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AN EMPIRICAL EQUILIBRIUM JOB SEARCH MODEL WITH CONTINUOUSLY DISTRIBUTED HETEROGENEITY OF WORKERS' OPPORTUNITY COSTS OF EMPLOYMENT AND FIRMS' PRODUCTIVITIES, AND SEARCH ON THE JOB .

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Abstract

In this paper we present and estimate a synthesis of previous equilibrium search models: allowing for continuous distributions of workers opportunity costs of employment as well as firms' productivities. The model allows for on-the-job search, and we assume that job offer arrival rates for workers are independent of their labor market state. We derive the theoretical implications of these assumptions. we provide simulations. and we develop a semi-parametric estimation procedure that we apply to a dataset of individual labor market histories.

Keywords: Labor market equilibrium, n-ages, heterogeneity, unemployment, unemployment benefits, frictions.

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1 Introduction

In the literature on unemployment and the effect of unemployment benefits, search theory has shown to be an adaptable tool. The analysis of partial search models, which focus on worker behavior and treat the wage offer distribution as given, is widespread. Such models are able to explain many stylized facts (see e.g. surveys by Devine and Kiefer (1991), Layard, Nickel1 and Jackman (1991) and Wolpin (1995)). However, a number of important issues can not be analyzed with these partial models. This of course includes all research issues related to wage determination, employer behavior, interaction between worker and employer behavior, and the effects of policies that directly affect wages.

All of the latter issues are of great importance for the analysis of the lower end of the labor market. Most economists and politicians agree that the situation at the lower end of the labor market constitutes one of the biggest problems of modern society. In Europe, long term unemployment of unskilled workers is dramatically high. In the U.S., unskilled workers often work at very low wages (for recent surveys, see e.g. Layard, Nickel1 and Jackman (1991), Krugman (1994), Bean (1994), Card and Krueger (1995) and Snower and De la Dehesa (1997)).

We feel that a search model that aims at giving an accurate and useful description of the lower end of the labor market should not treat the demand side of the labor market as given. In search theory, the optimal strategy of the workers usually has the reservation wage property. If employers know this, then in equilibrium their wage offers must equal the reservation wage of some (group of) worker(s). If the wage offer distribution depends on the distribution of reservation wages, then parameter changes that affect the reservation wages of job searchers also affect the wage offer distribution that they face. It is thus obvious that one has to take account of these interrelation between supply and demand if one wishes to use the model for policy analyses. For instance, an important question as the effect of unemployment benefits on job search by the unemployed cannot be answered satisfactorily without allowing for the possibility that employers respond to changes in the behavior of job seekers. In sum, we need an equilibrium model.

In this paper, we provide a theoretical and empirical analysis of a model that generalizes previous equilibrium search models. Recently, the theoretical and empirical literature on equilibrium search has made substantial progress. The present paper thus contributes to the literature to date, and it is useful for our purposes to briefly summarize this literature.[1] We will argue that our model generalization can be regarded as a step forward in the attempt to provide an accurate description of the lower end

[1] See Davidson (1990), Burdett (1990) and Ridder and Van den Berg (1997) for additional descriptions of this literature.

of the labor market.

Equilibrium search models provide a structural framework for labor markets in which the wage offer distribution which workers face in their search emerges as the equilibrium of a non-cooperative wage-search and wage-posting game between workers and employers. The seminal paper by Diamond (1971) shows that, in an economy where both workers and firms are homogeneous, with no possibility for workers to search while employed, the resulting equilibrium wage distribution is a mass point at the wage prevailing if labor demand is monopolized (which is the workers' opportunity cost of employment). As has been noted above, in the general case, equilibrium wage offers must equal the reservation wage of some (group of) worker(s). Intuitively, this is because otherwise a firm can reduce its wage offer without loss of potential workers. Thus, a model in which potential workers at a firm differ in their reservation wage values may generate wage dispersion. Basically, two approaches can be distinguished in the literature, depending on the source of this reservation wage heterogeneity. In both cases, the Diamond (1971) model serves as the point of departure.

In the first approach (MacMinn (1980), Albrecht and Axell (1984)), workers are heterogeneous by nature. In particular, they differ in their opportunity cost of employment. (The latter may be the result of heterogeneity in their value of leisure, their unemployment benefits, their search intensity, etc.). This implies heterogeneity of the unemployed workers' reservation wages. Allowing for such heterogeneity may generate wage dispersion. It can be shown that the support of the equilibrium wage offer distribution is a subset of (or coincides with) the set of unemployed workers' reservation wages. In this model, the observable exit rate out of unemployment displays negative duration dependence. The latter is an important stylized fact which accounts for the fact that a substantial fraction of the unemployed in Europe experience very long durations. In the model, the negative duration dependence is due to the unobserved heterogeneity in the unemployed workers' opportunity cost of employment. The model thus also predicts that at least some unemployed workers reject at least some of their job offers some of the time. In the absence of the latter, there would be no choice element in unemployment, and as a result the model would not be sufficiently rich to analyze policy measures aimed at modifying unemployed workers' behavior. In particular, if all workers accept all job offers all the time then there is no direct effect of unemployment benefits on unemployment duration. MacMinn (1980) and Albrecht and Axell (1984) also allow for firm heterogeneity in their model. In general, higher productivity firms offer higher wages and are therefore able to attract more workers.

Eckstein and Wolpin (1990) estimate this model. They conclude that the fit to unemployment durations is good while the fit to the wage data is not good. The latter is due to the fact that each point of support of the wage offer distribution necessarily equals the reservation wage of an unemployed worker type (see also the discussion by

Eckstein and Wolpin (1990) of their results).

In the second approach to model wage dispersion in an equilibrium search framework (Mortensen (1990), Burdett and Mortensen (1998)), *ex ante* identical workers are allowed to search for another job while working. Search on the job is by now regarded to be an important source of wage dispersion (see references below) as well as of individual wage growth. Of course, job-to-job transitions simply represent a substantial fraction of all labor market transitions, and they provide an option for unskilled workers to increase their income in the long run. In the equilibrium model, working individuals only change jobs if the wage offer exceeds a reservation wage value that is increasing in their current wage. A firm that sets a high wage is thus able to attract workers from firms offering lower wages. So, if individuals work at different wages then, from the point of view of an employer, the labor supply curve is upward sloping, and there is again a trade-off between the wage and the labor force of the firm, which in turn generates equilibrium wage dispersion.

This model makes some empirically sensible predictions, notably on the relations between job durations, wages, and the size of firms (see Ridder and Van den Berg (1997) for a survey). Yet, the main prediction of the homogeneous model is that the equilibrium wage density is increasing, which is at odds with the data. Additional heterogeneity seems therefore needed to make the Burdett-Mortensen model a reasonable empirical description of labor markets.

Bontemps, Robin and Van den Berg (1997) develop and estimate a version of the Burdett-Mortensen model in which workers are identical but firms differ in their marginal productivity of labor, which is assumed to be continuously distributed (note that the latter type of heterogeneity by itself is not able to generate wage dispersion in the Diamond (1971) model). They show that equilibrium is such that each firm offers a unique wage which is a one-to-one increasing function of its productivity type. They also show that there exists a minimum set of constraints on the shape of any distribution to be implementable as the equilibrium of this game. As a consequence, by choosing an appropriate distribution of productivity types, the model allows a perfect fit to the wage data density. Nevertheless, this model does not provide a satisfactory description of the unemployment duration distribution, and it is not sufficiently rich and flexible to study labor market policies like changes in unemployment benefits.

It is clear that each of the two modeling approaches discussed above captures important labor market phenomena. It is however also clear that, in order to obtain a model that is able to provide a sufficiently rich description of the lower end of the labor market, we need a comprehensive equilibrium search model that combines the two different modeling approaches. In the theoretical part of this paper, we develop such a model. We will be particularly concerned with *continuous* heterogeneity distributions of productivities across firms and of opportunity costs of employment across workers.[2] Qualitative features of the equilibrium solution are more transparent in the continuous case. Moreover, our model allows a feasible nonparametric estimation method for continuous heterogeneity distributions, whereas a similar method for discrete distributions does not exist.

Unfortunately, accounting for both kinds of heterogeneity (workers' search costs and firms' productivities) in the most general way rapidly makes the analysis extremely intractable. We therefore choose here to limit the generality of the model somewhat by assuming that job offer arrival rates are the same whether a worker is unemployed or employed. This may seem restrictive. However, the only other equilibrium search model with both kinds of heterogeneity that has ever been estimated on individual data is provided by Eckstein and Wolpin (1990), and they assume that employed workers do not receive alternative offers. Since other empirical studies based on equilibrium search models either accept the null hypothesis of equality of arrival rates (see Van den Berg and Ridder's **(1998)** analysis of Dutch data), or find that the arrival rate of job offers to employees is an order of magnitude smaller than that of unemployed job searchers[3], at least our model, together with Eckstein and Wolpin's model, could provide useful benchmarks.

We provide a comprehensive theoretical analysis of the model. We investigate whether equilibrium exists and is unique. It turns out that log-concavity of the distribution of workers' opportunity costs of employment is sufficient for uniqueness. We also derive expressions for the distributions of endogenous variables (reservation wages, spell durations, wages, profits, firm sizes) in terms of the primitives of the model. Part of the theoretical analysis concerns the derivation of qualitative features of the equilibrium solutions. For example, we derive necessary conditions for the wage distributions to display a peak at the minimum wage.

We also structurally estimate the model. We develop and apply an estimation method that estimates the frictional parameters as well as the heterogeneity distributions. The distribution of worker heterogeneity is estimated along with the frictional parameters, treating the distribution of observed wages as a nuisance "parameter". The productivity distribution is subsequently estimated nonparametrically, by way of an inversion from observed wages to productivities.

The estimation results are used to shed more light on the situation at the lower end of the labor market. For example, to what extent is unemployment due to large values of workers' opportunity costs of employment, and to what extent are the latter

[2] Mortensen (1990) examines model properties in the special case of discrete heterogeneity distributions with a finite number of points of support.

[3] This is evidence from Kiefer and Neumann's (1993) study of US data and Bontemps, Robin and Van den Berg's (1997) analysis of French data.

due to high unemployment benefits? To what extent do firms exploit the monopsony power that they derive from the presence of search frictions? What happens when the opportunity costs of employment increase?

The empirical analysis is the first analysis ever in which both sources of wage dispersion (that is, worker heterogeneity in the opportunity cost of employment, and on-the-job search) are simultaneously taken into account. As such it is the true synthesis between the empirical analyses in Eckstein and Wolpin (1990) on the one hand, and Bontemps, Robin and Van den Berg (1997) (and the other empirical analyses of (extensions of) the Burdett and Mortensen (1998) model) on the other. We are thus able to address the empirical importance of these two sources of wage dispersion as determinants of total wage variation. Also, since we allow for firm heterogeneity, we are able to address the importance of the latter in obtaining a good fit to the wage data. From the results in Bontemps, Robin and Van den Berg (1997) we expect that allowing for firm heterogeneity enables a perfect fit to the wage data. The distribution of worker heterogeneity can be thought of as being fitted to the unemployment duration data. This can be contrasted to Eckstein and Wolpin (1990), in which the distribution of worker heterogeneity had to account for the fit of both the wage and the duration data distribution.

In Sections 2, 3 and 4 of this paper we develop the theoretical model. The special case in which workers are homogeneous but productivities are continuously distributed has been examined in detail by Bontemps, Robin and Van den Berg (1997). It is also instructive to examine the special case in which firms are homogeneous but workers' opportunity costs are continuously distributed. This case has been studied in some detail by Burdett and Mortensen (1998). In Section 3, we extend their analysis of this special case, and we will pay particular attention to empirically relevant model properties. For the general case, we find that the equilibrium wage distribution cannot be obtained as an explicit expression. We therefore consider numerical examples. Section 5 is devoted to the construction of the estimation method, which is subsequently applied on a dataset of individual labor market histories. Section 6 concludes.

2 The model

The setup of the model we discuss in this paper is basically the same as in Bontemps, Robin and Van den Berg (1997), except that we now allow for heterogeneity in the opportunity cost of employment of workers.

2.1 The labor market setup, and the strategy of workers and firms

Individuals seek to maximize the expected steady-state discounted (at rate p) future income. The opportunity cost of employment is denoted b, which includes search costs and unemployment benefits. Its distribution in the population of individuals is denoted by H, and we assume that H is continuous on its support with infimum point $\underline{b}$ ($\underline{b} \geq 0$) and supremum $\overline{b}$ ($\overline{b} < +\infty$). Job offers accrue at the constant rate $\lambda > 0$. A job offer is characterized by a drawing from a wage distribution F ($\underline{w}$ and $\overline{w}$ denote the infimum and supremum point of the support of F; $\overline{w}$ can a priori be infinite). Note that neither the arrival rate nor the distribution of wage offers depends on the current state of the job searcher (employed or unemployed). Layoffs accrue at the constant rate δ. In the following, we denote $\kappa =$ X/S. The next result is well known,

Proposition 1 *The optimal strategy when unemployed is to accept any wage offer w such that $w \geq b$, where b is his opportunity cost of employment. The optimal strategy when employed is to accept any wage offer strictly greater than the present wage contract.*

Because job offers accrue at the same rate whatever the state of workers, the reservation wage is explicit and equal to the opportunity cost of employment. Thus, the strategy of workers is independent of the steady-state wage offer distribution. This rules out feedback from wages and firm behavior to workers' optimal equilibrium strategies.

Now let us turn to firm behavior. A standard assumption in equilibrium job search models is that workers draw job offers by randomly picking firms using a uniform sampling scheme (here and in the sequel, we use "worker" as a synonym for individual). If a given firm, whatever its size, offers only one wage (which is the maintained assumption in this paper), and if all firms want to expand by posting vacancies (which will be shown to be true below), then $F(w)$ must also be the measure of firms offering a wage less than w. Consider a firm with a flow p of marginal revenue generated by employing a worker. We assume that p does not depend on the number of workers at the firm, and consequently we will refer to this firm as a firm of type p and to p as *the* (labor) productivity of this firm. We assume that such a firm seeks to maximize its steadv-state profit flow:

$$\pi(p, w) = (p - w)l(w), \tag{1}$$

where l (w) is the size of the labor force that it can expect to employ if all workers behave so as to maximize their expected wealth and if all other firms' wage offers are distributed according to the distribution F.

The assumption that a firm offers one wage value to all its potential employees deserves some discussion, in the light of the fact that potential employees have different reservation wages. In particular, unemployed job applicants have different values of b, so their fallback options are of different magnitudes. If firms observe the values of b then they could exploit this by offering lower wages to applicants with lower values of b. It is conceivable that then equilibrium is such that a given firm pays different wages to workers with identical productivity in identical jobs, just because these workers had different reservation wages at the time they applied. We thus have to assume that firms are not able to observe the b-values of job applicants, or that they are not allowed to pay different wages to individuals with different b, or that the costs involved in paying different wages to individuals with different b are too high. The first assumption is an asymmetric-information assumption. It may be plausible if b consists mainly of the non-pecuniary utility of leisure, and if firms are not able to observe the applicant's unemployment duration (since the latter is informative on b). The second and third assumptions can be justified by within-firm fairness constraints or by costs considerations. Manning (1993) argues that costs considerations make the single-wage setting assumption less restrictive for anonymous markets with low-skilled workers than for markets with high-skilled workers. It should be noted that previous studies (e.g. Albrecht and Axell (1984), Eckstein and Wolpin (1990) and Albrecht and Vroman (1996)) also assume that firms do not offer different wages to workers with different b.

Firms may differ by their labor productivity p. The distribution function of p is $\Gamma(p)$, which is continuous; $\underline{p} \geq 0$ is the infimum point of the support of Γ (we assume that $\underline{p} > \underline{b}$; this ensures that there will be production in the economy) and $\overline{p} \leq +\infty$ the supremum. We also assume that p has a finite mean, i.e. $E_\Gamma(p) < \infty$. Note that a firm does not offer a wage exceeding its revenue product p because profits would be negative. As a result, $E_F(w) < \infty$.

At this stage it is already clear that firms do not offer a wage $w < \underline{b}$, because they would not hire anybody and, as a result, would have zero profit. Note that since $\underline{p} > \underline{b}$, it is always possible to operate at a positive profit by offering a wage in between the latter two values. Thus, $F(\underline{b}^-) = 0$ and $\underline{w} \geq 0$. (Here and in the following, $F(x^-) = \lim_{\varepsilon \downarrow 0} F(x - \varepsilon), \overline{F}(w) = 1 - F(w)$.)

We do not assume from the outset that all firms of the same type necessarily have the same strategy. If different wage values yield identical profit, two firms may choose different strategies (this is the case when both workers and firms are homogeneous, see Mortensen (1990)). An optimal strategy for a firm of type p will thus be a point in a set K_p of profit maximizing wages:

$$K_p = \arg\max_w \{\pi(p, w) \quad w \leq p\}$$

with $\pi(p, w)$ as in equation (1).

- Finally, the measure of participants is normalized to one and the (endogenous) measure of unemployed is the unemployment rate u. The measure of firms is also normalized to one.

2.2 The steady-state demographic equilibrium

In this subsection, we derive the implications of the maintained assumption that all demographic stocks do not change when time passes, exit flows being balanced by entry flows.

2.2.1 Distribution of workers' heterogeneity in various subsamples

Let H_u denote the distribution of b in the stock sample of unemployed and H_e the distribution of b in the stock sample of employed. Then, for all b :

$$H(b) = uH_u(b) + (1 - u)H_e(b).$$

Clearly, both probability measures H_u and H_e are dominated by H, hence they admit Radon-Nykodin densities dH_u/dH and dH_e/dH such that

$$1 = u\frac{dH_u(b)}{dH} + (1 - u)\frac{dH_e(b)}{dH}. \tag{2}$$

Clearly also, H_e is dominated by H_u: if a subset B of $\left[\underline{b}, \bar{b}\right]$ has probability $H_u(B)$ equal to **0**, then it is also true that $H_e(B) = 0$, because any employed worker has a positive probability of being laid off. Note however that H_u and H may not be dominated by H_e because no worker with opportunity cost of employment b greater than $\overline{w}$ will ever be willing to be employed (i.e. $H_e(\overline{w}) = 1$). Let us call $\tilde{b} = \min\left\{\bar{b}, \overline{w}\right\}$. This value will have some importance in the sequel. Only the fraction $H(b)$ of workers is really participating to the market. Note that $H(b)$ and $\tilde{b}$, via $\overline{w}$, are functions of the equilibrium wage offer distribution F.

