VU Research Portal

Search externalities in a Jackson queuing model for the labormarket

Gautier, P.

1994

document version
Publisher's PDF, also known as Version of record

Link to publication in VU Research Portal

citation for published version (APA)
Gautier, P. (1994). *Search externalities in a Jackson queuing model for the labormarket*. (Serie Research Memoranda; No. 1994-19). Faculty of Economics and Business Administration, Vrije Universiteit Amsterdam.

Serie research memoranda

Search Externalities in a Jackson Queuing Model for the Labormarket
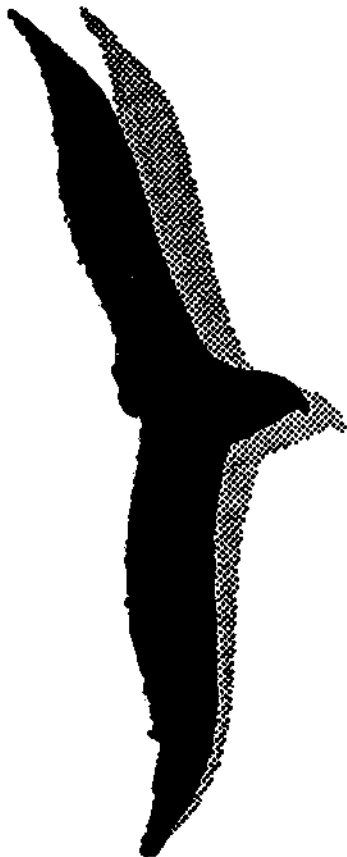
Pieter Gautier

|a| l |e|r|t|

applied
labour
economics
research
team

vrije Universiteit   *amsterdam*

Search Externalities in a Jackson Queuing Model for the Labormarket

Pieter Gautier

Tinbergen Institute and Free University Amsterdam

Comments invited

Abstract:

In this paper it is argued that unemployment is more related to waiting than to searching. In particular we focus on a congestion externality that employers and searchers place on the other searchers. This externality arises because the expected waiting time of unemployment increases with the amount of searchers relative to the amount of open jobs and the time employers need to screen the applicants. We have modeled the labor market as a Jackson queuing network which enabled us to make a distinction between the time a worker spends searching and the time he spends "waiting". Within this framework we were able to derive analytical solutions for steady state unemployment and average waiting time. In the model, unemployed workers can only partly influence the duration of unemployment. Because of this, the resulting equilibrium is unlikely to be efficient. In equilibrium workers may decide not to participate although their reservation wage can be lower than the social marginal product. We also give a rational explanation for the discouraged worker effect. In severe recessions when many persons start searching, expected unemployment duration will increase and hence the expected benefits of entering the labor market will decrease.

Introduction

In the standard literature of job search theory, a representative worker samples wage offers sequentially and decides on the basis of the sample obtained whether or not to continue searching, see e.g Lucas and Prescott (1974). Those models often ignore externalities that arise when many persons search at the same time. One type of such an externality is described in Diamond (1982 b). Diamond shows that an increase in the number of potential trading partners increases the probability of a successful match. Other externalities are related to congestion effects in the search process, see for example Tobin (1972). The probability for an unemployed individual to find a job is likely to be lower, the larger the stock of unemployed, is the basic message. This type of externality does not differ in its origin significantly from the type of externality that the seller of any good imposes on other sellers[1]. Other extensions of this idea can be found in Diamond (1981) where a worker can generate a negative externality if he reduces his reservation wage to fill a vacancy. This action reduces the stock of vacancies and hence the probability for other searches to find a job. This type of externality depends crucially on the assumption that the number of jobs is fixed. Another type of externality arises when productivity is match specific and revealed when a worker and a vacancy are matched. The meeting firm and worker are not likely to take into account the effect of their choices on other firms and workers still searching, this may lead to socially inefficient outcomes, see Diamond (1982:a), Mortenson (1982) and Pissarides (1990). If wages are determined by the outcome of a bilateral bargaining game, a match will only take place if the wage is within the interval determined by the reservation wage of the worker and the maximum wage the worker wants to pay. If there is a surplus, it is assumed that this surplus is shared between worker and employer. If the employer's share is high, this implies that the private return to a match is lower than the social return (marginal product) of a match. In the standard stopping model of search unemployment there will be too few searchers. But if the worker share is high, unemployment may well be above the social optimum because the private return to search effort is lower than the marginal product of the match. A higher worker share of the match reduces the number of jobs and it raises the expected wages. Pissarides (1990) shows that if the surplus of a match is shared in a way that maximizes the expected returns of the unemployed, the outcome is socially efficient. This outcome can be reached under a centralized bargaining system.

In this paper, we will focus on a different but related type of (congestion) externality. Given the fact that the application process takes some time, because for example application letters have to be read or interviews have to be taken, an increase in the number of searchers will influence the average waiting time of the other searchers and will indirectly lower the hazard rate out of unemployment. Moreover,

---

[1] See Lucas and Prescott (1974)

employers who let applicants wait before inviting them for an interview do not internalize the costs of the waiting applicant. An increase in the expected "waiting time" and a decrease of the hazard rate will lower the expected returns on search, hence the number of searchers will be lower than the social optimum.

In the model, three groups are distinguished; unemployed workers, employed workers and non participants. Every unit of time, people "flow", from one group to another. We will analyze those flows within a queuing framework. The advantage of this approach is that we can take a lot of factors into account that are important to explain the existence of unemployment for a relatively low price in terms of increasing complexity. The strategy that will be followed is not to derive all ideas from "first principles" but rather to refer to the existing literature in which those ideas have been derived rigorously.

We will distinguish two obstacles that the unemployed have to take before they become employed. The first one is caused by the time the unemployed worker spends on searching for vacancies and waiting to get invited for an interview, the second one is determined by the application process (interviews and waiting till the employer has made his decision). The more people there are in the system, the longer the average waiting time will be[2]. Thus the entrance of one worker will increase the waiting time of other workers. We will also take into account that people who are unemployed for a longer period of time loose human capital and become discouraged, because of that they face a lower probability of finding a successful match and devote fewer resources to search. This "persistence" of unemployment is what we observe in many European Countries. In addition we will assume, following Darby et al. (1985) that (older) workers with a lot of firm specific capital need to spend a longer time searching to find a successful match, once they have become unemployed. Another stylized fact of labor markets is that, especially in the U.S., the search behavior of young workers is characterized by short employment spells (job hopping). This aspect is taken into account by assuming that after a match has taken place, both employer and worker need to spend some time to learn about the real quality of the match. After that, a decision is made whether to continue the relationship or to separate. Separated workers have to start the search process again.

The distinction between voluntary and involuntary unemployment is not very relevant here. In this model people can speed up their search process by accepting a lower wage. On the other hand, the duration of the application process is determined by the employers and there is also some time involved between the moment of application and the moment the worker knows whether he will be

---

[2] If the supply of jobs would increase directly with the number of searchers, average waiting time can remain constant.

