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Convergence of a neutron-deuteron multiple-scattering series

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It is shown by means of an example that a multiple-scattering series for neutron-deuteron scattering, based on the Alt-Grassberger-Sandhas version of the Faddeev equations, can be made to converge by subtracting out the deuteron pole in the two-nucleon T matrix. The subtraction is carried out by means of Kowalski's generalized Sasakawa method. The interactions between the nucleons are rank one separable potentials with Gaussian form factors.

[NUCLEAR REACTIONS Scattering theory, neutron-deuteron scattering below breakup threshold.]

A few years ago Sloan[1] investigated the convergence of a multiple-scattering series for elastic neutron-deuteron scattering. The series was derived from the Alt, Grassberger, and Sandhas[2] version of the Faddeev equations,[3] and was studied for the separable Yamaguchi[4] interaction. One of the main results of Sloan's[1] analysis is that the doublet $L=0$ series diverges at low energies. We shall show here, using a slightly different separable interaction, that the multiple-scattering series can be made to converge by "subtracting out" the deuteron pole in the two-nucleon T matrix. This subtraction is carried out by means of Kowalski's[5] generalized Sasakawa[6] method. This method has already been applied successfully to nucleon-nucleon scattering.[7]

For the two-nucleon transition operator we use a spin-dependent s-wave separable interaction of the form

$$t_n(z)=|g_n\rangle\Delta_n(z)\langle g_n|, \quad n=1,2, \tag{1}$$

where $n=1$ and 2 refer to the triplet and singlet states, respectively. Here z is a complex energy parameter, and

$$\Delta_n^{-1}(z)=-\lambda_n^{-1}+\langle g_n|(H_0-z)^{-1}|g_n\rangle . \tag{2}$$

The "strength" of the interaction, which we have assumed to be attractive, is determined by λ_n. H_0 is the kinetic energy operator. The form factor $|g_1\rangle$ can be expressed in terms of the deuteron binding energy B and state vector $|B\rangle$ by means of the relation

$$|g_1\rangle=(-B-H_0)|B\rangle . \tag{3}$$

The parameter λ_1 is adjusted so that

$$\Delta_1^{-1}(-B)=0 , \tag{4}$$

which implies that

$$\Delta_1^{-1}(z)\to z+B, \quad z\to -B . \tag{5}$$

This is the deuteron pole referred to above.

After a partial wave analysis, the equations studied by Sloan[1,8] become

$$X_{Lnm}(q,k;s)=Z_{Lnm}(q,k;s)+\sum_{r=1}^{2}\int_0^\infty Z_{Lnr}(q,q';s)q'^2dq'\times\Delta_r(s-\tfrac{3}{4}q'^2)X_{Lrm}(q',k;s) , \tag{6}$$

where

$$Z_{Lnm}(q,k;s)=J_{nm}\int_{-1}^{1}dx\,P_L(x)\frac{g_n(|\tfrac{1}{2}\vec{q}+\vec{k}|)g_m(|\tfrac{1}{2}\vec{k}+\vec{q}|)}{s-q^2-\vec{q}\cdot\vec{k}-k^2}, \quad x=\hat{q}\cdot\hat{k} ,$$

and

$$s=E+i\epsilon=-B+\tfrac{3}{4}k^2+i\epsilon, \quad 0<\epsilon\ll k^2 . \tag{8}$$

The quantity J_{nm} is a spin-isospin factor,[1] $P_L(x)$ is a Legendre polynomial, and the normalization of the form factors is given by

$$\langle\vec{p}|g_n\rangle=g_n(p)/(4\pi)^{1/2} , \tag{9}$$

assuming that the momentum states have the normalization

$$\langle \vec{p} | \vec{p}' \rangle = \delta(\vec{p} - \vec{p}') \ . \tag{10}$$

E is the total three-particle energy. Using the analysis of Ref. 5, it is easy to show that the solution of Eq. (6), with $m=1$, can be related to the solution of the equation

$$\Gamma_{Ln}(q, k) = Z_{Ln1}(q, k; s) + \sum_{r=1}^{2} \int_0^\infty \Big\{ Z_{Lnr}(q, q'; s)\Delta_r(s - \tfrac{3}{4}q'^2) - Z_{Ln1}(q, k; s)\delta_{r1}[\gamma_L(k,q')/(\tfrac{3}{4}k^2 + i\epsilon - \tfrac{3}{4}q'^2)] \Big\} \cdot q'^2 dq' \Gamma_{Lr}(q',k;s) , \tag{11}$$

by means of the relation

$$X_{Ln1}(q, k; s) = \frac{\Gamma_{Ln}(q, k)}{1 - \int_0^\infty \frac{\gamma_L(k, q') q'^2 dq' \Gamma_{L1}(q', k)}{\frac{3}{4}k^2 + i\epsilon - \frac{3}{4}q'^2}} \tag{12}$$

