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Adaptive Network Modeling for Criterial Causation

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Abstract. Propagation of activation of neurons depends on settings of a number of intrinsic characteristics of the network of neurons, such as synaptic connection strengths and excitability thresholds for neurons. These settings serve as criteria on the incoming signals for a neuron to get activated. As part of the plasticity of the neural processing these network characteristics also change over time. Such changes can be slow compared to propagation of activation, like in learning from a number of experiences, but they can also be fast, like in memory formation. From the informational perspective on the criteria, this can be considered a form of information formation, and the firing of neurons as driven by this information. This is called criterial causation. In this paper, an adaptive network model is presented modeling such criterial causation. Moreover, it is shown how criterial causation in the brain relates to the more general temporal factorisation principle for the world's dynamics.

1 Introduction

Neural processing is much more than propagation of activation of neurons; e.g., [25]. Such propagation depends on settings for a number of intrinsic characteristics of the network of neurons, such as synaptic connection strengths and excitability thresholds for neurons. These settings form a configuration in the brain that serves as a set of criteria on the incoming patterns of signals for a neuron to get activated; by Tse [25, 26] this is called *criterial causation*. As put forward by Tse, the criteria can be considered a form of *information* realised in the concerning brain configuration: 'physically realised informational criteria', e.g., [26], p. 259; in future situations, only when these criteria are met, the neuron will fire. As part of the neural processing, not only activation of neurons, but also the network characteristics defining these criteria change over time. Such changes of the network characteristics depend on the patterns in the past that affect them. The changes can be slow compared to propagation of activation of neurons, like in learning from a larger number of experiences, but they can also happen almost instantly, like in memory formation; the latter is called *rapid resetting* of the criteria by [25, 26]. From the informational perspective on the criteria, this form of network adaptation can be considered as a form of *emerging information formation*, and the firing of neurons as driven by this information [25, 26].

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This paper presents a computational adaptive network model that makes the above more precise, and illustrates it by a simulation for an example scenario. Moreover, the paper shows how the perspective as sketched can be considered a special case of the more general perspective on the dynamics of the world based on temporal factorisation by mediating state properties [17, 18]. Mediating state properties on the one hand encode in the present world configuration, information on the past pattern in the world, and on the other hand they determine the possible future patterns for the world from there. This wider perspective generalises specific processes of emerging information formation and usage taking place in the brain, to more general emerging information formation and usage as an inherent characteristic of the world's dynamics. Viewed in this more general way, it may be argued that the brain's functioning by criterial causation is entailed by the more general principle of temporal factorisation of the world's dynamics, or at least makes clever use of that general principle. Then, viewed from an informational perspective, the temporal factorisation principle can be seen as a way in which in general the world's dynamics creates emergent information in its configuration, and the more specific principle of criterial causation describes particularly how the brain creates emergent information in its configuration. In both cases this emergent information determines the options for the future patterns.

2 Temporal Factorisation and Criterial Causation

The temporal factorisation principle [17, 18] states that any systematic temporal 'past pattern implies future pattern' relationship $a \Rightarrow b$ between a past pattern a and a future pattern b can be factorised in the form of two temporal relationships $a \Rightarrow p$ and $p \Rightarrow b$ for some state property p (called *mediating state property*) of the present world state; see Fig. 1, left hand side. More specifically, the principle claims that for any 'past pattern implies future pattern' relationship $a \Rightarrow b$ *there exists* a world state property p (describing some configuration of the present world state) such that temporal relationships 'past pattern implies present state property' $a \Rightarrow p$ and 'present state property implies future pattern' $p \Rightarrow b$ hold: $a \Rightarrow b \Rightarrow \exists\ p\ a \Rightarrow p\ \&\ p \Rightarrow b$. Note that the mediating state property p does not need to be one simple state propertyy; it can (and often will) be a combination of multiple state properties occurring at that time point.

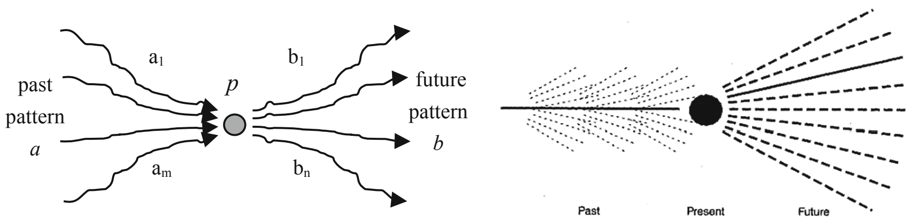

Fig. 1. Left hand picture: Mediating state property p in the present for the past pattern implies future pattern relation $a \Rightarrow b$, adopted from [17], p. 60, Fig. 1. Right hand picture: Criterial causation; adopted from [25], p. 125, Fig. 6.2.

The temporal factorisation principle claims that the present world state configuration p encodes sufficient information so that the world can forget about the temporal pattern a in the past if it makes temporal pattern b occur in the future; therefore it essentially is a claim that world state configurations are sufficiently rich to encode all (future-)relevant information on the past (which in theory could concern an almost infinite number of world states, with their temporal relations) in some condensed form in one state configuration. In [17] it is discussed in more detail how this principle relates to views on the world's dynamics from Descartes [8], Laplace [13], Ashby [2] and van Gelder and Port [27]. Descartes [8] puts forward that systematic relationships (laws of nature) exist for world states over time, in the sense that past world states imply future world states (called the clockwork universe). Laplace [13] claims: 'We may regard the present state of the universe as the effect of its past and the cause of its future'. In [27], following Ashby [2] the notion of state-determined system is taken as a basis for dynamics: '… its current state always determines a unique future behaviour … the future behaviour cannot depend in any way on whatever states the system might have been in *before* the current state' [27], p. 6. Note that the temporal factorisation principle relates to these views but, in contrast, does not assume an overall deterministic world. It only applies to aspects of the world that happen to be deterministic (as expressed by the conditional $a \Rightarrow b$). In [17, 18], the temporal factorisation principle is modeled in a formalised manner, and by many examples it is shown how the temporal factorisation principle plays its role in the world's dynamics, taken from Physics and from Cognitive Science.