2

invited for an interview or not. In this sense, the duration of unemployment is out of control of the unemployed worker.

The outline of the paper is as follows: In section 1, the model with its assumptions will be presented. In section 2, the number of unemployed in the steady state will be derived as a function of the parameters of the model. We will show how expected unemployment duration influences wages in section 3. Finally in section 4 we will investigate whether the resulting search equilibrium is likely to be efficient.

1. The Model

As mentioned before, in most labor market models the flows between unemployment and employment are induced by the desire to search. But as Blanchard and Diamond (1992) argue, endogenous search, while surely present, is not of the essence. What is of the essence is that there is an endogenous delay in finding another job. This is an important aspect to take into account when it comes to the determination of unemployment and wages.

In the model below we will take into account that unemployed workers spend most of their time "waiting" instead of searching. In this section we will analyze the factors that influence this delay in finding a job.

The assumptions

We will assume that both badly matched employed and unemployed workers are looking for jobs. Employers are assumed to search sequentially and workers are assumed to search non-sequentially[3]. Job creation and job destruction continuously take place and follow a Poisson process.

Once a match between worker and employer has been made, worker and employer only gradually learn about the quality of the match[4]. We will denote Θ to be the true value of the match (consisting of the wage and all other utility that the job offers). Both worker and firm initially observe a noisy version of this parameter, equal to $w = \Theta + \varepsilon$, where ε is a random noise that is uncorrelated with Θ. The

---

[3] Abbring en van Ours (1994) show that this is indeed the most likely case in the Netherlands.

[4] See Jovanovic (1979) and Wright (1986)

3

firm will offer to pay the worker $E(\Theta|\Theta+\varepsilon)$. The worker can either accept or reject this offer. If he rejects it, he will remain searching. After the contact has been made, firm and worker will spend some time learning about the true value of w. After this learning period, they decide whether to continue the relationship or not. We will abstract here from the possibility of renegotiating the labor contract after new information becomes available, as for example in MacLeod and Malcomson (1993).

The road to employment is not a smooth one. The first obstacle (node 1) is caused by the fact that the process of looking for a suitable job takes time. One has to read newspapers to look for vacancies, go to the labor office, balance the offered wage and the reservation wage against each other, write application letters, wait till one gets invited for an interview etc. Every unit of time, only a limited amount of workers will leave this first node, since it takes time for companies to handle all the applications. If $\mu 1$ is the amount of workers that has been screened and leave the first node, then a queue will be formed if in a certain time interval the arrival rate exceeds $\mu 1$. Thus the total waiting time can be divided in screening time (the time that a worker actually spends searching plus the time the employer spends on screening the applicant) and the time spent in the queue (the external effect of other searchers).

We will call the time spent at the first node, search time, although as mentioned before, most of the time is spent waiting and not searching. It will be assumed that $\mu 1$ follows a negative exponential distribution and that $\mu 1$ is a function of the wage w and the number of vacancies, v^5.

$$b(t) = \mu 1 e^{-\mu 1 t} \tag{1}$$

Where b(t) is search time and $\mu 1 = \mu 1(w,v)$ with $d\mu 1/dw > 0$ and $d\mu 1/dv > 0$.

$\mu 1w$ is positive because if the wage is high, the proportion of jobs for which the wage exceeds the reservation wage, is larger[6]. $\mu 1v$ is positive because the more vacancies there are the faster one can find a suitable vacancy.

We will assume that firms invite a fixed proportion of applicants for an interview, we will call this proportion, p. Thus, after the unemployed worker has decided to which jobs he wants to apply, he faces a probability p that he is invited into the application process and a probability (1-p) that he is rejected. If he is rejected, he will flow back into the pool of unemployed, we will take into account that search intensity decreases with the duration of unemployment and also that the long term

---

[5] In section 4, we will derive the number of vacancies in the steady state.

[6] We implicitly assume here that the reservation wage rises less than proportionally with the offered wage.

unemployed face lower probabilities to be invited for an interview. Reasons for this may be depreciation of human capital or being unemployed for a longer period is a negative signal towards the employer. We model this by introducing an extra node to which the workers flow after they have been rejected. Of course the model can be extended with n extra nodes, each with a lower escape probability, where workers who have been rejected i times, would flow to pool i, this would add however little to our insight of the matching process. As we will see, one important implication of this model is that small shocks that increase the number of unemployed can have strong and long lasting effects on future unemployment.

Once a worker is accepted to enter the application process, he will face a second obstacle (node 2). This obstacle is mainly determined by the firm and depends on items as selection time, the number of interviews the firm takes for a vacancy and the total amount of vacancies, v. We will assume again that the application time is exponentially distributed with average waiting time $1/\mu2$. If the arrival rate at node two, exceeds $\mu2$, again a queue will be formed.

$$a(t) = \mu2 e^{-\mu2t} \qquad (2)$$

Where a(t) is application time and $\mu2$ is a function of the number of vacancies, v. $d\mu2/dv$ is positive because an unfilled vacancy is costly (see Pissarides, 1990). So firms will speed up their selection process, the more vacancies they have got. In reality, $\mu2$ will vary between sectors and jobs, we will abstract from that here.

After the application process, the worker has got a probability r of getting an offer to enter the pool of employed and a probability of (1-r) to be rejected and forced to go back to the pool of unemployed, where he must start the whole process again (this time with a lower escape probability and longer waiting time). r can be considered to be the probability of a match after a contact is made between a worker and an employer.

Lindeboom et al. (1993) have estimated this probability for the Netherlands. They used a logit specification and studied the effects of education, occupation and region on the conditional match probability. For employed males, the conditional matching probability turned to be the highest, 15.4%. This is about two times as high as for unemployed males. The way the contact probability is modelled, differs however remarkably from the approach used in this paper. They followed the Blanchard and Diamond (1989) approach, using a Cobb-Douglas specification.

$$C = \lambda_1 U^{\alpha} V^{\beta} \qquad (3)$$

λ_1 is the contact probability and C is the number of contacts that have been made.

The conditional match probability, r, can be considered to be the result of the job offer probability and the acceptance probability. Lindeboom et al. used information on reservation wages, to calculate the acceptance probability, for employed male workers, they found an acceptance probability of 50%, so the job offer probability is 30%.

We will furthermore assume, following Darby, Haltiwanger and Plant (1985), that there are two types of employed workers. The first group consists of those workers who have just entered and have not built up any firm-specific human capital yet, this group consists of mostly young workers who are in the process of job shopping and individuals employed in sectors that involve low accumulation of firm specific capital, we will call this group E1. This group is characterized by high rates of entry into unemployment and are able to find a job relatively fast. Especially in the U.S., this group forms a large fraction of total unemployment. The second group consists of workers with a high degree of firm-specific capital, who only in severe recessions can become unemployed (E2) but once they have become unemployed they spend a long time searching. In addition it will be assumed that every unit of time, a number of jobs becomes unproductive. The rate at which jobs become unproductive will be considered to be a Poisson stream with rate $\tau 2$.

$$\tau 2 = \tau 2(c,w) \qquad\qquad \tau 2_c < 0, \quad \tau 2_w > 0 \qquad\qquad (4)$$

Where c is a business cycle or aggregate demand parameter.

