VU Research Portal

Classical general equilibrium models

Schoonenberg, W.J.M.

1991

Faculteit der Economische Wetenschappen en Econometrie

Serie Research Memoranda
Classical General Equilibrium Models
A Reinterpretation

W.J.M. Schoonenberg

Research Memorandum 1991-111
december 1991

vrije Universiteit *amsterdam*

CLASSICAL GENERAL EQUILIBRIUM MODELS

A REINTERPRETATION

by

W.M.J. Schoonenberg

Free University
Department of Econometrics
Faculty of Economics and Econometrics
De Boelelaan 1105
1081 HV Amsterdam
The Netherlands

December 1991

Abstract:

This paper shows that: (1) the distinction between the Walrasian and the Classical (Neoricardian) price theory is also a distinction between two types of general equilibrium models, and (2) these two types of models are conceptually incommensurable. This means it is only possible to derive the Classical model mathematically from the Walrasian by ignoring the conceptual differences between the two types of general equilibrium models.
In section 2 the analysis is started with the Sraffa-Leontief interpretation of the Ricardian production model, which is an efficient way to introduce the Classical general equilibrium model. The paper clarifies some concepts and extends the model by including domestic labour. In section 3 some nineteenth century stains are removed from it. After that, a Sraffian general equilibrium model is proposed in which both profit and preference maximization takes place. This more modern model still possesses the essential Classical characteristics like the separation of demand and supply and the surplus nature of profit.

1. INTRODUCTION

As Joan Robinson frequently said[1], two types of price theory can be distinguished: the Walrasian approach in which relative prices are determined by scarcity, i.e. by demand and supply conditions, and the Classical (or Neoricardian) approach in which relative prices are determined by technology. Behind both price theories a full general equilibrium model can be found. The important characteristics of the Classical general equilibrium (CGE) model compared to the Walrasian general equilibrium (WGE) model are:

1. Supply and demand are completely separated from each other: supply conditions (technology and distribution) determine prices, while demand conditions determine the scale of operation of the economy. This means prices are not determined by scarcity.
2. The wage and profit rate represent divisions of a surplus in terms of exchange values. They are not analogous to prices, i.e. they are not payments for the productive contributions of factors.[2]
3. Capital is not only conceived as a physical means of production or, alternatively, as an investment fund. Capital is *at one moment in time* both. As a result competition leads to an equilibrium characterized by equal rate of returns on invested capital in all sectors.
4. General equilibrium is not only defined as a competitive equilibrium where total demand and supply are equal for all commodities (market clearing), while demand and supply of individual agents are their "best choices" when they take the prices prevailing in the markets as given. A Classical general equilibrium is the state in which the system can reproduce itself unhindered. This condition is stronger than market clearing.

The Classical theory does not claim like the Walrasian that it is a "pure" theory, on the contrary it claims that it is a theory of the capitalist mode of production. This means that to compare the two approaches we have to examine the way in which both approaches view *production*. The WGE model[3] sees production as a *one-way avenue* from primary factors of production, the services of which are used in production, to commodities used for final consumption. The markets for factors and for final products are analyzed in a similar way. Furthermore, capital and labour are presented on the same footing. They are resources which give their owners a rent by hiring their services to firms. In the CGE model this mathematical elegance does not

---

[1] See for instance: Robinson 1961.

[2] Formally this comes to the surface in the fact that prices are an eigenvector of the technology matrix while the rate of profit is associated with an eigenvalue of this matrix (see below).

[3] It could be defended that Walras was a Ricardian (see: Morishima 1989, p. 1-8). In this paper we do not have Walras' original work in mind, but the Arrow-Debreu interpretation of Walras' model. See: Arrow & Debreu 1954 and Debreu 1959. In fact, Walras' theory of capital accumulation in which the theory of exchange and production is extended to a temporary equilibrium model of economic growth is neglected in the modern interpretation of Walras. The mathematical tour de force of Arrow and Debreu washed away the last Ricardian remnants in Walras' orginal work. It would be more correct to use the term *Postwalrasian*.

exist. But it is replaced by another elegance: production is viewed as a *circular process* by which commodities are transformed into each other through the application of labour. Capital goods are presented not essentially different from consumption goods: they are both produced through the application of labour. In this approach labour is essential: it is the only *human* contribution in production.