One example to illustrate the principle is as follows. Suppose in reality or in a virtual game context in the present state there is a door that was locked by someone in a past pattern a and therefore only can be opened in a future pattern b in which you bring the right key with you. From $a \Rightarrow b$, the temporal factorisation principle concludes that there is a mediating state property p that holds in the present state such that $a \Rightarrow p$ & $p \Rightarrow b$. Indeed this p is the state property of the door being locked; then $a \Rightarrow p$ expresses that if in the past someone locked the door, it is locked now, and $p \Rightarrow b$ expresses that (only) when you bring the right key with you in the future it can be opened. The informational perspective here is that within the world the lock represents some form of information, and only when in the future pattern b the right key (with the right key shape, according to that lock information) occurs, the door opens. This is a clear case illustrating the way in which the mediating state configuration encodes information, and it is this information what drives the world to future pattern b, in which when the key occurs, the door can be opened. In this case, humans are actors encoding the information in the world, as the lock and the matching key are human-made: humans informationalise the world, the world is becoming more informational due to human intervention. A similar example of human-made informationalisation of the world is when Little Thumb drops pebbles to find his way back. However, the temporal factorisation principle claims in general that the world (as a kind of actor) is doing a similar encoding of information concerning past patterns in present world state configurations without human intervention.

The temporal factorisation principle can also be illustrated by the behavioural notion of 'delayed response behaviour', that has a long tradition in the psychology literature concerning animal cognition and behaviour; e.g., [7, 9, 11, 15]. Consider c is food at location l_0 visible for an animal, d is the animal gets released, and e is that the animal gets at l_0. An example of a past pattern a is: for at least two different time points in the past, state c (food visible at l_0) occurred. An example of a future pattern b is: if in some future state d occurs (animal is released), then at some later time point state e will occur (animal at l_0). The temporal factorisation principle says: if $a \Rightarrow b$, then there is some state property p such that $a \Rightarrow p$ and $p \Rightarrow b$. In this example, the (mediating) state property p postulated by the temporal factorisation principle would refer to an internal cognitive state, functioning as a form of memory for the animal. More specifically, in this case, after observing in animal experiments many times that the past-future relationship $a \Rightarrow b$ holds, the temporal factorisation principle postulates that some kind of (memory) state is formed after a past pattern a occurred, and that this memory state drives the organism's future behaviour in the sense that b holds. Also in this case, such a memory state can be considered to encode information about the world, and this information drives the future behaviour. Here, the information formation is an emerging process taking place without any human intervention, and also for the animal probably it will not happen as an intentional process. So, the world itself does the information formation, in this case via the brain.

Criterial Causation as Temporal Factorisation
After Sect. 1 and the above explanation, it may already have become clear that temporal factorisation and criterial causation have a close relationship; even the two pictures shown in Fig. 1 for temporal factorisation (left hand side) and criterial causation (right hand side) have a high extent of similarity. The correspondence can essentially be formulated as follows. In the above explanation the mediating state property p in the present state for temporal factorisation corresponds to the locked door; this lock defines the criteria for criterial causation. Fulfilment of the criteria in a future pattern b correspond to the fitting of a key in the lock, after which in b the door opens. This fulfilment corresponds to the firing of a neuron and its consequences in future pattern b.

3 Criterial Causation in Temporal-Causal Networks

In the above general formulation of the temporal factorisation principle, world states and past and future world patterns are kept abstract. However, often a notion of causality is considered as a way to describe the world's dynamics. Also in Tse [25] 's perspective based on criterial causation, causal relations play an important role. Therefore it makes sense to analyse how temporal factorisation and criterial causation work in combination with a description of world dynamics by a temporal-causal network [19, 21]. A temporal-causal network is characterised by *connectivity characteristics* (the connections from nodes X to Y and their weights $\boldsymbol{\omega}_{X,Y}$), *aggregation characteristics* (for each node Y, by a combination function $\mathbf{c}_{\boldsymbol{Y}}(\ldots)$ some form of aggregation is applied to the causal impacts from the incoming connections), and *timing characteristics* (nodes Y have speed factors $\boldsymbol{\eta}_Y$ indicating how fast they change upon causal impact). The difference equations used for simulation and mathematical analysis incorporate these three types of network characteristics $\boldsymbol{\omega}_{X,Y}$, $\mathbf{c}_Y(\ldots)$, $\boldsymbol{\eta}_Y$: for any state Y it holds

$$Y(t+\Delta t) = Y(t) + \boldsymbol{\eta}_Y\left[\mathbf{c}_Y\left(\boldsymbol{\omega}_{X_1,Y}X_1(t),\ldots,\boldsymbol{\omega}_{X_k,Y}X_k(t)\right) - Y(t)\right]\Delta t \tag{1}$$

where $X_1,\ldots,X_k$ are the states from which Y gets incoming connections. These concepts enable to design networks with their dynamics in a declarative manner, by mathematically defined relations; see [19, 21] for more information on Network-Oriented Modeling based on temporal-causal networks.

