X6 X7 X8 X9 X10 X11 X12	initial values $Wx_{15}x_{16}$ $Wx_{3}x_{4}$	1 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
X1 X2 X3 X4 X5 X6 X7 X8 X0 X10 X11 X12 X13	$weights$ $W_{x_{15}x_{16}}$ $W_{x_{5}x_{4}}$ $W_{x_{5}x_{4}}$	alogistic 1 1 1 1 1 1 1 1 1 1 1 1 1		X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13	$w_{x_{15}x_{16}}$ $w_{x_{3}x_{4}}$ $w_{x_{7}x_{8}}$	1 5 18 3 3 18 3 3 18 3 3 18 3 3 18	$\begin{array}{c} 2\\ \tau\\ 0.2\\ 1.6\\ 0.3\\ 0.6\\ 0.2\\ 1.3\\ 0.3\\ 1\\ 0.2\\ 1.3\\ 0.3\\ 1\\ 0.2\\ \end{array}$	ř	X X1 X2 X3 X4 X5 X6 X7 X8 X6 X7 X8 X9 X10 X11 X12 X13	initial values $W_{X_{15}X_{16}}$ $W_{X_{3}X_{4}}$ $W_{X_{3}X_{4}}$	1 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X14	weights Wx ₁₃ x ₁₆ Wx ₃ x ₄ Wx ₇ x ₈	1 alogistic 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14	$w_{x_{15}x_{16}}$ $w_{x_{3}x_{4}}$ $w_{x_{7}x_{8}}$	1 σ 18 3 3 3 18 3 3 18 3 3 18 3 3 18 3 3 3 18 3 3 3 3 3 3 3 3 3 3 3 3 3	$ \begin{array}{c} 2 \\ \tau \\ 0.2 \\ 1.6 \\ 0.3 \\ 0.6 \\ 0.2 \\ 1.3 \\ 0.3 \\ 1 \\ 0.2 \\ 1.3 \\ 0.3 \\ 1 \\ 0.2 \\ 1.3 \\ 0.3 \\ $	ř	X X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X14 X14 X12 X13 X14 X14 X16 X6 X7 X8 X6 X7 X8 X8 X8 X8 X8 X8 X8 X8 X8 X8	initial values $Wx_{15}x_{16}$ $Wx_{5}x_{4}$ $Wx_{7}x_{8}$	1 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
X1 X2 X3 X4 X5 X6 X7 X8 X10 X11 X12 X13 X14 X15 X14	weights W _{x13} x ₁₆ W _{x3} x ₄ W _{x5} x ₈ W _{x5} x ₈	alogistic alogis		X ₁ X ₂ X ₃ X ₄ X ₅ X ₆ X ₇ X ₈ X ₉ X ₁₀ X ₁₁ X ₁₂ X ₁₃ X ₁₄ X ₁₅ X ₁₄	$w_{x_{15}x_{16}}$ $w_{x_{3}x_{4}}$ $w_{x_{7}x_{8}}$	1 σ 18 3 3 3 18 3 3 3 18 3 3 3 3 3 3 3 3 3 3 3 3 3	2 r 1.6 0.3 0.6 0.2 1.3 0.3 1 0.2 1.3 0.3 1 0.2 1.3 0.3 1 0.2 1.3 0.3 0.6 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5		X X1 X2 X3 X4 X5 X6 X7 X8 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X-4 X15 X-4 X-5 X-6 X-7 X-7 X-7 X-7 X-7 X-7 X-7 X-7	initial values $W_{X_1 s X_{16}}$ $W_{X_2 s X_4}$ $W_{X_7 x_8}$	1 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Box 8.1 Role matrices for the first Strange Loop reified network example

8.4 Simulation Example of the Four Level Strange Loop Reified Network Model

Using the dedicated software environment described in Chap. 9, simulation has been performed based on the data shown in Box 8.1. The overall graph is shown in Fig. 8.7.

To get a bit more insight four groups of states are depicted separately in Fig. 8.8. Each group consists of four states from each of the four levels. The first, upper graph in Fig. 8.8 shows the independent external input each level gets at different points in time (the lower levels earlier than the higher levels). The second graph shows the states that are mutually connected by the upward connections (the blue arrows in Fig. 8.6). The third graph shows the states for which the outgoing connection is adapted from one reification level higher.