The Classical approach is formalized in Sraffa's *Production of Commodities by Means of Commodities.*[4] The theoretical importance of this work is often neglected or misunderstood.[5] It is not only an important step towards the translation of Ricardo's model into mathematical economics, but also a modern reinterpretation of the CGE model hidden in the work of Ricardo.[6] In this paper the mathematics of Leontief's input-output model will be used in describing the CGE model. This multisectoral model is often described as a special case of the WGE model, but Leontief's input-output model makes more sense as a mathematical reinterpretation of Quesnay's *Tableau Economique*; its *logical* structure is that of the CGE model.[7]

2. THE RICARDIAN MODEL

2.1 Introduction

In this section the analysis is started with the Sraffa-Leontief interpretation of the Ricardian production model, which is an efficient way to introduce the CGE model. This model is a linear economic model in which linearity is as important as differentiability in Neoclassical models. Each includes production technologies that the other excludes. Although the analysis could be generalized to convex technologies[8], this is not necessary

---

[4] Sraffa 1960. The Classical approach, "rehabilitated" by Sraffa (see: Dobb 1973, p. 257), is an essential part of the theoretical structure of the Postkeynesian (or Cambridge) school, founded by Joan Robinson (see, for instance, Robinson 1956), together with for instance Eatwell, Kaldor, Nuti, and Pasinetti. Joan Robinson was not inspired by Ricardo, like Sraffa, but by Keynes. But she was also inspired by Sraffa: "I had picked up the clue from Piero Sraffa's Preface to Ricardo's Principles and my analysis (errors and omissions excepted) was a preview of his" (Robinson 1970, p. 234). More recent contributions in the Classical or Postkeynesian tradition are for instance Pasinetti 1977, 1980, 1981, 1989, Roncaglia 1978, Abraham-Frois & Berrebi 1979, Steenge 1980, Salvadori 1988, Bidard 1988, 1990, Schefold 1989, Bharadwaj & Schefold 1989, and Van den Noord 1990.

[5] See: Levine 1974, 1985.

[6] See also: Morishima 1989, p. 3.

[7] Walras identifies capital goods and the services they provide, in which case a circular process is avoided. Leontief on the other hand has a circular process in mind in which produced products itself are used as inputs in production. This is a Classical approach. See: Phillips 1955 and Barna 1975.

[8] See, for instance, in a slightly different context: Roemer 1980.

for our purposes. This paper shows that equilibrium prices can be determined in a quite different *conceptual* framework of general equilibrium analysis.[9] Although this paper is not about the theoretical structure of WGE models, a few points may be useful for comparative purposes.

The WGE model of exchange and production is an extension of Walras' pure exchange model. Production in a WGE model is not as important and essential as exchange in these models. This is because production is conceived as another form of exchange, which is certainly in the spirit of Walras' own thinking. In the words of Arrow and Starrett, the founders of the Neoclassical school "recognized the importance of production, but Menger and Jevons especially put stress on the notion of exchange as expressing the essence of the economic system; production to some extent appeared merely as an indirect way of exchanging initial holdings."[10] Production is conceived in terms of the allocation of given resources, *endowments*, of commodities and factors of production. In a generalized WGE model the price formation is the outcome of the maximization of subjective preferences under the constraints of initial endowments and technical conditions. This is an essentially static process. Time in a WGE model is simply another physical characteristic by which commodities are distinguished from each other.[11] In an intertemporal WGE model agents trade in all these commodities, also distinguished by time and location, at one moment in time. In other words, in an intertemporal equilibrium there is a sequence of trading through time. Production in a CGE model on the other hand is viewed as a circular, dynamic process. We do not have a "one-way avenue"[12] from given resources (factors of production) to end products (consumption goods), but production of commodities by means of commodities which takes time.

