Heterogeneous beliefs and asset price dynamics

Verschoor, W.F.C.; ter Ellen, Saskia

published in
Uncertainty, expectations and asset price dynamics
2018

DOI (link to publisher)
10.1007/978-3-319-98714-9_3

document version
Publisher's PDF, also known as Version of record

document license
Article 25fa Dutch Copyright Act

Heterogeneous Beliefs and Asset Price Dynamics: A Survey of Recent Evidence

Saskia ter Ellen and Willem F. C. Verschoor

Abstract This contribution reviews the empirical literature on heterogeneous beliefs and asset price dynamics that challenges the traditional rational agent framework. Emphasis is given to the validation and estimation of (dynamic) heterogeneous agent models that have their roots in the agent-based literature. Heterogeneous agent models perform well in describing, explaining and often forecasting asset markets dynamics, such as equities, foreign exchange, credit, housing, derivatives and commodities. Our survey suggests that heterogeneous agent models have the ability to produce important stylised facts observed in financial time series and to replicate important episodes of financial turmoil.

1 Introduction

In recent decades, we have seen an increase in the number of studies that attempt to explain asset price dynamics in financial markets. Expectations are crucial in this respect, and theories of the expectations formation process have been at the forefront of economic research in the financial economic literature. Muth's (1961) 'rational expectations hypothesis' (REH) has attracted the greatest attention and states that market participants have equal access to information and form their expectations

This chapter should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. We would like to thank Elisabeth Werenskiold and Lein M. Hansen for providing research assistance, and seminar participants at Norges Bank for valuable comments and discussion.

S. ter Ellen (✉)
Norges Bank, Oslo, Norway
e-mail: Saskia.ter-Ellen@Norges-Bank.no

W. F. C. Verschoor
Tinbergen Institute and VU University, Amsterdam, Netherlands

F. Jawadi (ed.), *Uncertainty, Expectations and Asset Price Dynamics*,
Dynamic Modeling and Econometrics in Economics and Finance 24,
https://doi.org/10.1007/978-3-319-98714-9_3

about future events in a uniform, rational manner based on the 'true' probability of the state of the economy. Whereas classical economic models often assume these expectations to be rational and therefore conveniently summarised by a representative, perfectly rational agent, there is an interesting and promising new literature in the direction of bounded rationality, and the accompanying heterogeneity of agents' expectations. The notion of rational expectations is losing more and more ground and new insights on how economic agents form their expectations is therefore warranted. As it turns out, economic models that incorporate a behavioural, agent-based approach are better able to explain financial market asset price dynamics than are models based on a representative rational agent.

In this work, we will provide an overview of the empirical literature that acknowledges and incorporates the heterogeneous agents approach that challenges the traditional rational agent framework. More specifically, our focus is on the validation and estimation of (dynamic) heterogeneous agent models (HAM) that have their roots in the agent-based literature. This branch of behavioural finance assumes that agents are at least boundedly rational (Simon, 1957), and that they use certain rules of thumb in order to form expectations about future asset prices. This setup goes back to Zeeman (1974), and was further advanced by, among others, Frankel and Froot (1987), Chiarella (1992), Brock and Hommes (1997, 1998), Lux (1998) and De Grauwe and Grimaldi (2006). Although different names are being used in the literature for different forecasting strategies, they roughly come down to two or three types of agents. One typical type of agent uses past (price) information in order to predict future returns. The strategy this agent uses is referred to as (trend) extrapolation, technical analysis, bandwagon (for positive trend extrapolation), contrarian (for trend reversion) or chartism. The second type of agent bases his expectations on the deviation of the asset price from its fundamental value. This agent is said to be mean reverting, regressive or fundamentalist. Third and fourth types differ among studies and markets, such as carry traders (Pojarliev and Levich, 2008; Spronk et al., 2013).

Although several studies survey the theoretical work on this type of models (Hommes, 2006; LeBaron, 2000; Chiarella et al., 2009, among others), there is a gap in the literature when it comes to surveying empirical work. Our purpose is to present a comprehensive review of the empirical findings and recent developments of estimation designs put forth over the past two decades. Heterogeneous agent models perform very well in describing, explaining, and often forecasting (financial) market's dynamics: they have been used to explain asset price dynamics in equities, foreign exchange, bonds, housing, derivatives, commodities and even macroeconomic variables.[1] In order to make the results comparable, Ter Ellen et al. (2017) estimate a generic heterogeneous agent model on a variety of asset classes and find support for heterogeneity of market participants for all asset classes but equities.

[1]They have also proven to be very well able to explain and replicate certain stylised facts of financial markets (Lux, 2009), such as volatility clustering, fat tails, and bull and bear markets.

Moreover, they find that heterogeneity is more pronounced for macroeconomic variables and that these are more prone to behavioural bubbles than financial assets.

The remainder of this chapter is as follows. Section 2 provides a brief description of how the field developed from rational agent models to models with boundedly rational, heterogeneous agents. Section 3 presents the first theoretical contributions that have been made and some of the empirical support from experiments and survey studies. In Sect. 4, the focus of attention is turned to the challenges in empirically measuring heterogeneous agent models for a variety type of asset classes and estimation methods. Section 5 concludes the survey.

2 From Rational Expectations to Bounded Rationality

2.1 Efficient Markets

The rationality of agents' expectations has been at the forefront of economic research in the financial economics literature. As such, expectations are the driving force in the (financial) marketplace. Modelling these expectations as rational has the convenient attribute that in such a case '[expectations] are essentially the same as the predictions of the relevant economic theory' (Muth, 1961). Fama (1965) argued that financial markets are efficient because of rational behaviour and expectations of economic agents, and that market efficiency (EMH) requires that actual prices (or rates of return) follow a 'fair game' process relative to expected equilibrium prices (or rates of return). The assumption of rational agents implies that agents incorporate all available information in their decision-making process and that they are able to do this in an efficient way because they have full knowledge about the economic models underlying financial markets. This means that all agents should have the same expectations and that all prices of (financial) products should reflect their fundamental values. It is acknowledged that some agents might not be rational and that therefore mispricing may occur. However, overreaction of some agents will be offset by underreaction of other agents. Moreover, according to Friedman (1953), possible mispricing caused by the so-called noise traders will soon vanish through the actions of rational agents. He argues that in such a way, speculators keep foreign exchange markets stable and efficient in case of a flexible exchange rate system. The concept of arbitrage, as described by Friedman, is one of the main fundaments of the EMH. It entails that rational agents will observe mispricing and take actions upon it. Therefore, noise traders do not have a significant effect on prices, and it is impossible to consistently beat the market and earn riskless returns. If arbitrage opportunities exist, rational agents would pick upon these and trade upon them. In other words, 'there's no such thing as a free lunch'.

Although the efficient market hypothesis has been the conventional way of thinking about asset pricing on financial markets at least since the seventies, it has also been a target of criticism since its publication. An important reason for the criticism is that the theory has some internal contradictions. If agents are rational and thus have the same expectations, there would be no trade in financial securities at all. With transaction costs taken into account and prices being perfect reflections of all (available) information no agent would either want to sell or buy its assets, since no extra returns can be made with that transaction. Milgrom and Stokey (1982) show that even when some agents have private information, this 'no-trade theorem' applies. The fact that trade does take place, and in large and growing amounts, is one of the observations that weaken the EMH.

2.2 Limits of the EMH

The debate regarding the validity of the efficient market hypothesis is a long and standing one. With the arrival of several anomalies that are puzzling from the perspective of purely rational models, such as the forward premium puzzle, the equity premium puzzle or the excess trade volume, the notion of the rational expectations hypothesis is losing more and more ground. The finding of excessive trading (Milgrom and Stokey, 1982) poses a challenge to the hypothesis that investors are rational. Other observed market anomalies that are difficult to explain in the conventional setup are, for example, momentum effect (Jegadeesh, 1990, on the short term recent losers tend to underperform the market, recent winners tend to outperform the market), post-earnings announcement drift (Ball and Brown, 1968, prices do not adjust to information immediately but adjust slowly, causing a positive drift after positive news and a negative drift after disappointing news), long-term reversal (De Bondt and Thaler, 1985, extreme past losers tend to outperform the market, past winners tend to underperform the market), size effect (Black et al., 1972, small-firm stocks outperform stocks of large companies), excess volatility (Shiller, 1981) and foreign exchange rate puzzles (e.g. reversed evidence on purchasing power parity and interest parity).

Another explanation for the persistence of mispricing that can be found in the literature is that there are serious limits to arbitrage. Grossman and Stiglitz (1980) argue that if arbitrage is costly (which it inherently is), it cannot be the case that a competitive economy is always in equilibrium, as that would mean that arbitrageurs would not be able to make returns. Among others, De Long et al. (1990) introduce noise trader risk to explain why arbitrage opportunities cannot always be fully exploited. They argue that the existence of noise traders (i.e. traders whose trading decisions are based on non-fundamental information: noise) in the market brings along a significant amount of uncertainty that affects the riskiness of arbitrage. After all, if the effect of noise traders was strong enough to create the mispricing, these traders could as well increase the gap even further. Therefore, noise traders can heavily destabilise the market. According to the EMH, mispricing cannot persist because it creates the possibility of a riskless return that would immediately be exploited. However, if the profit opportunity is not riskless because of the unpredictable behaviour of noise traders, the mispricing can persist. This limit to arbitrage is usually labelled 'noise trader risk', but there can be other risks that limit arbitrage opportunities.