Criteria for Criterial Causation in Temporal-Causal Networks

Based on (1) the firing criterion of state Y can be expressed by putting that the aggregated impact on Y is higher than 0.5:

$$\mathbf{c}_Y\left(\boldsymbol{\omega}_{X_1,Y}X_1(t),\ldots,\boldsymbol{\omega}_{X_k,Y}X_k(t)\right) > 0.5 \tag{2}$$

So, this (2) is taken as the general criterion for criterial causation for a state Y in a temporal-causal network. Often used combination functions $\mathbf{c}_Y(..)$ are the simple logistic $\mathbf{slogistic}_{\boldsymbol{\sigma},\boldsymbol{\tau}}(\ldots)$ and advanced logistic sum function $\mathbf{alogistic}_{\boldsymbol{\sigma},\boldsymbol{\tau}}(\ldots)$, both with steepness parameter $\boldsymbol{\sigma} > 0$ and excitability threshold parameter $\boldsymbol{\tau}$:

$$\mathbf{slogistic}_{\boldsymbol{\sigma},\boldsymbol{\tau}}(V_1,\ldots,V_k) = \frac{1}{1+\mathrm{e}^{-\boldsymbol{\sigma}(V_1+\ldots+V_k-\boldsymbol{\tau})}} \tag{3}$$

$$\mathbf{alogistic}_{\boldsymbol{\sigma},\boldsymbol{\tau}}(V_1,\ldots,V_k) = \left[\frac{1}{1+\mathrm{e}^{-\boldsymbol{\sigma}(V_1+\ldots+V_k-\boldsymbol{\tau})}} - \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma}\boldsymbol{\tau}})}\right](1+\mathrm{e}^{-\boldsymbol{\sigma}\boldsymbol{\tau}}) \tag{4}$$

Here the V_i denote the single impacts $\boldsymbol{\omega}_{X_i,Y}X_i(t)$ on state Y for each of the incoming connections from states $X_1,\ldots,X_k$. For the simple logistic sum function (3), criterion (2) is equivalent to

$$\frac{1}{1+\mathrm{e}^{-\boldsymbol{\sigma}(V_1+\ldots+V_k-\boldsymbol{\tau})}} > 0.5$$

with the V_i denoting the single impacts $\boldsymbol{\omega}_{X_i,Y}X_i(t)$ on state Y. By rewriting (see Box 1 left), this is equivalent to

$$\boldsymbol{\omega}_{X_1,Y}X_1(t) + \cdots + \boldsymbol{\omega}_{X_k,Y}X_k(t) > \boldsymbol{\tau} \tag{5}$$

So, (5) is the more specific criterion for criterial causation for the simple logistic combination function. For the advanced logistic combination the following similar but more complicated criterion for criterial causation can be derived (see Box 1 right).

$$\boldsymbol{\omega}_{X_1,Y}X_1(t) + \cdots + \boldsymbol{\omega}_{X_k,Y}X_k(t) > \tau - \log\left(\frac{1}{\frac{0.5}{1+\mathrm{e}^{-\sigma\tau}} + \frac{1}{1+\mathrm{e}^{\sigma\tau}}} - 1\right)/\boldsymbol{\sigma} \tag{6}$$

$1 + \mathrm{e}^{-\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau})} < 2 \Leftrightarrow$ $\mathrm{e}^{-\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau})} < 1 \Leftrightarrow$ $\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau}) > 0 \Leftrightarrow$ $V_1 + \cdots + V_k > \boldsymbol{\tau} \Leftrightarrow$ $\omega_{X_1,Y}X_1(t) + \cdots + \omega_{X_k,Y}X_k(t) > \boldsymbol{\tau}$	$\left[\frac{1}{1+\mathrm{e}^{-\boldsymbol{\sigma}(V_1+\cdots+V_k-\boldsymbol{\tau})}} - \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}\right](1+\mathrm{e}^{-\sigma\tau}) > 0.5 \Leftrightarrow$ $\left[\frac{1}{1+\mathrm{e}^{-\boldsymbol{\sigma}(V_1+\cdots+V_k-\boldsymbol{\tau})}} - \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}\right] > \frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} \Leftrightarrow$ $\frac{1}{1+\mathrm{e}^{-\boldsymbol{\sigma}(V_1+\cdots+V_k-\boldsymbol{\tau})}} > \frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}} \Leftrightarrow$ $1 + \mathrm{e}^{-\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau})} < \frac{1}{\frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}} \Leftrightarrow$ $\mathrm{e}^{-\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau})} < \frac{1}{\frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}} - 1 \Leftrightarrow$ $-\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau}) < \log(\frac{1}{\frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}} - 1) \Leftrightarrow$ $\boldsymbol{\sigma}(V_1 + \cdots + V_k - \boldsymbol{\tau}) > -\log(\frac{1}{\frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}} - 1) \Leftrightarrow$ $V_1 + \cdots + V_k > \boldsymbol{\tau} - \log(\frac{1}{\frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}} - 1)/\boldsymbol{\sigma} \Leftrightarrow$ $\omega_{X_1,Y}X_1(t) + \cdots + \omega_{X_k,Y}X_k(t) > \boldsymbol{\tau} - \log(\frac{1}{\frac{0.5}{1+\mathrm{e}^{-\boldsymbol{\sigma\tau}}} + \frac{1}{1+\mathrm{e}^{\boldsymbol{\sigma\tau}}}} - 1)/\boldsymbol{\sigma}$

Box 1. Deriving criteria (5) (left) and (6) (right) for criterial causation for combination functions **slogistic**$_{\sigma,\tau}(\ldots)$ and **alogistic**$_{\sigma,\tau}(\ldots)$

Note that for these combination functions the criteria are expressed as linear inequalities for state values $X_1, \ldots, X_k$ with as coefficients expressions in terms of network characteristics $\boldsymbol{\omega}$, $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$. The criteria for criterial causation for other combination functions (scaled maximum $\mathbf{smax}_\lambda(\ldots)$ and minimum $\mathbf{smin}_\lambda(\ldots)$, scaled sum $\mathbf{ssum}_\lambda(\ldots)$, Euclidean $\mathbf{eucl}_{n,\lambda}(\ldots)$ and scaled geometric mean $\mathbf{sgeomean}_\lambda(\ldots)$) found are in Table 1.