A number of authors, e.g. Blanchard and Diamond (1990), Davis and Haltiwanger (1990) have studied the cyclical behavior of jobs and worker flows. Surprisingly it turns out that job creation is not very pro-cyclical, while as we would expect, the rate of job destruction is strongly counter cyclical. At first, this result might seem somewhat counter intuitive, because if job creation is associated with new firms entering an existing market, one would expect the opposite. Since new firms have to cover average costs while existing firms only have to cover marginal costs. A possible explanation is that the timing of job destruction is endogenous and takes place in recessions. Firms will reallocate labor in recessions which are periods of low productivity. This is consistent with evidence from the United States manufacturing data which show that the proportion of job destruction, due to plant closings decreases in recessions. Firms just want to reallocate labor and don't want to go bankrupt. Blanchard and Diamond (1990) suggest that not bankruptcy but the threat for bankruptcy is the reason for the high rates of job destruction during recessions. They argue that in good times slack (x-inefficiency) enters the operation of firms and that in recessions when there is the threat of bankruptcy, the slack is squeezed out. In terms of equation (4), this means that $\tau 2c$, will be negative.

$\tau 2\Theta$ will be positive, since the higher wages are, the greater will be the proportion of jobs that are

unprofitable, see also Broersma (1994). This is one of the external effects that individual wage bargaining can have on equilibrium unemployment. In what follows we will assume that the effect of Θ on $\tau 2$ is the same as the effect of w (the biased wage) on $\tau 2$.

In our framework, (E1), consists of the workers who have just entered employment and who flow in with a rate that equals r times the arrival rate of node 2. If the match is successful (with probability, s), the worker flows from E1 to E2 and starts to build up firm specific human capital. The probability that the true value of the match Θ turns out to be too low for one of the parties is thus equal to (1-s). The rate at which workers flow out of E1 is again assumed to be exponential, with rate $\mu 4$. We will not explicitly model employed job searchers but it is easy to see that search by employed will increase congestion.

People are unemployed if they are not working and are looking for a job. If L is the number of people in the system, who are either waiting or who are in the search or the application process, then the number of unemployed equals: $U = L - E1$

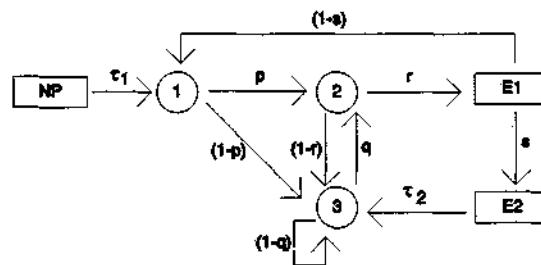Figure 1 *The matching process in terms of a Jackson Network*

Figure 1 shows us the interactions between all flows. Every worker that joins a queue increases the average waiting time of the others. We see that all non participants who start searching flow to node 1. We will assume that this flow follows a Poisson process, with parameter $\tau 1$. The probability function for exactly n arrivals in period t is:

$$f(n) = \frac{\tau 1 t^n}{n!} e^{-\tau 1 t} \tag{5}$$

where, $\tau 1 = \tau 1(w,v)$
With $d1\tau / dw > 0$, $d1\tau / dv > 0$

The more vacancies there are and the higher the wage, the more people will start looking for work. If more people flow to nodes 1 and 2, the queue length and the waiting time will increase and the

duration of unemployment will also increase. The direct effect of wages on vacancies will be ignored.

In the next section we will calculate the steady state properties of the system.

2 Steady State properties

The system as represented in figure 1 can be considered to be a Jackson Network, (see Jackson, 1957). This is so, because the following assumptions are met: The external streams are Poisson distributed, the search time and the application time are exponential distributed, there are fixed transition probabilities to reach one of the other nodes and there is unlimited waiting space. Jackson Networks are a special class of Markovian queuing networks for which the steady state[7] queue length can be found relatively easy.

To find the steady state values of the model, we first have to find the rates $\alpha 1$, $\alpha 2$ and $\alpha 3$ at which arrivals occur at each of the three nodes. At any moment in time, the state of this waiting line process is completely described by the number of people in the system. Let P_n be the probability that there are n people in the system in the steady state. Because we have a Poisson input of unemployed workers and a negative exponential search and application time, the probabilities that an arrival occurs or an application process is completed in an interval of time h, do not depend on the history of the system before the start of the interval. In this sense the system is Markovian. In the steady state, the probability of finding the system in the various states, will be the same at two randomly chosen moments in time, separated by a short interval of time h. This requires that, for each state, the probability of being in that state and leaving it during an interval of time h, has to be equal to the probability of being in other states and entering that state during h.

We can now calculate the rates $\alpha 1$, $\alpha 2$, $\alpha 3$ and $\alpha 4$ at which arrivals occur at each of the nodes (E1 can be interpreted as node 4).

$$\begin{pmatrix} \alpha 1 \\ \alpha 2 \\ \alpha 3 \\ \alpha 4 \end{pmatrix} = \begin{pmatrix} \tau 1 \\ 0 \\ \tau 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & (1-s) \\ p & 0 & q & 0 \\ (1-p) & (1-r) & (1-q) & 0 \\ 0 & r & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha 1 \\ \alpha 2 \\ \alpha 3 \\ \alpha 4 \end{pmatrix} \qquad (6)$$

---

[7] The steady state is the most interesting case to study because if the average inflow into the system would exceed the average outflow, queues would go to infinity whereas if average outflow exceeds the average inflow, queue length will go to zero. In section 4 we will derive the wage rate that will bring the economy in the steady state.
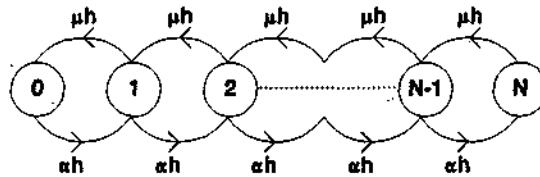
This gives the following solutions for α1, α2 and α3.

$$\alpha 1 = \frac{\tau 1 + (1-s)\tau 2}{s} \tag{7}$$

$$\alpha 2 = \frac{\tau 1 + \tau 2}{rs} \tag{8}$$

$$\alpha 3 = \frac{(1-rp)\tau 1 + (1-rp(1-s))\tau 2}{rqs} \tag{9}$$

$$\alpha 4 = \frac{\tau 1 + \tau 2}{s} \tag{10}$$

The steady state solution procedure will be illustrated with the solution for a simple M│M│1 system (Markovian waiting line model with one service station). For each node it will hold that in the steady state, the inflow is equal to the outflow, see figure 2.

2 Transition diagram for M│M│1 Figure

Let P(k) be: Prob.(k units in the system). The probability of being in state 1 (there is one person in the system) and leaving it, has to be equal to the probability of being in state 0 or state 2 and flowing into state 1. Hence, P1 αh + P1 μh = P0 αh + P2 μh.