Still, limits to arbitrage are no explanation of exchange rate puzzles, the inefficiency of markets and the inherent mispricing. After all, it does not explain how mispricing can occur in the first place. Results from psychology and sociology have given some insight in the non-rational beliefs of investors which may help to understand the observed anomalies in financial markets.

2.3 Survey Evidence and Bounded Rationality

Although these contributions from the field of psychology are an important insight in the actual behaviour of people and clearly show that agents do not behave in a rational way, they have generated quite some skepticism. After all, most economists already knew from the start that not all investors behave fully rationally, but they consider this as a necessary assumption to include investor behaviour in sophisticated economic models. They argued that behavioural economics and behavioural finance were impractical bifurcations of economics, since it was impossible to model the complex behaviour of human beings. On top of that, the results from psychology were mainly generated by laboratory experiments which did not always replicate the real world in a very accurate way. These difficulties were reinforced by the problem that we could only observe price reactions to human behaviour instead of observing actual expectations of future asset prices.

The latter problem was partly overcome in the eighties, when companies like Money Market Services International (MMSI) and Consensus Economics started to gather investors' expectations of future asset prices by means of surveys. The use of survey data allows researchers to directly observe investors' expectations about future prices and exchange rates,[2] therefore making it easier for them to test investor rationality and information efficiency and to detect possible expectation formation mechanisms that are used by institutional investors. Early work by Blake et al. (1972), Dominguez (1986) and Frankel and Froot (1987) utilises such survey-based expectations, and many studies have used some form of survey measures of expectations in explaining foreign exchange rate puzzles after that. For example, MacDonald (1990a), MacDonald and Marsh (1996), Cavaglia et al. (1993) and Ito (1990) have used foreign exchange rate survey data in examining the rationality of exchange rate expectations and have concluded that respondents give biased forecast that do not efficiently capture publicly available information such as past interest rate movements.

The EMH incorporates the joint hypothesis that expectations are formed rationally and that market participants are risk neutral with respect to investing in

[2]Jongen et al. (2008) provide an excellent overview of the literature on expectations in foreign exchange markets.

domestic or foreign assets (Jongen et al., 2008). Several possible explanations for the failure of the forward rate as an unbiased estimate for future spot rates have been put forward in the financial economics literature (see Engel (1996), MacDonald (1990b) and Jongen et al. (2008), for instance). The main competing views are that the unbiasedness stems from irrational behaviour of exchange rate forecasters (Bilson, 1981; Cumby and Obstfeld, 1984, for instance), versus the existence of a time-varying risk premium (Fama, 1984; Hsieh, 2017; Wolff, 1987). However, the inherently necessary use of joint tests of rationality and for the existence of a risk premium made it impossible to distinguish between these causes of the forward premium bias. Survey-based expectations are a useful tool in this respect, as they allow us to decompose the forward premium into an 'irrational expectations' component and a 'time-varying risk premium' component. The literature suggests that both irrational expectations and time-varying risk premiums account for the forward discount anomaly (Froot, 1989; Froot and Thaler, 1990; Cavaglia et al., 1994, for instance). With the arrival of irrational expectations, the focus is shifting in the direction of expectation formation mechanisms. Three alternative models of expectation formation are mainly considered in the literature—the extrapolative, the regressive and the adaptive—against the null hypothesis that expectations are static. Whereas many of the studies focus on expectations following one of these specifications at a time, Prat and Uctum (2007) show that survey respondents use a combination of these rules.

When analysing the process of expectations formation, it appears that the longer the forecast horizon, the more exchange rate expectations reverse recent price trends. At horizons exceeding one month, expectations appear to stabilise and regress towards their equilibrium values. However, at horizons up to approximately one month agents extrapolate the most recent trend and diverge from their hypothesised long-run equilibrium values (Frankel and Froot, 1987, 1990a; Cavaglia et al., 1993; Ito, 1990). Prat and Uctum (2015) find that although the share of fundamentalists indeed increases with forecasting horizon, chartists always dominate.

2.4 Boundedly Rational Heterogeneous Agents Models

Although survey studies provided evidence to reject the assumptions of rational expectation formation and information efficiency, the problem of modelling behaviour persisted. As a response, some authors started incorporating certain aspects of the investors' behaviour in their models. In their contribution, Barberis et al. (1998) propose a parsimonious model of how investors form beliefs that is consistent with the available statistical and psychological evidence. In their 'model of investor sentiment', they include conservatism and representativeness to explain under- and overreaction of stock prices. Almost parallel to that, boundedly rational heterogeneous agents models (BRHA models, or HAM) were developed. This heterogeneous agents theory, originally founded by Zeeman (1974), Beja and Goldman (1980) and Frankel and Froot (1987) and further developed by, among others, Brock and Hommes (1997, 1998), Day and Huang (1990), Chiarella (1992) and De Grauwe et al. (1993) rejects the idea that investors behave rationally.

With some exceptions, these investigations have in common that the distinction they make is one between a fundamental approach in forming expectations and an extrapolative approach, which is usually referred to as 'technical analysis' or 'chartist behaviour'. Furthermore, some of the models assume that agents switch between the two strategies, depending on the forecasting performance or profitability of a certain strategy.

Fundamentalists base their expectations on economic theory about future asset prices and their trading strategy upon market fundamentals. They believe that the market price will revert to the intrinsic value of an asset and therefore bases expectations on the deviation of the market price from the fundamental economic value. In contrast, technical traders, or chartists, base their expectations on past price behaviour and try to extrapolate the trend in the most recent period(s). They expect trends to continue in the same direction and exploit these historical patterns in their investment decisions. Fundamentalist behaviour is generally found to have a stabilising effect on prices, while chartists tend to have a destabilising effect driving asset prices away from the intrinsic value of the asset.

3 Early Contributions and Supporting Evidence

3.1 Early Contributions

One of the earliest examples of a heterogeneous agent model that we can find in the literature is Zeeman (1974). He recognises and distinguishes two types of agents in the stock market, similar to the ones used in the 'modern-day' heterogeneous agent models. One group, chartists, chases trends, therefore buying when prices go up and selling when prices go down. The other group, fundamentalists, is aware of the true fundamental value, and buys (sells) when the stock is currently undervalued (overvalued). Zeeman explains the slow feedback flow observed in the stock market by the fact that the rate of change of stock market indices responds to chartist and fundamentalist demand faster than their demand responds to the return changes of these indices. In other words, while chartists and fundamentalists demand has a direct effect on returns, fundamentalists may only start selling when a stock is overvalued by a certain amount, thereby causing bull (chartists driving the price up) and bear (both chartists and fundamentalists selling stocks) markets. Although Zeeman's model is very similar in terms of set-up and implications to the heterogeneous agent models as we know them now, it lacked clear micro-foundations (Hommes, 2006) and his theory was not picked up at the time.

Another important contribution came from Beja and Goldman (1980). According to them, it is obvious that a man-made market where people interact and respond to each other cannot be fully efficient. Therefore, discrepancies will exist and human beings will naturally respond to these discrepancies by speculating on their expected direction of the market. Since this is bound to lead to different price dynamics than would occur under the efficient markets hypothesis, they propose an alternative theory. In line with Zeeman (1974), Beja and Goldman (1980) assume a mechanism where the speed of price changes and the speed of demand changes are not in line. Furthermore, they propose a market which consists of fundamental demand (based on the expectations of future equilibrium prices) and speculative demand (based on the state of the market). Dynamics in the aggregate demand especially occur due to relative sizes of the fundamental and speculative demand (which becomes larger if the price change is larger than expected) and the flexibility of the trend followers. The market will be stable if the impact of the fundamental demand is sufficiently high or if the impact of the trend followers is sufficiently low.

The heterogeneous agents literature has thereafter benefitted a lot from contributions from, among others, Frankel and Froot (1987, 1990a,b) and Brock and Hommes (1997, 1998). Frankel and Froot showed, by using survey data, that expectations could be classified as extrapolative, regressive and adaptive (1987), or as chartist and fundamentalist (1990a). Brock and Hommes (1997, 1998) introduced an intuitive switching rule, effectively implying that investors would switch to the rule with the best recent performance. HAM have been very well able to explain and replicate certain stylised facts of financial markets (Lux, 2009), such as volatility clustering, fat tails, and bull and bear markets. For comprehensive overviews of the (theoretical) HAM literature, see, for example, Hommes (2006), Chiarella et al. (2009) and LeBaron (2000).[3]

3.2 Supporting Evidence on the Micro-Level

Over the years, studies have collected empirical evidence in favour of the chartist–fundamentalist approach in various ways. In this section, we will discuss some of the evidence collected on the micro-level, of which the majority comes from laboratory experiments and survey studies.