An important Ricardian theme, pushed aside in economic thinking after the Neoclassical revolution of the 1870's, is the distinction between *produced* commodities and *scarce* commodities. Ricardo says: "Possessing utility, commodities derive their exchangeable value from two sources: from their scarcity, and from the quantity of labour required to obtain them. There are some commodities, the value of which is determined by their scarcity alone. (...) These commodities, however, form a very small part of the mass of commodities daily exchanged in the market. By far the greatest part of those goods (...) are procured by labour; and they may be multiplied, (...) almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them."[13] Important here is the distinction between scarce

---

[9] There is no need to deny that the CGE model is *mathematical* equivalent with a special case of the intertemporal WGE model. In such a model there are several rates of interest relating to different goods and different periods. But under the assumptions of constant returns to scale and balanced growth all rates of interest are equal (this is demonstrated by Malinvaud 1972, ch. 10), in which case the model is mathematical equivalent with the CGE model.

[10] Arrow & Starrett 1973, p. 133.

[11] See: Debreu 1959, ch. 2 and 7.

[12] Sraffa 1960, p. 93.

[13] Ricardo 1978, p. 5-6. The labour theory of value lurks behind these words, but that is not important here. For Ricardo capital is nothing else then a

and reproducible commodities. The former are commodities of the *natural endowment type*: "They are given (by nature or by (...) skills) in a fixed quantity and they have to be accepted as they are. They become only economically relevant when they are scarce, i.e. when their given quantity is insufficient to meet all needs."[14] In Walrasian economics all economic goods with positive prices are *defined* as "scarce goods". This makes sense in an exchange model. In such a model all things in nature are goods and only goods with positive prices are traded: they are not *free*, so they must be *scarce*. Of course this is merely a definition. In Classical analysis the problem of scarce commodities is not considered. All commodities in this analysis are commodities of the *production type*: they are not given but produced - and reproducible - by industries: "All commodities considered are produced, and can be made in practically whatever quantity may be wanted, provided that they are devoted that amount of effort they technically require"[15] (labour, natural recources). These commodities are commodities with positive prices but they are not scarce. Of course this is merely a definition too, but a quite different one. These differences show that the WGE model is about exchange in markets, while the CGE model is about production for markets (but neither ignores the other). Mathematical economists always seek elegant symmetries, but they cannot (in fact, should not) obscure this theoretically important fact.[16]

Of course such a choice of objects of study is no coincidence. The Walrasians study a society, consisting of atomistic individuals (households) with particular preferences and endowments. They engage in trading to satisfy their needs and they are selfish. Schumpeter regarded this as "methodological individualism". Classical analysis on the other hand is not about a general trading society but about a modern, capitalist industrial society. It views society as a "class society" (in Marxian terminology) in which, in Ricardo's famous words, "the produce of the earth (...) is divided among three classes of the community, namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated (...). To determine the laws which regulate this distribution is the principal problem in Political

---

wagefund. Moreover, the labour theory of value is removed from modern CGE models (see: Steedman 1977).

[14] Pasinetti 1981, p. 7.

[15] Pasinetti 1981, p. 24. The approach does not deny the role of particular resources in limiting the production. In Ricardian economics this was "land"; in some Classical models it is generalized to primary, non-reproducible resources, for instance natural resources. This means there could be scarce resources in the model, but it is not on them that the analysis is focused. In fact, in this paper there are no scarce resources at all. The analysis is about the internal operation of the productive system as a circular process.

[16] The distinction between scarce or reproducible commodities leads to a more fundamental point. The importance of scarce goods in Walrasian analysis highlights the main objective of mainstream economics: to study, in the famous definition of Lord Robbins, "the relationship between ends and scarce means that have alternative uses". Neoricardians object to this definition of the economic science.