If h is a very small interval of time, the following relations will hold in the steady state:

P0αh = P1μh

P1αh + P1μh = P0αh + P2μh

..   ..

..   ..

Pn αh + Pn μh = Pn-1 αh + Pn+1 μh

We can solve this system recursively, this gives:

$$P_n = (\frac{\alpha}{\mu})^n P_0 \qquad (11)$$

Taking into account that:

$$\sum_{n=0}^{\infty} (\frac{\alpha}{\mu})^n P_0 = 1 \rightarrow P_0 = (1-\frac{\alpha}{\mu}) \qquad (12)$$

So:

$$P_n = (\frac{\alpha}{\mu})^n (1-\frac{\alpha}{\mu}) \qquad (13)$$

A Jackson Network has got the special property that: *whenever an equilibrium condition exists, each node in the network behaves as if it were an independent M/M/s queue with Poisson input*. Thus:

$$P(k1, k2,...kn) = P(k1)\, P(k2)\, P(k3)...\, P(kn) \qquad (14)$$

Where P(k1, k2, k3) is the probability to find k1 unemployed at node 1, k2 unemployed at node 2, k3 unemployed at node 3 and k4 employed workers at node 4[8].

For a proof of this, see Jackson (1957).

From (7-10), (13) and (14), it follows that:

$$P(k_1,k_2,k_3,k_4) = \left(1 - \frac{\tau 1 + (1-s)\tau 2}{\mu 1 s}\right)\left(\frac{\tau 1 + (1-s)\tau 2}{\mu 1 s}\right)^{k_1}\left(1 - \frac{\tau 1 + \tau 2}{\mu 2 rs}\right)\left(\frac{\tau 1 + \tau 2}{\mu 2 rs}\right)^{k_2}$$
$$\left(1 - \frac{(1-pr)\tau 1 + (1-pr(1-s))\tau 2}{\mu 3 rqs}\right)\left(\frac{(1-pr)\tau 1 + (1-pr(1-s))\tau 2}{\mu 3 rqs}\right)^{k_3} \qquad (15)$$
$$\left(1 - \frac{\tau 1 + \tau 2}{\mu 4 s}\right)\left(\frac{\tau 1 + \tau 2}{\mu 4 s}\right)^{k_4}$$

Provided that:

$$\frac{\tau 1 + (1-s)\tau 2}{s} < \mu 1 \ \wedge\ \frac{\tau 1 + \tau 2}{rs} < \mu 2 \ \wedge\ \frac{(1-pr)\tau 1 + (1-pr(1-s))\tau 2}{rqs} < \mu 3 \ \wedge\ \frac{\tau 1 + \tau 2}{s} < \mu 4 \qquad (16)$$

Economically those restrictions mean that the amount of new workers who flow to the different nodes does not exceed the rate at which people leave the different nodes.

---

[8] Note that E1 can also be viewed as a node.

If condition (16) is not met, no steady state will exists and the number of people in the system (the unemployed) will go to infinity.

The joint steady state queue line distribution is given by (15). It gives the probability that (k1+k3) people are in the search process and k2 people are in the application process and k4 people employed, but searching. Of course we are most interested in the total number of unemployed in the steady state equilibrium.

The average number of units in the system for a simple $M|M|1$ model (Markovian waiting line model with one service station) is:

$$L = \sum_{n=1}^{\infty} n P_n = \sum_{n=1}^{\infty} n \left(\frac{\alpha}{\mu}\right)^n \left(1 - \frac{\alpha}{\mu}\right) \qquad (17)$$

$$= \left(1 - \frac{\alpha}{\mu}\right) \frac{\alpha}{\mu} \sum_{n=1}^{\infty} n \left(\frac{\alpha}{\mu}\right)^{n-1} = \frac{\frac{\alpha}{\mu}}{1 - \frac{\alpha}{\mu}} \qquad (18)$$

Since $\sum_{n=1}^{\infty} n \left(\frac{\alpha}{\mu}\right)^{n-1}$ is the derivative of $\sum_{n=1}^{\infty} \left(\frac{\alpha}{\mu}\right)^n = \frac{\frac{\alpha}{\mu}}{1 - \frac{\alpha}{\mu}}$

For our model this means that:

$$L = \sum_{n=1}^{\infty} n \left(1 - \frac{\tau 1 + (1-s)\tau 2}{\mu 1 s}\right) \left(\frac{\tau 1 + (1-s)\tau 2}{\mu 1 s}\right)^n + \sum_{i=1}^{\infty} i \left(1 - \frac{\tau 1 + \tau 2}{\mu 2 rs}\right) \left(\frac{\tau 1 + \tau 2}{\mu 2 rs}\right)^i$$
$$+ \sum_{j=1}^{\infty} j \left(1 - \frac{(1-pr)\tau 1 + (1-pr(1-s))\tau 2}{\mu 3 rqs}\right) \left(\frac{(1-pr)\tau 1 + (1-pr(1-s))\tau 2}{\mu 3 rqs}\right)^j$$
$$+ \sum_{i=1}^{\infty} i \left(1 - \frac{\tau 1 + \tau 2}{\mu 4 s}\right) \left(\frac{\tau 1 + \tau 2}{\mu 4 s}\right)^i \qquad (19)$$

This is simply the sum of workers "waiting" at the different nodes. Simplifying yields:

$$L = \frac{\tau 1 + (1-s)\tau 2}{\mu 1 s - \tau 1 - (1-s)\tau 2} + \frac{\tau 1 + \tau 2}{\mu 2 rs - \tau 1 - \tau 2}$$
$$+ \frac{(1-pr)\tau 1 + (1-pr(1-s))\tau 2}{\mu 3 rqs - (1-pr)\tau 1 - (1-pr(1-s))\tau 2} + \frac{\tau 1 + \tau 2}{\mu 4 s - \tau 1 - \tau 2} \qquad (20)$$

equation (20) shows us the number of workers in the system in the steady state. Since U = (L - E1) it follows that the total number of unemployed in the system can be represented by:

11

$$U = \frac{\tau 1(w,v)+(1-s)\tau 2(c,w)}{\mu 1(w,v)s-\tau 1-(1-s)\tau 2(c,w)} + \frac{\tau 1(w,v)+\tau 2(c,w)}{\mu 2(v)rs-\tau 1(w,v)-\tau 2(c,w)}$$
$$+ \frac{(1-pr)\tau 1(w,v)+(1-pr(1-s))\tau 2(c,w)}{\mu 3(w,v)rqs-(1-pr)\tau 1(w,v)-(1-pr(1-s))\tau 2(c,w)} + \quad (21)$$

Thus the number of unemployed workers in the system is equal to the total number of workers, who are looking for a job, minus the employed workers.