Table 1. Overview of the criteria for criterial causation for different combination functions

Combination function		Criterion for criterial causation
Name	Formula	
$\mathbf{c}_Y(V_1, \ldots, V_k)$	$\mathbf{c}_Y(V_1, \ldots, V_k)$	$\mathbf{c}_Y\left(\boldsymbol{\omega}_{X_1,Y}X_1(t), \ldots, \boldsymbol{\omega}_{X_k,Y}X_k(t)\right) > 0.5$
$\mathbf{slogistic}_{\sigma,\tau}(V_1, \ldots, V_k)$	$\frac{1}{1+\mathrm{e}^{-\sigma(V_1+\ldots+V_k-\tau)}}$	$\boldsymbol{\omega}_{X_1,Y}X_1(t) + \ldots + \boldsymbol{\omega}_{X_k,Y}X_k(t) > \boldsymbol{\tau}$
$\mathbf{alogistic}_{\sigma,\tau}(V_1, \ldots, V_k)$	$\left[\frac{1}{1+\mathrm{e}^{-\sigma(V_1+\ldots+V_k-\tau)}} - \frac{1}{1+\mathrm{e}^{\sigma\tau}}\right](1+\mathrm{e}^{-\sigma\tau})$	$\boldsymbol{\omega}_{X_1,Y}X_1(t) + \ldots + \boldsymbol{\omega}_{X_k,Y}X_k(t) > \tau - \log(\frac{1}{\frac{0.5}{1+\mathrm{e}^{-\sigma\tau}} + \frac{1}{1+\mathrm{e}^{\sigma\tau}}} - 1)/\boldsymbol{\sigma}$
$\mathbf{smax}_\lambda(V_1, \ldots, V_k)$	$\mathbf{max}(V_1, \ldots, V_k)/\boldsymbol{\lambda}$	$\boldsymbol{\omega}_{X_i,Y}X_i(t) > 0.5\ \boldsymbol{\lambda}$ for some i
$\mathbf{smin}_\lambda(V_1, \ldots, V_k)$	$\mathbf{min}(V_1, \ldots, V_k)/\boldsymbol{\lambda}$	$\boldsymbol{\omega}_{X_i,Y}X_i(t) > 0.5\ \boldsymbol{\lambda}$ for all i
$\mathbf{ssum}_\lambda(V_1, \ldots, V_k)$	$\frac{V_1+\ldots+V_k}{\boldsymbol{\lambda}}$	$\boldsymbol{\omega}_{X_1,Y}X_1(t) + \ldots + \boldsymbol{\omega}_{X_k,Y}X_k(t) > 0.5\boldsymbol{\lambda}$
$\mathbf{eucl}_{n,\lambda}(V_1, \ldots, V_k)$	$\sqrt[n]{\frac{V_1^n+\ldots+V_k^n}{\boldsymbol{\lambda}}}$	$\boldsymbol{\omega}^{\boldsymbol{n}}_{X_1,Y}X_1(t)^{\boldsymbol{n}} + \ldots + \boldsymbol{\omega}^{\boldsymbol{n}}_{X_k,Y}X_k(t)^{\boldsymbol{n}} > 0.5^{\boldsymbol{n}}\boldsymbol{\lambda}$
$\mathbf{sgeomean}_\lambda(V_1, \ldots, V_k)$	$\sqrt[k]{\frac{V_1*\ldots*V_k}{\boldsymbol{\lambda}}}$	$\boldsymbol{\omega}_{X_1,Y}X_1(t) * \ldots * \boldsymbol{\omega}_{X_k,Y}X_k(t) > 0.5^{\boldsymbol{k}}\boldsymbol{\lambda}$

Emerging Criteria for Criterial Causation in Adaptive Temporal-Causal Networks

Networks considered for real world domains are often adaptive, so that some or all of the above network characteristics can change over time as well. This is the way in which the criteria for criterial causation are set dynamically, and the information based on the criteria is not fixed but emerges. For example, in the above criteria (2), (5), (6) the connection weights $\boldsymbol{\omega}$, excitability threshold $\boldsymbol{\tau}$ and steepness $\boldsymbol{\sigma}$ can change over time, thereby changing the criterion. Then the overall dynamics is an interaction (or co-evolution) of two types of dynamics, one of which (dynamics of the nodes) is modeled in a declarative mathematical manner from a Network-Oriented Modeling perspective, and the other one (dynamics of the characteristics and the criteria they define) is usually described in a different, nondeclarative (procedural or algorithmic) manner. This leads to a kind of hybrid model. By using the notion of *network reification*, the Network-Oriented Modeling perspective can also be used to design *adaptive* networks in a declarative manner by mathematically defined relations. This works by adding the adaptive network characteristics (in a reified form) to the (base) network as nodes at a second level, called *reification level*, while the original network forms the *base level*. In this way an extended, reified network is obtained, which is again a temporal-causal network. As for any temporal-causal network model, the dynamics of such a reified network is described in a declarative mathematical Network-Oriented manner by the nodes and their connections, including causal interlevel connections for the impact from one level to the other. This can iteratively be applied to obtain multiple reification levels to model multiple orders of adaptation of a network. For more details, see [20, 23], or the forthcoming book [24].

Using a reified temporal-causal network model to describe the world's dynamics, in a relatively easy manner causality can be modeled for criterial causation and the temporal factorisation principle. For example, then a mediating state configuration p can be described by the state values of a number of nodes, which each can be at the base level, or at any reification level. And also the past and future patterns a and b are described as patterns of state values for a number of nodes of any level over time. In particular, for the criteria expressed by linear inequalities (5) and (6), the coefficients are based on reification states at the reification level, whereas the states $X_1, \ldots, X_k$ to which the criteria are applied are at the base level.