Schmalensee (1976) was one of the first to use experimental methods to reveal expectation formation processes for time series, in particular with respect to technical rules. Smith et al. (1988) are able to replicate bubbles and crashes in a laboratory environment. De Bondt (1993) and Bloomfield and Hales (2002) use classroom experiments and find evidence of trend-following behaviour, where the latter also find support for the assumption in Barberis et al. (1998) that investors perceive past trend reversals as an indicator for the probability of future reversals

[3] Not all papers on HAM estimation are positive about the use and appropriateness of such models. Amilon (2008) uses maximum likelihood and efficient method of moments and finds that the models generally have a poor fit and do not generate all the stylised facts that some of the simulation studies are able to match.

even though they are aware of the random walk character. A laboratory experiment is used by Hommes et al. (2005) to evaluate how subjects form expectations when all they know is dividend yield, interest rates and past realised prices. The authors find that participants make use of very similar linear rules, such as autoregressive or adaptive strategies, in forming expectations. Assenza et al. (2014) provide an excellent summary of the relevant experimental work in this field.

As (laboratory) experiments are, in general, not fully able to replicate the real world situation, and their generalisability has therefore been questioned, attempts have been made to directly measure investor expectations and expectation formation rules. To this end, both quantitative and qualitative surveys have been conducted. Taylor and Allen (1992) show, based on a questionnaire survey, that 90% of the foreign exchange dealers based in London use some form of technical analysis in forming expectations about future exchange rates, particularly for short-term horizons. The foreign exchange dealers further stated that they see fundamental and technical analyses as complementary strategies for making forecasts and that technical analysis can serve as a self-fulfilling mechanism. Menkhoff (2010) gathered similar data from fund managers in five different countries. In line with the findings of Taylor and Allen, he finds that 87% of the fund managers surveyed use technical analysis. About 20% of the fund managers consider technical analysis as more important than fundamental analysis. Various quantitative surveys have been evaluated as well. For a more extensive overview, see Jongen et al. (2008). Frankel and Froot (1987, 1990a,b) have had a substantial impact on the foreign exchange literature and the further development of heterogeneous agent models. They were among the first to show that survey data reveals non-rationality and heterogeneity of investors. They also find evidence for the chartist–fundamentalist approach employed in many of the heterogeneous agent models. Others have confirmed these findings in later years, and with various datasets. Dick and Menkhoff (2013) use forecasters' self-assessment to classify themselves as chartists, fundamentalists or a mix. They find that forecasters who classify their forecasting tools as chartist use trend-following strategies and who classify as fundamentalist have a stronger preference for purchasing power parity (PPP). They also find that chartists update their forecasts more frequently than fundamentalists.

Ter Ellen et al. (2013) are among the first to estimate a full dynamic heterogeneous agent model (HAM) on survey data, meaning that the expectations of investors can be dynamic in various ways. They find that three forecasting rules fit the survey data very well: a PPP rule (fundamentalist), a momentum rule (chartist) and an interest parity rule. They confirm the earlier finding from Frankel and Froot (1990a,b) that investors use more speculative strategies for shorter horizons (1 month) and more fundamental strategies for longer horizons (12 months). Moreover, investors switch between forecasting rules depending on the past performance of these rules. Goldbaum and Zwinkels (2014) find that a model with fundamentalists and chartists can explain the survey data well. As in Ter Ellen et al. (2013), they find that fundamentalists are mean reverting and that this model is increasingly used for longer horizons. Chartists have contrarian expectations. A model with time-varying weights on the different strategies outperforms a static version of this model. Jongen et al. (2012) also allow the weights on different strategies to vary depending on market circumstances. However, instead of directly explaining the survey expectations, they analyse the dispersion between forecasts. They find that the dispersion is caused by investors using heterogeneous forecasting rules and having private information. This is in line with the earlier findings of Menkhoff et al. (2009) for a dataset on German financial market professionals.

Zwinkels and co-authors have collected evidence for heterogeneous beliefs from data on fund managers' exposure. Verschoor and Zwinkels (2013) show that foreign exchange fund managers behave like heterogeneous agents. They find that fund managers allocate capital to a momentum, carry and value strategy depending on the past performance of these strategies. They make money by employing a negative feedback strategy: shifting money from recent winning strategies to recent losing strategies. Schauten et al. (2015) apply a heterogeneous agent model to hedge fund risk exposure. Because of the non-linear trading strategies that hedge fund managers employ, a non-linear model with dynamic weights seems to be appropriate to capture the hedge fund risk exposure. The heterogeneity of the hedge funds lies in the dynamic weighting of exposure to different risk factors.

3.3 An Example

We will now provide an example of a heterogeneous agent model with chartists, fundamentalists, and dynamic weighting of the two groups. Many of the models employed can be simplified to this model. The form of the model we show here is mostly related to some of our own applications of HAM (e.g. De Jong et al., 2010; Ter Ellen and Zwinkels, 2010; Chiarella et al., 2014), which are largely based on the functional form from Brock and Hommes (1997, 1998) and Boswijk et al. (2007).

The base of the model is the price of an asset. The price of an asset tomorrow, P_{t+1}, equals the price of today, P_t, and the weighted demand of different types of agents, typically chartists and fundamentalists[4]:

$$P_{t+1} = P_t + W_t D_t^c + (1 - W_t) D_t^f \tag{1}$$

Here, W_t is the chartist weight in the market, D_t^c is the chartist demand, $(1 - W_t)$ is the weight of fundamentalists in the market, and D_t^f is the demand function of fundamentalists. The demand functions can be specified as the difference between the current asset price and the expected asset price under chartist ($E_t^c[P_{t+1}]$) or fundamentalist ($E_t^f[P_{t+1}]$) expectations:

$$D_t^c = a^c (E_t^c[P_{t+1}] - P_t) \tag{2}$$

[4]Note that this simple linear function can follow from mean-variance optimising agents and zero outside supply, see Brock and Hommes (1998) and Hommes (2001), for example.

$$D_t^f = a^f(E_t^f[P_{t+1}] - P_t) \tag{3}$$

The demand is naturally positively related to the expected price change for both chartists and fundamentalists. In other words, when agents expect the price to increase in the coming period, they will increase their demand for that asset today. However, chartists and fundamentalists differ in the way they form expectations about future prices. Chartists form their expectations based on some form of technical analysis. Commonly used rules are moving average (MA) rules and AR(n) rules. For simplicity, we will focus on a simple AR(1) rule for chartists:

$$E_t^c[P_{t+1}] = P_t + \beta_c(P_t - P_{t-1}). \tag{4}$$

According to this rule, chartists expect price movements to continue if $\beta_c > 0$ or to reverse if $\beta_c < 0$. This often depends on the time horizon, i.e. whether t denotes a week, month or year, for example. Fundamentalists form their expectations based on their perception of a fundamental value of the asset, $(\overline{P_t})$, and the current price deviation thereof:

$$E_t^f[P_{t+1}] = P_t + \beta_f(\overline{P_t} - P_t). \tag{5}$$

Often, fundamentalists are a stabilising force, which means that they expect prices to revert to their fundamental levels. In such a case, $\beta_f > 0$. Computing a fundamental value as input for the model is one of the most challenging tasks of estimating a HAM. For some markets, there are multiple competing models, for example, in the foreign exchange market (PPP, UIP, monetary model, etc.), at other times there are no obvious candidates at all (for example, in commodity markets).

In many applications, the dynamics of the market can be best explained with time-varying weights for chartists and fundamentalists (in other words, when agents can 'switch' between the strategies). Switching functions may vary. For an evaluation of different switching functions, see Baur and Glover (2014). The example we show is an adapted multinomial logit rule from Brock and Hommes (1997, 1998) and similar to Ter Ellen and Zwinkels (2010). In this case, the weight of the chartists depends on the recent forecasting accuracy of the chartist forecasting rule, Π_t^c, relative to the recent forecasting accuracy of the fundamentalist rule, Π_t^f:

$$W_t = \left[1 + \exp\left(\gamma\left[\frac{\Pi_t^c - \Pi_t^f}{\Pi_t^c + \Pi_t^f}\right]\right)\right]^{-1} \tag{6}$$

In this setup, W_t is the proportion of chartists in the market (or the weight put on the chartist forecasting rule), and $1 - W_t$ is the proportion of fundamentalists. The forecasting accuracy of chartists (fundamentalists) is measured as the mean squared error of the chartists (fundamentalists) over the past period. Note that it is also possible that the agents evaluate the rule over more than one period.

$$\Pi_t^c = [(E_{t-1}^c[P_t] - P_{t-1}) - \Delta P_t]^2 \tag{7}$$

$$\Pi_t^f = [(E_{t-1}^f[P_t] - P_{t-1}) - \Delta P_t]^2 \tag{8}$$

As in Ter Ellen and Zwinkels (2010), Eq. (6) differs slightly from the weighting mechanism originally proposed by Brock and Hommes (1997). Instead of using the absolute difference in forecasting accuracy of the two rules, $\Pi_t^c - \Pi_t^f$, weights are calculated by using the relative forecasting (in)accuracy $\left(\frac{\Pi_t^c - \Pi_t^f}{\Pi_t^c + \Pi_t^f}\right)$. Ter Ellen and Zwinkels 2010 and Ter Ellen et al. 2017 argue that this method has the advantages of ease of estimation and comparability between different markets. The coefficient γ is called the intensity of choice and represents the investors' speed of switching. If $\gamma = 0$, investors do not adapt the importance given to the two rules and $W_t = 0.5$. The other extreme is when $\gamma = \infty$ where investors are perfectly adaptive and immediately adjust all weights to the rule with the smallest forecast error. A small positive γ can be an indication of status quo bias, introduced by Kahneman et al. (1982). If investors suffer from this bias, they are reluctant to change their status quo belief, which results in a slower updating of beliefs.