Moreover, in a steady-state demographic equilibrium, assuming that unemployed accept any wage greater *or equal* to the reservation wage, the flow of layoffs must be equal to the flow into unemployment:

$$\delta(1 - u)H_e(b) = \lambda u H_u(b)$$

if $b \leq \underline{w}$, and

$$\delta(1 - u)H_e(b) = \lambda u H_u(\underline{w}) + \lambda u \int_{\underline{w}}^{b} \overline{F}(x^-)dH_u(x)$$

if $b > \underline{w}$. Hence,

$$\frac{dH_e(b)}{dH_u} = \frac{u\kappa}{1-u}\overline{F}(b^-). \tag{3}$$

Combining equalities **(2)** and **(3)** yields:

$$(1-u)H_e(b) = \frac{\kappa}{1+\kappa}H(b) \tag{4}$$

if $b \leq \underline{w}$, *and*

$$(1-u)H_e(b) = \frac{\kappa}{1+\kappa}H(\underline{w}) + \int_{\underline{w}}^{b} \frac{\kappa\overline{F}(x^-)}{1+\kappa\overline{F}(x^-)}dH(x) \tag{5}$$

if $b >$ w. Moreover,

$$uH_u(b) = \frac{1}{1+\kappa}H(b) \tag{6}$$

if $b \leq \underline{w}$, and

$$uH_u(b) = \frac{1}{1+\kappa}H(\underline{w}) + \int_{\underline{w}}^{b} \frac{1}{1+\kappa\overline{F}(x^-)}dH(x) \tag{7}$$

if $b > \underline{w}$.

2.2.2 Distribution of wages in the sample of employed

Let G(w) be the measure of individuals with a wage lower or equal to w in the sample of employed. In a steady-state demographic equilibrium, the flow of layoffs in an interval $(t, t+dt]$ is $\delta(1-u)G($ w). The measure of upgraded wages is, assuming that only wages *strictly greater* than present wage are accepted, equal to $\lambda\overline{F}(w)(1-u)G(w)$. The measure of unemployed individuals of tvpe b accepting a wage *greater or equal* to the reservation wage is λ [F(w) $- F(b^-)]\, u$. At equilibrium, one must have equal flows in and out of unemployment. Hence,

$$\left[\delta + \lambda\overline{F}(w)\right](1-u)G(w) = \lambda u F(w)H_u(\underline{w}) + \lambda u \int_{\underline{w}}^{w} \left[F(w) - F(x^-)\right] dH_u(x) \tag{8}$$

$H_u(\underline{w})$ is the fraction of unemployed who are willing to work for any wage. Integrating by part the right-hand side of this equation yields the following equivalent form of the steady-state employment equation flow:

$$\left[1 + \kappa\overline{F}(w)\right](1-u)G(w) = \kappa u \int_{\underline{w}}^{w} H_u(x)dF(x). \tag{9}$$

Note that using the equations from the preceding subsection, one also obtains that

$$\left[1 + \kappa\overline{F}(w)\right](1-u)G(w) = H(w) - \left[1 + \kappa\overline{F}(w)\right] uH_u(w) \tag{10}$$

The next proposition follows immediately:

Proposition 2 *In a steady-state equilibrium,, one must have:*

$$u = \left[\frac{1}{1+\kappa}H(\underline{w}) + \int_{\underline{w}}^{\overline{w}} \frac{1}{1+\kappa\overline{F}(x^-)}dH(x)\right] + [1 - H(\overline{w})]$$

$$G(w) = \frac{H(w) - \left[1+\kappa\overline{F}(w)\right]\left[\frac{1}{1+\kappa}H(\underline{w}) + \int_{\underline{w}}^{w} \frac{1}{1+\kappa\overline{F}(x^-)}dH(x)\right]}{\left[1+\kappa\overline{F}(w)\right](1-u)}$$

The equation above for u splits the unemplovment rate u into three components that can be associated with three subgroups of individuals: (i) those who accept any job offer while unemployed, (ii) those who accept some job offers and reject others, and (iii) those who reject all job offers. Individuals in the third subgroup are permanently unemployed.

In the model with homogeneous workers, all unemployed workers accept all job offers, so F(w) is equal to the distribution of accepted wages in the inflow at a given point of time into to employment. In the present model, for any given F, the distribution of accepted wages in this inflow first-order stochastically dominates F, because individuals with high b only accept high wages. It is not difficult to show that if F has density f, then the density of accepted wages in the inflow is proportional to $f(w)H_u(w)$. The latter density is of importance in the empirical analysis, as the data provide drawings from the corresponding distribution, and we mav require a good fit to such data.

2.2.3 Measure of workers per firm paying a certain wage

Because of the assumption that workers draw offers by sampling firms using a uniform sampling scheme then, in a steady state equilibrium, the measure of workers $l(w)$ employed by a firm offering a wage w is the Radon-Nykodin density of distribution $(1-u)G$ with respect to F.

Proposition 3 *Any set of F-probability zero is also of G-probability zero and any wage w which is a mass point of F is a mass point of G, and conversely. Moreover, for all wages w in the support of F, the Radon-Nykodin density of distribution $(1-u)G$ with respect to F is:*

$$\begin{aligned} l(w) &= \frac{d(1-u)G(w)}{dF} \\ &= \frac{\kappa(1-u)G(w) + \kappa u H_u(w)}{1+\kappa\overline{F}(w^-)} \\ &= \frac{\kappa H(w)}{\left[1+\kappa\overline{F}(w^-)\right]\left[1+\kappa\overline{F}(w)\right]} \end{aligned} \quad (11)$$

Proof. By definition of the Stieltjes integral the function $\int_{-\infty}^{w} H_u(x)dF(x)$ defines a measure which is absolutely continuous with respect to F, and its Radon-Nykodin density is $H_u(w)$. Moreover, for all positive ε, using equation (9):

$$\begin{aligned}
& \frac{\kappa u}{1-u}\int_{w-\varepsilon}^{w} H_u(x)dF(x) \\
= & \left[1+\kappa_1\overline{F}(w)\right]G(w)-\left[1+\kappa_1\overline{F}(w-\varepsilon)\right]G(w-\varepsilon) \\
= & \kappa_1\left[\overline{F}(w)-\overline{F}(w-\varepsilon)\right]G(w)+\left[1+\kappa_1\overline{F}(w-\varepsilon)\right]\left[G(w)-G(w-\varepsilon)\right].
\end{aligned}$$

It follows that any mass point of F is a mass point of G, and conversely. Moreover, the Radon-Nykodin density of $(1-u)G$ with respect to F is given by the formula stated in the proposition. Note that here we treat u as a constant. □

The expression for l (w) given F equals the product of (i) the expression for l (w) given F in the model in which individuals are homogeneous and (ii) *H(w)*. This makes sense, as *H(w)* measures the proportion of individuals who are willing to participate on a market where only wages smaller than or equal to w are offered.

The following properties of l(w) immediately follow from equation (11), because H is assumed to be continuous:

Proposition 4 *l(w) is a non-decreasing function of w. Moreover, it is right-discontinuous at any point w that is a mass point of F (or, equivalently, G).*

Equilibrium search models therefore predict that the high-wage firms are also the large firms. A prediction which has some empirical content. Note that this is true whatever the equilibrium wage distribution. This is due to the fact that high-wage firms attract more workers. It is important to realize that in our model there are two channels by way of which this works. First, high-wage firms attract more unemployed workers. In particular, they attract workers with relatively high b, who are not attracted by low-wage firms. This is the channel modeled by Albrecht and Axe11 (1984). Secondly, high-wage firms attract more employed workers. In particular, they attract employed workers with relatively high wages, who are not attracted by low-wage firms. This is the channel modeled by Burdett and Mortensen (1998). In the empirical analysis we are able to address the empirical relevance of both channels as means to increase the labor force of a firm, for different labor markets.

2.3 Definition and properties of market equilibrium

The following definition of the economic equilibrium, as well as certain of the propositions below, are adapted from Mortensen (1990) who considers the case of discrete heterogeneity distributions, and from Bontemps, Robin and Van den Berg (1997) who consider the case of a non-degenerate, continuous productivity distribution. The aim of this subsection is to derive analytical characteristics of the equilibrium wage distribution when there is a continuum of workers indexed by their opportunity cost of employment b drawn from a continuous distribution H, and a non-degenerate, continuous productivity distribution Γ.

Definition. A market equilibrium is defined by $(F(\cdot;\, p),\, p > \underline{p})$ such that, simultaneously,

1. the distribution of wage offers in the economy is

$$F(\cdot) = \int F(\cdot; p) d\Gamma(p),$$

2. Each worker whose opportunity cost of employment is b follows the strategy explained in Proposition 1.

3. The strategy of each type-p firm is to randomly draw a wage from a probability distribution $F(\cdot;\, p)$ which puts probability 1 on the set K_p of profit maximizing wages of type-p firms given other firms' and workers' strategies, i.e.

$$K_p = \arg\max_{w} \{\pi(p, w) \quad w \leq p\},$$

with

$$\pi(p,\, w) = (p - w) \frac{\kappa H(w)}{\left[1 + \kappa\overline{F}(w)\right]\left[1 + \kappa\overline{F}(w^-)\right]}$$

Recall that, because of the assumption that the job offer arrival rate in employment has exactly the same value as in unemployment, the optimal strategy of unemployed individuals does not depend on the expression of F. This makes the definition of equilibrium recursive in the specific sense that there is no feedback from firms' behavior to workers' behavior. This simplifies the analysis in comparison to Bontemps, Robin and Van den Berg (1997)[4]. However, as will be shown below, the presence of $H(w)$ in the expression for $\pi(p,\, w)$ complicates the analysis in comparison to Bontemps, Robin and Van den Berg (1997).

Proposition 5 *A wage offer w that attracts workers is profit maximizing for employers of type $p > w$ only if no mass of other employers offer w. Consequently, the equilibrium wage offer distribution has no mass point.*

[4]Note that the optimal strategy of employed individuals does not depend on F regardless of whether the arrival rates are equal.

Proof. see Bontemps, Robin, Van den Berg (1997). This comes from the fact that if there is a mass point in w, then l (w) is also right-discontinuous (and strictly increasing) in w. A firm which offers w can therefore deviate and offer a little more to make more profit. cl

Corollary 1 *l (w) is continuous.*

Proof. This is a simple consequence of the fact that both F and H are continuous. □

Proposition 6 *(a) At the equilibrium, the set K_p of profit maximizing wages of type-j firms is closed.*
(b) Let $p_1 > p_2$ be two productivities, and let $w_1 \in K_{p_1}$ and $w_2 \in K_{p_2}$. Then $w_1 \geq w_2$.

Proof. see Bontemps, Robin and Van den Berg (1997).

In equilibrium, high-productivity firms offer high wages, have higher profits and employ more workers.

Proposition 7 *If the distribution Γ of labor productivities is continuous, then there exists a function K mapping $supp(\Gamma)$ into $supp(F)$, such that K_p = {K(p)} r-almost surely.*

Proof. see Bontemps, Robin and Van den Berg (1997).

It thus follows that, when there is a continuous distribution of firms' types, the only equilibrium strategies are pure strategies. At the equilibrium, only one wage can be profit maximizing for a firm of a given type. The distribution of wage offers is $F(w) = \Gamma(K^{-1}(w))$ where K is an increasing function on $[\underline{p}, \bar{p}]$ (K is increasing because of Proposition 6). Note that $K'(p)$ need not exist everywhere, e.g. because the density $\gamma(p)$ associated with $\Gamma(p)$ need not be continuous. If $\gamma(p)$ is not continuous then K' is to be interpreted as the left-derivative of K. For convenience, we will assume in the sequel that the density $\gamma(p)$ is continuous and positive on the support of I'.

As will be shown below, the support of F is not necessarily connected. This is a consequence of the following. If there is a concentration of workers whose b is close to a certain value b_0, then anv firm offering a wage slightly below b_0 may be better off bv offering a wage larger than or equal b_0. In the latter case, profits per worker will be slightly lower, but the inflow of unemployed workers will be much higher, resulting in a higher over-all profit. This phenomenon was pointed out by Mortensen (1990) for the model with discretely distributed b. In case of a continuous distribution of b, a concentration of workers at sav b_0 means that the density $h(b)$ displays a peak at $b = b_0$. Such peaks may be induced by the welfare system. For example, if the non-pecuniary utility of unemployment is zero and if welfare recipients receive a fixed amount of benefits, depending on their household composition, then the density of b may display peaks at these amounts.

From both a theoretical and an empirical point of view, equilibrium wage (offer) distributions with a non-connected support are unattractive. Such distributions cannot easily be characterized in a transparent way, and this hampers comparative statics analysis. Moreover, estimation of such distributions is complicated by the dependence of the support on unknown parameters. Below we derive sufficient conditions on the shape of H(b) for the support of F to be connected.

3 The special case of homogeneous firms

3.1 The equilibrium

The special case in which workers are homogeneous but productivities are continuously distributed has been examined in detail by Bontemps, Robin and Van den Berg (1997). It is also instructive to examine the special case in which firms are homogeneous but workers' opportunity costs are continuously distributed. This case has been studied in some detail by Burdett and Mortensen **(1998).** Here, we extend their analysis, and we will pay particular attention to empirically relevant model properties.

We start by characterizing the equilibrium wage distribution for the particular case of homogeneous firms, each having the same productivity parameter p. At the equilibrium, each firm necessarily makes the same profit. The wage offer distribution (and consequently l (w)) are continuous, and the profit function is:

$$\pi(w) = (p - w)l(w) = \kappa\frac{(p - w)\, H(w)}{\left[1 + \kappa\overline{F}(w)\right]^2} \qquad (12)$$

Proposition 8 *The support of F is closed.*

Proof. This comes straight from the expression of $\pi(w)$ which is a continuous function of w. □

Proposition 9 *$\underline{w}$ is unique, and it follows from:*

$$\underline{w} = \sup\left\{\underset{w}{\operatorname{argmax}}(p - w)H(w)\right\}$$

Proof. We know from equation (12) that:

$$\pi(\underline{w}) = \frac{\kappa\,(p - \underline{w})\,H(\underline{w})}{(1+\kappa)^2}$$

so:

$$\underline{w} \in \left\{\operatorname*{argmax}_w (p - w)H(w)\right\}$$

Suppose that $w_1 < w_2$ both maximize $(p - w)H(w)$, then we show that $\underline{w}$ cannot be equal to w_1. Suppose the contrary, then $F(w_2) > F(w_1) = 0$. Moreover,

$$\begin{aligned}\pi(w_2) &= \kappa\frac{p - w_2}{\left[1 + \kappa\overline{F}(w_2)\right]^2}H(w_2) = \kappa\frac{p - w_1}{\left[1 + \kappa\overline{F}(w_2)\right]^2}H(w_1)\\ &= \left[\frac{1+\kappa}{1+\kappa\overline{F}(w_2)}\right]^2 \pi(w_1) > \pi(w_1),\end{aligned}$$

which shows that w_1 cannot be profit maximizing. □

Note that Burdett and Mortensen (1998), when deriving $\underline{w}$, omit the fact that $\underline{w}$ is the *supremum* of $\{\arg\max_w (p - w)H(w)\}$ rather than any value of the latter set.

In Subsection 2.1 we assumed that $\underline{b} < \underline{p}$, which here amounts to $\underline{b} < p$. From equation (12) it then follows that for all $w \in (\underline{b}, p)$, $\pi(w) > 0$. Thus, necessarily, $\pi(\underline{w}) > 0$, and $\underline{b} < \underline{w} < p$. This implies that $H(\underline{w}) > 0$. To put this in another way: a mass of workers is willing to work for the minimum wage in the market.

Since $(p - w)H(w)$ decreases for $w > \overline{b}$, we also have that. $\underline{w} < \overline{b}$. This is intuitively plausible, as any firm offering a wage $\underline{w} > \overline{b}$ could increase its profits at no cost by reducing its wage offer. In sum,

Proposition 10 *There holds that* $\underline{b} < \underline{w}$, $\underline{w} < \overline{b}$ *and* $\underline{w} < p$

Now let us turn to the equilibrium solution for F.

Proposition 11 *The equilibrium wage distribution F is unique, and it satisfies*

$$\underline{w} = \sup\left\{\operatorname*{argmax}_w (p - w)H(w)\right\} \tag{13}$$

$$\overline{w} = \sup\left\{w \;\; \text{such that } (p - w)H(w) = \frac{(p - \underline{w})\,H(\underline{w})}{(1+\kappa)^2}\right\} \tag{14}$$

$$F(w) = \frac{1+\kappa}{\kappa} - \frac{1+\kappa}{\kappa}\sqrt{\frac{(p - w)H(w)}{(p - \underline{w})H(\underline{w})}}, \qquad (w \in Supp(F)) \tag{15}$$

The support of F is defined recursively.
First denote

$$L_w = \{x \in]\underline{w}, w[\text{ such that }]x, w[\subset L_w \text{ and } (p - w)H(w) \text{ is decreasing on }]x, w[\}$$
$$s(w) = Inf_w\,(L_w)$$

Take first $w_1 = \overline{w}$:
**if* $s(\overline{w}) = \underline{w}$ *then* $Supp(F) = [\underline{w}, \overline{w}]$
**else* $w_2 = s(w_1), w_3 = Sup\ \{w < w_2 \text{ such that } (p - w)H(w) = (p - w_2)H(w_2)\}$.
**Go back to the first step with* w_3 *instead of* w_1 *until* $s(w_{2i+1}) = \underline{w}$.

$$Supp(F) = \cup[w_{2i}; w_{2i-1}]$$

Proof. By the condition of equal profit we know that for all w on the support of F, there holds that $\pi(w) = \pi(\underline{w})$, which gives equation (15). However, $(p - w)H(w)$ (which we call q(w) for convenience) may increase on $[\underline{w}, p]$.