The fourth graph at the bottom in Fig. 8.8 shows the states with incoming connection the one that is adapted from one reification level higher, and, besides, as reification states they represent the corresponding connection weight for one level lower; that's because these are the states at the different levels that have the mutual downward connections (the pink arrows in Fig. 8.6).

Note that the upward connections (the blue arrows) make that X_2 , X_6 , X_{10} , X_{14} in principle cyclically affect each other (clockwise interlevel interaction): upward cyclic causality. However, it can be seen that at the lowest level state X_2 does not respond much on state X_1 (due to a relatively high threshold of X_2), so without the other levels that lowest level would not have become very active. It can also be seen that at the second and the third level states X_6 and X_{10} and later X_{14} at the fourth



Fig. 8.7 Overall view on the strange loop example simulation





level respond much more actively on their input (their threshold is a bit lower). Finally, also X_2 becomes more active because of the other levels, in particular via X_{14} at the fourth level. Within that first level X_2 will affect X_4 more now; this is a first causal path by which X_4 is affected (via clockwise interlevel interaction via X_2).

The downward connections (the pink arrows) make that X_4 , X_{16} , X_{12} , X_8 in principle cyclically affect each other in that order (anti-clockwise interlevel interaction), via the connection weights of their adaptive incoming connection that is affected from one level higher: downward causality. In the second graph also the effect of the less responsive X_2 (because of its higher threshold value $\tau = 1.6$) is seen. It only goes up after time point 70, later that the other three states in the second graph. What happens is that X_{16} at the fourth level (having a relatively low threshold value $\tau = 0.6$) saves the situation, and in turn, in a domino effect pushes the other states in this group X_4 , X_{16} , X_{12} , X_8 upward by first increasing the weight of the incoming connection to X_{12} one level lower which then in turns affect X_8 which then affects X_4 ; this is a second causal path by which X_4 is affected (via anti-clockwise interlevel interaction).

In Fig. 8.9 the match of this four level cyclic reified network and Escher (1960)'s design of his Ascending and Descending lithograph is shown.



Fig. 8.9 The match between Escher (1960)'s ascending and descending design and the design of the four level cyclic reified network model

8.5 A Drawing Hands Reified Network Model for Adaptive Decision Making

As another example of a reified network model for a Strange Loop, an example adaptive decision making model is considered, designed according to Escher's Drawing Hands lithograph shown in Fig. 8.3. This decision making takes place in the lower (base) plane in Fig. 8.10, where two options a_1 and a_2 are modeled. Recall the quote of Hofstadter in Sect. 8.2.2 about rules at different levels, and the adaptation made by replacing the word 'rule' by 'causal connection'. The idea of rules at different levels was also explored in (Davis 1980), although in that case no Strange Loop was modeled. For the network model in the current section, a Strange Loop is incorporated, and following Escher's Drawing Hands design, just two levels are used, where one level is a reification level for (and controlling) the other and vice versa. And, of course, no rules are used but causal connections. So, instead of priorities for rules, here the connection weights used in the decision making process (at the lower level) are adaptive and controlled by the other (upper) level. In turn, the connections in the upper level are controlled by the lower decision making level, in particular, based on the extent of successfulness of a decision in the given context. The network model's conceptual representation is depicted in Fig. 8.10. In Table 8.2 an overview of all states is shown.



Fig. 8.10 Mutually reified temporal-causal network model for adaptive decision making according to Escher (1948)'s drawing hands design

	State	Explanation	Level
X_1	C1	First level context factor	
X_2	a_1	Action option 1	
X_3	a_2	Action option 2	Base
X_4	$e_1 = \mathbf{W}_{X_6, X_7}$	Positive evaluation for a_1 / Reified representation state for weight from c_2 to p_1	level
X_5	$e_2 = \mathbf{W}_{X_6, X_8}$	Positive evaluation for a_2 / Reified representation state for weight from c_2 to p_2	
X_6	C2	Second level context factor	First
X_7	$p_1 = \mathbf{W}_{X_1, X_2}$	Priority for a_1 / Reified representation state for weight from c_1 to a_1	reification
X_8	$p_2 = \mathbf{W}_{X_1, X_3}$	Priority for a_2 / Reified representation state for weight from c_1 to a_2	level