4 Estimation

Due to the complex and nonlinear nature of the bounded rationality heterogeneous agent models, most of the early papers in this field were restricted to theoretical explanations and simulations of these models. These simulations produced interesting results and were able to reproduce many of the stylised facts observed in (financial) markets. Therefore, direct confrontation of the model with real financial data was desirable. Vigfusson (1997) was the first to make an attempt to estimate the parameters of a model with chartists and fundamentalists to financial data.

Given that the dynamic weighting of the two strategies is unobserved, Vigfusson applied the Markov regime switching approach to the foreign exchange market, where chartist and fundamentalist behaviour can be seen as different states. After him, several other authors used this approach for the foreign exchange market (Ahrens and Reitz, 2003) and the stock market (Alfarano et al., 2006; Chiarella et al., 2012). Baak (1999) and Chavas (2000) suggested an approach with General Method of Moments (GMM) and Kalman filtering to estimate a chartist–fundamentalist model for the beef market. Not much later, Winker and Gilli (2001) and Gilli and Winker (2003) used a simulation-based indirect estimation approach by minimising loss functions based on the simulated moments and the realised moments from foreign exchange data. Westerhoff, Reitz and Manzan use a STAR-GARCH approach in several papers. An important characteristic of this estimation technique is that only one type of agents can have a deterministic time-varying weight. Westerhoff and Reitz (2003, 2005) incorporate dynamic weighting in one of the two types of agents by means of a STAR-GARCH estimation for the foreign exchange market (2003, time-varying fundamentalist impact) and the commodity market (2005, time-varying chartist impact). Manzan and Westerhoff (2007) also apply this method with time-varying weights on the chartist impact for the foreign exchange market, whereas Reitz and Slopek (2009) apply it to the oil market.

An important contribution in the estimation of heterogeneous agents models came from Boswijk et al. (2007). They use nonlinear least squares estimation combined with a multinomial logit switching rule to empirically validate a heterogeneous agents model for the S&P500. The main improvements of their method over estimating based on Markov switching are the smaller number of parameters to be estimated and the deterministic nature of their switching process, in contrast to a stochastic Markov process. Many empirical papers on heterogeneous agents models have successfully used, and sometimes adapted, the techniques from Boswijk et al. (2007) for stock markets (De Jong et al., 2009; Chiarella et al., 2014) and foreign exchange markets (De Jong et al., 2010), but also for less obvious asset classes, such as oil (Ter Ellen and Zwinkels, 2010), housing (Kouwenberg and Zwinkels, 2014), gold (Baur and Glover, 2014), options (Frijns et al., 2010), hedge funds (Schauten et al., 2015) and credit markets (Chiarella et al., 2015).

A recent survey study by Lux and Zwinkels (2018) extensively covers various techniques for estimating agent-based models. Here, we rather focus on the results from estimating heterogeneous agent models.

4.1 Results

Most empirical studies on heterogeneous agent models use the classification of chartists and fundamentalists as found in the theoretical literature, where chartists base their expectations either on an autoregressive or on a moving average rule, and fundamentalists choose a fundamental value that is appropriate for the asset class under consideration. According to the theory on chartists and fundamentalists, chartists generally play a destabilising role by extrapolating and enforcing trends, whereas fundamentalists have a stabilising impact on the asset price due to their mean reverting expectations. This presumption is confirmed by many empirical validations of the model (Table 1).

4.1.1 Stock Market

One of the most widely used methods for estimating a heterogeneous agents model (HAM) is with nonlinear least squares or maximum likelihood, combined with a multinomial logit switching rule which is inspired by the work of Brock and Hommes (1997, 1998). This method was introduced by Boswijk et al. (2007),

Table 1 Overview of empirical validations of heterogeneous agent models

Study	Market	Sample	Freq	Estimation	Fundamental	Agents
Equity						
Alfarano et al. (2006)	ASX & AUDUSD	1980(/1983)–2004	D	ML	Implied	Noise & fund
Boswijk et al. (2007)	S&P500	1871–2003	A	NLS	GG	Chart & fund
Amilon (2008)	S&P500	1980–2000	D	EMM / ML	GG	Chart & fund
De Jong et al. (2009)	Hang S. & B-SET	1980–2007	Q	ML	GG	Chart & fund & int
Chiarella et al. (2012)	S&P500	2000–2010	M	Markov RS	GG	Chart & fund & noise
Chiarella et al. (2014)	S&P500	1970–2012	M	ML	GG	Chart & fund & noise
Lof (2014)	S&P500	1871–2011	A	NLS	GG	fund & rat spec & cont spec
Frijns and Zwinkels (2016b)	Can. firms	2010–2011	HF	VECM	–	Chart & arb
Hommes and in 't Veld (2017)	S&P500	1950–2012	Q	NLS	GG	Chart & fund
Huang and Tsao (2018)	Taiwan SE	2010–2011	HF	ML	CAPM	Chart & fund & liq
Forex						
Vigfusson (1997)	CAD	1983–1992	D	Markov RS	PPP & TOT	Chart & fund
Winker and Gilli (2001)	DM	1991–2000	D	SMM	–	Chart & fund
Gilli and Winker (2003)	DM	1991–2000	D	SMM	–	Chart & fund
Westerhoff and Reitz (2003)	GBP, DM, JPY	1980–1996	D	STAR-GARCH	PPP	Chart & fund
Manzan and Westerhoff (2007)	DM, JPY, CAD, FF, GBP	1974–1998	M	OLS	PPP	Chart & fund
Menkhoff et al. (2009)	EUR, GBP, JPY	1992–2006	M	OLS	PPP & MM	Chart & fund
De Jong et al. (2010)	7 EMS currencies	1979–1998	M	ML	EMS parity	Chart & fund
Jongen et al. (2012)	EUR, JPY, GBP	1989–2009	M	NLS with panel	PPP	Chart & fund & carry
Ter Ellen et al. (2013)	JPY, GBP, EUR	2003–2008	W	NLS	PPP	Chart & fund & IP
Dick and Menkhoff (2013)	EUR	1991–2011	M	OLS	PPP	Chart & fund & inter
Goldbaum and Zwinkels (2014)	JPY	1995–2007	M	OLS	MM	Chart & fund

Commodities						
Westerhoff and Reitz (2005)	Corn	1973–2003	M	STAR-GARCH ML	LRA	Chart & fund
Reitz and Westerhoff (2007)	6 commodities	1973–2003	M	STAR-GARCH ML	LRA	Chart & fund
Reitz and Slopek (2009)	Oil (WTI)	1986–2006	M	STAR-GARCH	Demands	Chart & fund
Ter Ellen and Zwinkels (2010)	Oil (Brent & WTI)	1983–2009	M	ML	2Y MA	Chart & fund
Baur and Glover (2014)	Gold	1970–2012	M	NLS	EWMA	Chart & fund
Housing						
Kouwenberg and Zwinkels (2014)	US	1960–2012	Q	ML	PV of rents	Chart & fund
Kouwenberg and Zwinkels (2015)	US	1960–2014	Q	ML	PV of rents	Chart & fund
Eichholtz et al. (2015)	Amsterdam	1649–2005	A	ML	Inflation	Chart & fund
Bolt et al. (2014)	US, UK, NL, JP, CH, ES, SE, BE	1970–2013	Q	NLS	PV of rents	Chart & fund
Credit						
Chiarella et al. (2015)	13 European CDS	2004–2013	W	ML	Hazard rate	Chart & fund
Frijns and Zwinkels (2016a)	European bonds & CDS	2008–2015	D	ML	Latent factor	Arb & chart & liq
Other						
Frijns et al. (2010)	DAX 30 Volatility	2000–2000	D	GJR-GARCH	LRA	Chart & fund
Frijns et al. (2013)	US equity MF	1998–2004	D	ML	–	–
Verschoor and Zwinkels (2013)	FX FM	2000–2009	M	ML	–	Chart & fund & carry
Schauten et al. (2015)	HF exposure	1996–2009	M	ML	–	–
Cornea-Madeira et al. (2017)	US inflation	1968–2015	Q	NLS	Real MC	Fund & RW