4 An Example Reified Network for Criterial Causation

As discussed in Sect. 3, a temporal-causal network model involves three main characteristics connectivity, aggregation, and timing of the network structure, modeled by $\boldsymbol{\omega}_{X,Y}$, $\mathbf{c}_Y(\ldots)$, $\boldsymbol{\eta}_Y$. The difference equations used for simulation and mathematical analysis incorporate these three types of network characteristics as expressed in (1) above. For the sake of practicality, for each application from a library basic combination functions $\mathrm{bcf}_i(\ldots)$, $i = 1, \ldots, m$ can be selected according to weights $\boldsymbol{\gamma}_{i,Y}$, so that the combination function used for any state Y is the weighted average

$$\mathbf{c}_Y(\ldots) = \left(\gamma_{1,Y}\mathrm{bcf}_1(\ldots) + \ldots + \gamma_{m,Y}\mathrm{bcf}_m(\ldots)\right)/\left(\gamma_{1,Y} + \ldots + \gamma_{m,Y}\right)$$

Moreover, parameters of these combination functions can be considered, so that $\mathrm{bcf}_i(\ldots) = \mathrm{bcf}_i(\mathbf{p}, \mathbf{v})$ with $\mathbf{p}$ a list of parameters and $\mathbf{v}$ a list of values. For reified network models additional reification states are introduced in the network that explicitly represent characteristics of the network such as *connectivity*, *aggregation*, and *timing*, and makes them adaptive; these reification states are indicated by $\mathbf{W}_{X,Y}$, $\mathbf{C}_{i,Y}$, $\mathbf{P}_{i,j,Y}$, and $\mathbf{H}_Y$:

- **Adaptive connection weight $\boldsymbol{\omega}_{X,Y}$**: reified connection weight representations $\mathbf{W}_{X,Y}$
- **Adaptive combination function weight $\gamma_{i,Y}$ for $\mathbf{c}_Y(\ldots)$**: reified combination function weight representations $\mathbf{C}_{i,Y}$ (for the i^{th} combination function used)
- **Adaptive combination function parameter $\mathbf{p}_Y$ for $\mathbf{c}_Y(\ldots)$**: reified combination function parameter representations $\mathbf{P}_{i,j,Y}$ (the j^{th} parameter of the i^{th} combination function for Y)
- **Adaptive speed factor $\boldsymbol{\eta}_Y$**: reified speed factor representations $\mathbf{H}_Y$

Example Scenario

The considered scenario is as follows. A person who is new in an organisation has to recognize a colleague from seeing his face, modeled by stimulus s. There are two options, colleagues a_1 and a_2. Deciding for one of them is represented by preparation states ps_{a_i}. A belief bs_1 suggests that it should be collegue a_1, and a belief bs_2 that it should be colleague a_2; however, these beliefs are indicative (for example, based on the location at which the person is seen), but not sufficient to firmly decide for one of the two. The beliefs and s are generated from independent circumstantial environmental factors; for the model they just happen. Two types of adaptive network characteristics are involved: the weights of the connections from the sensory state srs_s for s to ps_{a_1} and ps_{a_2}, and the excitability thresholds for states ps_{a_1} and ps_{a_2}. During the scenario these characteristics are adapted so that a decision results. The obtained settings define the criteria for criterial causation of the recognition. Based on them, in future situations any encounter with s (also at unexpected locations, such as a supermarket or during holidays) leads to fulfilment of the criteria and as a consequence to recognition.

In a graphical representation the reification states are depicted in a 3D format in a second plane, above the (pink) plane for the base network; see the blue plane in the example reified network model depicted in Fig. 2, also indicated as reification level; see Table 2 for an explanation of the states. Three types of causal connections are distinguished: upward causal connections, downward causal connections and leveled (horizontal) causal connections. The downward causal connections have their own fixed role and meaning in the sense that as their special effect they are causally effectuating one of the four types of adaptive values listed above. In the reified network model for criterial causation described here, for almost all states logistic function $\mathbf{alogistic}_{\sigma,\tau}(\ldots)$ is used; see (4) above. As an exception, for Hebbian learning [10] the following combination function is used for the reification states $\mathbf{W}_{X,Y}$ at the reification level:

$$\mathbf{hebb}_{\boldsymbol{\mu}}(V_1, V_2, W) = V_1 V_2 (1 - W) + \boldsymbol{\mu} W \tag{7}$$

where V_1, V_2 indicate the single impacts from the connected states (base states in the bottom plane in Fig. 1) and W the connection weight (represented by reification state $\mathbf{W}_{X,Y}$ in the upper plane in Fig. 1), and $\boldsymbol{\mu}$ is a persistence parameter. In Fig. 1 only the following reification states are used:

- $\mathbf{W}_{X,Y}$ plays the role of connection weight for the connection from X to Y [10]
- $\mathbf{T}_Y$ plays the role of combination function parameter value for the excitability threshold parameter $\boldsymbol{\tau}$ for state Y [5]

The roles of the different base and reification states are specified by *role matrices* **mb** (base connection role), **mcw** (connection weight role), **ms** (speed factor role), **mcfw** (combination function weight role), and **mcfp** (combination function parameter role); e.g., [22–24]. In these role matrices (see Box 2) at each row for the given state it is specified which other states have impact on it (incoming arrows in Fig. 2).