If q(w) is decreasing on $[\underline{w}; p]$ then:

$$\overline{w} = Arg\left\{w > \underline{w}, \quad q(w) = \frac{q(\underline{w})}{(1+\kappa)^2}\right\} \tag{16}$$

which exists because of $q(p) = 0$, so $Supp(F) = [\underline{w}, \overline{w}]$.

Now let us assume that there is a unique interval of $[\underline{w}; p]$, $[w_0; w_1]$, where q is increasing (we will prove the result in this case, the proof being the same in the general case). There are two possibilities.

- $q(w_0) > q(\underline{w})/(1+\kappa)^2$ (see Figure 1). Then $\overline{w}$ is uniquely defined by (16). Define

$$\tilde{w}_0 = \{w \in]\underline{w}; w_0[, \quad q(w) = q(w_1)\}, \quad \tilde{w}_1 = \{w \in]w_1; \overline{w}[, \quad q(w) = q(w_0)\}$$

It is obvious that $([\underline{w}; \tilde{w}_0] \cup [\tilde{w}_1; \overline{w}]) \subset Supp(F)$. However:

$$1 + \kappa\overline{F}(\tilde{w}_0) = (1+\kappa)\sqrt{\frac{q(\tilde{w}_0)}{q(\underline{w})}} > (1+\kappa)\sqrt{\frac{q(\tilde{w}_1)}{q(\underline{w})}} = 1 + \kappa\overline{F}(\tilde{w}_1)$$

Take $w' \in]\tilde{w}_0; w_0]$, $w'' \in]w_1; \tilde{w}_1]$ such that $q(w') = q(w'')$:
$[\underline{w}; w'] \cup [w''; \overline{w}]$ seems to be a good candidate for $Supp(F)$ as q is decreasing on this interval and $1 + \kappa\overline{F}$ decreases from $1 + \kappa$ to 1. However, for ε such that $w'' - \varepsilon > w_1$:

$$\pi(w'' - \varepsilon) = \kappa\frac{q(w'' - \varepsilon)}{\left[1 + \kappa\overline{F}(w'')\right]^2} > \kappa\frac{q(w'')}{\left[1 + \kappa\overline{F}(w'')\right]^2} = \pi(w'')$$

and a firm which offers w'' would have an incentive to offer $w'' - \varepsilon$. Consequently, such a situation can not be an equilibrium and $Supp(F) = [\underline{w}; \tilde{w}_0] \cup [w_1; \overline{w}]$.

- $q(w_0) < q(\underline{w})/(1+\kappa)^2$ (see Figure 2). Then the set $\{w > \underline{w}$ such that $q(w) = q(\underline{w})/(1+\kappa)^2\}$ contains two points x' and x" (with $x' < x''$).

  Following our previous results leads to two possibilities: either $Supp(F) = [\underline{w}; x']$ or $Supp(F) = [\underline{w}; \tilde{w}_0] \cup [w_1; x'']$ (where $\tilde{w}_0$ is defined as before: $\tilde{w}_0 = \{w \in]\underline{w}; w_0[, \; q(w) = q(w_1)\}$). But if $Supp(F) = [\underline{w}; x']$:

$$\begin{aligned}\pi(w_1) &= \kappa\frac{q(w_1)}{\left[1+\kappa\overline{F}(w_1)\right]^2} = \kappa\frac{q(w_1)}{\left[1+\kappa\overline{F}(x')\right]^2} \\ &> \kappa\frac{q(x')}{\left[1+\kappa\overline{F}(x')\right]^2} = \pi(x')\end{aligned}$$

  Consequently, $Supp(F) = [\underline{w}; \tilde{w}_0] \cup [w_1; x'']$

c l

According to Proposition 11, equilibrium is unique if both sources of wage dispersion (search on the job and dispersion of workers' opportunity costs of employment) are present, workers' search technology is invariant across the labor market state ($\lambda_0 = \lambda_1$), and firms are homogeneous. Burdett and Mortensen (1998) show that equilibrium is unique in the case in which workers as well as firms are homogeneous while search on the job is possible. However, Bontemps, Robin and Van den Berg (1997) show that if the Burdett and Mortensen (1998) model is generalized to allow for dispersion of firms' productivities, then multiple equilibria are possible.

Note that Proposition 11 provides a recursive algorithm for F. This can be used to obtain explicit expressions for F given any H.

3.2 Empirically relevant properties

We now discuss some empirically relevant qualitative features of the equilibrium. First of all, Proposition 11 implies that the support of F may have gaps. Specifically, either the support is connected, or it consists of a countable set of non-connected intervals between $\underline{w}$ and p. As explained in the previous section, gaps may occur if there are concentrations of workers with more or less the same value of b. This is also clear from the proof above: in order to generate gaps, $(p - w)\,H(w)$ must increase for some w.

To be more specific, if $H(w)$ is such that $(p - w)\,H(w)$ is non-increasing in w for all $w \in (\underline{w}, p)$, then the support of F is connected. Note that (i) $(p - w)H(w)$ is zero at $\underline{b}$ and at p, (ii) it is positive in between these values, and (iii) $\underline{w}$ is the highest value at which this function attains its global maximum.

Proposition 12 *If the distribution $H(b)$ is log-concave then F has a connected support.*

Proof. There holds either that $\underline{w} < \bar{b}$ or that $\underline{w} = \bar{b}$. We start with the first case. Then $\underline{w}$ satisfies the first-order condition $(p - w)h(w) - H(w) = 0$. Since $H(\underline{w}) > 0$, this is equivalent to

$$\frac{h(w)}{H(w)} = \frac{1}{p-w} \quad \text{at} \quad w = \underline{w} \tag{17}$$

If $H(b)$ is log-concave then $h(b)/H(b)$ is non-increasing in b on $(\underline{b}, \infty)$. So, for all $w \in (\underline{w}, p)$, the left-hand side of **(17)** is non-increasing in w whereas the right-hand side increases in w. This implies that $(p - w)\, H(w)$ decreases on the support of F.

Now consider the case $\underline{w} = \bar{b}$. It is obvious from Proposition **11** that then F is like in the fully homogeneous model, and, in particular, that F has a connected support. □

It is well known that sufficient for a distribution function $H(b)$ to be log-concave is that its density function h(b) is log-concave. Examples of families of distributions with log-concave densities are the exponential, normal and uniform families. For other examples, see Van den Berg **(1994).** Log-concavity of H (or h) rules out that the support of H itself has gaps. Moreover, it ensures that h does not have pronounced spikes (see Van den Berg **(1994)).** Note that the log-concavity condition is by no means necessary to obtain a connected support for F.

Note that from the proof of this proposition it follows that if $H(b)$ is log-concave then the set $\{\arg\max_w (p - w)H(w)\}$ contains exactly one element $w \in (\underline{b}, p)$, so then $\underline{w}$ is uniquely described by this set.

Now that we have examined the connectedness of the support of F, let us turn to differentiability properties of F on its support, and the behavior of F and G close to $\underline{w}$. We should point out immediately that $F(w)$ need not be differentiable everywhere on the interior of its support. Specifically, if $\bar{b} < \overline{w}$ and if $\lim_{b\uparrow\bar{b}} h(b) > 0$ then $F(w)$ is not differentiable at $w = \bar{b}$ (see equation **(15)).**

Proposition 13 (a) Differentiability. *If H is C^n then F is C^n as well, except possibly at $w = \bar{b}$. Moreover, if H is C^1 then we can express the wage earnings density $g(w)$ as follows,*

$$(1 - u)g(w) = \kappa \frac{f(w)H(w)}{\left[1 + \kappa \overline{F}(w)\right]^2} \tag{18}$$

(b) Lowest wage. *If $\underline{w} < \bar{b}$ then*

$$f(\underline{w}) = g(\underline{w}) = 0$$

($f(\underline{w})$ means $\lim_{w \downarrow \underline{w}} f(w)$, etc.) while if $\underline{w} = \bar{b}$ then

$$f(\underline{w}) = \frac{1+\kappa}{2\kappa(p-\bar{b})} = g(\underline{w})(1+\kappa) > 0$$

Proof. Straight from Proposition 2 and equations (15) and (13). □

As a result, it follows that $f(w)$ and $g(w)$ do not display a peak at $\underline{w}$. If $\underline{w} = \bar{b}$ then sure enough $f(\underline{w})$ and $g(\underline{w})$ are both positive, but it is straightforward to show that in such a case both f and g are increasing on their support (the equilibrium is then equivalent to that in a homogeneous model). This inability to generate a peak at $\underline{w}$ is an undesirable property of the model, since such spikes are often observed (see Card and Krueger (1995)). Note that the model with productivity dispersion is able to generate such peaks (see Bontemps, Robin and Van den Berg (1997)).

We now turn to a discussion of the shapes of F that can be generated from the model with homogeneous firms. This turns out to be useful for empirical analyses. First of all, consider the shape of $F(w)$ on $w \in (\bar{b}, \overline{w})$, which is relevant only if $\overline{w} > \bar{b}$. It is obvious from Proposition 11 that then the shape of $F(w)$ on $(\bar{b}, \overline{w})$ is the same as in the homogeneous model. Now consider the shape of $F(w)$ below $\bar{b}$, which is the generic case. In that case, h(w) > 0. Rewrite equation (15) as follows,

$$H(w) = \left(\frac{1+\kappa\overline{F}(w)}{1+\kappa}\right)^2 \cdot \frac{(p-\underline{w})H(\underline{w})}{p-w}$$

By differentiating this with respect to w we obtain that h(w) > 0 iff

$$\frac{f(w)}{1+\kappa\overline{F}(w)} < \frac{1}{2\kappa(p-w)}$$

Now integrate this inequality over an interval (w_1, w) in the interior of the support of F. We obtain

Proposition 14 *There holds that*

$$\overline{F}(w) > -\frac{1}{\kappa} + \frac{1}{\kappa}(1+\kappa\overline{F}(w_1)) \cdot \sqrt{\frac{p-w}{p-w_1}} \tag{19}$$

But the right-hand side of the latter inequality is nothing but the expression for $\overline{F}(w)$ in the homogeneous model in which $w_1 = \underline{w}$. Somewhat loosely we may therefore say that there is *more* probability mass at high w than in the homogeneous model, and that the distribution of F is more skewed to the left than in the homogeneous model. (Recall though that there may be gaps in the support given certain H.)

In sum, the shape of the wage offer distribution is at least as skewed to the left as in the homogeneous model. Now it may be argued that the shape of F is not of importance for the reason that in the model with heterogeneous workers, F does not correspond to a random variable that is readily observed in longitudinal labor supply data.[5] First of all, the density of wages in the flow from unemployment to employment is proportional to f(w) $H_u(w)$. But this means that this densitv is even more skewed to the left than f(w). Secondly, the density of cross-sectional wages is given by g(w). But from equation (18) it follows that g(w) first-order stochastically dominates g(w) in the homogeneous model, for any given F. So, g(w) is even more skewed to the left than it would be in the homogeneous model, for any given F. Thus, the conclusions regarding the shape of F are reinforced for the distributions of wages in the inflow into employment and cross-sectional wages. Now, it is a well-established fact that the homogeneous model gives a bad fit to data on such wages, precisely for the reason that F is skewed to the left and accordingly has an increasing density (see Van den Berg and Ridder (1998) and Ridder and Van den Berg (1997)). Thus, we conclude that the model with worker heterogeneity cannot be expected to give a satisfactory fit to the wage data.

In contrast, in the model with heterogeneous firms, f(w) and g(w) cannot increase as fast as in the homogeneous model. Indeed, in that model, the set of admissible wage (offer) distributions is characterized by the property that their densities do not increase as fast as in the homogeneous model (see Bontemps, Robin and Van den Berg (1997) for technical details). This can be regarded as an advantage from an empirical point of view. For example, the model with heterogeneous firms is able to provide a perfect fit to wage (offer) distributions with decreasing densities. We therefore conclude that, to obtain a good fit to wage data, it is necessary to allow for heterogeneity in firms' productivities.

It should be noted that the inequality (19) on $\overline{F}(w)$ above does not preclude that f (w) decreases for *some* w. (Technically, the inequality is on F(w) instead of f(w) .) We will now examine brieflv to what extent it is possible to obtain a wage offer density which is decreasing for large w if firms are homogeneous. To obtain a density that decreases for all w sufficiently close to $\overline{w}$, it is necessary that $\overline{w} \leq \overline{b}$, since otherwise the shapes of f and g near $\overline{w}$ are as in the fully homogeneous model (i.e. increasing in W).

Using equations (14) and (15), it can be shown that, in general,

$$f(\overline{w}) = \frac{1}{2\kappa}\left[\frac{1}{p-\overline{w}} - \frac{h(\overline{w})}{H(\overline{w})}\right]$$

[5]Firm data may however provide observed random drawings from F. Bontemps, Robin and Van den Berg (1997) find that such data do not support distributions that are skewed to the left.

$$g(\overline{w}) = \frac{1}{2(1-u)}\left[\frac{H(\overline{w})}{p-\overline{w}} - h(\overline{w})\right]$$

A first idea may be to increase κ, but this will push up wages to p, as frictions vanish. Formally,

$$(p-\overline{w})H(\overline{w}) = \frac{(p-\underline{w})\,H(\underline{w})}{(1+\kappa)^2}$$

then:

$$f(\overline{w}) = \frac{1}{2\kappa}\left[\frac{(1+\kappa)^2\,H(\overline{w})}{(p-\underline{w})\,H(\underline{w})} - \frac{h(\overline{w})}{H(\overline{w})}\right]$$

$$\text{and} \qquad f(\overline{w})_{\kappa\to+\infty}\longrightarrow +\infty$$

From equation (15), if f is decreasing in w then

$$(p-w)^2(2H(w)h'(w) - h^2(w)) \geq H^2(w)$$

(where h' is the second derivative of H), which implies that necessarily the density of workers heterogeneity is increasing in w (because $H(w) \geq (p-w)h(w)$ on the support of F). In fact it seems that the best way to make f decreasing near the maximum wage is to take a very small value of κ, in order to make the ratio $\frac{(1+\kappa)^2}{\kappa}H(\overline{w})$ as small as possible (but as $\kappa \to 0$, $\overline{w} \to \underline{w}$). It is therefore very restrictive to obtain such case and impossible to have $f(\overline{w}) = 0$.

Figure 3 shows three cases to illustrate this discussion (the left-hand side displays the density of the workers' opportunity costs of employment, and the right-hand side the equilibrium wage offer density). Figure 3.a is the usual case (we choose $\delta = 0.02$, $\kappa = 2$, $p = 20000$). Figure 3.b shows us that, with the right parameters, it is possible to have a f(w) which decreases somewhere (we choose a bimodal distribution for H, and $\delta = 0.02$, $\kappa = 1$, $p = 20000$). In Figure 3.c we diminish κ (0.1 instead of 1) in order to reduce the value of $\overline{w}$. Other examples for which f(w) increases for low w but decreases for high w can be obtained with $H(b) = 3b + 2(1-b)^{3/2} - 2$ with support $b \in (0,1)$ (take e.g. $p = 1.33$ and $\kappa = 0.002324$; then $\underline{w} = 0.9561$ and $\overline{w} = 1$). However, this does not provide an appealing shape for F either. As a conclusion, we reaffirm that heterogeneity in firms' productivities is necessary for a good fit to wage data.

We end this section bv making a remark on the fit to unemployment duration data. In the homogeneous model and in the model with only firm heterogeneity, all unemployed individuals accept all job offers, and the unemployment duration t_u has an exponential distribution with parameter λ. However, if workers' opportunitv costs of employment are dispersed then, conditional on b, t_u has an exponential distribution with parameter $\lambda\overline{F}(b)$ (note that this parameter is zero if $b \geq \overline{w}$, so in that case t_u is degenerate at infinity). If b is unobservable to a certain extent, then the distribution of observed unemployment durations is obtained by integrating this exponential distribution with respect to b. This gives a mixture of exponentials, so the observed exit rate out of unemployment displays negative duration dependence. This can be regarded as an advantage of the model with worker heterogeneity, since observed exit rates out of unemployment often display such negative duration dependence (see e.g. the survey in Devine and Kiefer (1991)).

4 Characterization of equilibrium

4.1 The firm's optimal strategy

From Subsection 2.3 it follows amongst other things (i) that a higher productivity p is associated with a higher wage offer K(p), for all p, with K(p) continuous, (ii) that the distribution F(w) is continuous, and (iii) that $\underline{w} = K(\underline{p})$.

Proposition 15 $\underline{w} \in \arg\max_x \left((\underline{p} - x)H(x)\right)$ *with* $\underline{b} < \underline{w} < \underline{p}$ *and* $w \leq \overline{b}$.

Proof. Let $\pi(p)$ denote the profit flow of a firm with productivity p, if this firm offers its optimal wage K(p). For $\underline{w} = K(\underline{p})$,

$$\pi(\underline{p}) = \kappa \frac{(\underline{p} - \underline{w})H(\underline{w})}{(1 + \kappa)^2}$$

and the maximization of the profit for a type $\underline{p}$ firm gives the result. The final line follows directly from the previous sections. □

As a first remark, the result that $\underline{w} < p$ is in contrast to the result in the model with homogeneous workers (see Bontemps, Robin and Van den Berg (1997)) that both variables can be equal. Here, as long as $\underline{p} > \underline{b}$, any firm with the least profitable production technology is able to earn a positive profit per worker by offering a wage in between $\underline{p}$ and $\underline{b}$. As we will see below, this can be expected to have implications for the fit of the model.

As a second remark, by analogy with the proof of Proposition 12, if $H(b)$ is log-concave then the set $\{\arg\max_x (p - x)H(x)\}$ contains exactly one element in $(\underline{b}, \underline{p})$, so then $\underline{w}$ is uniquely described by this set. We return to this below.