Notes: Estimation methods refer to Maximum Likelihood (ML), Nonlinear Least Squares (NLS), Method of Moments (MM), and Markov regime shifting (Markov RS). Agent types refer to chartists (chart), fundamentalists (fund), arbitrageurs (arb), internationalists (int), liquidity traders (liq), noise traders (noise), rational speculators (rat) and contrarian speculators (cont). More detailed description of these papers can be found in Sect. 4.1

who directly estimate a HAM on stock returns (S&P500). In their model, there are heterogeneous agents with access to the fundamental value of a risky asset, but with different beliefs about the persistence of the deviation between the spot price and the fundamental price of the asset. Switching between the different beliefs takes place based on the relative past profitability of that strategy. Chiarella et al. (2014) estimate a heterogeneous agents model for the S&P500 with three types of agents: fundamentalists, chartist and noise traders. Consistent with most of the other empirical studies, fundamentalists play a stabilising role with respect to the fundamental value of the asset. Chartists trade based on a moving average rule given by a geometric decay process, while most empirical studies rely on an AR(1) rule. While the relative weight of fundamentalists and chartists in the market changes over time based on the relative performance of these rules, the impact of noise traders is assumed to be constant. Noise traders have no specific expectations of future returns, and their demand is driven by a noisy signal that depends on volatility. Both Boswijk et al. (2007) and Chiarella et al. (2014) find support for mean reversion in fundamentalists' expectations and trend extrapolation in chartists' expectations of the S&P500. The model with time-varying weights has a significantly better fit than the static model.

Lof (2014) also estimates a heterogeneous agent model on S&P500 data. The types of agents he distinguishes are fundamentalists, rational speculators and contrarian speculators. The latter two types have exactly opposing beliefs to one another. He finds that the existence of contrarians can explain some of the most volatile episodes of the S&P500. De Jong et al. (2009) also distinguish three types of agents, to shed light on the Asian crisis in the context of heterogeneous agents. Besides chartists and fundamentalists, they distinguish internationalists, who condition their expectations on foreign market conditions. In a two-country model (with Hong Kong and Thailand) for the stock market, chartists and fundamentalists base their expectations on past price changes and the price deviation from the fundamental value, respectively, whereas internationalists base their expectations on the past price changes of the foreign market. Market dynamics occur due to switching between the different groups conditional on their past forecasting performance. Their estimation method is in many ways comparable to the one in Boswijk et al. (2007), yet De Jong et al. (2009) use maximum likelihood techniques instead of nonlinear least squares. All these studies compute a fundamental stock price by taking the discounted value of expected future dividends, which comes down to a simple Gordon growth model when a constant growth rate of dividends is assumed. Given the earlier critique on the use of a benchmark fundamental value with constant risk premium, Hommes and in 't Veld (2017) also calculate a fundamental value based on the Campbell–Cochrane consumption-habit model that allows for variation in the risk premium. Even with this model as a benchmark, they find substantial behavioural heterogeneity for the S&P500.

Alfarano et al. (2006) use Markov switching to estimate a HAM for Australian stock and FX data. They recognise the complexity of the agent-based models and the fact that this makes it difficult to directly estimate all the underlying parameters. They simplify the model to a closed-form solution for returns to overcome this problem. Although their model is highly simplified compared to some of the earlier agent-based models for financial markets, the authors are still able to reproduce some of the stylised features of stock returns. The two groups of traders are labelled as fundamentalists and noise traders, and switching between the two groups occurs based on asymmetric switching probabilities, inspired by Kirman's herding mechanism. The switching is asymmetric because the transition probability of an agent switching from the group of noise traders to the group of fundamentalists differs from the transition probability of a switch in the opposite direction. Chiarella et al. (2012) use Markov regime switching to explain the market dynamics of the S&P500. In their model, investors' beliefs about returns are regime dependent, and regimes (a bull state of the market with positive returns and low volatility or a bust state of the market with negative returns and high volatility) are generated by a stochastic process.

Recent contributions have used the heterogeneous agent framework to explain very high frequency stock price movements. Frijns and Zwinkels (2016b) look at cross-listed Canadian firms to find out where price discovery takes place. The model shows time variation in price discovery that is driven by agents switching between an arbitrage and a speculative strategy. Huang and Tsao (2018) use intraday data on three stocks listed on the Taiwan Stock Exchange to investigate whether there is evidence of heterogeneity of beliefs. They find that fundamentalists are stabilising, given that they expect mispricing to reduce in the next period. Chartists (technical analysts) behave as contrarians, but extrapolate buyer-initiated trades as a sign that prices will rise, and seller-initiated trades as a sign that prices will decline. Interestingly, they also find that chartists perform slightly better than fundamentalists.

4.1.2 Foreign Exchange Market

Vigfusson (1997) is the first to empirically test the chartist–fundamentalist approach for the foreign exchange market, and does this by means of a Markov switching approach. He tests two different specifications for fundamentalists and two for chartists. He finds that more important than the functional form of the types of agents is the different variances in the two regimes. He concludes that the USDCAD market is certainly characterised by quite regular regime shifts, but that it is not straightforward to conclude that this directly stems from the presence of chartists and fundamentalists in the market.

De Jong et al. (2010) estimate a full heterogeneous agents model with switching on exchange rates. By estimating the chartist–fundamentalist model on EMS rates, they circumvent the problem of having to choose a fundamental rate. Instead, they can use the 'parity' rate. With a survey dataset from Consensus Economics London, Goldbaum and Zwinkels (2014) directly test investor heterogeneity and expectation formation for the Japanese yen and the euro against the US dollar. The authors estimate three different models with chartists and fundamentalists. In the first model, both rules are estimated for the full sample of respondents and time.

In the second model, every forecaster is labelled as being either fundamentalist or chartist, based on the sum of the relative difference between the forecast and the outcome of the respective forecasting strategy. Finally, the respondents are allowed to switch their strategy. Every single forecast is labelled as resulting from either the fundamentalist or chartist strategy. The authors use the monetary model to compute a fundamental value for the exchange rates. Another paper that evaluates investor expectations for the foreign exchange market with survey data comes from Ter Ellen et al. (2013). They estimate a full heterogeneous agent model with dynamic weights of PPP traders (fundamentalists), momentum traders (chartists) and interest parity traders on forecasts for the euro, pound sterling and Japanese yen against the US dollar and the Japanese yen against the euro. One of their main findings is that they find forecasters to use rather 'speculative' models, such as momentum and carry, to predict exchange rates for short horizons, and rather 'fundamental' models, such as PPP and UIP, to predict exchange rates for longer horizons. The same strategies are identified by Verschoor and Zwinkels (2013) by looking at currency trader indices. They further find that FX fund managers apply a negative feedback strategy, moving capital from strategies with high past performance to low past performance.

Winker and Gilli (2001) and Gilli and Winker (2003) use a simulation-based indirect estimation approach to find the parameter values of a HAM applied to the US dollar–German mark exchange rate. The parameter values of the model are obtained by minimising a loss function based on the model simulated moments and the moments from the real data. The 2001 paper serves as an introduction of this method and therefore only focuses on two moments: kurtosis and ARCH-effects. The authors only estimate the random switching probability parameter and the probability that an agent will switch after interacting with another agent. In the 2003 paper, the optimisation algorithm is improved and a third parameter, the standard deviation of noise in the majority assessment, is estimated.

Westerhoff and Reitz (2003) estimate a STAR-GARCH model where the impact of fundamentalists depends on the strength of their belief in fundamental analysis. If the misalignment of the exchange rate with the fundamental value increases, fundamentalists lose their faith in fundamental analysis and leave the market. Therefore, the dynamics in the fundamentalists' behaviour further destabilise the exchange rate. This is in stark contrast to the findings in Manzan and Westerhoff (2007). They find that fundamentalists play an increasingly stabilising role in the event of a larger misalignment of the exchange rate. However, chartists play a destabilising role only within a certain range. When the past appreciation or depreciation of the exchange rate is larger than the threshold value, their behaviour becomes stabilising. De Jong et al. (2010) find evidence of stabilising behaviour of all types of agents for EMS rates, a result they assign to the investors' trust in the monetary authorities.

Finally, rather than explaining price movements or expectations directly, a few papers explain the dispersion of beliefs by a model with chartists and fundamentalists (Menkhoff et al., 2009; Jongen et al., 2012). They provide further evidence that agents in the foreign exchange market are heterogeneous due to the use of these different forecasting approaches.