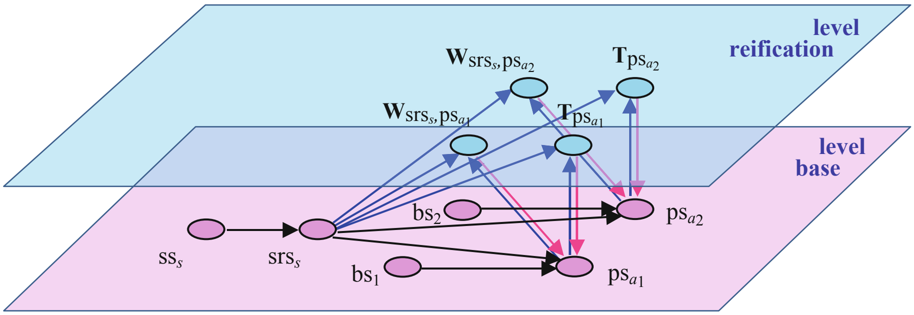

Fig. 2. Overview of the example reified network model for criterial causation, with: (1) *base level* for face recognition (lower plane, pink), (2) *reification level* (upper plane, blue) for the criteria represented by the weights $\boldsymbol{\omega}$ of the base connections from srs_s to ps_{a_1} and ps_{a_2} (reified by the two **W** states) and the excitability thresholds $\boldsymbol{\tau}$ of these two base states ps_{a_1} and ps_{a_2} (reified by the two **T** states)

This is distinguished according to their role: *base* or *non-base* connections, from which for the latter a distinction is made for the roles *connection weight*, *speed factor*, *combination function weight* and *combination function parameter reification*. The matrices all have rows according to the numbered states X_1, X_2, X_3, …. For a given application a limited sequence of combination functions is specified by **mcf** = […], for the example this is **mcf** = [2 3 35], where the numbers 2, 3 refer to the numbering in the function library, the first three being $\mathbf{eucl}_{n,\lambda}(\ldots)$, $\mathbf{alogistic}_{\sigma,\tau}(\ldots)$, $\mathbf{hebb}_{\boldsymbol{\mu}}(\ldots)$. In Box 2 the role matrices for the reified network model for criterial causation are shown.

Table 2. Overview of the states in the example reified network model

state nr	state name	explanation
X_1	ss_s	Sensor state for external stimulus s (seeing a face)
X_2	srs_s	Sensory representation state for stimulus s
X_3	bs_1	Belief state 1 (belief that it is Person 1)
X_4	bs_2	Belief state 2 (belief that it is Person 2)
X_5	ps_{a1}	Preparation state for recognition as Person 1
X_6	ps_{a2}	Preparation state for recognition as Person 2
X_7	$\mathbf{W}_{\mathrm{srs}_s,\mathrm{ps}_{a1}}$	Reification state for the weight of the connection from srs_s to ps_{a1}
X_8	$\mathbf{W}_{\mathrm{srs}_s,\mathrm{ps}_{a2}}$	Reification state for the weight of the connection from srs_s to ps_{a2}
X_9	$\mathbf{T}_{\mathrm{ps}_{a1}}$	Reification state for the excitability threshold of ps_{a1}
X_{10}	$\mathbf{T}_{\mathrm{ps}_{a2}}$	Reification state for the excitability threshold of ps_{a2}

The first role matrix **mb** for *base connectivity* specifies on each row for a given state from which states at the same or a lower level it has incoming connections. For example, in the fifth row it is indicated that state X_5 (= ps_{a1}) has two incoming base connections, from state X_2 (= srs_s), and from state X_3 (= bs_1). As another example, the 7th row indicates that state X_7 (= $\mathbf{W}_{\mathrm{srs}_s,\mathrm{ps}_{a1}}$) has incoming base connections from X_2 (= srs_s), X_5 (= ps_{a_1}) and from X_7 itself, and in that order, which is important as the Hebbian combination function $\mathbf{hebb}_{\boldsymbol{\mu}}(\ldots)$ used here is not symmetric in its arguments.

In a similar way the four types of role matrices for *non-base connectivity* (i.e., connectivity from reification states at a higher level of reification: the downward arrows in Fig. 2), were defined: role matrices **mcw** for connection weights and **ms** for speed factors, and role matrices **mcfw** for combination function weights and **mcfp** for combination function parameters (see Box 2).

Within each of the role matrices **mcw**, **mcfw**, **mcfp** and **ms** a difference is made between cell entries indicating (in red) a reference to the name of another state that as a form of reification represents in a dynamic manner an adaptive characteristic, and entries indicating (in green) fixed values for nonadaptive characteristics. Indeed, in Box 1 it can be seen that the red cells of the non-base role matrices are filled with the (reification) states X_7 to X_{10} of the first reification level. For example, in Box 1 the name X_7 in the red cell row-column (5, 1) in role matrix **mcw** indicates that the value of the connection weight from srs_s to ps_{a1} (as indicated in role matrix **mb**) can be found as value of the seventh state X_7. In contrast, the 1 in green cell (7, 1) of **mcw** indicates the static value of the connection weight from X_2 (= srs_s) to X_7 (= $\mathbf{W}_{\mathrm{srs}_s,\mathrm{ps}_{a1}}$).