Now consider the steady state profit flow $\pi(p, w)$ of a firm with productivity p offering w: there holds that $\pi(p, w) = (p - w)l(w)$, with $l(w)$ as specified earlier. The optimal $w = K(p)$ given p and F follows from first order conditions: by taking $\partial\pi(p, w)/\partial w = 0$ we obtain:

$$- \left[1 + \kappa\overline{F}(w)\right] H(w) + \left(2\kappa f(w)H(w) + h(w) \left[1 + \kappa\overline{F}(w)\right]\right) (p - w) = 0 \qquad (20)$$

under the restriction that $w \geq \underline{w}$, where $w = K(p)$.[6] Equation (20) is an implicit equation for w given p and F, for given "parameters" (or primitives) κ and H. Although we do not use equation (20) in the upcoming derivation of the expressions, it turns out to be useful for other purposes later on.

The profit flow $\pi(p)$ of a firm with productivity p offering $K(p)$ equals $(p-K(p))l(K(p))$. Differentiation with respect to p, using the Envelope Theorem, gives that $\pi'(p) = l(K(p))$. Note that this, together with the facts that l and K are increasing, implies that the profit flow $\pi(p)$ is convex in p. This suggests that high-productivity firms have much more monopsonv power in the labor market than low-productivity firms. We return to this issue later on.

We have

$$\pi(p) = \pi(\underline{p}) + \int_{\underline{p}}^{p} l(K(x))dx.$$

with $\pi(p)$ following from equation **(4.1).** Since $\overline{F}(K(x)) = \overline{\Gamma}(x)$, we can write $l(K(p))$ as

$$l(K(p)) = \frac{\kappa.H \circ K(p)}{\left[1 + \kappa_1\overline{\Gamma}(x)\right]^2}.$$

As a result,

$$\pi(p) = \kappa \left[\frac{(\underline{p} - \underline{w})H(\underline{w})}{(1+ \kappa)^2} + \int_{\underline{p}}^{p} \frac{H \circ K(x)dx}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2}\right] \tag{21}$$

Equation (21) expresses $\pi(p)$ in terms of p, Γ, $\underline{w}$, H and κ. Apart from $\underline{w}$, these are true primitives of the model. Note that $\lim_{p\downarrow p} \pi'(p) = \kappa H(\underline{w})/(1 + \kappa)^2 < \kappa H(\overline{w}) = \lim_{p\uparrow \overline{p}} \pi'(p)$.

From $\pi(p) = (p - K(p))l(K(p))$ it follows that $K(p) = p - \pi(p)/l(K(p))$. Substitution of $\pi(p)$ and $l(K(p))$ gives $w = K(p)$ in terms of p, Γ, $\underline{w}$, H and κ. As a result,

Proposition 16 *The wuge offer $w \equiv K(p)$ of a firm with productivity p satisfies the following implicit equation*

$$K(p) = p - \left[\frac{\underline{p} - \underline{w}}{\left[1 + \kappa\right]^2} H(\underline{w}) + \int_{\underline{p}}^{p} \frac{H \circ K(x)}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2}dx\right] \frac{\left[1 + \kappa\overline{\Gamma}(p)\right]^2}{H \circ K(P)} \tag{22}$$

The profit flow satisfies equation (21), which is an increasing, convex function of p.

[6] To keep the exposition relatively brief and readable, we here omit details concerning the possible occurrence of gaps in the support of F and the possible non-differentiability of F at $\overline{b}$.

Note that, for $w > \bar{b}$, the solution for $K(p)$ is qualitatively the same as in the model with homogeneous workers (see Bontemps, Robin and Van den Berg (1997)).

It should also be noted that the support of the equilibrium F need not be connected. A gap corresponds to a value of p for which $K'(p) = \infty$.

Proposition 17 *If $H(b)$ is log-concave then*
(a) *the support of F is connected,*
(b) *the equilibrium exists and is unique.*

Proof. That the support of F is connected if H is log-concave follows by a simple rewriting of the expression for $K'(p)$ below, which follows from equation (20),

$$K'(p) = \left[\frac{2\kappa\gamma(p)}{1+\kappa\bar{\Gamma}(p)} + \frac{h(w)K'(p)}{H(w)}\right](p - w)$$

Now consider (b). If $H(b)$ is log-concave then $\underline{w}$ is unique (this follows from the previous section). The function $K(p)$ given $\underline{w}$ is given recursively in Proposition 16. This gives F given the primitives of the model. Note that there is no feedback from F to the workers' strategies. □

We briefly consider the case of the introduction of a legal (or mandatory) minimum wage w_l in the economy. Two cases are possible. First, suppose that $w_l < \underline{p}$. Then:

$$\underline{w} \in \operatorname*{argmax}_{x \geq w_l} (\underline{p} - x)H(x)$$

and $K(p)$ follows as above. It should be noted that $\underline{w}$ is not necessarily the legal minimum wage even if w_l exceeds the minimum wage that would prevail in absence of a legal minimum wage. In other words, an increase in the legal minimum wage w_l may result in a new actual lowest wage in the market $\underline{w}$ that exceeds the value of w_l. However, it can be shown that if $H(b)$ is log-concave then the latter cannot occur, and the new actual lowest wage $\underline{w}$ equals w_l.

In general, a sufficiently large increase in w_l shifts up wages, which makes it easier for unemployed workers with high b to find jobs, which in turn reduces unemployment.

Now consider $w_l \geq \underline{p}$. Then a fraction $\Gamma(w_l)$ of firms will disappear because of the introduction of such a minimum wage. The new distribution of the firms' productivities will be the old one truncated at the minimum wage. The new minimum productivity in the economy will also be the minimum legal wage which becomes the minimum wage offered at the new equilibrium.

In Subsection 4.3 we return to the effects of an increase in w_l.

4.2 Tail properties

Proposition 18 ***The upper bound $\overline{w}$ of the support of F(w) and G(w) is finite.***

Proof. This is trivially true if $\overline{p} < \infty$. Let $\overline{p} = \infty$ and assume the contrary. Consider p_0 such that $K(p_0) \geq \overline{b}$ (which implies that $H \circ K(p_0) = 1$). For any p between p_0 and $\overline{p}$, equation (22) can be written as:

$$
\begin{aligned}
K(p) &= p - \left[\frac{\underline{p} - \underline{w}}{(1+\kappa)^2} H(\underline{w}) + \int_{\underline{p}}^{p} \frac{H \circ K(x)}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2} dx\right] \left[1 + \kappa\overline{\Gamma}(p)\right]^2 \\
&= p - \left[1 + \kappa\overline{\Gamma}(p)\right]^2 \left[\int_{p_0}^{p} \frac{1}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2} dx\right] \\
&\quad - \left[\frac{\underline{p} - \underline{w}}{(1+\kappa)^2} H(\underline{w}) + \int_{\underline{p}}^{p_0} \frac{H \circ K(x)}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2} dx\right] \left[1 + \kappa\overline{\Gamma}(p)\right]^2 \\
&= p\left(1 - \left[1 + \kappa\overline{\Gamma}(p)\right]^2\right) + p_0 \left[1 + \kappa\overline{\Gamma}(p)\right]^2 \\
&\quad + \left[1 + \kappa\overline{\Gamma}(p)\right]^2 \int_{p_0}^{p} 1 - \frac{1}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2} dx \\
&\quad - \left[\frac{\underline{p} - \underline{w}}{(1+\kappa)^2} H(\underline{w}) + \int_{\underline{p}}^{p_0} \frac{H \circ K(x)}{\left[1 + \kappa\overline{\Gamma}(x)\right]^2} dx\right] \left[1 + \kappa\overline{\Gamma}(p)\right]^2
\end{aligned}
$$

The limit as $p \to \infty$ of the second and the fourth terms equals respectively p_0 and $(K(p_0)-p_0)/[1+\kappa\overline{\Gamma}(p_0)]^2$. The assumption that $E_\gamma(p) < \infty$ implies that $\lim_{p\to\infty} p\overline{\Gamma}(p) = 0$. Therefore, the limit as $p \to \infty$ of the first term is 0.

Now consider the third term. The integral I in it can be rewritten as

$$I = \int_{p_0}^{p} \overline{\Gamma}(x) \frac{2\kappa + \kappa^2\overline{\Gamma}(x)}{1 + 2\kappa\overline{\Gamma}(x) + \kappa^2\overline{\Gamma}^2(x)} dx.$$

It can be shown that the ratio within the integral is strictly smaller than 2κ, for any $x < \overline{p}$. Therefore, for any $p < \overline{p}$,

$$I \leq \int_{p_0}^{p} \overline{\Gamma}(x) 2\kappa dx \leq \int_{\underline{p}}^{\overline{p}} \overline{\Gamma}(x) 2\kappa dx = 2\kappa \left[E_\gamma(p) - \underline{p}\right] < \infty.$$

In sum, $\lim_{p\to\infty} K(p) < \infty$ which is in contradiction with our assumption. Consequently $\overline{w}$ is finite. □

The finiteness of $\overline{w}$ implies that firms with a high productivity have very high monopsony power. Indeed, if $\overline{p} = \infty$ then the monopsony power index $(p - w)/p$ tends to its maximum value 1 for high values of p. Although on-the-job search and high b-values boost worker power, this is not sufficient to prevent firms with high productivities to take a large part of the rents. The firms at the upper end of the market do not have to fear much competition, as most other firms have lower productivity and lower wages. This is in sharp contrast to the position of low productivity firms. Their situation is restricted by the fact that they must pay more than the wage floor.

Proposition 19 (a) *Suppose that the lowest wage offered in the market is not the legal minimum wage. If* $\underline{w} < \overline{b}$ *then*

$$f(\underline{w}) = g(\underline{w}) = 0$$

whereas if $\underline{w} = \overline{b}$ *then both* $f(\underline{w}) > 0$ *and* $g(\underline{w}) > 0$.
(b) *If* $\overline{p} = +\infty$ *then*

$$f(w) = g(\overline{w}) = 0 \quad \text{and} \quad \overline{w} \geq \overline{b}$$

(c) If $\overline{p} < +\infty$ *then in general both* $f(\overline{w}) > 0$ *and* $g(\overline{w}) > 0$ *(although in special cases both can be zero).*

Proof. (a) This comes straight from equation (20), using $H(\underline{w}) = (p - \underline{w})h(\underline{w})$ and $\underline{p} - \underline{w} > 0$.
(b) Let $\overline{p} = +\infty$. We know that $\overline{w} < \infty$. By taking the limit in equation (20), it follows that both $f(\overline{w})$ and $h(\overline{w})$ tend to zero. Because of the latter, $\overline{w} \geq \overline{b}$.
(c) This also follows from equation *(20)*. In fact,

$$f(\overline{w}) = \frac{1}{2\kappa}\left[\frac{1}{\overline{p} - \overline{w}} - \frac{h(\overline{w})}{H(\overline{w})}\right]$$
$$g(\overline{w}) = \frac{1}{2(1-u)}\left[\frac{1-H(\overline{w})}{\overline{p} - \overline{w}} - h(\overline{w})\right]$$

□

Result (b) implies that if $\overline{p} = \infty$ then there are no permanently unemployed workers. In this case, if $\overline{w} > \overline{b}$, all results in Bontemps, Robin and Van den Berg (1997) on the behavior of the right tails of f and g are also valid for the current model.

Result (a) above may have a strong implication for the fit of the model. Suppose that the density of cross-sectional wage data has a nonzero limit at the lowest wage (so $g(\underline{w}) > 0$). This is only compatible to $\overline{b} = \underline{w}$. This implies that unemployed workers always accept all job offers, so there cannot be any observed duration dependence in the exit rate out of unemployment. (Note that in case of a legal minimum wage this implication does not follow.)

4.3 Simulations

As it is impossible to derive general comparative statics results, we use simulations to shed more light on how the structural parameters affect, the equilibrium solutions. As our baseline model, we take $\delta = 0.005$ and $\kappa = 20$, and we take H to be a normal[7] distribution with mean 2500 and standard error 1000. Note that this H is log-concave. Finally, the productivity distribution Γ is Pareto with a minimum value equal to 3000 and a rate of decrease equal to 2.8 (so $\overline{\Gamma}(p) = (3000/p)^{2.8})$). Figure 4 plots the equilibrium wage offer and cross-sectional wage (or wage earnings) densities (f(w) and g(w)., respectively), the one-to-one function $K(p)$ between p and w, and the density h(b). The equilibrium unemployment rate is 7.3% and the lowest wage offer $\underline{w}$ equals 2078.

4.3.1 Change in κ

First, consider the effect of a change in κ, holding the others parameters constant. Figure 5 displays the evolution of some quantities as κ changes. Figure 5a plots the unemployment rate as a function of κ, as compared to the unemployment rate without worker heterogeneity (which is equal to $1/(1 + \kappa)$). Figure 5b and Figure 5c plot the average monopsony power (defined as the average of $(p - w)/p$) and the wage function, for five cases: $\kappa = 0.1$ (in solid line), $\kappa = 5$ (dots and dashes), $\kappa = 20$ (short dashes), $\kappa = 50$ (dotted) and $\kappa = 500$ (dashed). Figure 5d plots the wage offer density for three cases: $\kappa = 0.1$ (in solid line), $\kappa = 5$ (dots and dashes) and $\kappa = 500$ (dashed). As κ increases, firms lose monopsony power: the monopsony power decreases whereas the wage function increases, and the wage offer density shifts to the right. The unemployment rate decreases because workers accept offers more often (as the wage function increases) and these offers accrue at a higher rate.

4.3.2 Imposition and change of the legal minimum wage

Now let us examine how the equilibrium solution changes when a legal minimum wage w_l is imposed (or changes). Note that an increase of the legal minimum wage may reduce the measure of active firms. We make κ dependent on that measure by way of the relation $\kappa = 20\overline{\Gamma}(p)$. This captures the idea that the job offer arrival rate is linear in the measure of active-firms (i.e., firms that actively search for workers). As a result, if $w_l = \underline{w} = \underline{p}$ then an increase in w_l results in destruction of firms, which in turn results in a decrease of the job offer arrival rate. Figure 6a plots the unemployment rate (in solid line) as a function of the legal minimum wage, as compared to the unemployment rate without worker heterogeneity. Figure 6b plots the evolution of the mean unemployment

[7]Formally, this is incompatible with $\overline{b} < \infty$, but this can easily be accommodated for by truncating the normal distribution from above at $\overline{w}$.

duration, whereas Figures 6c and 6d plot the wage offer density and the wage function for five cases: the standard case (in solid line) and increases of w_l of 10% (dots and dashes), 25% (short dashes), 50% (dotted) and 75%(dashed).

Consider increases in w_l such that w_l stays below the ex-ante minimum productivity. Then the wage function K(p) increases and the wage offer density shifts to the right, whereas the value of this density at, the legal minimum wage increases. As the wage function increases, the unemployment rate among the workers whose opportunity cost of employment is between the old minimum wage and the new one decreases, because they accept offers more often. As a result, the unemployment rate converges to the value in the model in which all unemployed workers accept all wage offers. As long as w_l is smaller than the ex-ante minimum productivity, the actual unemployment rate thus decreases with w_l. The decrease of the mean duration of unemployment can be explained analogously.

When w_l passes the ex-ante minimum productivity then the new minimum productivity becomes equal to w_l. Therefore κ decreases. We have two effects here: the first one (κ constant) is the same as in the previous paragraph; it makes the unemployment rate and the mean unemployment duration decrease; the wage function increases and the wage offer density shifts further to the right (although the density tends to infinity at the minimum wage because of $\underline{p} = \underline{w}$). The second effect is due to the decrease of κ; this effect is opposite to the first. The net result depends on the way κ is specified as a function of p. Here, except for the unemployment rate, the first effect is more important than the second one. As for the unemployment rate, the net effect is positive, with a value more closer to the value in the absence of worker dispersion. We then have an increasing unemployment rate combined with a decreasing mean unemployment duration.

4.3.3 Change in the layoff rate

Now consider what happens when δ changes, holding λ constant (so κ changes too). Figure 7a plots the unemployment rate (in solid line) as a function of 6, as compared to the unemployment rate without worker heterogeneity. Figure 7b plots the evolution of the mean unemployment duration, and Figure 7c plots the wage function for five cases: $\delta = 0.001$ (in solid line), $\delta = 0.005$ (dots and dashes), $\delta = 0.009$ (short dashes), $\delta = 0.01$ (dotted) and $\delta = 0.1$ (dashed). Figure 7d plots the wage offer density for three cases: $\delta = 0.001$ (in solid line), $\delta = 0.009$ (short dashes) and $\delta = 0.1$ (dashed). As δ increases, κ decreases. The wage function decreases and the wage offer density shifts to the left (the minimum wage offer remaining constant, there are more wage offers near this minimum wage). Thus, the mean unemployment duration for a type-b worker increases (since F(b) increases too). The unemployment rate increases because of two effects: first the decrease of κ, and secondly the fact that workers of type-b accept job offers less often. The second effect is reflected in the increasing distance between the two curves in Figure 7a.

4.3.4 Change in the distribution of the opportunity cost of employment

Finally, we simulate changes in the mean opportunity cost of employment μ. Figure 8a plots the unemployment rate (in solid line) as a function of μ, as compared to the unemployment rate without worker heterogeneity. Figure 8b plots the evolution of the lowest wage offer in the market (in solid line), as compared to μ itself (in dashed line). Figure 8c plots the wage function for five cases: $\mu = 256$ (in solid line), $\mu = 1250$ (dots and dashes), $\mu = 2625$ (short dashes), $\mu = 3125$ (dotted) and $\mu = 5000$ (dashed). Figure 8d plots the wage offer density as a function of the difference of the wage and the lowest wage, for three cases: $\mu = 256$ (in solid line), $\mu = 2625$ (short dashes) and $\mu = 5000$ (dashed). As μ increases, the wage function increases, the wage offer density shifts to the right ($\underline{w}$ increases, but less than μ, as we can see in Figure 8b), but it becomes more and more concentrated around the lowest wage. There are more and more job offers near the lowest wage, and those offers are unacceptable for more and more workers (as the difference between μ and $\underline{w}$ increases). As a result, the unemployment rate and the mean unemployment duration increase.