4.1.3 Commodities

Prat and Uctum (2011) describe the expectation formation process for WTI oil prices as a combination of the extrapolative, regressive and adaptive expectation formation processes, based on survey data obtained from Consensus Economics. Reitz and Slopek (2009) explain the large price swings observed in the oil market by stabilising fundamentalists, who have a larger impact the larger the misalignment of the oil price is, and chartists, who are dominant and play a destabilising role when the price of oil is close to its fundamental value. While Reitz and Slopek (2009) take a STAR-GARCH approach with heterogeneous agents to explain large oil price swings, Ter Ellen and Zwinkels (2010) employ maximum likelihood with a multinomial logit switching rule. In their approach, the market impact of trend-extrapolating chartists and mean-reversion fundamentalists is time varying, based on the relative past forecasting accuracy of the strategies. Fundamentalists believe in mean reversion of the WTI and Brent price of crude oil to a long-term moving average of the oil price, whereas chartists extrapolate the price movement from the previous period. Considering that there is no consensus on the fundamental value of oil and computing one can be costly, the authors use a 2-year moving average as a proxy for the fundamental value. They confirm the destabilising (stabilising) effect of chartists (fundamentalists) and additionally find asymmetry in the responses of both chartists and fundamentalists. Furthermore, high weights for the chartist strategy coincide with different price spikes in the sample period, suggesting that they contributed to an oil price bubble in these periods. The model has a good out-of-sample fit. The authors show that the heterogeneous agent model outperforms the random walk model and a VAR(1,1) model.

Baur and Glover (2014) find that investors in the gold market are heterogeneous. They find that whereas both chartists and fundamentalists help to explain the price of gold, it was mostly the extrapolative behaviour of chartists that contributed to the large and persistent increase in the price of gold in the early 2000s. However, the coefficients they obtain for chartist and fundamentalist behaviour are somewhat different from what is commonly found in other financial markets. One such surprising results is that in some specifications, fundamentalists in the market for gold play a destabilising role, i.e. they behave more like the chartists in the original model of Brock and Hommes (1997).

Westerhoff and Reitz (2005) estimate a model for the US corn market with constant stabilising fundamentalist behaviour and dynamic technical trading activity, which is time varying depending on the misalignment of the corn price. They find that chartists play a highly destabilising role, and that this effect becomes stronger the further the price of corn is away from its fundamental, or long-run equilibrium, price. They estimate a similar model, but with time variation in fundamentalists beliefs, in Reitz and Westerhoff (2007) for cotton, lead, rice, soybeans, sugar and zinc, and find that for these commodities, fundamentalists play a stabilising role when the misalignment is sizable enough.

4.1.4 Credit

Chiarella et al. (2015) analyse the large deviations from fundamental levels of credit risk for some European countries during the European sovereign debt crisis and find that these can be partly explained by a combination of increased global risk aversion and the dynamics between momentum traders (chartists) and fundamentalists. Although the increase in credit spreads for peripheral European countries during the sovereign debt crisis was initially caused by deteriorating fundamentals, a large part of the surge can be explained by momentum traders further extrapolating these trends of higher CDS spreads. Frijns and Zwinkels (2016a) jointly model the bond and CDS market for a very similar sample. Rather than calculating the underlying fundamental value, they treat the fundamental process as an unobservable factor driving both markets. They find that, on average, only 5.5% of spread variation can be explained by speculation, but that the effect varies over time.

4.1.5 Housing

Kouwenberg and Zwinkels (2014, 2015) show that even the price movements in the US housing market can be well explained by a dynamic heterogeneous agent model. The model is estimated with maximum likelihood, including fundamentalists who believe in mean reversion of house prices to a rents-based fundamental value and chartists who destabilise the market by extrapolating trends. Agents switch between strategies based on the past forecasting accuracy of the respective strategies. They further find that the dominance of chartists in the housing market from 1992 to 2005 can explain the bubble-like behaviour of house prices in that period. Their model with time-varying impact of fundamentalists, who believe in mean reversion to a fundamental value based on rents, and chartists, who extrapolate past price trends, explains the house price for the in-sample period, and is also able to predict the decline in house prices from 2006 onwards.

Bolt et al. (2014) estimate a heterogeneous agent model on housing data for eight countries, including the USA. Different from Kouwenberg and Zwinkels, Bolt et al. (2014) include (the possibility of) a risk premium in the fundamental value calculation. Also, their chartists extrapolate price misalignments rather than price trends. Overall, they find that the housing markets in all countries studied are prone to behavioural bubbles. They also suggest some policies that can help stabilise prices.

Whereas the aforementioned studies start their samples in the 1960s and 1970s, Eichholtz et al. (2015) study house prices in Amsterdam, the Netherlands, from the seventeenth century onwards. They find that expectation formation depends on the stage of the economic cycle: during economic slowdowns, agents focus more on fundamentals, whereas they are more prone to follow trends during booms.

4.1.6 Other Asset Classes

The evidence in favour of heterogeneous agents extends more and more to other (financial) markets. Frijns et al. (2010) propose a way to model heterogeneous expectations of volatility by applying a heterogeneous agent model to the option market, where volatility is priced and traded. Fundamentalists believe that conditional volatility will revert to the level of the unconditional volatility and chartists trade based on recently observed unexpected shocks. Their heterogeneous agent model simplifies to a GJR-Garch(1,1) model with time-varying coefficients, which depend on the time-varying market impact of chartists and fundamentalists.

Frijns et al. (2013) estimate a switching model on 400 US equity mutual funds where investors can switch between cash and stocks depending on the expected relative performance of stocks or cash, and evaluate the market timing ability of these funds. Strikingly, they find that less than 5% of the mutual funds in their study have positive market timing skills, versus more than 40% with negative timing skills.

Schauten et al. (2015) consider style investing hedge funds, and find that there is time variation in their exposure to certain investment styles. The time variation depends on the recent relative performance of the styles, as is common in the heterogeneous agent literature. Hedge funds display positive feedback trading, but could do better by doing this more aggressively.

As it turns out, housing is not the only macro-variable that can be explained by heterogeneous agents. Cornea-Madeira et al. (2017) estimate a HAM on the US inflation data. Fundamentalists expect inflation to revert back to a fundamental value, which is based on the relation between inflation and real marginal costs. The other group of firms, which they call random walk believers, have naive expectations, and are thus backward-looking. They find that the majority of firms follows such a backward-looking strategy when forming inflation expectations, but that there are also occurrences of the dominance of fundamentalists.

5 Conclusion

Although the rational paradigm has been at the forefront of financial markets research since the seventies, rejections of this paradigm and attempts to model investor behaviour in a different way are gaining ground. Boundedly rational heterogeneous agent models (HAM) are an example of such models. In these models, agents are allowed to form expectations using relatively simple rules of thumb. In the empirical applications, this often boils down to two to four different agent types: fundamentalists, who expect market prices to revert to the fundamental value of the respective assets, chartists, who extrapolate price trends, and third and fourth types that often differ among various applications. In this contribution, we have provided an overview of papers estimating such models and their main results.

We have learned from this literature that investors are not only heterogeneous, they also do not use stable, unconditional, forecasting rules to form their expectation on future movements of exchange rates. Instead, they may change the way they form expectations based on various factors, such as the past performance of different forecasting rules or the horizon for which they form their expectations. The dynamics between the different types of investors can cause periods of severe mispricing and disruption of financial markets.

There is ample micro-evidence that agents indeed do not form rational expectations but use rules of thumb to forecast (financial) variables. Survey datasets that contain analysts' forecast are an important tool to unravel investor expectation mechanisms and dynamics that can otherwise not always be directly observed in the data. Studies based on such data have shown that expectations are not unbiased and do sometimes not even incorporate all available public information. Furthermore, the expectation formation rules that are found to explain the data well can be summarised by extrapolative, adaptive, and regressive rules, much in line with the rules chartists and fundamentalists use in heterogeneous agent models.

More micro-evidence on the behaviour of economic agents has come from experimental studies. Although a common critique of such studies is often the potential lack of external validity, many experimental studies have confirmed the behavioural rules found in survey responses. These rules are very much in line with behavioural rules in heterogeneous agent models: economic agents use (approximate) linear forecasting rules, such as autoregressive, mean reverting or adaptive strategies.

As surveyed in this chapter, heterogeneous agent models typically explain the stylised facts of financial markets well, and they are able to replicate important episodes of turmoil. However, empirically obtained results for various asset markets are often hard to compare, due to the researcher's choice of sample, fundamental value, set of behavioural rules and functional form of the switching function. Some efforts have been made to increase comparability by estimating a generic model on several (asset) prices, based on the same sample, switching function and behavioural rules, and based on a similar model for the fundamental value. In more general terms though, the degrees of freedom of behavioural (asset pricing) models needs to be taken seriously. It is the reason that the models can produce a very good fit of the data, but it can also lead to ad hoc modelling decisions that lack micro-foundations. One reason that the rational expectations paradigm is and has been the dominant one for so long is that there is only one way to be rational (and thus to model rationality), while there are infinite ways to deviate from rationality. When deviating from the rational expectations paradigm, it is important to keep in mind that there needs to be clear evidence on the micro-level for the way expectations are modelled.