Table 8.2 Overview of the states in the drawing hands adaptive decision model

In the base plane, a context state c_1 is shown with two decision options a_1 and a_2 , triggered in this context according to the two connection weights. Whether or not an option a_i is successful, is modeled by state e_i , which as a kind of reality check gets an incoming connection from the context state c_1 . For the adaptation process, the upper plane uses three states c_2 , p_1 and p_2 . Here c_2 is another state representing (aspects of) the current context, and p_1 and p_2 can be considered as indicating priorities for the choice for a_1 or a_2 , respectively. These states p_1 and p_2 play the role of connection weight reification states for the incoming connections to a_1 and a_2 , as indicated by X_7 and X_8 in the red cells in the second and third row in role matrix **mcw** in Box 8.2. In the picture the pink or red arrows indicate what is usually called a downward causal connection. This is a connection from a reification state to the related base state.

The pattern described this far can be considered an adaptive hierarchical decision model. However, a Strange Loop comes in by assigning states e_1 and e_2 the role of reification state for the connections from c_2 to p_1 and p_2 , as indicated in role matrix **mcw** in Box 8.2 by X_4 and X_5 in the red cells in the last two rows for p_1 and p_2 . In this way evaluation of decisions made at the 'lower' level provides feedback to the 'higher' level, so that at that level adaptation can take place. However, due to the cyclic structure of the levels, there is no lower and higher level anymore.

The role matrices and initial values for this mutually reified network model are shown in Box 8.2. The simulations shown in Sect. 8.6 are based on these role matrices and initial values.