In order for the deuteron pole to be removed [see Eqs. (5) and (8)], the function γ_L must be normalized so that

$$\gamma_L(k, k) = 1 \ . \tag{13}$$

From Eq. (12) it follows that the on-shell scattering amplitude can be written in the form

$$X_{L11}(k, k; s) = -\frac{3}{2\pi k} e^{i\delta_L} \sin\delta_L \ , \tag{14}$$

where δ_L is the phase shift for the Lth partial wave.

For the separable interaction, we use Bakker's[9] Gaussian, separable potential, since it gives a better fit to the two-nucleon data than the Yamaguchi[4] potential. The form factor is given by

$$g(p) \propto \exp(-p^2/\beta^2) \ , \tag{15}$$

and the parameters λ and β are given in Table 2 of Ref. 10. The parameters were fitted to a deuteron binding energy of 2.2246 MeV, a triplet effective range of 1.747 fm, and a singlet scattering length and effective range of −23.715 and 2.73 fm, respectively.

Equation (11) has been solved by iteration, using the subtraction function

$$\gamma_L(k, q) = (q/k)^L \left(\frac{k^2 + \sigma^2}{q^2 + \sigma^2} \right)^{n+L} \tag{16}$$

There is no fundamental reason for choosing this form. The factor $(q/k)^L$ was introduced by analogy to the two-body problem,[7] and the parameters σ and n where varied to optimize the convergence rate. The results given in Tables I–III are for $\sigma = 2\ \text{fm}^{-1}$ and $n = 3$. In the tables the zeroth order iteration refers to the results obtained by keeping only the inhomogeneous term on the right hand side of Eq. (11). The resulting amplitude is not just the Born approximation, because of the denominator in Eq. (12). In each table the list was terminated when convergence to three significant figures was achieved. It can be seen from the tables that the convergence for the quartet state is rapid for all cases considered. For the doublet state the convergence is rapid, except for $L = 0$. A triton binding energy of 9.41 MeV was found by locating the zero in the denominator of Eq. (12). It was necessary to iterate Eq. (11) 25 times to obtain accuracy to three significant figures with $n = 2$.

We have also carried out calculations of the triton binding energy and doublet scattering length with the potential parameters of Ref. 9. The con-

TABLE I. Neutron-deuteron scattering lengths in fm.

Iteration	Doublet	Quartet
0	14.9	4.30
1	5.15	6.22
2	2.74	6.28
3	1.42	6.30
4	0.819	
5	0.538	
6	0.424	
7	0.382	
8	0.367	
9	0.362	
10	0.361	

TABLE II. Doublet phase shifts in degrees at a neutron lab energy of 2.45 MeV.

Iteration / L	0	1	2	3	4	5
0	66.4	−7.48	1.88	−0.338	0.0718	−0.0150
1	−62.5	−6.86	1.89	−0.342	0.0715	
2	−45.2	−5.83	1.87	−0.341		
3	−31.9	−6.05	1.86			
4	−26.6	−6.04				
5	−25.3					
6	−25.3					
7	−25.6					
8	−25.8					
9	−25.9					

TABLE III. Quartet phase shifts in degrees at a neutron lab energy of 2.45 MeV.

Iteration L	0	1	2	3	4	5
0	−41.6	31.6	−3.02	0.725	−0.140	0.0302
1	−64.8	22.1	−3.36	0.695	−0.142	0.0301
2	−64.7	22.5	−3.38			
3	−65.2					
4	−65.0					
5	−65.1					

vergence pattern is very similar to that reported here, and moreover, our results agree with those of Ref. 9 to better than 0.5%, and in most cases to three significant figures.

It is interesting to note that the number of iterations we need to get convergence for the scattering lengths and phase shifts is comparable to the number needed when Pade techniques are applied to the conventional multiple-scattering series.[11,12] In conclusion, we point out that the iteration scheme we have presented here can be used above the breakup threshold by using the subtraction techniques of Ref. 12 to treat the moving logarithmic singularities that occur in the effective potential given by Eq. (7).

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