The average duration of unemployment in the steady state is:

$$D = \frac{1}{\mu 1-\alpha 1} + \frac{1}{\mu 2-\alpha 2} + \frac{1}{\mu 3-\alpha 3} \qquad (22)$$

Which is equal to:

$$D = \frac{s}{\mu 1s-\tau 1-(1-s)\tau 2} + \frac{rs}{\mu 2rs-\tau 1-\tau 2}$$
$$+ \frac{rqs}{\mu 3rqs-(1-pr)\tau 1-(1-pr(1-s))\tau 2} \qquad (23)$$

An individual worker can only influence $\mu 1$, the time to find and apply for a suitable vacancy. $\mu 2$ and $\mu 3$ and the transition probabilities are totally determined by the other workers and the employers. As we will see in the next section, expected unemployment duration will influence the decision to enter the labormarket.

Now some comparative statics for unemployment will be presented. It is useful to define the following expressions first.

$$\begin{aligned}
\Delta 1 &= \tau 1+(1-s)\tau 2 \quad (>0) \\
\Delta 2 &= \mu 1s-\tau 1-(1-s)\tau 2 \quad (>0 \ \text{by eq. 16}) \\
\Delta 3 &= \tau 1+\tau 2 \quad (>0) \\
\Delta 4 &= \mu 2rs-\tau 1-\tau 2 (>0 \ \text{by eq }) \\
\Delta 5 &= (1-pr)\tau 1+(1-pr(1-s))\tau 2 \quad (>0) \\
\Delta 6 &= \mu 3rqs-(1-pr)\tau 1-(1-pr(1-s))\tau 2 \quad (>0 \ \text{by eq. 16}) \\
\Delta 1 &+ \Delta 2 = \mu 1s \quad (>0) \\
\Delta 3 &+ \Delta 4 = \mu 2s \quad (>0) \\
\Delta 5 &+ \Delta 6 = \mu 3rqs \quad (>0)
\end{aligned} \qquad (24)$$

The effect of a change in wages on unemployment is given by:

$$\frac{dU}{dW} = \frac{s(\mu 1(\tau 1_w+(1-s)\tau 2_w)-\mu 1_w)}{(\Delta 2)^2} + \frac{\mu 2s(\tau 1_w+\tau 2_w)}{(\Delta 4)^2} +$$
$$\frac{rqs(\mu 3(1-pr)(\tau 1_w+prs\tau 2_w)-(\Delta 5)\mu 3_w)}{(\Delta 6)^2} \qquad (25)$$

The sign depends on four positive terms and one negative term and is thus ambiguous. An increase in

12

the wage level will increase the inflow from the non-participants into unemployment and will increase the rate of job destruction, but it will also increase the search intensity (which reduces unemployment).

The effect of vacancies on steady state unemployment is:

$$\frac{dU}{dV} = \frac{s(\mu1(\tau1_v)-(\Delta1)\mu1_v)}{(\Delta2)^2} + \frac{s(\mu2(\tau1_v)-(\Delta3)r\mu2_v)}{(\Delta4)^2} + \frac{rqs((1-pr)\mu3\tau1_v-\mu3_v(\Delta5))}{(\mu6)^2} \quad (26)$$

An increase of vacancies leads to an increase of the inflow into unemployment but also to an increase of the number of unemployed workers who leave the search and application process (which reduces unemployment). The total sign is again ambiguous. It is however not very plausible that an increase in the vacancy rate will lead to more net inflow into unemployment. So an increase of vacancies will probably reduce unemployment.

The business cycle effect on unemployment is:

$$\frac{dU}{dc} = \frac{s(1-s)\mu1\tau2_c}{(\Delta2)^2} + \frac{s\mu2\tau2_c}{(\Delta4)^2} + \frac{rqs\mu3(1-pr(1-s))\tau2_c}{(\Delta6)^2} \quad (27)$$

An increase of economic activity decreases the amount of unproductive jobs ($d\tau2/dc < 0$) and will lower unemployment.

An important policy issue in the Netherlands is whether an increase of participation, increases employment. The effects are given below:

$$\frac{dU}{d\tau1} = \frac{s\mu1}{(\Delta2)^2} + \frac{s\mu2}{(\Delta4)^2} + \frac{(1-pr)(rqs)\mu3}{(\Delta6)^2}$$
$$\frac{dE}{d\tau1} = \frac{s\mu4}{(\mu4s-\tau1-\tau2)^2} \quad (\geq0) \quad (28)$$

An increase in the inflow into unemployment or of participation, leads to a higher level of unemployment but also to a higher level of employment.

It is also interesting to see the effect of participation on average unemployment duration.

$$\frac{dD}{d\tau1} = \frac{s}{(\mu1s-\tau1-(1-s)\tau2)^2} + \frac{rs}{(\mu2rs-\tau1-\tau2)^2}$$
$$+ \frac{rqs}{(\mu3rqs-(1-pr)\tau1-(1-pr(1-s))\tau2)^2} \quad (29)$$

This effect is positive.

13

$$\frac{dU}{d\mu 1} = \quad (\leq 0)$$
$$\frac{dU}{d\mu 2} = \quad (\leq 0)$$

(30)

An increase of the rates at which people leave the search and application processes will lead to lower unemployment.

3 The Determination of the wage in the steady state

In this section we will calculate the money wage that is consistent with steady state equilibrium. Our main goal is to show how unemployment duration will affect the wage and the decision to enter the laborforce. To keep things as simple as possible, we will make a number of additional simplifying assumptions. Firstly we will make no difference between non participants and unemployed workers. In that case we can assume $\tau 1$ to be equal to 0. In the steady state, employment inflow and outflow are equal to each other. We will assume that this flow is equal to a fraction λ of E, in our model this is τ_2 /s. Ultimately we are interested in the efficiency of the market equilibrium. We do this by comparing the expected life time earnings of an entrant on the labor market with the (social) marginal product of that worker. Diamond (1982), Mortenson (1989) and Pissarides (1990) have taken similar approaches but neither has taken the effects of expected unemployment duration on expected life time earnings explicitly into account. In the previous section, we showed that unemployment duration is for the largest part out of control of the worker, but it will influence the expected benefits of entering the labormarket. In the next section, the expected benefits of entering the labormarket as a function of expected unemployment duration will be derived.

The average worker has got a probability of $\lambda E/U$ of finding a job. Similarly, the probability for an employed worker to loose his job is equal to λ. We will assume that the expected duration of a job is exogenous and equal to D1 and that the expected duration of unemployment is equal to D2, given by equation (23). The expected unemployment duration depends on the number of employed and unemployed searchers and on the transition probabilities. We will assume that in total, there are J jobs, F filled jobs and V vacant jobs. The number of filled jobs is equal to the number of employed workers, thus the following relationship holds: N - U = E = F = J - V. A filled vacancy is assumed to produce output y each period. Since we are in the steady state, the future path of the economy will not be discounted.

14

$$\frac{dU}{d\mu 1} = \quad (\leq 0)$$

$$\frac{dU}{d\mu 2} = \quad (\leq 0) \tag{30}$$

An increase of the rates at which people leave the search and application processes will lead to lower unemployment.