	mb	base	1 2	mcw		1	2	ms	speed	1
	connectivity		connection weights				factors			
	X_1	c_1	X_1	X_1	c_1	1		X_1	c_1	0.05
	X_2	a_1	X_1	X_2	a_1	X_7		X_2	a_1	0.5
	X_3	a_2	X_1	X_3	a_2	X_8		X_3	a_2	0.5
	X_4	$e_1 = \mathbf{W}_{X_6, X_7}$	$X_2 X_1$	X_4	$e_1 = \mathbf{W}_{X_6, X_7}$	1	0.6	X_4	$e_1 = \mathbf{W}_{X_6, X_7}$	0.005
	X_5	$e_2 = \mathbf{W}_{X_6, X_8}$	$X_3 X_1$	X_5	$e_2 = \mathbf{W}_{X_6, X_8}$	0.2	0.5	X_5	$e_2 = \mathbf{W}_{X_6, X_8}$	0.007
	X6	C2	X_1	X_6	C2	1		X6	Ca	0.5
	X7	$p_1 = \mathbf{W}_{X_1, X_2}$	X_6 X_2	X7	$p_1 = \mathbf{W}_{X_1, X_2}$	X_4	1	X_7	$p_1 = \mathbf{W}_{X_1, X_2}$	0.004
	X_8	$p_2 = \mathbf{W}_{X_1, X_3}$	X ₆ X ₃	X	$p_2 = \mathbf{W}_{X_1, X_3}$	X_5	1	X_8	$p_2 = \mathbf{W}_{X_1, X_3}$	0.005
			2			12	-			
				mcfp	function		1			
	·			combi	nation func-	alo	gistic			
	mcfw combination 1		tion p	arameters	1	2	iv	initial	1	
	fu	nction weights	alogistic		parameter	σ	τ		values	
	X_1	c_1	1	X_1	<i>c</i> ₁	18	0.2	X_1	C_1	0.1
	X_2	a_1	1	X_2	a_1	4	0.5	X_2	a_1	0
	X_3	a_2	1	X_3	a_2	4	0.5	X_3	a_2	0
	X_4	$e_1 = \mathbf{W}_{X_1, X_2}$	1	Χ.	$e_1 = \mathbf{W}_{X,X}$	2	12	X_{4}	$e_1 = \mathbf{W}_{X_c, X_n}$	0.3
		6'7	-	214	6.7	5	1.2	214	6 /	
	X_5	$e_2 = \mathbf{W}_{X_6, X_8}^{6^{-7}}$	1	X ₄ X ₅	$e_2 = \mathbf{W}_{X_6, X_8}$	3	1.2	X5	$e_2 = \mathbf{W}_{X_6, X_8}^{6, X_8}$	0.5
	X5 X6	$e_2 = \mathbf{W}_{X_6, X_8}$ c_2	1	X ₄ X ₅ X ₆	$e_2 = \mathbf{W}_{X_6, X_8}$ c_2	3 18	1.2 1.2 0.4	X ₅ X ₆	$e_2 = \mathbf{W}_{X_6, X_8}$ c_2	0.5
	X5 X6 X7	$e_{2} = \mathbf{W}_{X_{6},X_{8}}^{6^{-7}}$ $p_{1} = \mathbf{W}_{X_{1},X_{2}}^{C_{2}}$	1 1 1 1	$ \begin{array}{c} X_4 \\ X_5 \\ \hline X_6 \\ X_7 \end{array} $	$e_2 = \mathbf{W}_{X_6, X_8}$ $p_1 = \mathbf{W}_{X_1, X_2}$	3 18 6	1.2 1.2 0.4 0.5		$e_2 = \mathbf{W}_{X_6, X_8}^{6/7}$ $p_1 = \mathbf{W}_{X_1, X_2}^{6/7}$	0.5 0 0.3
	X5 X6 X7 X8	$e_{2} = \mathbf{W}_{X_{6} \times X_{8}}^{C \cdot 7}$ $p_{1} = \mathbf{W}_{X_{1} \times X_{2}}^{C \cdot 2}$ $p_{2} = \mathbf{W}_{X_{1} \times X_{3}}^{C \cdot 2}$	1 1 1 1 1	$\begin{array}{c} X_4 \\ X_5 \\ \hline X_6 \\ X_7 \\ X_8 \end{array}$	$e_{2} = \mathbf{W}_{X_{6}X_{8}}$ $e_{1} = \mathbf{W}_{X_{1}X_{2}}$ $p_{2} = \mathbf{W}_{X_{1}X_{3}}$	3 18 6 6	1.2 1.2 0.4 0.5 0.5	X ₅ X ₆ X ₇ X ₈	$e_{2} = \mathbf{W}_{X_{6}X_{8}}^{c_{7}}$ $p_{1} = \mathbf{W}_{X_{1}X_{2}}$ $p_{2} = \mathbf{W}_{X_{1}X_{3}}$	0.5 0 0.3 0.6
	X5 X6 X7 X8	$e_{2} = \mathbf{W}_{X_{6}X_{8}}^{6-7}$ $p_{1} = \mathbf{W}_{X_{1},X_{2}}^{C2}$ $p_{2} = \mathbf{W}_{X_{1},X_{3}}$	1 1 1 1	X ₄ X ₅ X ₆ X ₇ X ₈	$e_{2} = \mathbf{W}_{X_{6}X_{8}}$ $e_{1} = \mathbf{W}_{X_{1}X_{2}}$ $p_{2} = \mathbf{W}_{X_{1}X_{3}}$	3 3 18 6 6	1.2 1.2 0.4 0.5 0.5	$ \begin{array}{c} X_{4} \\ X_{5} \\ \overline{X_{6}} \\ X_{7} \\ \overline{X_{8}} \end{array} $	$e_{2} = \mathbf{W}_{X_{6}X_{8}}^{c_{7}}$ $p_{1} = \mathbf{W}_{X_{1}X_{2}}^{c_{2}}$ $p_{2} = \mathbf{W}_{X_{1}X_{3}}$	0.5 0 0.3 0.6
2	X5 X6 X7 X8	$e_{2} = \mathbf{W}_{X_{6}X_{8}}^{6-7}$ $p_{1} = \mathbf{W}_{X_{1},X_{2}}^{C2}$ $p_{2} = \mathbf{W}_{X_{1},X_{3}}$	1 1 1 1	$\begin{array}{c} X_4 \\ X_5 \\ \hline X_6 \\ X_7 \\ X_8 \end{array}$	$e_{2} = \mathbf{W}_{X_{6}X_{8}}$ $e_{1} = \mathbf{W}_{X_{1}X_{2}}$ $p_{2} = \mathbf{W}_{X_{1}X_{3}}$	3 18 6 6	1.2 1.2 0.4 0.5 0.5	$ \begin{array}{c} X_{5} \\ \overline{X_{6}} \\ \overline{X_{7}} \\ \overline{X_{8}} \end{array} $	$e_{2} = \mathbf{W}_{X_{6}X_{8}}^{c_{7}}$ $p_{1} = \mathbf{W}_{X_{1}X_{2}}^{c_{2}}$ $p_{2} = \mathbf{W}_{X_{1}X_{3}}$	0.5 0 0.3 0.6

Box 8.2 Role matrices for the mutually reified network for adaptive decision making

Figure 8.11 shows how the mutually reified network model matches to Escher (1948)'s fully symmetric Drawing Hands design.