Economy".[17] In modern language, the Classical theory distinguishes several categories of income (wage income, profit income, etc.) and defines economic agents around these categories.[18] The theory makes a distinction between income from work and income from property. There are no apologetic theories about "waiting", "abstinence", etc. Moreover, the main drama in the capitalist society, as conceived in Classical analysis, is capital accumulation, as a consequence of profit maximization by capitalists. Not consumption but accumulation is seen as the driving force in capitalist society.

2.2 Production without a surplus

The Classical model is macro-economic in nature, but has a micro-economic base which is explored in this paper. This base consists of a model of industries in which each industry produces commodities by means of produced and non-produced (primary) commodities. The economy is a circular process: the output good of an industry is an input good of another industry (or possibly the industry itself). The industries are independent subsystems, only connected to each other by flows of commodities. There are various technical conditions of production that need to be taken into account, like the number of commodities, the number of methods of production available, the time pattern of production, the durability of capital goods, etc. We simplificate the analysis by assuming that each industry produces a single, homogeneous output good. The analysis is also restricted to a single unchanging technique of production, so there is by definition no problem of choice of technique. We start with a pure circulating capital model, i.e. all inputs are entirely used up in one production period. This assumption excludes fixed capital. To summarize:

A.1: The economic system is a disaggregated system: commodities are produced by industries.
A.2: The economic system is a circular system: commodities are produced by means of commodities, including homogeneous labour.[19] No natural resources are used.
A.3: There is no joint production.[20]
A.4: There is a finite number of produced commodities, say n, with $1 \leq n < \infty$
A.5: Every industry is represented by a given unchanging method of

---

[17] Ricardo 1978, original preface. As this definition of the Classical economic science shows, early Classical economics is only appropriate to a simple agrarian system. Modern Classical economics, with its more up-to-date terminology and its more advanced mathematics, analyses a complicated industrial system in which land is not important. For this reason this paper ignores Ricardo's Theory of Differential Rent. Furthermore, in this paper is abstracted from "land".

[18] Classical analysis does not consider social, but economic "classes", which distinguishes it from Marxian analysis.

[19] This assumption of homogeneous labour is essential in a CGE model with a uniform wage rate: it is assumed workers are mobile between industries and competition between workers equalizes the wage rate.

[20] This assumption is not necessary for the Classical analysis. See Pasinetti 1980 for some interesting essays on Classical joint production models.

production.[21]
A.6: Production takes one period, the same for all commodities, but when an input is required it is immediately available.
A.7: All capital is pure circulating capital.

Because of A.3, A.4 means there are exactly n industries. There is a 1-1 correspondence between industries and commodities.

Because of A.2, CGE models regard capital quite different from WGE models. Capital in WGE models is regarded as a *stock* of resources, a factor of production endowed by individuals on the same footing as labour. These capital stocks can generate a *flow* of services. In CGE models capital itself is regarded as a *flow* of produced commodities from industries to industries. This means there is no strict distinction between capital goods and consumption goods in a CGE model. A good is a capital good if and only if the available technique makes it necessary as an input in production. But capital goods are produced goods and they are quite different from labour. This distinction is implicit in CGE models but is missing in WGE models. In Nuti's words: "Labour is different from all other inputs and this difference, that should be the starting point of a satisfactory theory of production and distribution, is completely ignored in this version of neo-classical theory".[22] This is the price to be paid for mathematical elegance.

We now formulate a price model of a non-productive system. Ricardo closed this price model by assuming the Malthusian principle of population. This principle would in the long run keep the real wage close to the subsistence level. Such a theory is maybe in our part of the world no longer valid but it is one method of closing the model.[23] We assume:

A.8: The economic system is not productive: no industry produces a positive surplus.
A.9: Workers are paid a subsistence basket of wage goods, i.e. exactly the wage goods necessary to reproduce their labour time for the next period.