5 Empirical Application

In this section we structurally estimate the model using a set of professional histories drawn from the 1991-1993 wave of the longitudinal French Labor Survey. We start with a brief discussion of these data. After that, we present the estimation method and we a discuss the results.

5.1 Data

For the estimation of the model we use data from the French Labor Force Survey collected by the INSEE. In March every year, around 60,000 French households are interviewed. The sample is partially renewed each year (a third is dropped) so that an individual is interviewed in three consecutive years. We use the data of those who entered the survey in 1991. From this survey we obtain a sequence of labor market states occupied by the individuals and the sojourn times in these states. In particular, we have such trajectories from March 1991 to March 1994. A number of individual characteristics is recorded at the first interview of the respondent.

We restrict our sample to adults aged between 25 and 55. Individuals who were self-employed, nonparticipant or part-time worker for some period during the time span covered by the survey are omitted, since the behavior of such individuals, at least in a certain period, may well deviate from the behavior as described by the model. We stratify the data by industry, using the first digit of the industry code, and we exclude individuals associated to the agricultural, energy, housing, insurance and public sector of the economy, because the number of observations in these sectors is too small, and/or because it can be argued that firms in these sectors are not profit maximizing. (For more details on the construction of the sample, see Bontemps, Robin and Van den Berg (1997).) We estimate our model separately for each of the remaining 8 industries. For each respondent, the first spell used is the spell which is in progress at the date of the first interview. For computational reasons, information on the durations of any subsequent spells is not used.

The French labor force survey data have been used in a large number of studies to estimate reduced-form unemployment duration models (see for example Magnac and Robin (1994); Van den Berg and Van der Klaauw (1998) provide a survey of this literature). These studies often find negative duration dependence of the observed exit rate out of unemployment, even if the model controls for a large number of observed covariates. In our sample, we do not find strong evidence of negative duration dependence. Estimation of a reduced-form Weibull unemployment duration model (i.e., exit rate at duration t proportional to $t^{\alpha-1}$) results in an estimate of α that is insignificantly smaller than one. Note that significant negative duration dependence would suggest that H(b) has a non-trivial effect on the wage distribution. However, it has been a stylized fact of the French labor market (and, indeed, of the labor markets of other European countries) that unemployment durations are insensitive to changes in unemployment benefits (Bonnal, Fougere and Serandon (1997), Van den Berg (1990), Devine and Kiefer (1991)). To the extent that b is related to unemployment benefits, this suggests that H(b) does not have a non-trivial effect on the wage distribution.

The mandatory minimum wage w_l is known from institutional sources; it equals 4500 French Francs per month. All monetary variables are pre-income tax. We do not impose b to equal the observed pecuniary unemployment insurance benefits, since b also includes search costs and the non-pecuniary utility of being unemployed. Thus, the structural model determinants to be estimated are κ, δ, Γ and IT.

The data contain wages below the legal minimum wage. Such observations correspond to jobs that are exempt from the minimum wage constraint. These are mostly apprenticeship positions. These are intended to help schoolleavers or unemployed workers to build up the skills necessary for a more regular job (see Bonnal, Fougere and Serandon (1997) for more details on such positions). Our equilibrium search model does not account for such positions. One may argue that these positions are fundamentally different from regular jobs, and that therefore such observations should be dropped from the data. On the other hand, there is evidence that employers use these positions to get around the minimum wage constraint. In that case, the legal minimum wage is not, (fullv) enforced.

We feel that it would be restrictive to choose unambiguously for either one of these two points of view. We therefore perform the empirical analysis with two different samples. The first one (sample A) is the same as in Bontemps, Robin and Van den Berg (1997). In this sample. all wage observations below the legal minimum wage were regarded as missing observations. In the second sample (sample B), we include all the wage observations from the survey. Table 1 presents, for both samples and each industry, sample statistics on the endogenous labor market variables, notably cross-sectional wages at the first interview. Figure 10 plots the wage densities g(w) of the w_1 data in sample B.

5.2 Likelihood of the observations

It is useful to derive the likelihood of the different types of observations that we use to estimate the model. The dependent variables are elements of the individual labor market histories,

- Individual labor market position at the time of the first interview:

$$x = 1 \text{ if unemployed,}$$
$$= 0 \text{ if employed.}$$

- Elapsed and residual duration in that position:

t_{0b} = elapsed unemployment duration

t_{0f} = residual unemployment duration

d_{0b} = 1 if unemployment duration left-censored, otherwise = 0

d_{0f} = 1 if unemployment duration right-censored, otherwise = 0

t_{1b} = elapsed employment duration

t_{1f} = residual employment duration

d_{1b} = 1 if job duration left-censored, otherwise = 0

d_{1f} = 1 if job duration right-censored, otherwise = 0

- Earned and accepted wages:

w_0 = wage accepted by unemployed individuals

d_0 = 1 if w_0 unobserved, otherwise = 0

w_1 = wage of employees at time of first interview

d_1 = 1 if w_1 unobserved, otherwise = 0

- First transition if employed at first interview:

$$
\begin{aligned}
v &= 1 \text{ if job-to-unemployment transition} \\
&= 0 \text{ if job-to-job transition}
\end{aligned}
$$

First, let us derive the likelihood of the observations from an individual who is unemployed at the first interview. The distribution of elapsed (or residual) durations t_{0b} in the stock of unemployed at the first interview is a mixture distribution. Conditional on the worker's opportunity cost of employment b, the exit rate from unemployment is $\lambda\overline{F}(b)$ and the accepted wage is drawn from the wage offer distribution truncated at b: $f(.)/\overline{F}(b)$. The marginal duration distribution has a single mass point at infinity, corresponding to the permanently (or structurally) unemployed who have $b \geq \overline{w}$.

It is not difficult to derive that if $d_0 = 0$ (which entails $d_{0f} = 0$) then the likelihood is

$$
\begin{aligned}
&\lambda^{2-d_{0b}} \exp\left[-\lambda(t_{0b} + t_{0f})\right] \frac{H(\underline{w})}{1+\kappa} f(w_0) \\
&+ \int_{\underline{w}}^{w_0} \left[\lambda\overline{F}(b)\right]^{2-d_{0b}} \exp\left[-\lambda\overline{F}(b)\left(t_{0b} + t_{0f}\right)\right] \frac{f(w_0)}{\overline{F}(b)} \frac{dH(b)}{1+\kappa\overline{F}(b)}
\end{aligned}
$$

The first part of this expression corresponds to workers with $b \leq \underline{w}$; these accept all job offers. The second part corresponds to workers with $\leq b < \overline{w}$; these accept some offers and reject others.

If $d_0 = 1$ and $d_{0b} + d_{0f} < 2$, the likelihood equals

$$
\begin{aligned}
&\lambda^{2-d_{0b}-d_{0f}} \exp\left[-\lambda\left(t_{0b} + t_{0f}\right)\right] \frac{H(\underline{w})}{1+\kappa} \\
&+ \int_{\underline{w}}^{\overline{w}} \left[\lambda\overline{F}(b)\right]^{2-d_{0b}-d_{0f}} \exp\left[-\lambda\overline{F}(b)\left(t_{0b} + t_{0f}\right)\right] \frac{dH(b)}{1+\kappa\overline{F}(b)}.
\end{aligned}
$$

If $d_{0b} + d_{0f} = 2$ then $d_0 = 1$ and the likelihood equals

$$
\begin{aligned}
&\exp\left[-\lambda\left(t_{0b} + t_{0f}\right)\right] \frac{H(\underline{w})}{1+\kappa} \\
&+ \int_{\underline{w}}^{\overline{w}} \exp\left[-\lambda\overline{F}(b)\left(t_{0b} + t_{0f}\right)\right] \frac{dH(b)}{1+\kappa\overline{F}(b)} + \left[1 - H(\overline{w})\right]
\end{aligned}
$$

Here, the third term corresponds to workers with $b \geq \underline{w}$; these reject all job offers and are permanently unemployed.

Now let us derive the likelihood for an individual who is employed at the first interview. The probability of observing an individual in the state of employment equals $1 - u$. If $d_1 = 0$ then the likelihood is

$$(1-u)g(w_1)\left[\delta+\lambda\overline{F}(w_1)\right]^{1-d_{1b}}\exp\left[-\left(\delta+\lambda\overline{F}(w_1)\right)\left(t_{1b}+t_{1f}\right)\right]\left[\delta^v\left(\lambda\overline{F}(w_1)\right)^{1-v}\right]^{1-d_{1f}}$$

whereas if $d_1 = 0$ then the likelihood simply equals $1 - u$.

5.3 Estimation procedure

The basic idea of the estimation method is the same as in the three stage method developed in Bontemps, Robin and Van den Berg (1997).

1. We estimate $\underline{w}$ and $\overline{w}$ as the minimum and maximum observation on w_1 in the sample (note that these are super-efficient estimators). The density g(w) and distribution G(w) are estimated by a non parametric (kernel) procedure from the w_1 data.
2. We assume a parametric form for H with a parameter vector θ. By integrating equation (18) (which also holds in the general model; see equation (11), we obtain

$$\frac{1}{\left[1+\kappa\overline{F}(w)\right]}=(1-u)\int_{\underline{w}}^{w}\frac{g(t)}{H(t)}dt+\frac{1}{1+\kappa}$$

$$(1 \quad u)=\frac{\kappa}{(1+\kappa)\int_{\underline{w}}^{\overline{w}}\frac{g(t)}{H(t)}dt}$$

   The second equation follows from the first by substituting $w = \underline{w}$. These two equations express F and u in terms of g, κ and H. Substitution of these equations into the likelihood function enables estimation of the structural parameters λ, δ and θ from individual labor market histories by maximum likelihood, considering the (estimated) g as a nuisance parameter. Note that *H(b)* is only identified for $b \in [\underline{w}, \overline{w}]$. If $H(\underline{w}) = 1$ then θ is not identified. The estimation of the remaining model parameters then reduces to the estimation of the model of Bontemps, Robin and Van den Berg (1997) with the restriction that the job offer arrival rates in employment and unemployment are equal.

3. From equation (20) it follows that

$$K^{-1}(w) = w + \frac{\left(1+\kappa\overline{F}(w)\right)H(w)}{2\kappa f(w)H(w) + h(w)\left(1+\kappa\overline{F}(w)\right)}$$
$$= w + \frac{H(w)}{2(1-u)g(w)\left(1+\kappa\overline{F}(w)\right) + h(w)}$$

With this we can reconstruct the distribution of productivities, using the estimates of the previous steps.

We obtain consistent standard errors by bootstrapping the three step estimation procedure.

In order to obtain some feeling on the performance of this estimation method, we estimate the model with simulated data. We take true parameter values that are the same as in the baseline simulation of Subsection 4.3. Next, we draw 200 samples of 2000 observations each, and we estimate the model for each of these 2000 samples. The table below shows a 95% confidence interval for each parameter and for the unemployment rate. Figure 9 displays 95% confidence bounds for the wage function K(p), the productivity density y(p), and the wage density g(w) (the true functions are plotted as solid lines).

	True value	95% confidence interval
δ	0.005	$[0.0049, 0.0051]$
κ	20	$[17.7, 21.4]$
μ	2500	$[2070, 2560]$
σ	1000	$[966, 1260]$
u	7.3	$[6.6, 8.0]$

We conclude that the estimation method works well in this case, with a sample of moderate size. The parameters μ and a are sometimes a bit underestimated and overestimated, respectively. This is not surprising, given that H is only fitted by way of information on its right tail (that is, for $b > \underline{w}$, which equals 2078).

5.4 Estimation results

We assume that $H(b)$ is a normal distribution with unknown parameters μ and σ. This H is log-concave, so we impose that the equilibrium exists and is unique, and the support of F is connected. The structural model determinants we estimate are κ, δ, μ, σ and I'. Table 2 reports the results.

We first discuss the results for sample A. In this sample, $\underline{w}$ equals the legal minimum wage w_l. For each industry, we estimate that $H(\underline{w}) = 1$.[8] As a result, the model is equivalent to a model in which workers are homogeneous. The estimates of the remaining parameters equal those for the model of Bontemps, Robin and Van den Berg (1997) with the restriction that the job offer arrival rates in employment and unemployment are equal.

The estimates based on sample A imply that no unemployed individual would reject an offer of a job that pays the legal minimum wage, and that unemployment benefits do not affect the duration of unemployment. The latter is consistent with Bonnal, Fougère and Sérandon (1997), who estimate reduced-form unemployment duration models using multiple spell data.[9]

However, recall that there is evidence that employers circumvent the legal minimum wage constraint by offering "apprenticeship jobs". In that case, the support of the wage (offer) distribution extends to lower wages, and it is conceivable that workers with a value of b just below the legal minimum wage reject such job offers. To investigate this, we estimate the model with sample B, within which such jobs are included (see Table 2 for the parameter estimates, Figures 10–13 for the estimated g(w), f(w), K(p) and $\gamma(p)$,[10] and Table 3 for the estimated u and mean durations). Note that the legal minimum wage w_l does not enter the model in this case, and that the lowest wage in the industry equals the value reported in Table 1. With this sample, for most industries, we estimate $H(\underline{w}) < 1$.

However, in general, the probability mass of H above the legal minimum wage w_l ranges from zero to just two percent. This means that H is still not very important for the shape of the wage (offer) distribution for jobs which comply with the legal minimum wage. Moreover, it means that the vast majority of workers accept all wage offers at or above the legal minimum wage, when unemployed. From Table 1 we know that the latter type of jobs concern more than 90% of all jobs. In sum, the vast majority of workers accept almost all job offers, when unemployed. This is in line with results from structural empirical analysis of partial job search models with European data (see Devine and Kiefer (1991) for a survey). Clearly, this means that a shift in the distribution of b (e.g. because of a shift in the average unemployment benefits level)

[8]During the maximum likelihood iterations of the second estimation step, μ and σ tended to values for which the probability that b exceeds $\underline{b}$ became too small to be detectable.

[9]We also estimated the model using Dutch labor force survey data. These resemble sample A in the sense that they contain almost no wage observations below the legal minimum wage (and any observations below it can be attributed to measurement error), and there is no strong evidence of negative duration dependence in the unemployment duration data. For this sample we also estimate that $H(\underline{w}) = 1$.

[10]We plot $\gamma(p)$ as a function of $\Gamma(p)$. Because of the skewedness of the density, a plot as a function of p is not informative.

does not have a sizeable effect on equilibrium unemployment or wages.

One might argue that therefore it is not important to take account of dispersion in the opportunity cost of employment. To investigate this further, we also estimate a model that does not allow for worker heterogeneity: using sample B (see Table 2). For most industries, this results in significantly lower estimates of λ. Note that if a substantial amount of unemployed workers reject wage offers below the legal minimum wage then a model that imposes the contrary can only be reconciled with the data if the job offer arrival rate decreases. In fact, the strong decrease in the estimates of λ may indicate that the restriction that. job offer arrival rates are the same in employment and unemployment is too strong. If the arrival rate for the employed is smaller, then a decrease in λ may improve the fit of our model to the job duration data.

6 Conclusion

This paper provides a theoretical synthesis of previous equilibrium search models, and an empirical synthesis of previously estimated equilibrium search models. We allow for inherent heterogeneity on both sides of the market: firms have heterogeneous production technologies and workers have different opportunity costs of employment. We show that such a model is consistent with stylized facts on (the lower end of) the labor market, and we derive a number of characteristics of equilibrium. Notably, we show that if the shape of the distribution of worker heterogeneity has a certain smoothness, then equilibrium exists and is unique, and the wage (offer) distribution has a connected support. We also show that if there is no mandatory minimum wage then the wage (offer) distribution does not have a spike at the lowest wage. We perform simulations to study the effects of changes in the layoff rate, the degree of search frictions, the mean opportunity cost of employment and other parameters, on unemployment and wages. The model is structurally estimated with French labor force survey data.

It turns out that heterogeneity of firms, in combination with workers searching on the job as well as off the job, can explain most of the variation in observed cross-sectional wages. Indeed, a model with worker heterogeneity but no firm heterogeneity gives an even worse fit (in a certain sense) to wage data than a model without any heterogeneity at all. In the empirical analysis, the shape of the distribution of workers' opportunity costs of employment is not of importance for the distribution of wages above the legal minimum wage. Most unemployed workers accept most job offers. Moreover, changes in the distribution of workers' opportunity costs (which includes changes in the unemployment benefits levels) do not have a big impact on job acceptance behavior or unemployment. However, many unemployed workers do reject offers of jobs that are exempt from paying the legal minimum wage, and although such jobs constitute a small fraction of all jobs, ignoring them results in underestimation of the job offer arrival rate.

One may question whether there are not other types of worker heterogeneity that are of more importance for the distribution of wages. Recent empirical studies on matched worker-firm data suggest that the productivity of a job has a strong individual-specific component (see Abowd, Kramarz and Margolis (1998)). The literature on equilibrium search models with wage setting has mostly ignored this type of heterogeneity. We feel that this creates a challenge for further research.