Finally, one needs to keep in mind that models based on the heterogeneous beliefs of agents still abstract from reality in many other respects. In reality, it is very likely that agents do not only differ in the way they form beliefs but also in the preferences they have, the shocks that they are hit by and the information set they have access to. Especially on a macro-level, it is very hard to pin down whether people behave different from our model because they are irrational, or because we do not capture their preferences well. Currently, there is ample evidence that heterogeneous agent models beat a random walk model in forecasting financial variables. However, as of yet there is very little work that compares the performance of these models to other deviations of the efficient markets hypothesis, such as full versus limited information/attention, heterogeneous preferences or financial (market) frictions. This can be a promising line of future research.

References

Ahrens, R., & Reitz, S. (2003). Heterogenous expectations in the foreign exchange market: Evidence from daily dollar/DM exchange rates. *Journal of Evolutionary Economics, 15*(1), 65–82.

Alfarano, S., Lux, T., & Wagner, F. (2006). Estimation of a simple agent-based model of financial markets: An application to Australian stock and foreign exchange data. *Physica: Statistical Mechanics and Its Applications, 370*(1), 38–42.

Amilon, H. (2008). Estimation of an adaptive stock market model with heterogeneous agents. *Journal of Empirical Finance, 15*(2), 342–362.

Assenza, T., Bao, T., Hommes, C. H., & Massaro, D. (2014). Experiments on expectations in macroeconomics and finance. In J. Duffy (Ed.), *Experiments in macroeconomics* (pp. 11–70). Bingley: Emerald Group Publishing Limited.

Baak, S. J. (1999). Tests for bounded rationality with a linear dynamic model distorted by hetereogeneous expectations. *Journal of Economics Dynamics and Control, 32*(9), 1517–1573.

Ball, R., & Brown, P. (1968). An empirical evaluation of accounting income numbers. *Journal of Accounting Research, 6*(2), 159–178.

Barberis, N., Shleifer, A., & Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics, 49*(3), 307–345.

Baur, D. G., & Glover, K. J. (2014). Heterogeneous expectation in the gold market: Specification and estimation. *Journal of Economic Dynamics and Control, 40*(C), 116–133.

Beja, A., & Goldman, M. B. (1980). On the dynamic behaviour of prices in disequilibrium. *Journal of Finance, 35*(2), 235–248.

Bilson, J. (1981). The speculative efficiency hypothesis. *Journal of Business, 54*, 435–451.

Black, F., Jensen, M. C., & Scholes, M. (1972). The capital asset pricing model: Some empirical tests. In M. C. Jensen (Ed.), *Studies in the theory of capital markets* (pp. 79–124). New York: Praeger.

Blake, D., Beenstock, M., & Brasse, V. (1972). The performance of UK exchange rate forecasters. *The Economic Journal, 96*(384), 986–999.

Bloomfield, R., & Hales, J. (2002). Predicting the next step of a random walk: Experimental evidence of regime-switching beliefs. *Journal of Financial Economics, 65*(3), 397–414.

Bolt, W., Demertzis, M., Diks, C., Hommes, C., & van der Leij, M. (2014). *Identifying Booms and Busts in House Prices Under Heterogeneous Expectations*. De Nederlandsche Bank Working Paper (450).

Boswijk, H. P., Hommes, C. H., & Manzan, S. (2007). Behavioral hetreogeneity in stock prices. *Journal of Economic Dynamics and Control, 31*(6), 1938–1970.

Brock, W. A., & Hommes, C. H. (1997). A rational route to randomness. *Econometrica, 65*(5), 1059–1095.

Brock, W. A., & Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control, 22*(8), 1235–1274.

Cavaglia, S., Verschoor, W. F. C., & Wolff, C. C. P. (1993). Further evidence on exchange rate expectations. *Journal of International Money and Finance, 12*(1), 78–98.

Cavaglia, S., Verschoor, W. F. C., & Wolff, C. C. P. (1994). On the biasedness of forward foreign exchange rates: Irrationality or risk premia? *The Journal of Business, 67*(3), 321–343.

Chavas, J.-P. (2000). On information and market dynamics: The case of the US beef market. *Journal of Economic Dynamics and Control, 25*(5), 833–853.

Chiarella, C. (1992). The dynamics of speculative behavior. *Annals of Operational Research, 37*(1), 101–123.

Chiarella, C., Dieci, R., & He, X.-Z. (2009). Heterogeneity, market mechanisms and asset price dynamics. In T. Hens & K. R. Schenk-Hoppe (Eds.), *The handbook of financial markets: Dynamics and evolution* (pp. 277–344). Burlington: Elsevier.

Chiarella, C., He, X.-Z., Huang, W., & Zheng, H. (2012). Estimating behavioural heterogeneity under regime switching. *Journal of Economic Behaviour and Organization, 83*(3), 446–460.

Chiarella, C., He, X.-Z., & Zwinkels, R. C. J. (2014). Heterogeneous expectations in asset pricing: Empirical evidence from the S&P500. *Journal of Economic Behaviour and Organization, 105*, 1–16.

Chiarella, C., Ter Ellen, S., He, X.-Z., & Wu, E. (2015). Fear or fundamentals? Heterogeneous beliefs in the European sovereign CDS market. *Journal of Empirical Finance, 32*, 19–34.

Cornea-Madeira, A., Hommes, C., & Massaro, D. (2017, forthcoming). Behavioral heterogeneity in US inflation dynamics. Journal of Business & Economic Statistics.

Cumby, R., & Obstfeld, M. (1984). International interest rate and price level linkages under flexible exchange rates. In: J. F. O. Bilson & R. C. Marston (Eds.), *Exchange rate theory and practice* (pp. 121–152). Chicago: University of Chicago Press.

Day, R. H., & Huang, W. (1990). Bulls, bears and market sheep. *Journal of Economic Behaviour and Organization, 14*(3), 299–329.

De Bondt, W. F. M. (1993). Betting on trends: Intuitive forecasts of financial risk and return. *International Journal of Forecasting, 9*(3), 355–371.

De Bondt, W. F. M., & Thaler, R. (1985). Does the stock market overreact? *The Journal of Finance, 40*(3), 793–805.

De Grauwe, P., Dewachter, H., & Embrechts, M. (1993). *Exchange rate theory: Chaotic models of foreign exchange markets*. Oxford: Blackwell.

De Grauwe, P., & Grimaldi, M. (2006). Exchange rate puzzles: A tale of switching attractors. *European Economic Review, 50*(1), 1–33.

De Jong, E., Verschoor, W. F. C., & Zwinkels, R. C. J. (2009). Behavioural heterogeneity and shift-contagion: Evidence from the Asian crisis. *Journal of Economic Dynamics and Control, 33*(11), 1929–1944.

De Jong, E., Verschoor, W. F. C., & Zwinkels, R. C. J. (2010). Heterogeneity of agents and exchange rate dynamics: Evidence from the EMS. *Journal of International Money and Finance, 29*(8), 1652–1669.

De Long, J. B., Shleifer, A., Summers, L., & Waldmann, R. (1990). Noise trader risk in financial markets. *Journal of Political Economy, 98*(4), 703–738.

Dick, C. D., & Menkhoff, L. (2013). Exchange rate expectations of chartists and fundamentalists. *Journal of Economic Dynamics and Control, 37*(7), 1362–1383.

Dominguez, K. (1986). Are foreign exchange rate forecasts rational? New evidence from survey data. *Economics Letters, 21*(3), 277–281.

Eichholtz, P., Huisman, R., & Zwinkels, R. C. J. (2015). Fundamentals or trends? A long-term perspective on house prices. *Applied Economics, 47*(10), 1050–1059.

Engel, C. (1996). The forward discount anomaly and the risk premium: A survey of recent evidence. *Journal of Empirical Finance, 3*(2), 123–192.

Fama, E. F. (1965). Random walks in stock market prices. *Financial Analysts Journal, 51*(1), 55–59.

Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics, 14*, 319–338.

Frankel, J. A., & Froot, K. A. (1987). *Understanding the US Dollar in the Eighties: The Expectations of Chartists and Fundamentalists*. NBER Working Paper, No. R0957.

Frankel, J. A., & Froot, K. A. (1990a). Chartists, fundamentalists, and trading in the foreign exchange market. *American Economic Review, 80*(2), 181–185.

Frankel, J. A., & Froot, K. A. (1990b). *Exchange Rate Forecasting Techniques, Survey Data, and Implications for the Foreign Exchange Market*. IMF Working Paper.

Friedman, M. (1953). The case for flexible exchange rates. In *Essays in positive economics* (pp. 157–203). Chicago: University of Chicago Press.

Frijns, B., Gilbert, A., & Zwinkels, R. C. (2013). Market timing ability and mutual funds: A heterogeneous agent approach. *Quantitative Finance, 13*(10), 1613–1620.

Frijns, B., Lehnert, T., & Zwinkels, R. C. J. (2010). Behavioral heterogeneity in the option market. *Journal of Economic Dynamics and Control, 34*(11), 2273–2287.

Frijns, B., & Zwinkels, R. C. (2016a). *Speculation in European Sovereign Debt Markets*. Working paper.