3 The Determination of the wage in the steady state

In this section we will calculate the money wage that is consistent with steady state equilibrium. Our main goal is to show how unemployment duration will affect the wage and the decision to enter the laborforce. To keep things as simple as possible, we will make a number of additional simplifying assumptions. Firstly we will make no difference between non participants and unemployed workers. In that case we can assume $\tau 1$ to be equal to 0. In the steady state, employment inflow and outflow are equal to each other. We will assume that this flow is equal to a fraction λ of E, in our model this is τ_2 /s. Ultimately we are interested in the efficiency of the market equilibrium. We do this by comparing the expected life time earnings of an entrant on the labor market with the (social) marginal product of that worker. Diamond (1982), Mortenson (1989) and Pissarides (1990) have taken similar approaches but neither has taken the effects of expected unemployment duration on expected life time earnings explicitly into account. In the previous section, we showed that unemployment duration is for the largest part out of control of the worker, but it will influence the expected benefits of entering the labormarket. In the next section, the expected benefits of entering the labourmarket as a function of expected unemployment duration will be derived.

The average worker has got a probability of $\lambda E/U$ of finding a job. Similarly, the probability for an employed worker to loose his job is equal to λ. We will assume that the expected duration of a job is exogenous and equal to D1 and that the expected duration of unemployment is equal to D2, given by equation (23). The expected unemployment duration depends on the number of employed and unemployed searchers and on the transition probabilities. We will assume that in total, there are J jobs, F filled jobs and V vacant jobs. The number of filled jobs is equal to the number of employed workers, thus the following relationship holds: $N - U = E = F = J - V$. A filled vacancy is assumed to produce output y each period. Since we are in the steady state, the future path of the economy will not be discounted.

14

Wage Bargaining

Once the worker and firm have come together they will evenly share the surplus that results from the match. As a result, the wage will be between the wage that makes the worker indifferent between working and further searching and the wage that makes the firm indifferent between filling the vacancy or wait for the next applicant. The expected duration of unemployment and employment influences those bounds. We can now write down the expected benefits for a worker to be unemployed or employed and for a firm to have a filled job or a vacancy. An employed worker earns wage w for a period D1 and has got a probability λ of getting unemployed for a period D2, and loose wealth (W_E-W_U). So the expected benefits of being employed are given by:

$$W_E = D1w - D2\lambda [W_E - W_U] \tag{31}$$

To discount future profits, we should interpret D1 and D2 as:

$$D_j = \int_{t=0}^{D_j} e^{-rt} \tag{32}$$
$$j = 1,2$$

For an unemployed worker, the expected benefits are equal to the money value of being unemployed, b plus the probability of getting employed times expected employment duration times the expected wealth change resulting from this transition:

$$W_U = bD2 + D1\lambda E/U [W_E - W_U] \tag{33}$$

For firms, the expected value of a vacant job is equal to the probability that the vacancy is filled, times the expected duration that the vacancy is filled times the change in wealth resulting from filling the vacancy minus the fixed costs c of holding a vacancy.

$$W_V = D1 \lambda E/V [W_F - W_V] - c \tag{34}$$

The value of a filled job is equal to the difference between the output per worker minus the wage multiplied by the expected duration of the job, and the probability that the job closes times the

difference in wealth of an open and a closed job[9].

$$W_F = D1 \ (y - w) - D2\lambda[W_F - W_V] \tag{35}$$

Solving this system gives us the equilibrium wage:

$$w = \frac{D1D2bV^{-1}+D2^2b+yD1D2+yD1^2EU^{-1}+c(D2+D1EU^{-1})}{D1(2D2+D1(EU^{-1}+VE^{-1})} \tag{36}$$

From this wage we can see directly that higher benefits have a positive impact on wages because it strengthens the bargaining position of workers:

$$\frac{dw}{db} = \frac{D2[D1\frac{E}{V}+D2]}{D1[2D2+D1(\frac{E}{U}+\frac{E}{V})]} > 0 \tag{37}$$

To keep things as simple as possible we will assume b and c to be zero in what follows. Hence:

$$w = \frac{(D2+D1\frac{E}{U})y}{2D2+D1(\frac{E}{U}+\frac{E}{V})} \tag{38}$$

When U and V are equal, the wage becomes equal to: 1/2 y. Both firms and workers receive an equal share of the wage surplus. When there are more unemployed workers than vacancies, the wage share of the surplus will be smaller than 1/2 because the bargaining power of the unemployed is reduced. Similarly, when there are more vacancies than unemployed workers, the wage share is larger than 1/2 y.

4 The efficiency of search equilibrium

To find out whether search equilibrium is efficient, we will follow the same strategy as Diamond (1982): First we will derive the expected benefits of searching, W_U. We will compare those with the social marginal product of a new worker. It will turn out that those are in general not equal.

To keep things as simple as possible, we will assume now that both workers and employers receive half of the surplus, since the sharing rule always involves some arbitrariness, this is acceptable. We

---

[9] A natural assumption, is that in equilibrium all profit opportunities from new jobs are exploited, this means that the value of holding a vacancy (W_V) is zero in equilibrium). Hence from eq (34) and (35) it follows that in equilibrium, $V = D1^2 \ (y-w)\lambda E / (1+D2\lambda)$

will just keep in mind that when there are more unemployed than vacancies, w is likely to be higher than 1/2y. For different sharing rules see Diamond (1982) Mortenson (1982) and Pissarides (1990).

From (31) and (33) and (38) it follows that the expected benefits of entering the labor market are:

$$W_U = \frac{EyD1^2}{2[U(\lambda^{-1}+D2)+D1E]} \tag{39}$$

Now we will calculate the social marginal product of a new worker. We therefore have to describe how the economy moves out of steady state. This is necessary because the addition of an additional worker by itself implies a movement out of steady state.

If employment inflow is determined by an aggregate matching function $g(U,V)=g(N-E,J-E)$ [10] with: $-g_E=(g_N+g_J)$ and employment outflow is equal to λE.

Then from any starting point, the path of W (the present discounted value of aggregated output) and E can be described by the following equations:

$$W = \int_0^\infty e^{-rt} E_t y \, dt$$
$$\frac{dE}{dt} = g(U,V) - \lambda U \tag{40}$$
$$E_0 = E$$

From this system we can derive the marginal product of a worker as[11]:

$$\frac{\partial W}{\partial N} = \frac{y}{r}\left[\frac{g_U}{r+\lambda+g_U+g_V}\right] \tag{41}$$

If we substitute $g=\lambda E$ in equation (39), we see that the expected rewards to search are equal to:

$$W_U = \frac{y}{2}\left[\frac{\frac{g}{U}D1^2}{1+D2\lambda+\frac{g}{U}D1}\right] \tag{42}$$

In the simplifying case of a constant elasticity to scale matching function $(Ug_U + Vg_V = g)$,

---

[10] In the steady state $g(U,V)=\lambda E=r2/s$

[11] For a proof, see the appendix 2.

equation (42) becomes:

$$W_U = \frac{y}{2}\left[\frac{(g_U + \frac{V}{U}g_V)D1^2}{1 + D2 + (g_U + \frac{V}{U}g_V)D1}\right] \tag{43}$$

If we compare the social marginal benefits to search with the private rewards to search it appears that the social benefits are more likely to exceed the private rewards to search, the longer expected unemployment duration, D2 is. This gives us a rational explanation for the discouraged worker effect. From (23) it follows that unemployment duration increases with the number of employed and unemployed searchers and the search intensity of those workers. A rational worker will only enter the labormarket if he expects to gain from it. As the number of other searchers increases, it becomes rational for him not to enter the labor market, while it is still possible that he is willing to accept a wage equal to his marginal product.