As yet another example, in role matrix **mcfp** for the combination function parameters, in cell (5, 2) it is indicated that the value of the excitability threshold of ps_{a1} is represented by reification state X_9 (= $\mathbf{T}_{\mathrm{ps}_{a_1}}$). For more explanation about this role matrix specification format, see [22, 23], or the forthcoming book [24].

mb	base connectivity	1	2	3
X_1	ss_s	X_1		
X_2	srs_s	X_1		
X_3	bs_1	X_3		
X_4	bs_2	X_4		
X_5	ps_{a1}	X_2	X_3	
X_6	ps_{a2}	X_2	X_4	
X_7	$\mathbf{W}srs_s,ps_{a1}$	X_2	X_5	X_7
X_8	$\mathbf{W}srs_s,ps_{a2}$	X_2	X_6	X_8
X_9	$\mathbf{T}ps_{a1}$	X_2	X_5	X_9
X_{10}	$\mathbf{T}ps_{a2}$	X_2	X_6	X_{10}

mcw	connection weights	1	2	3
X_1	ss_s	1		
X_2	srs_s	1		
X_3	bs_1	1		
X_4	bs_2	1		
X_5	ps_{a1}	X_7	0.5	
X_6	ps_{a2}	X_8	0.5	
X_7	$\mathbf{W}srs_s,ps_{a1}$	1	1	1
X_8	$\mathbf{W}srs_s,ps_{a2}$	1	1	1
X_9	$\mathbf{T}ps_{a1}$	-0.2	-0.2	1
X_{10}	$\mathbf{T}ps_{a2}$	-0.2	-0.2	1

mcfw	combination function weights	1 alo-gistic	2 hebb	3 step-mod
X_1	ss_s			1
X_2	srs_s	1		
X_3	bs_1			1
X_4	bs_2			1
X_5	ps_{a1}	1		
X_6	ps_{a2}	1		
X_7	$\mathbf{W}srs_s,ps_{a1}$		1	
X_8	$\mathbf{W}srs_s,ps_{a2}$		1	
X_9	$\mathbf{T}ps_{a1}$	1		
X_{10}	$\mathbf{T}ps_{a2}$	1		

mcfp	function	1 alogistic		2 hebb		3 stepmod	
	parameter	1 σ	2 τ	1 μ	2	1 rep	2 init
X_1	ss_s					50	25
X_2	srs_s	5	0.8				
X_3	bs_1					70	60
X_4	bs_2					50	25
X_5	ps_{a1}	5	X_9				
X_6	ps_{a2}	5	X_{10}				
X_7	$\mathbf{W}srs_s,ps_{a1}$			0.95			
X_8	$\mathbf{W}srs_s,ps_{a2}$			0.95			
X_9	$\mathbf{T}ps_{a1}$	5	0.4				
X_{10}	$\mathbf{T}ps_{a2}$	5	0.4				

ms	speed	1
X_1	ss_s	2
X_2	srs_s	0.5
X_3	bs_1	2
X_4	bs_2	2
X_5	ps_{a1}	0.2
X_6	ps_{a2}	0.5
X_7	$\mathbf{W}srs_s,ps_{a1}$	0.3
X_8	$\mathbf{W}srs_s,ps_{a2}$	0.3
X_9	$\mathbf{T}ps_{a1}$	0.07
X_{10}	$\mathbf{T}ps_{a2}$	0.07

initial values		
X_1	ss_s	0
X_2	srs_s	0
X_3	bs_1	0
X_4	bs_2	0
X_5	ps_{a1}	0
X_6	ps_{a2}	0
X_7	$\mathbf{W}srs_s,ps_{a1}$	0.3
X_8	$\mathbf{W}srs_s,ps_{a2}$	0.3
X_9	$\mathbf{T}ps_{a1}$	0.8
X_{10}	$\mathbf{T}ps_{a2}$	0.8

Box 2. Specification in role matrices format for the reified example network for criterial causation

5 Example Simulation of Criterial Causation

Using a dedicated modeling environment [22] for reified network models, simulations have been performed. In Fig. 3 simulation results are shown for the example Scenario described in Sect. 4. Here the settings shown in Box 2 were used. Like in the temporal factorisation principle, for the overall process a past pattern a, a future pattern b, and a mediating (present) state property p, are distinguished, each of which will be briefly discussed.

Past Pattern *a* (From Time Point 0 to 100)
During the past pattern *a*, the stimulus *s* (the observed face) occurs twice: from time 25 to 50 and from time 75 to 100. In these time periods also belief state bs_2 occurs. The upper and middle graph in Fig. 3 display the past pattern *a* and show how the recognition of stimulus *s* as Person 2 is emerging: during the first encounter, ps_{a_2} (the red line) increases relatively slowly, and during the second encounter this happens faster; apparently already a more adequate informational criterion has been set for the recognition.