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Table 1: Descriptive statistics

	Food		Intermediary goods		Equipment		Current consumption	
	Set A	Set B	Set A	Set B	Set A	Set B	S e t	Set B
Number of observations	489	487	1179	1175	1361	1359	1047	1046
Unemployed	69	63	74	71	88	82	130	118
Employed	420	424	1105	1104	1273	1277	917	928
Transitions Unemp. → Emp.	59	55	51	51	66	64	85	84
Transitions Emp. → Emp.	19	19	38	43	38	40	37	38
Transitions Emp. → Unemp.	26	26	73	66	81	80	77	79
Wages at 1st interview':								
lowest	4500	1283	4500	2003	4500	2200	4500	2000
P_{10}	4836	4583	5158	5020	5405	5300	4700	4567
Q_1	5500	5400	6000	5900	6300	6250	5250	5073
Q_2	6700	6500	7256	7200	7800	7726	6500	6374
Q_3	8667	8500	9185	9128	10500	10333	9208	9000
P_{90}	10933	10833	12000	12000	15000	15000	13612	13000
mean	7837	7600	8313	8226	9135	9041	8152	7890
standard deviation	3885	3908	3727	3749	4213	4197	4302	4191

	Construction		Trade		Transport telecom.		Services	
	Set A	Set B	Set A	Set B	Set A	Set B	Set A	Set B
Number of observations	1235	1235	1729	1724	787	788	2833	2823
Unemployed	160	148	206	187	54	52	347	310
Employed	1075	1087	1523	1537	733	736	2486	2513
Transitions Unemp. → Emp.	130	123	162	157	42	42	283	260
Transitions Emp. - Emp.	80	82	74	76	33	33	164	173
Transitions Emp. → Unemp.	111	113	110	109	26	25	251	250
Wages at 1st interview':								
lowest	4500	1800	4500	1300	4500	2312	4497	1280
P_{10}	5000	4800	4918	4600	5612	5500	5000	4500
Q_1	5700	5500	5580	5313	6500	6438	5694	5308
Q_2	6631	6500	6808	6500	7750	7670	7042	6800
Q_3	8125	8000	9225	8992	9750	9700	9898	9417
P_{90}	10761	10500	13700	13047	13000	12900	14000	13500
mean	7538	7350	8195	7865	8743	8663	8440	8020
standard deviation	3115	3160	3946	3949	3645	3676	4070	4124

1: $P_{10}, Q_1, Q_2, Q_3, P_{90}$ are the 10th, 25th, 50th, 75th and 90th percentile of the wage distribution.

Table 2: Estimation results

	δ	κ	μ	σ		δ	κ	μ	σ
Food					Construction				
Set A	0.0041	9.19	$H(\underline{w}) = 1$			0.0052	6.2	$H(\underline{w}) = 1$	
Set B	0.0043	28.13	319	1799		0.0053	18.4	316	2297
	(2e−4)[1]	(6.5)	(25)	(321)		(2.2e−4)	(3.5)	(18)	(22)
No Heterogeneity[2]	0.0041	11.1	***	***		0.0052	6.7	***	***
Intermediary goods					Trade				
Set A	0.0033	8.9	$H(\underline{w}) = 1$			0.0050	7.5	$H(\underline{w}) = 1$	
Set B	0.0033	16.43	254	2608		0.0053	38.0	253	1904
	(1e-4)	(1.2)	(19)	(520)		(1.4e−4)	(1.7)	(5.8)	(126)
No Heterogeneity	0.0032	9.7	***	***		0.0052	9.1	***	***
Equipment					Transport / telecom				
Set A	0.0032	9.9	$H(\underline{w}) = 1$			0.0036	11.0	$H(\underline{w}) = 1$	
Set B	0.0032	14.62	251	2197		0.0036	11.7	$H(\underline{w}) = 1$	
	(1e-4)	(1.4)	(15)	(226)		(1.4e−4)	(1.1)		
No Heterogeneity	0.0031	10.7	***	***		0.0036	11.7	***	***
Current consumption					Services				
Set A	0.0044	5.5	$H(\underline{w}) = 1$			0.0064	7.2	$H(\underline{w}) = 1$	
Set B	0.0045	25.6	260	2570		0.0069	17.2	295	1855
	(1.6e−4)	(1.2)	(7)	(13)		(1.8e−4)	(1.0)	(4.2)	(10)
No Heterogeneity	0.0044	6.8	***	***		0.0065	7.5	***	***

1: Bootstrap standard error.

2: We estimate the model without worker heterogeneity on set B.

Table 3: Estimation implications

	Food			Intermediary goods			Equipment			Current consumption		
	Data	With heter.	Without heter.	Data	With heter.	Without heter.	Data	With heter.	Without heter.	Data	With heter.	Without heter.
unemployment rate	12.9	5.7	83	6.0	10.7	9.3	6.0	10.1	8.5	11.3	12.7	12.8
mean unemployment duration	10.9	12.8	[illegible]	20.15	26.3	31.8	17.3	24.8	29.9	19.5	29.6	33.5
mean job duration	117.1	127.2	131.3	139.2	173.1	169.2	143.0	172.8	172.8	109.7	128.4	128.3

	Construction			Trade			Transport / telecom			Services		
	Data	With heter.	Without heter.	Data	With heter.	Without heter.	Data	With heter.	Without heter.	Data	With heter.	Without heter.
unemployment rate	12.0	12.5	13.0	10.8	10.6	9.9	6.6	7.9	79	11.0	10.2	11.8
mean unemployment duration	16.1	26.0	29.1	12.2	16.2	21.3	10.3	24.9	24.9	13.3	15.5	20.5
mean job duration	98.9	104.6	109.4	93.4	108.8	106.7	131.5	154.2	154.2	77.6	80.2	85.6

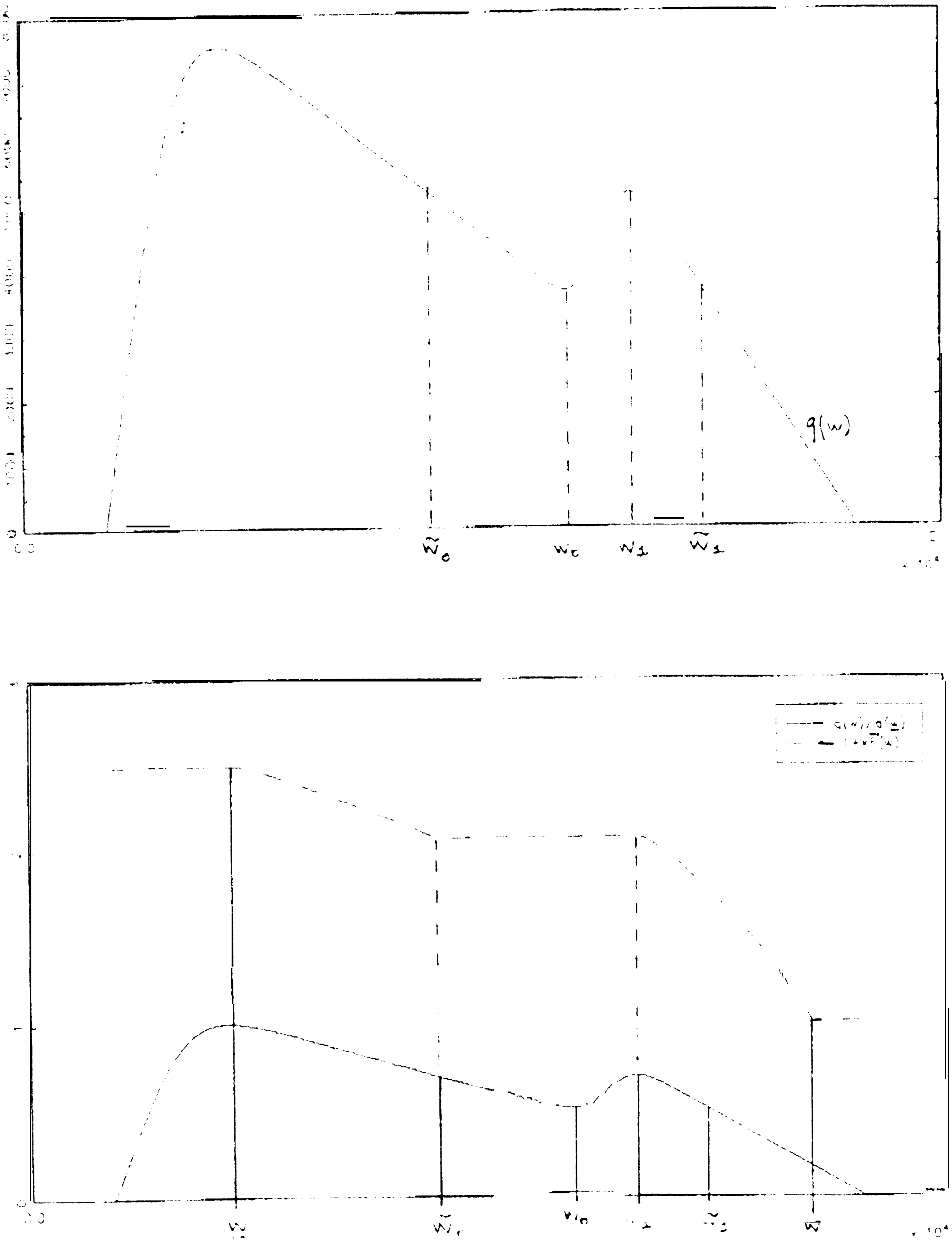
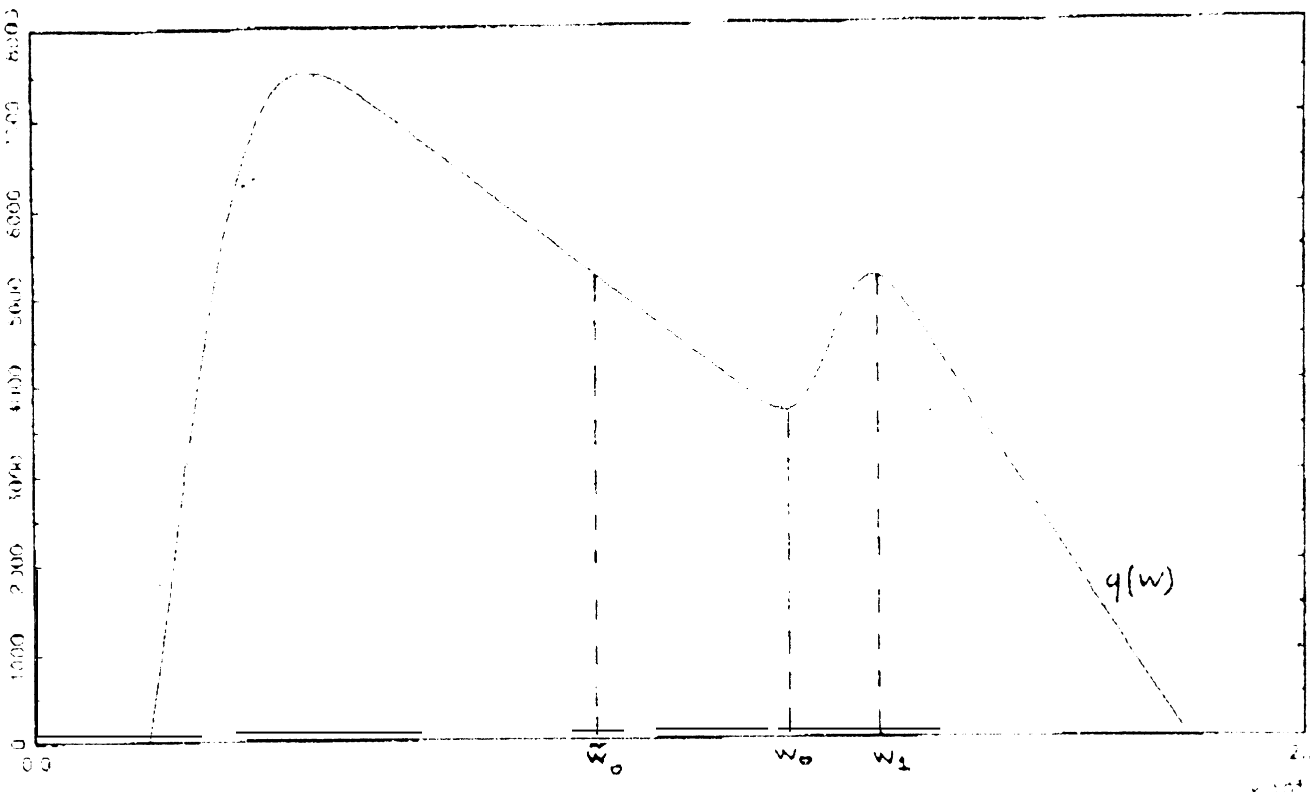


Figure **1.** Proof of Proposition 11; first case



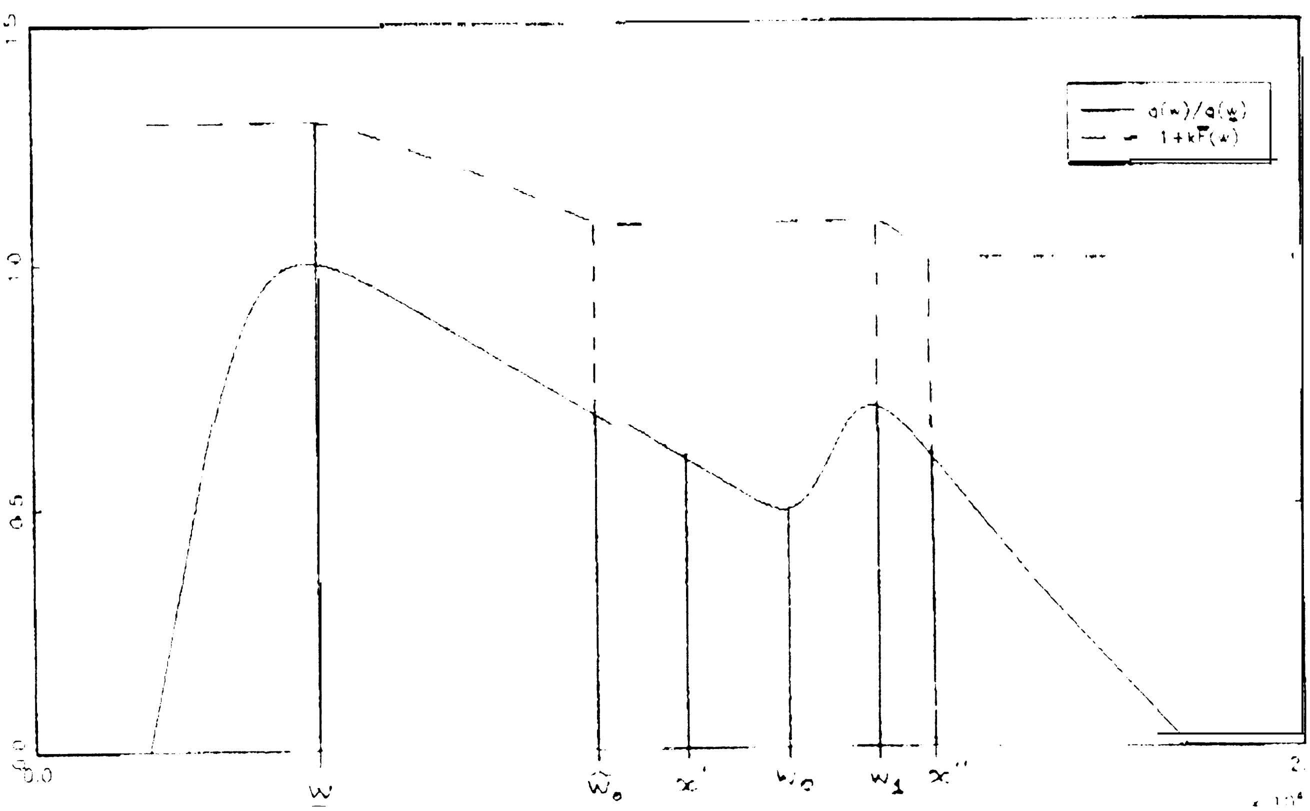


Figure 2. Proof of Proposition 11; second case

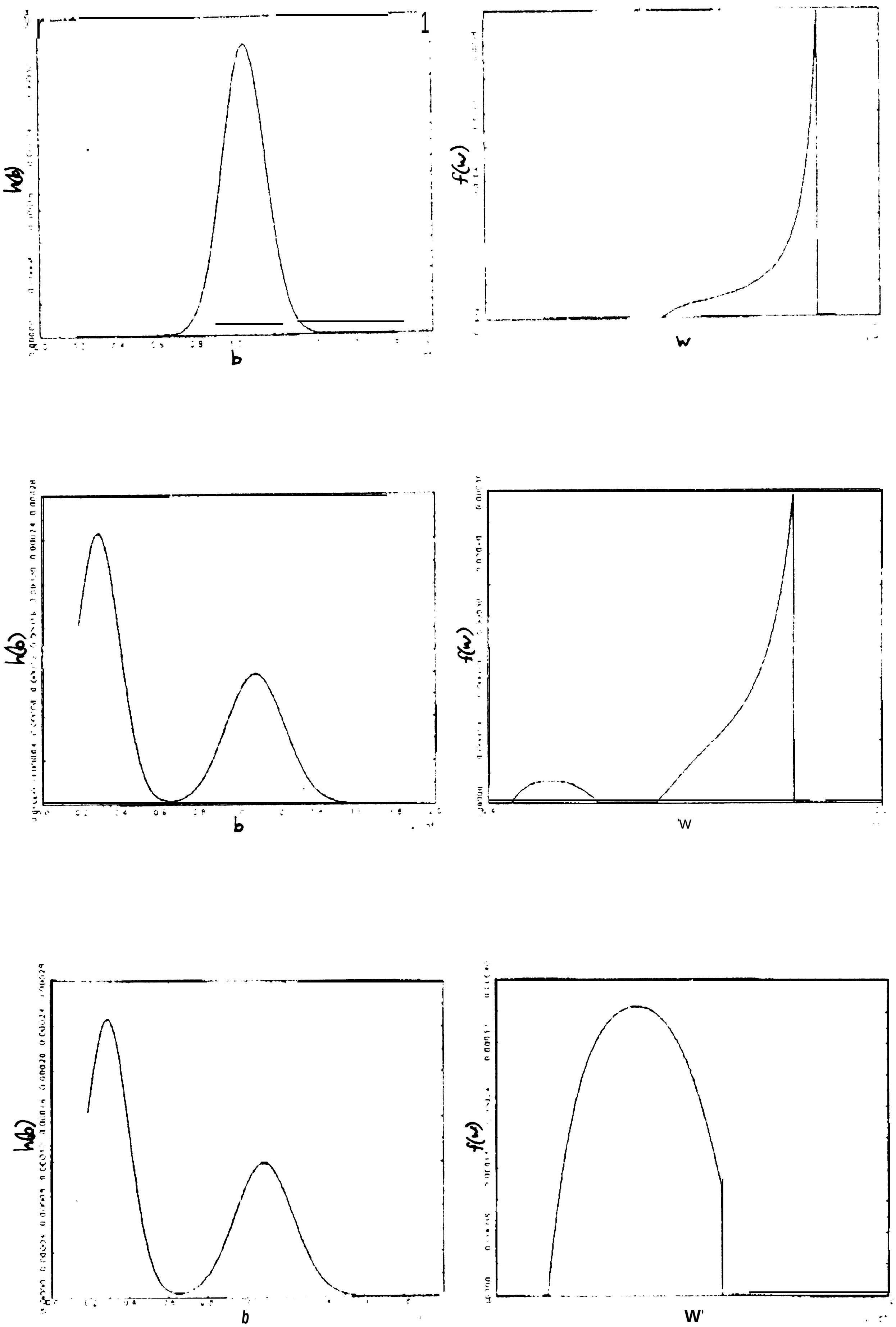


Figure 3. Examples of equilibrium with homogeneous firms

Figure 4. Baseline simulation

delt =0 005 l =20 ト ~N(2500 1000) pmin=3000 › pareto(2.8)