The Diamond (1982) and Mortenson (1982) type of externality caused by the fact that wages are always smaller than the match specific productivity (if employers receive a non-zero share of the match) is also captured in equations (41) and (43). If $g_U > V/U\, g_V$, then the social marginal product of a worker is more likely to exceed the private expected earnings of a new entrant. In other words there are too few searchers in equilibrium.

Final Remarks

This paper focused on a particular type of congestion externality namely one in which an increase of the search intensity of one person, increases the average waiting time of the others and hence reduces the expected benefits of search. In many papers, unemployment is considered to be the same as enjoying leisure. But someone who is actively searching is much more restricted in his (leisure activities) than is someone who is not actively searching, because he has to be available for an interview any time. There is many micro-evidence e.g. Moylan et al. (1982) that unemployment is indeed more associated with "waiting" than with "searching", most unemployed workers do not spend more than five hours a week on search activities. Still economists have paid little attention to the "waiting costs" of unemployment. In this paper I have described a queuing model in which unemployment duration is for a large part out of control of the worker and in the hands of other searchers and employers. The advantage of this approach is that many useful ideas in the field of labor economics could at very low costs be combined in one model. Examples of issues that we took into account are:

18

"learning about the true value of a match", hysteresis through loss of skill and discouraged worker effects and the asymmetric effects of the business cycle on employment inflow and outflow. We were able to derive analytical solutions for steady state unemployment and average duration as a function of the time spent on searching for a suitable vacancy and "waiting" for a reaction of the employer (this depends on the number of other searchers and their search intensity), wages, the business cycle, and the transition probabilities to flow to a new state. Finally it was shown how expected unemployment duration affected the equilibrium wage and the decision to enter the labormarket. When the expected waiting time of unemployment is large, the rewards to search and the number of searchers, may become lower than the socially optimal amount of searchers. In that case, policymakers should stimulate labormarket participation.

References:

Abbring J.H., J.C. van Ours (1994), Sequential or non-sequential employers' search?, *Economics Letters* 44, pp. 323-328

Akerlof G.A., A.K. Rose, J.L. Yellen (1990), Waiting for work, *NBER Working Paper* No.3385,41 p.

Becker G.S. (1965) A theory of the allocation of time, *Economic Journal*, pp. 493-517 .

Blanchard, O.J. and P. Diamond, (1989), The Beveridge curve, *Brookings Papers on Economic Activity*, nr 1, pp. 1-76.

Blanchard, O.J. and P. Diamond, (1990), The Cyclical Behavior of the Gross Flows of U.S. Workers, *Brookings Papers on Economic Activity*, 2, pp. 85-155.

Blanchard, O.J. and P. Diamond, (1992), The Flow Approach to Labor Markets, *American Economic Review*, pp. 354-359.

Broersma, L. (1993), "The Effect of Wage Restraint on Labour Market Flows", mimeo, Vrije Universiteit, Amsterdam.

Burke, P.J., (1956), The Output of a Queueing System, *Operations Research,* Vol. 4, pp. 699-704.

Butter F.A.G. den and J.H. Abbring, (1993), Dynamic Labour Market Equilibria with Heterogeneous Unemployment, *VU Research Memorandum* (1993), 23 p.

Darby M., J. Haltiwanger and M. Plant, (1985), "Unemployment Rate Dynamics and Persistent Unemployment under Rational Expectations,", *American Economic Review*, 614-637.

Davis S.J. and J. Haltiwanger, (1990), Gross Job Creation and Destruction: Micro Economic Evidence and Macro Economic Implications: *NBER Macro Economic Annual* (5), pp.123-68.

Diamond, P.A. (1980), An Alternative To Steady State Comparisons, *Economics Letters* 5, pp. 5-9.

Diamond, P.A. (1981), Mobility costs, frictional unemployment, and efficiency, Journal of Political Economy, 89, 789-812.

Diamond P.A. (1982 a) Wage determination and efficiency in search equilibrium, *Review of Economic Studies*, 49, pp. 217-227.

Diamond, P.A. (1982 b), Aggregate-Demand Management in Search Equilibrium, *Journal of Political Economy*, 5, pp. 881-894.

Gautier P.A. and L. Broersma (1994), The Timing of Labor Reallocation and the Business Cycle, Mimeo, Tinbergen Institute.

Jackson, J.R., (1957), Networks of Waiting Lines, *Operations Research*, Vol. 5, pp.518-521.

Jovanovic B., (1979), Job Matching and the Theory of Turnover, *Journal of Political Economy*, 87, pp. 972-990.

Jovanovic, B. (1984), Matching, turnover and unemployment. *Journal of Political Economy* 92, pp. 108-122.

Layard R. S. Nickell, R. Jackman. (1991), *"Unemployment"*, Oxford University Press.

Lemoine, A.J., 1977, Networks of Queues- A survey of Equilibrium Analysis, *Management Science*,

Vol 24, No. 4, pp. 464-481.

Lindeboom, M., J.C. van Ours and G. Renes, (1993), Matching Job Seekers and Vacancies, in: *Labor Demand and Equilibrium Wage Formation*, Eds. J.C. van Ours, G.A. Pfann, and G. Ridder, Amsterdam, North Holland, pp. 279-296.

Lucas, R.E. Jr, (1978), Unemployment Policy, *American Economic Review*, 68, pp. 353-357.

Mortenson, D.T. (1982) "The matching process as a noncooperative bargaining game", in J.J. McCall, ed, *The economics of information and uncertainty*. Chicago: NBER, University of Chicago Press.

Mortenson, D.T. (1986), Job Search and Labor Market Analysis, *Handbook of Labor Economics* ch 15, Volume II, Ed. by O. Ashenfelter and R. Layard, Elsevier Science Publishers.

Moylan, S., Millar, J, and Davies, R. (1982), "Unemployment-The Year After", Dep. of Employment, *Employment Gazette*, 90(8):334-40.

Pissarides C.A. (1984), "Search intensity, advertising, and efficiency", *Journal of Labor Economics*, 2-:pp. 128-143.

Pissarides C.A. (1993), "Loss of Skill During Unemployment and the Persistence of Employment Shocks", *Quarterly Journal of Economics*, pp.1371-1391.

Pissarides, C.A., (1990), *Equilibrium Unemployment Theory*, (Basil Blackwell, London).

Stiglitz, J.E., (1974), Wage Determination and Unemployment in L.D.C.'s: The Labor Turnover Models, *Quarterly Journal of Economics*, May '88, pp. 194-227.