Assumption A.9 means we deal here with some kind of Malthusian economy. The reproduction of labour time is a technological condition like all other technological conditions. Wages are "the necessary subsistence of the workers and thus entering the system on the same footing as the fuel for the engines or the feed for the cattle".[24] The worker is a machine: you feed it with wheat and it provides you with labour services. Assumptions A.1 - A.9 give Sraffa's "extremely simple society" producing just enough to maintain itself, a pure subsistence, pure circulating capital model:

280 qr. wheat + 12 t. iron -> 400 qr. wheat
120 qr. wheat + 8 t. iron -> 20 t. iron

---

[21] This assumption means fixed *coefficients* of production only if we specify a quantity system (see below).

[22] See: Nuti 1970, p. 231.

[23] Below a more modern method of closing the model will be considered.

[24] Sraffa 1960, p. 9.

It is now supposed that these produced commodities are exchanged at an annual market "heid after the harvest". Sraffa asserts: "There is a unique set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated; such values spring directly from the methods of production".[25] Production is not a black box, but a well-described process. In fact, the description of this process is all the analysis is about. Exchange takes place, but as a necessary consequence of multisectoral production. Moreover, the analysis searches a "repeatable", that is *reproducible*, solution. Equilibrium is not only defined as the state in which all markets clear, but also as the state in which the system can reproduce itself: whether it can produce enough output to replace the inputs used, and to reproduce the workers for another period of production. Market clearing is a necessary condition for reproducibility, but reproducibility is not a necessary condition for market clearing, i.e. the Classical definition of equilibrium is stronger than the Walrasian one.

If we denote the matrix of flows as Q, we have, by definition:

(2.2.1) $\quad x = Qi, \quad x \geq 0, \; Q \geq 0$

where x is the output vector (a column vector) and i is a vector with unit elements, $i = [1,1,..,1]'$. This system is only reproducible if the exchange at the "annual market" obeys the following equations:

(2.2.2) $\quad \hat{x}p = Q'p$

where $\hat{x} = \text{diag}(x)$, and p is the price vector (a column vector). We define:

$$A \equiv Q\,\hat{x}^{-1}, \quad A \geq 0$$

where A is the input-output matrix (assumed nonnegative and indecomposable). The coefficient a_{ij} represents the *portion* of the output of the i-th good that is being used in the production of the j-th good. If each sector has a unique input structure, i.e. the portions of output being used in production are not a scalar multiple of the portions used in another sector, matrix A has full rank. This highlights the difference between the way in which commodities in WGE and CGE models are viewed. In Classical analysis commodities are defined by the technology to produce them; they are unique if and only if they are *technically* unique. There is a complete correspondence between a commodity and the technology to produce it. In Walrasian analysis goods are defined by their physical characteristics, including time and location of their availability.[26] In this approach all things in nature are goods, even the air we breath. Furthermore, a change in technical coefficients (because of for instance factor substitution) would mean that another product is produced if we would include these coefficients into the physical characteristics of the good. A Walrasian model with production defines a commodity only by its physical characteristics; the technical characteristics of its production are excluded. This stresses the

---

[25] Sraffa 1960, p. 3.

[26] Debreu: "Summing up, a commodity is a good or service completely specified physically, temporally, and spatially" (Debreu 1959, p. 32).

importance of exchange instead of production in a Walrasian model.