Frijns, B., & Zwinkels, R. C. (2016b). *Time-Varying Arbitrage and Dynamic Price Discovery*. Working paper.

Froot, K. A. (1989). New hope for the expectations hypothesis of the term structure of interest rates. *Journal of Finance, 44*(2), 283–305.

Froot, K. A., & Thaler, R. H. (1990). Anomalies: Foreign exchange. *Journal of Economic Perspectives, 4*(3), 179–192.

Gilli, M., & Winker, P. (2003). A global optimization heuristic for estimating agent based models. *Computational Statistics and Data Analysis, 42*(3), 299–312.

Goldbaum, D., & Zwinkels, R. C. J. (2014). An empirical examination of heterogeneity and switching in foreign exchange markets. *Journal of Economic Behavior & Organization, 107*, 667–684.

Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *American Economic Review, 70*(3), 393–408.

Hommes, C. H. (2001). Financial markets as nonlinear adaptive evolutionary systems. *Quantitative Finance, 1*(1), 149–167.

Hommes, C. H. (2006). Heterogeneous agent models in economics and finance. In L. Tesfatsion & K. L. Judd (Eds.), *Handbook of computational economics* (Vol. 2, pp. 1109–1186). Amsterdam: North-Holland.

Hommes, C. H., & in 't Veld, D. (2017). Booms, busts and behavioural heterogeneity in stock prices. *Journal of Economic Dynamics and Control, 80*, 101–124.

Hommes, C. H., Sonnemans, J., Tuinstra, J., & van de Velden, H. (2005). Coordination of expectations in asset pricing experiments. *Review of Financial Studies, 18*(3), 955–980.

Hsieh, D. A. (2017). Tests of rational expectations and no risk premium in forward exchange markets. *Journal of International Economics, 17*, 173–184.

Huang, Y.-C., & Tsao, C.-Y. (2018). Discovering traders' heterogeneous behavior in high-frequency financial data. *Computational Economics, 51*(4), 821–846.

Ito, T. (1990). Foreign exchange rate expectations: Micro survey data. *The American Economic Review, 80*(3), 434–449.

Jegadeesh, N. (1990). Evidence of predictable behavior of security returns. *The Journal of Finance, 45*(3), 881–898.

Jongen, R., Verschoor, W. F. C., & Wolff, C. C. P. (2008). Foreign exchange rate expectations: Survey and synthesis. *Journal of Economic Surveys, 22*(1), 140–165.

Jongen, R., Verschoor, W. F. C., Wolff, C. C. P., & Zwinkels, R. C. J. (2012). Explaining dispersion in foreign exchange expectations: A heterogeneous agent approach. *Journal of Economic Dynamics and Control, 36*(5), 719–735.

Kahneman, D., Slovic, P., & Tversky, A. (1982). *Judgment under uncertainty: Heuristics and biases*. Cambridge: Cambridge University Press.

Kouwenberg, R., & Zwinkels, R. C. (2015). Endogenous price bubbles in a multi-agent system of the housing market. *PloS One, 10*(6), 1–10.

Kouwenberg, R., & Zwinkels, R. C. J. (2014). Forecasting the U.S housing market. *International Journal of Forecasting, 30*(3), 415–425.

LeBaron, B. (2000). Agent-based computational finance: Suggested readings and early research. *Journal of Economic Dynamics and Control, 24*(5–7), 679–702.

Lof, M. (2014). Rational speculators, contrarians and excess volatility. *Management Science, 61*(8), 1889–1901.
Lux, T. (1998). The socio-economic dynamics of speculative markets: Interacting agents, chaos and fat tails of return distributions. *Journal of Economic Behavior and Organization, 33*(2), 143–165.
Lux, T. (2009). Stochastic behavioral asset pricing models and the stylized facts. In T. Hens & K. R. Schenk-Hoppe (Eds.), *The handbook of financial markets: Dynamics and evolution* (pp. 161–215). Burlington: Elsevier.
Lux, T., & Zwinkels, R. C. J. (2018). Empirical validation of agent-based models. In *Handbook of Computational Economics*. Amsterdam: Elsevier.
MacDonald, R. (1990a). Are foreign exchange market forecasters 'rational'?: Some survey-based tests. *The Manchester School, 58*(3), 229–241.
MacDonald, R. (1990b). Expectations formation and risk in three financial markets: Surveying what the surveys say. *Journal of Economic Surveys, 14*(1), 69–100.
MacDonald, R., & Marsh, I. W. (1996). Currency forecasters are heterogeneous: Confirmation and consequences. *Journal of International Money and Finance, 15*(5), 665–685.
Manzan, S., & Westerhoff, F. H. (2007). Heterogeneous expectations, exchange rate dynamics and predictability. *Journal of Economic Behavior and Organization, 64*(1), 111–128.
Menkhoff, L. (2010). The use of technical analysis by fund managers: International evidence. *Journal of Banking and Finance, 34*(11), 2573–2586.
Menkhoff, L., Rebitsky, R. R., & Schröder, M. (2009). Heterogeneity in exchange rate expectations: Evidence on the chartist-fundamentalist approach. *Journal of Economic Behavior and Organization, 70*(1), 241–252.
Milgrom, P., & Stokey, N. (1982). Information, trade and common knowledge. *Journal of Economic Theory, 26*(1), 17–27.
Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica, 29*(3), 315–335.
Pojarliev, M., & Levich, R. (2008). Do professional currency managers beat the benchmark?. *Financial Analyst Journal, 64*(5), 18–30.
Prat, G., & Uctum, R. (2007). Switching between expectation processes in the foreign exchange market: A probabilistic approach using survey data. *Review of International Economics, 15*(4), 700–719.
Prat, G., & Uctum, R. (2011). Modelling oil price expectations: Evidence from survey data. *The Quarterly Review of Economics and Finance, 51*(3), 236–247.
Prat, G., & Uctum, R. (2015). Expectation formation in the foreign exchange market: A time-varying heterogeneity approach using survey data. *Applied Economics, 47*(34–35), 3673–3695.
Reitz, S. & Slopek, U. (2009). Non-linear oil price dynamics: A tale of heterogeneous speculators?. *German Economic Review, 10*(3), 270–283.
Reitz, S., & Westerhoff, F. (2007). Commodity price cycles and heterogeneous speculators: A STAR-GARCH model. *Empirical Economics, 33*, 231–244.
Schauten, M., Willemstein, R., & Zwinkels, R. C. J. (2015). A tale of feedback trading by hedge funds. *Journal of Empirical Finance, 34*, 239–259.
Schmalensee, R. (1976). An experimental study of expectation formation. *Econometrica, 44*(1), 17–41.
Shiller, R. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review, 71*(3), 421–436.
Simon, H. (1957). *Models of man*. New York: Wiley.
Smith, V. L., Suchanek, G. L., & Williams, A. W. (1988). Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica, 56*(5), 1119–1151.
Spronk, R., Verschoor, W. F., & Zwinkels, R. C. (2013). Carry trade and foreign exchange rate puzzles. *European Economic Review, 60*(C), 17–31.
Taylor, M. P., & Allen, H. (1992). The use of technical analysis in the foreign exchange market. *Journal of International Money and Finance, 11*(3), 304–314.

Ter Ellen, S., Hommes, C. H., & Zwinkels, R. C. J. (2017). *Comparing Behavioural Heterogeneity Across Asset Classes*. Norges Bank working paper, 12/2017.

Ter Ellen, S., Verschoor, W. F. C., & Zwinkels, R. C. J. (2013). Dynamic expectation formation in the foreign exchange market. *Journal of International Money and Finance, 37*, 75–97.

Ter Ellen, S., & Zwinkels, R. C. J. (2010). Oil price dynamics: A behavioural finance approach with heterogeneous agents. *Energy Economics, 32*(6), 1427–1434.

Verschoor, W. F. C., & Zwinkels, R. C. J. (2013). Do foreign exchange fund managers behave like heterogeneous agents? *Quantitative Finance, 13*(7), 1125–1134.

Vigfusson, R. J. (1997). Switching between chartists and fundamentalists: A Markov regime-switching approach. *International Journal of Economics and Finance, 2*(4), 291–305.

Westerhoff, F., & Reitz, S. (2003). Nonlinearities and cyclical behaviour: The role of chartists and fundamentalists. *Studies in Nonlinear Dynamics and Econometrics, 7*(4), 1558–3708.

Westerhoff, F., & Reitz, S. (2005). Commodity price dynamics and the nonlinear market impact of technical traders: Empirical evidence for the US corn market. *Physica A: Statistical Mechanics and its Applications, 349*(3), 641–648.

Winker, P., & Gilli, M. (2001). *Indirect Estimation of the Parameters of Agent Based Models of Financial Markets*. FAME Research Paper, 38.

Wolff, C. C. (1987). Forward foreign exchange rates, expected spot rates, and premia: A signal-extraction approach. *Journal of Finance, 42*(2), 395–406.

Zeeman, E. C. (1974). The unstable behavior of stock exchange. *Journal of Mathematical Economics, 1*(1), 39–49.