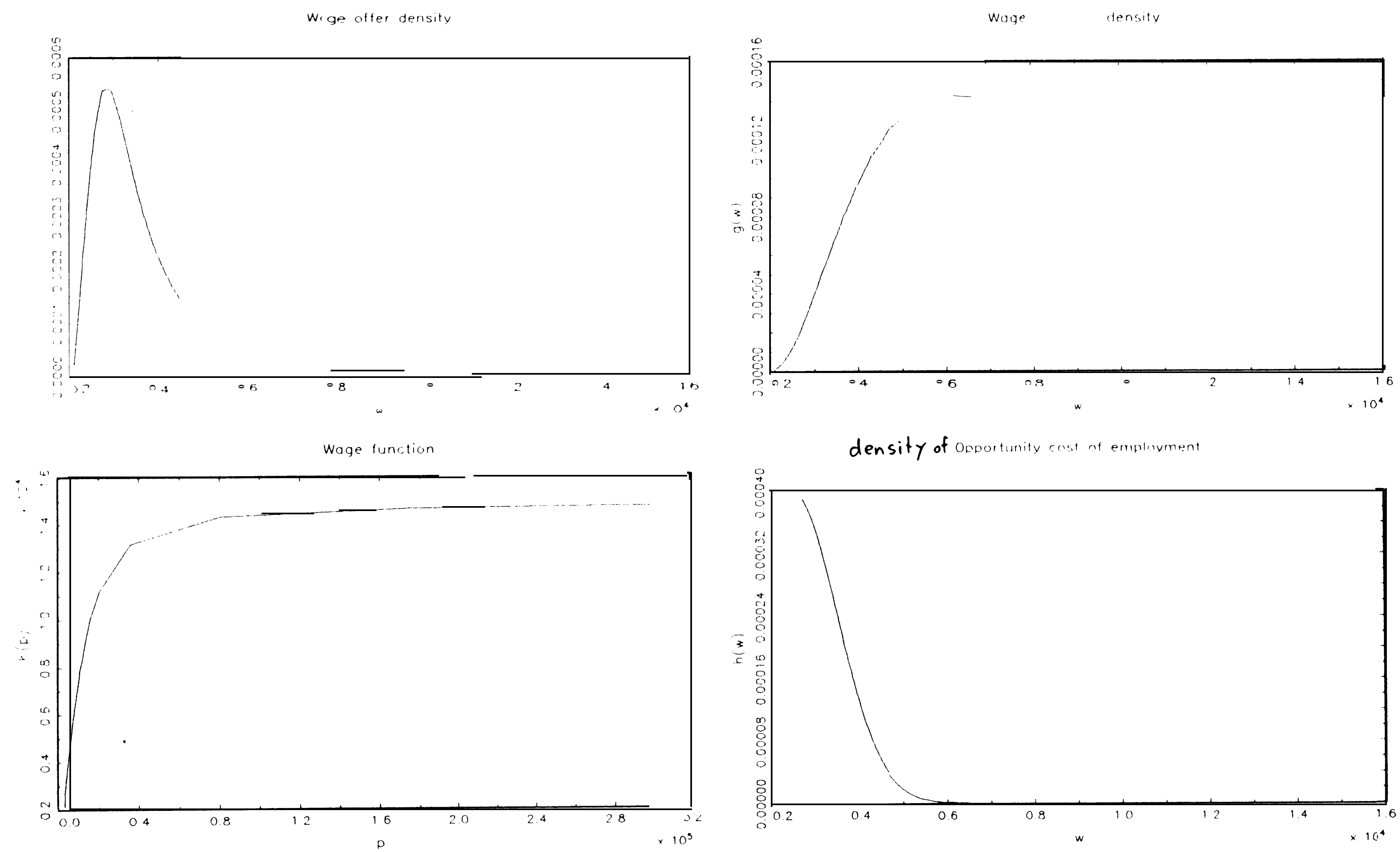


Figure 5. Change in κ

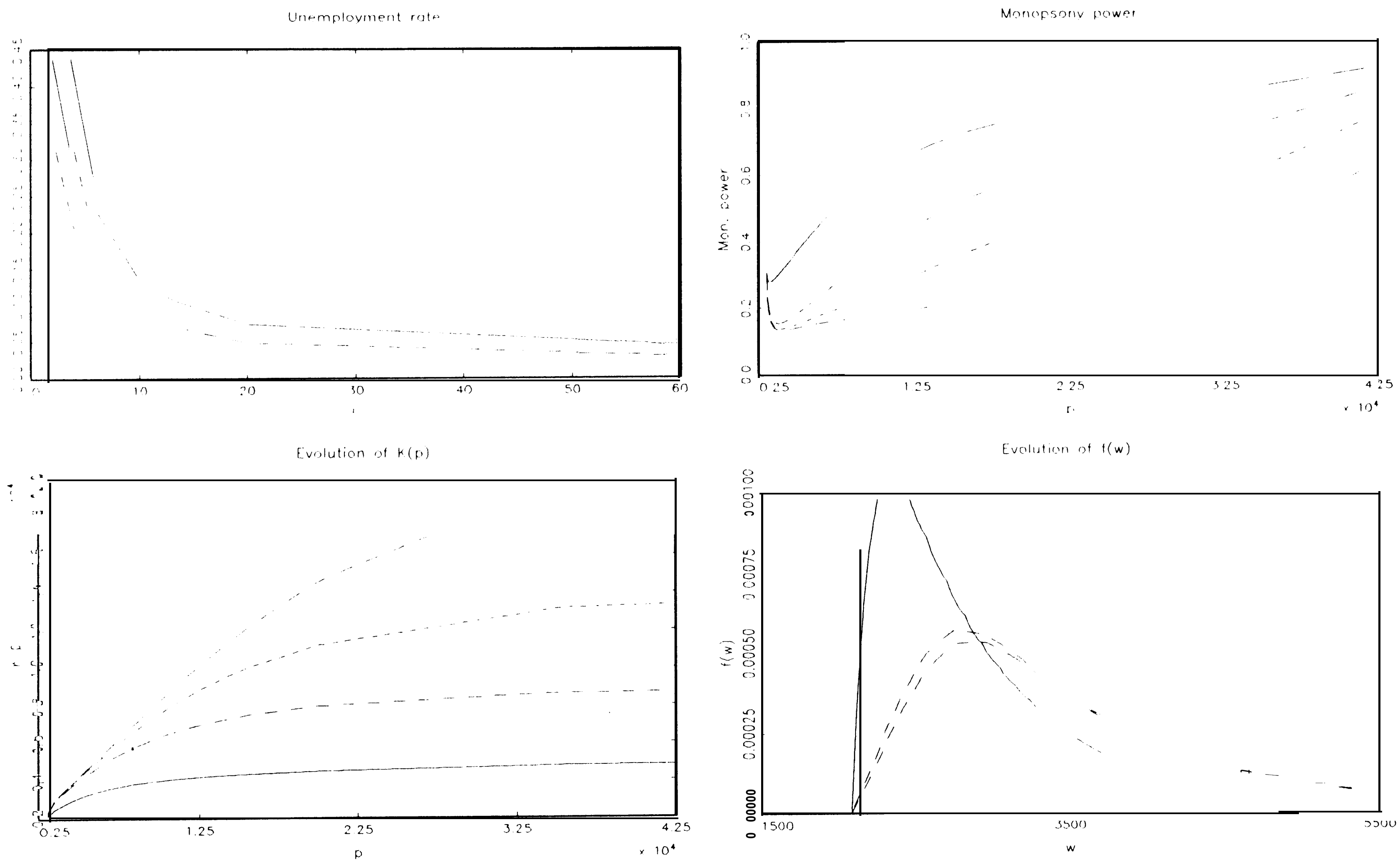


Figure 6. Increase in the legal minimum wage

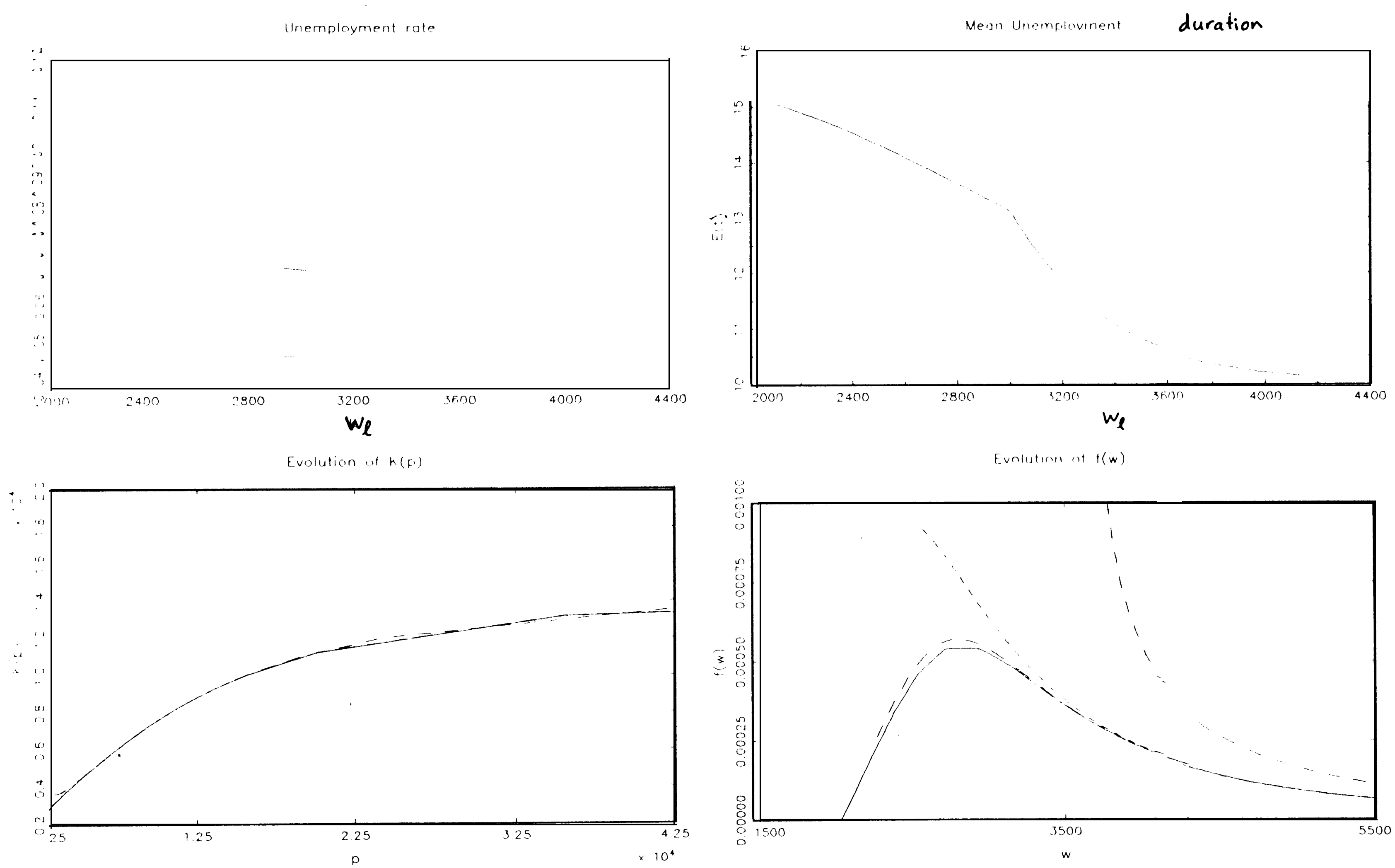


Figure 7. Change in δ

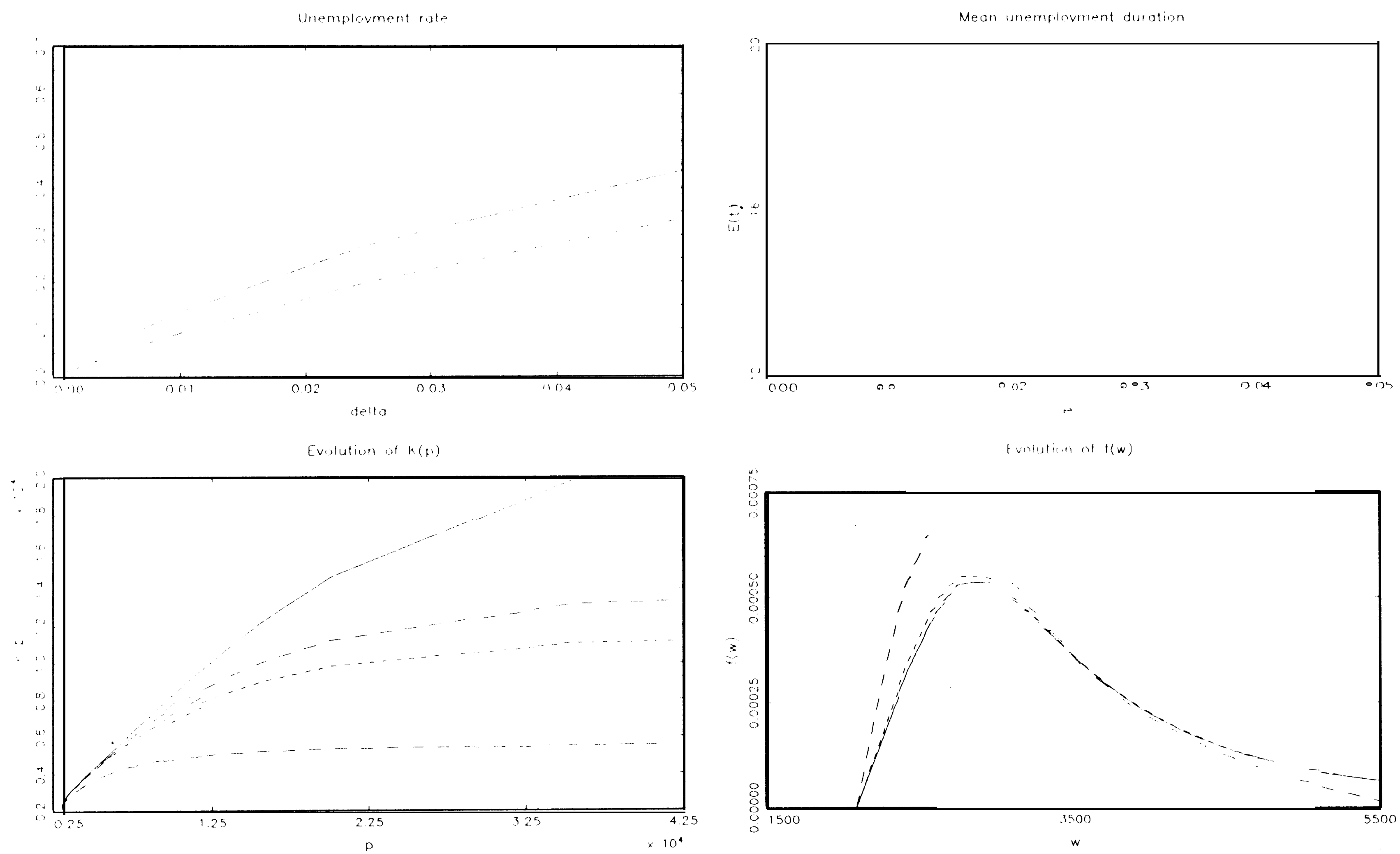


Figure 8. Change in μ

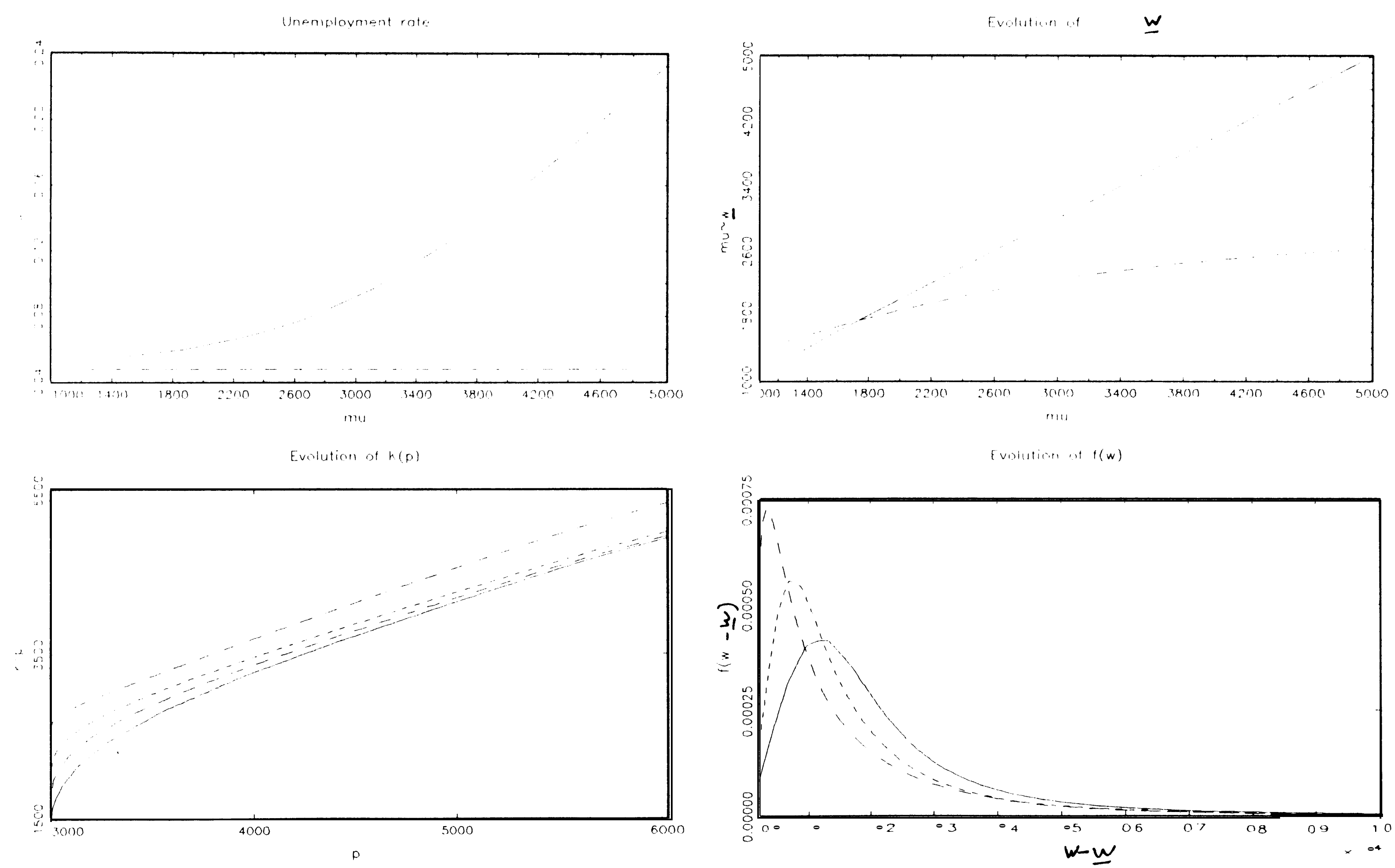


Figure 9. Estimation based on artificial data