Wright, R., (1986), Job Search and Cyclical Unemployment, *Journal of Political Economy*, vol 94, pp. 38-55.

Appendix 1: Examples

To get some feeling about the working of the model, some numerical examples will be given in this section. In reality, search strategies and escape probabilities will probably differ remarkably between sectors and types of labor. For low schooled workers, the search and application time will probably be very short. For high schooled labor, search and application time will be longer and they will probably not search sequentially. In this model we did not take this into account. Nevertheless it will be instructive to see how steady state unemployment varies with the parameters in the model.

First it is necessary to find out what the critical values (given by (16)) for $\mu1$, $\mu2$, $\mu3$ and $\mu4$ are. In this example we set $\tau1$ and $\tau2$ equal to their average values (x1000) in the Netherlands[12], see Gautier and Broersma (1994). r (the probability of a match after a contact has been made), is set at 0.15, the value found in Lindeboom et al. The values for s (fraction of people who remain at their new job after one period), p (probability for a new worker to be invited for an interview) and q (the fraction of long term unemployed who make contact with an employer), are set arbitrary at respectively: 0.8, 0.3 and 0.2. Table 1. shows the "critical" values for the μ's. Those values represent the minimum number of finished actions (e.g. applications interviews, acceptances, rejections, etc) that is necessary for an equilibrium to exist. If those values are not met in the long run, the number of unemployed will go to infinity. The most striking result is the extreme high critical value for $\mu3$. This value suggests that each year more than 21 mln people must leave the third node. At first, this may seem ridiculous, but if we take a look at figure 1, we see that at node 3, all the long term unemployed and the people with high firm-specific capital arrive (who need a relatively long time to find a job that is in agreement with their capacities). We also see that 80% of the applications of those people is rejected, which means that 80% of the people who leave the queue, immediately flow back to the beginning of the queue. Another way to look at this high number is to interpret it as 58000 applications a day (or 10 applications of 5800 people[13], if we would allow for non-sequential search by the unemployed) of which 20% is being invited for an interview and 80% is rejected directly and has to join the queue again. Of course in reality, people apply to different jobs and for each job a queue will be formed then. This could be easily modelled with a multiple "server" queuing system.

---

[12] $\tau2$ is set equal to the average flow from employment to unemployment (70-91) and $\tau1$ is set equal to the average unemployment inflow minus the flow from employment to unemployment.

[13] The more application letters a person writes, the greater the external effect (s)he places on the other searchers since each letter has to be studied by the employers and if s(he) gets invited to more interviews, this will increase the waiting time of the others even more.

Table 1. Critical values for $\mu1..\mu4$.

	critical value
$\mu1$	371293
$\mu2$	2810080
$\mu3$	21396811
$\mu4$	658613

τ1=239570, τ2=287320, p=0.3 r=0.15, s=0.8, q=0.2.

Tables 2,3,4 and 5 show how sensitive the expected stock of unemployment is to changes in $\mu1$, $\mu2$, $\mu3$ and $\mu4$. For the baseline projection, we will give $\mu1,\mu2$ and $\mu3$ high values, respectively:500, 2850, 21400) so that steady state unemployment is low (2342). As can be seen in table 1, unemployment becomes high when $\mu1$ approaches its critical value.

Table 2 Expected Unemployment and Average Duration for different values of $\mu1$.

	$\mu1=372$	$\mu1=375$	$\mu1=400$	$\mu1=500$
Expected Unemployment	524189	99540	12328	2278
Average Duration	1.73	0.58	0.35	0.32

Table 3 Expected Unemployment and Average Duration for different values of $\mu2$

	$\mu2=2811$	$\mu2=2815$	$\mu2=2850$	$\mu2=3000$
Expected Unemployment	2349	2342	2278	1971
Average Duration	0.32	0.32	0.32	0.32

Table 4 Expected Unemployment and Average Duration for different values of $\mu3$

	$\mu3=21397$	$\mu3=21400$	$\mu3=21450$	$\mu3=22000$
Expected Unemployment	2280	2279	2267	2146
Average Duration	5.31	0.32	0.03	0.01

23

Appendix 2 Derivation of equation (41) [14]

We will derive a simple expression for the present discounted value of the change in aggregate output along the convergent path. As Diamond (1980) shows, taking into account that moving from one steady state to another takes time, does not lead to more complicated expressions than assuming that the economy adjusts instantaneously to a new steady state. We will assume aggregate output to follow the following differential equation:

$\dot{E}y = (g(U,V) - \lambda E)y$

Since this equation is independent of time, we can write the solution at time s, $E_s y$ as a function of (s-t) and a previous value of Ey. If we assume that the differential equation is stable and that in the initial situation, the economy is in a steady state, $g(U,V) = \lambda E$, the present discounted value of E satisfies:

$$W_t = \int_t^\infty e^{-r(s-t)} E(s)y \ ds \equiv W(E_t y) \qquad\qquad i$$

The asset value of W can be written as:

$$rW = Ey + \frac{\partial W}{\partial E}(g(N-E,J-E)-\lambda E) \qquad\qquad (ii)$$

Differentiating (ii) with respect to N gives:

$$r\frac{\partial W}{\partial N} = \frac{\partial E}{\partial N} + \frac{\partial^2 W}{\partial E \partial N}g + \frac{\partial W}{\partial E}\left[\frac{\partial g}{\partial N} - \frac{\partial \lambda E}{\partial N}\right] \qquad\qquad (iii)$$

Differentiating (ii) with respect to E gives:

$$r\frac{\partial W}{\partial E} = y + \frac{\partial^2 W}{\partial E^2}(g-\lambda E) + \frac{\partial W}{\partial E}\left[\frac{\partial g}{\partial E} - \lambda\right] \qquad\qquad (iv)$$

Since we assumed that at time zero, the economy was in the steady state $(g - \lambda E = 0)$, Evaluating (iv) at t=0 makes $g=\lambda E$ and (iv) becomes:

---

[14] This proof draws on Diamond (1980)

24

$$\frac{\partial W}{\partial E} = \frac{y}{\left[r + \lambda - \frac{\partial g}{\partial E}\right]} = \frac{y}{\left[r + \lambda + \frac{\partial g}{\partial U} + \frac{\partial g}{\partial V}\right]} \qquad \text{(v)}$$

Substituting in (iii) gives us:

$$\frac{\partial W}{\partial N} = \frac{y}{r}\left[\frac{\frac{\partial g}{\partial U}}{r + \lambda + \frac{\partial g}{\partial U} + \frac{\partial g}{\partial V}}\right] \qquad \text{(vi)}$$

which is equal to equation (41). In general terms the marginal product of labor can be written as (see Diamond 1982):

$$\frac{\partial W}{\partial N} = \left(\frac{y}{r}\right)\left[\frac{\frac{\partial f}{\partial N}}{r + \lambda - \frac{\partial f}{\partial E}}\right]$$

Where f gives the aggregate outcome of the search process.