8.6 An Example Simulation of the Drawing Hands Reified Network Model

In this section, an example is shown of a simulation of the adaptive decision making process modeled by the mutually reified network model. For a relatively short term the outcome looks as shown in Fig. 8.12.

Here it can be seen that action option a_2 (the green line) is preferred over option a_1 (the red line), as the latter seems not to reach a level that is much higher than 0.2. However, in a longer term, for the same simulation, the adaptation process does its work. This is shown in Fig. 8.13 (note the 10 times longer time scale). Now it is



Fig. 8.11 The match between Escher's drawing hands design and the design of the mutually reified network for adaptive decision making



Fig. 8.12 Simulation for a relatively short term

clear that ultimately action option a_1 is the preferred one, as its value reaches a level above 0.8, whereas action option a_2 drops to 0.1.

So, although initially, a responsive process triggers the options with a preference for a_2 , the evaluation e_2 based on the reality of the context shows that for the given context this choice is not adequate (whereas a choice for a_1 would be more adequate). This leads after time point 150 or 200 subsequently to the following adaptations:



Fig. 8.13 Simulation for a longer term

- Change of values of evaluation states e_1 and e_2 ; see the upward trend of $X_4 = e_1$ (the grey line ending up between 0.8 and 0.7) and the downward trend of $X_5 = e_2$ (the blue-green line ending just below 0.1)
- An adaptive adjustment of the connections in the blue plane from context state c_2 to the states p_1 and p_2 , based on the 'downward' causal connections from e_1 and e_2 to p_1 and p_2 , depicted in Fig. 8.10 by the pink upward arrows
- Change of values of p_1 and p_2 ; see the upward trend of $X_7 = p_1$ (the blue line finally exceeding 0.9) and the downward trend of $X_8 = p_2$ (the brown line ending below 0.1)
- effect on the connections to a_1 and a_2 , based on the 'downward' causal connections from p_1 and p_2 to a_1 and a_2 , depicted in Fig. 8.10 by the red downward arrows
- a changed effect on the choices for a_1 and a_2 ; see the upward trend of $X_2 = a_1$ (the red line ending up above 0.8) and the downward trend of $X_3 = a_2$ (the green line ending below 0.1).

8.7 Discussion

In this chapter, like in Chap. 7, the challenge to find and explore a plausible reified network model of order higher than 2 was addressed. To this end, the notion of Strange Loop was considered, which from a philosophical perspective is claimed by Hofstadter (1979, 2007) to be at the basis of human intelligence and consciousness. Reified adaptive network models using a Strange Loop of order higher than 3 were

shown. Due to the Strange Loop they can even be interpreted as being of infinite order, as by a Strange Loop the reification levels form a (closed) cycle with no beginning or end.

The first network model as presented is mainly meant to show how the interaction between the levels works. The processes within each level were kept a bit toylike. Its design follows Escher (1960)'s design of his lithograph Ascending and Descending. The second example addresses adaptive decision making in which two levels have mutual reification relations. This second design follows Escher (1948)'s design of his lithograph Drawing Hands.

A future step may be to explore more complex real-world reified network models using a Strange Loop, for example, by using further inspiration from the literature relating to architectural design such as (Gannon 2017), to advertising such as (Hendlin 2019), or to psychotherapeutic understanding such as (Strijbos and Glas 2018).

Another challenge to explore adaptive networks of order higher than 2 was to focus the application scope for reified networks on evolutionary processes, which was addressed in Chap. 7 for the adaptive evolutionary processes leading to disgust in the first trimester of pregnancy (Fessler et al. 2005, 2015; Fleischman and Fessler 2011; Jones et al. 2005); also see Chap. 1, Sect. 1.3.2. Together Chaps. 7 and 8 provide different positive answers on the question of whether or not adaptive networks of order higher than two may have interesting applications as posed in Chap. 1, Sect. 1.3.

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