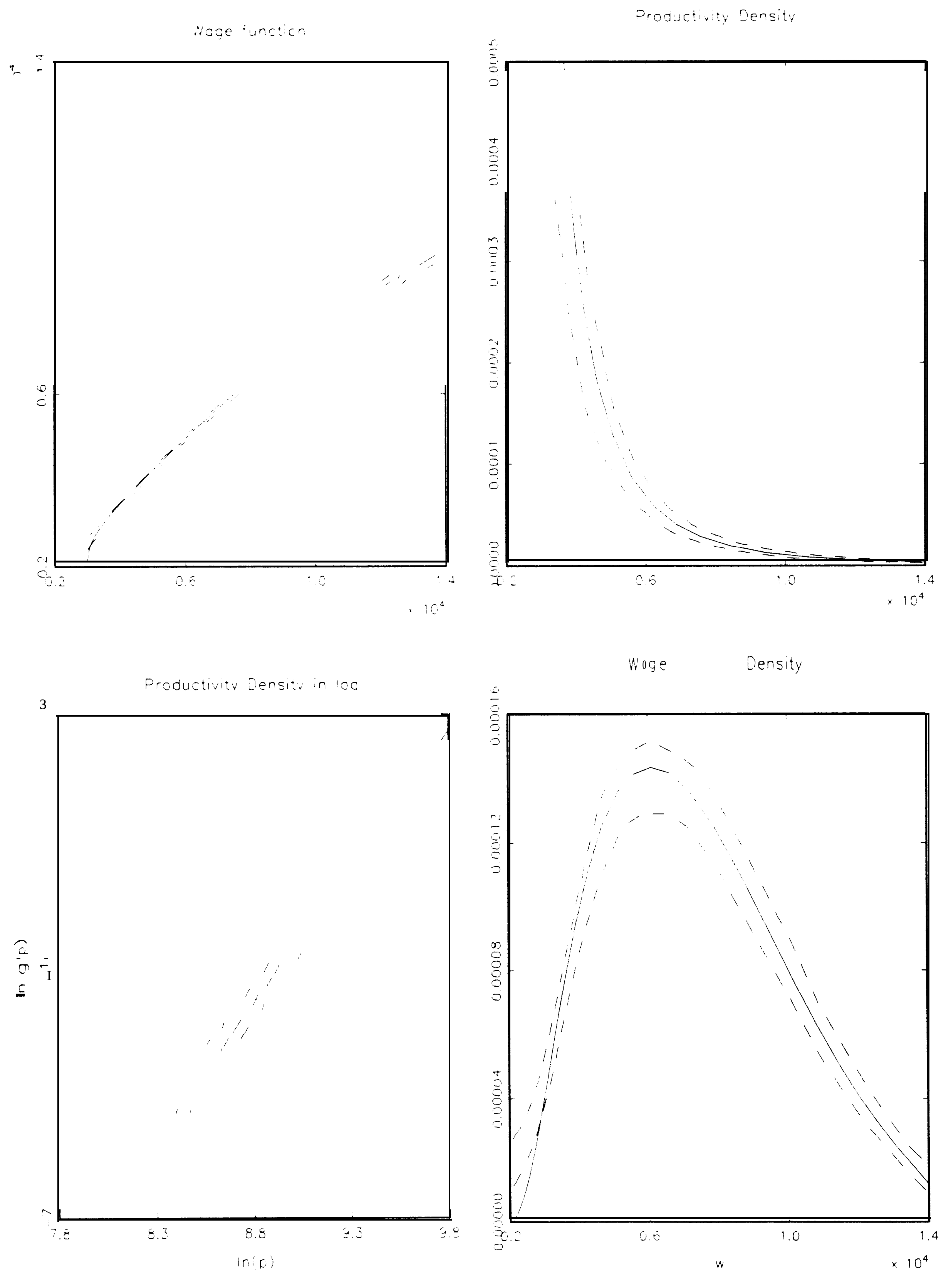


Figure 10. Estimated wage density $g(w)$

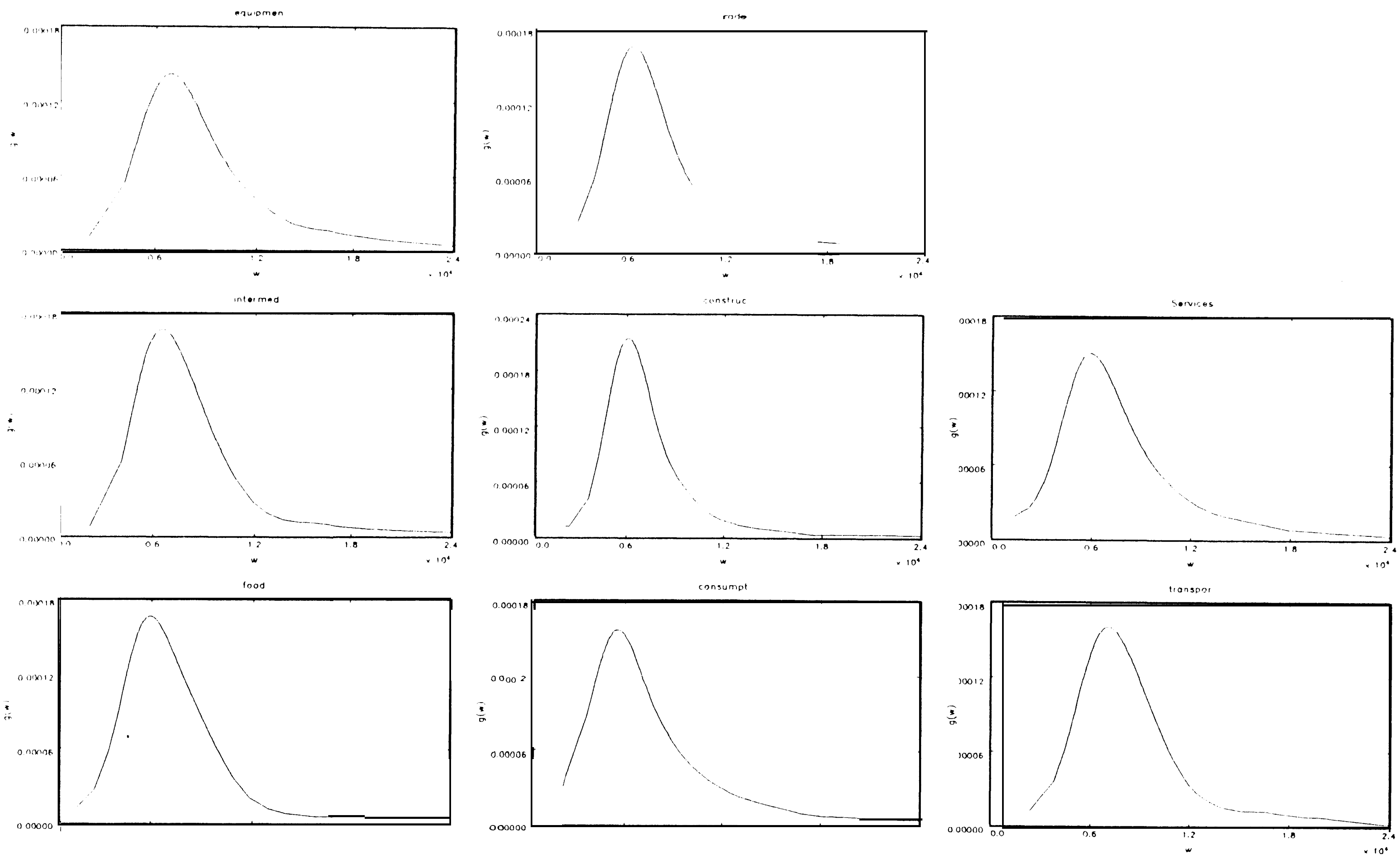


Figure 11 Estimated wage offer density $f(w)$

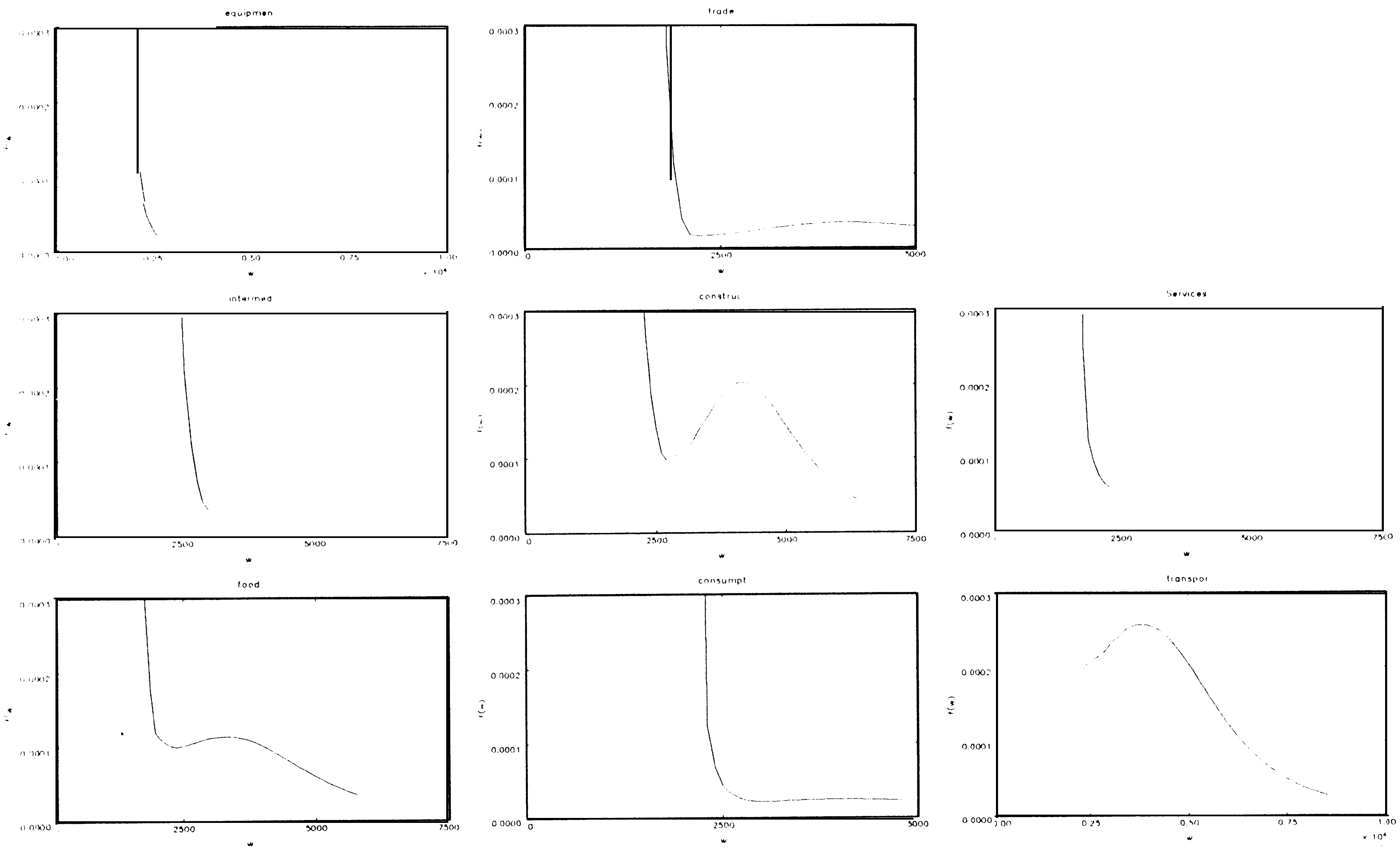


Figure 12. Estimated wage function $K(p)$

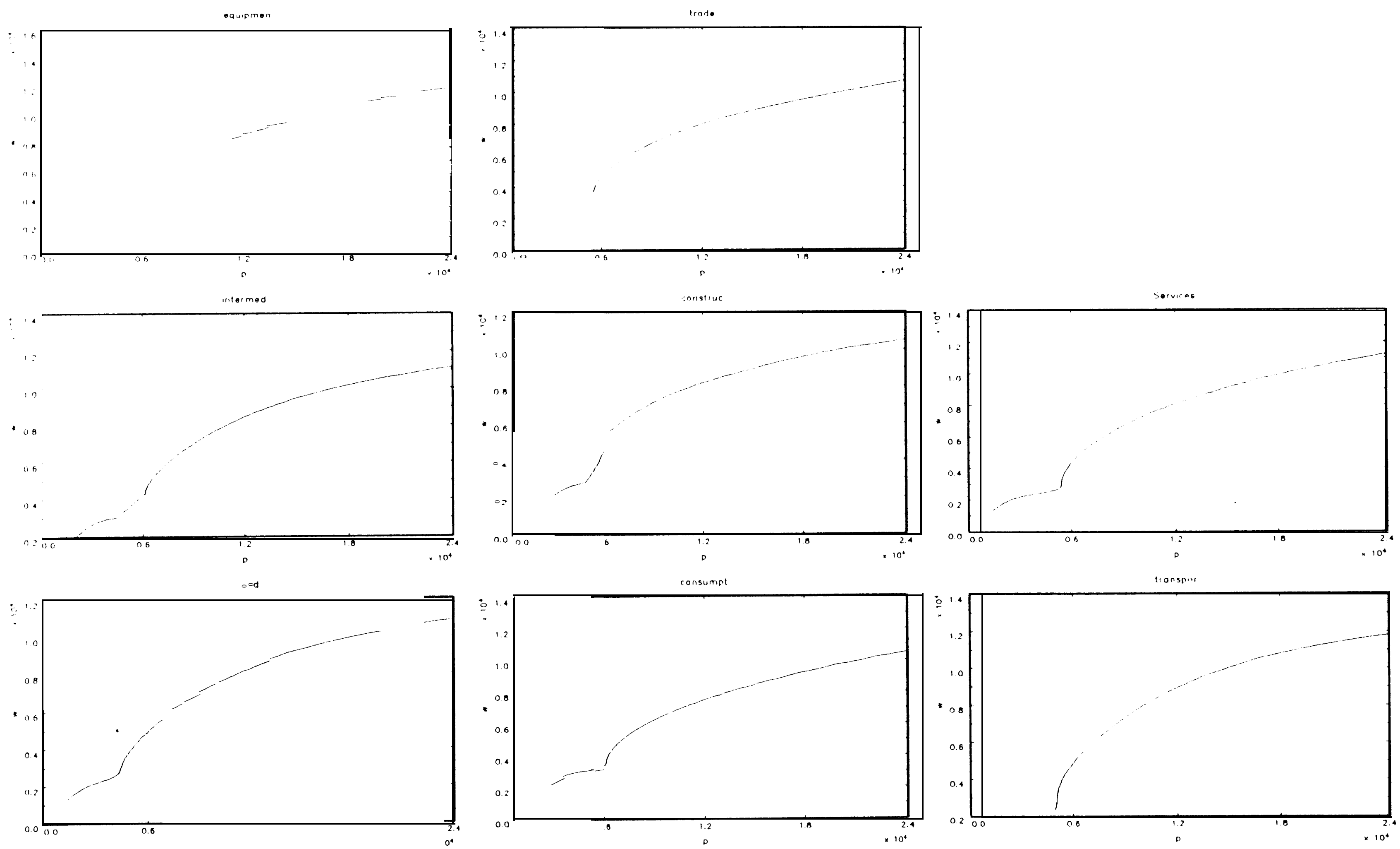


Figure 13. Estimated productivity density $\gamma(p)$