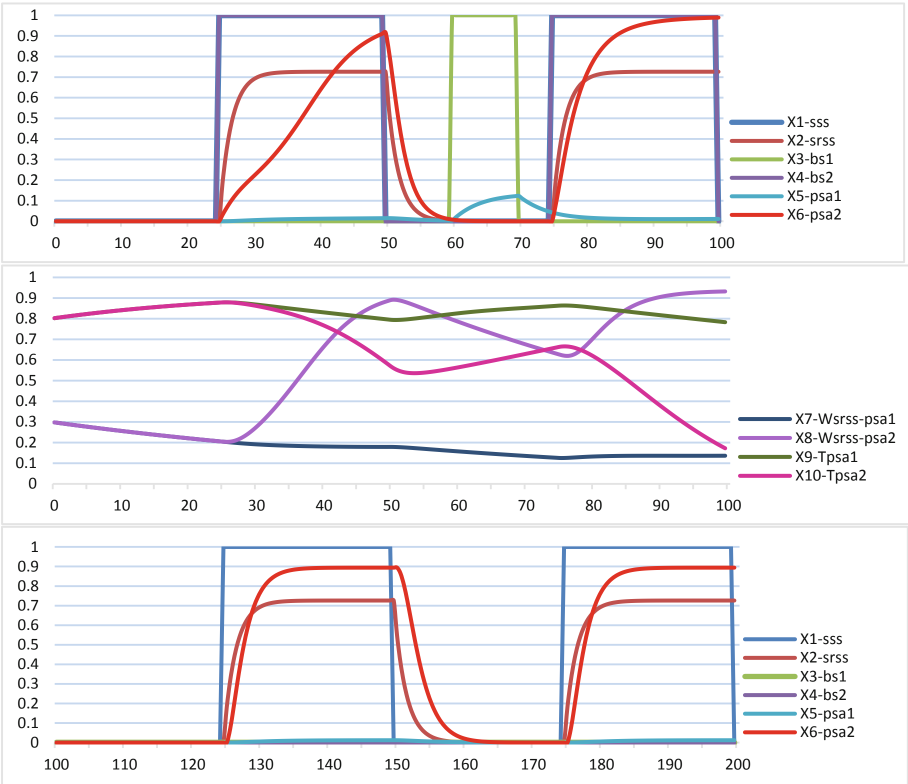

Fig. 3. A past pattern displayed in the upper graph and middle graph, and a future pattern displayed in the lower graph, where the criterion set as a mediating (present) state by the past pattern drives the future pattern.

In the middle graph, the emergence of the characteristics for this criterion are shown. In particular, it is shown that (due to belief state bs_2) the reified adaptive connection weight $\mathbf{W}_{srs_s, ps_{a1}}$ from srs_s to ps_{a_2} (purple line) becomes stronger, and the reified excitability threshold $\mathbf{T}_{ps_{a_2}}$ of ps_{a_2} (pink line) becomes lower. Note that between time 60 and 70 for a short period belief state bs_1 occurs, but this disturbance has no substantial consequences.

Criterion Set in the Mediating State Property *p* at Time Point 100
The mediating state property p describes criterion (6) for criterial causation. In this example scenario, the mediating state property consists of the values of the following relevant characteristics: the reified weights $\mathbf{W}_{\text{srs}_s,\text{ps}_{a1}}$ and $\mathbf{W}_{\text{srs}_s,\text{ps}_{a1}}$ of the connections from srs_s to ps_{a1} and ps_{a2}, and the reified excitability thresholds $\mathbf{T}_{\text{ps}_{a_1}}$ and $\mathbf{T}_{\text{ps}_{a_2}}$ for ps_{a_1} and ps_{a_1}, so the reification states X_7 to X_{10}. Note that all of them are at the reification level. At time point 100 they have the following values: $X_7 = 0.136275$, $X_8 = 0.93172$, $X_9 = 0.78281$, $X_{10} = 0.17232$. So, this configuration described at the reification level, defines the mediating state property, and, equivalently, the coefficients of criterion (6) for criterial causation. It can be seen that the connection weight from srs_s to ps_{a_1} is low (0.136275) and the excitability threshold for ps_{a_1} is high (0.78281). Therefore, for the choice for Person 1 the criterion for firing cannot be met in a reasonable way. For ps_{a_1} it is the opposite: the connection weight from srs_s to ps_{a_2} is high (0.93171758) and the excitability threshold for ps_{a_2} is low (0.17232). This means that the criterion for this choice for Person 2 is easy to fulfill by the causal impact coming from srs_s. Indeed, using (6) from Sect. 3 substituted by the values of the **W** states and **T** states (reifying $\boldsymbol{\omega}$ and $\boldsymbol{\tau}$, respectively) at time point 100, and the value 5 of $\boldsymbol{\sigma}$, and 0.5 for the $\boldsymbol{\omega}$ from the belief state, the criterion for firing becomes

$$0.5\,\text{bs}_2(t) + 0.93172\ \text{srs}_s(t) > 0.0857$$

Not assuming any positive value of $\text{bs}_2(t)$, this is already fulfilled if

$$\text{srs}_s(t) > 0.0857/0.93172 = 0.092$$

So already a very weak sensory representation signal of the observed face as low as 0.1 would be enough to recognize the face.

Future Pattern *b* (From Time Point 100 to 200)
In the lower graph in Fig. 3 the future pattern is displayed; it is shown how based on the emerged criterion (represented by the present mediating state at time 100), indeed instant recognition takes place in the absense of any of the belief states. Here the criterion is based on the (constant) values for the reified connection weights and excitability thresholds defining the mediating state property: the values $X_7 = 0.136275$, $X_8 = 0.93172$, $X_9 = 0.78281$, $X_{10} = 0.17232$. In the future pattern at times 125 and 175 the person is seen again (ss_s whereby the belief state bs_2 is kept 0), and the criterion becomes fulfilled so that indeed firm recognition ps_{a_2} as Person 2 takes place.

6 Discussion

Criterial causation as introduced by Tse [25] describes how as a form of plasticity, in the brain, configurations emerge that provide informational criteria for future processing and behaviour. In the current paper, first it was shown how this notion relates to the more general notion of temporal factorisation based on mediating state properties to describe the world's dynamics as introduced in [17]. The core of both of these two notions is that by some adaptive process, over time a (past) brain or world pattern leads to the formation of a present brain or world configuration that in turn drives the (future) brain or world pattern. From an informational perspective, this configuration in the present encodes emergent information based on the past that is relevant for the future. Choices made in the future are based on this information, by a person, or by the world. In this paper it has been shown how these processes can be modeled by an adaptive temporal-causal network.

For future work, it will be explored how the notion of Extended Mind [3, 6, 16] can be addressed in a similar manner and how a notion of representational content [4, 12] known from Philosophy of Mind can be used to describe the information in the emerging brain or world configurations. Moreover, it will be explored how also metaplasticity [1, 23] can be incorporated in the adaptive processes for criterial causation.

On purpose, in the current paper any link to notions such as the free will problem or the mental causation problem from Philosophy of Mind (as discussed extensively by Tse) has been left aside. The criterial causation perspective of Tse [25] has much value independent of such links (as also Levy [14] emphasizes), and that value has been the focus here. However, in future work, such philosophical links might be considered as well.

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