If we substitute the definition of the A matrix into expression (2.2.2) the following homogeneous system of equations follows:

(2.2.3) $\quad p = A'p, \quad p \geq 0$

It is well known that given A is a nonnegative, indecomposable matrix, there exists a positive price vector p which is unique up to scale. This follows from the Perron-Frobenius theorems. The theoretical significance is that these prices follow directly from the methods of production represented in the matrix A. Moreover, as long as we are interested in the existence, positiveness and uniqueness of the equilibrium prices, no assumption of constant returns to scale (CRS) is needed. The analysis involves only given states of the system: the output is constant (the matrix A consists of portions, not coefficients).[27] Problems of scale do not exist, so an assumption like CRS is irrelevant.[28]

We can make subsistence labour explicit by specifying the subsistence wage goods vector. To do this we have to redefine the above vectors and matrices. We obtain:

(2.2.4) $\quad p = A_+'p \qquad$ with: $A_+ \equiv A + b\, a_o$

where b is the subsistence wage goods vector (a column vector) and a_o is the labour coefficient vector (a row vector). A_+ is called the labour augmented input-output matrix.[29]

In this simple model it is possible to derive equilibrium prices entirely from technology (including subsistence which is a technological condition). There is no room for demand conditions. Of course, this is not surprising: the consumer is cut down to size from the very start. But it is quite surprising a CRS assumption is not needed.

Things change if we specify a quantity system. Equation (2.2.1) combined with the definition of the matrix A gives:

(2.2.5) $\quad x = A\, x$

If we make subsistence labour explicit, we have to redefine A and x again to get:

(2.2.6) $\quad x = A_+ x \qquad$ with: $A_+ \equiv A + b\, a_o$

Now CRS is required. So if we want to combine the price and quantity system

---

[27] So, $A = A(x)$ and $a_o = a_o(x)$ is no problem at all.

[28] In Sraffa's words: "No question arises as to the variation or constancy of returns" (Sraffa 1960, preface). Several articles are written about this. See, for instance: Levine 1985.

[29] See: Morishima 1989, p. 27.

to get a full general equilibrium model, we have to assume CRS. In that case equations (2.2.4) and (2.2.6) are a dual problem:

$$A_+'p = p, \quad p \geq 0$$
$$A_+ x = x, \quad x \geq 0$$

Nothing has been said about the price and quantity equations of the labour market. In concordance with Ricardo's theories the labour market is always in equilibrium. If we use the subsistence money wage (an equilibrium rate) as the numéraire and also normalize the total available labour resources (i.e. the fully employed "output" of the labour sector) we obtain:

(2.2.7) $b'p = 1$
(2.2.8) $a_o x = 1$

The general equilibrium system consisting of the equations (2.2.4), (2.2.6), (2.2.7) and (2.2.8) is completely determined. Given A is a nonnegative, indecomposable matrix, A_+ is so too, and then the first two equations yield a positive price vector p and a positive output vector x which are unique up to scale. The last two equations fix the absolute price and output levels. There is no room for demand equations, except that the demand for subsistence goods fixes the scale of the economy. We now state:

Proposition 1: In a Classical non-productive economy, with subsistence labour, prices are determined by the conditions of production. No CRS assumption is necessary.

2.3 Production with a surplus

Now we consider a productive system, which is considered more relevant in the analysis of a capitalist society. Instead of A.8 we now assume:

A.8′: The economic system is productive: at least one industry produces a positive surplus.

Sraffa gives the following example, in which again the input requirements of labour are included:[30]

280 qr. wheat + 12 t. iron -> 575 qr. wheat
120 qr. wheat + 8 t. iron -> 20 t. iron

It is important to remark that the price system under assumption A.8′ cannot be equation (2.2.4). To see this we first specify a quantity system. If there is a surplus, we have instead of (2.2.6):

(2.3.1) $x \geq A_+ x$

This equation could be regared as a feasibility condition for the reproduction of the system. Let z be the surplus vector, or in other words the final output vector (a column vector). By definition $z \geq 0$. Then:

---

[30] Sraffa 1960, p. 7.

(2.3.2) $\quad x = A_+ x + z$

Premultiply (2.3.2) by p' and premultiply (2.2.4) by x' to get $p'z = 0$ which contradicts the assumption that $z \geq 0$. The only possible case is a trivial one: $p = 0$. So, equation (2.2.4) cannot be combined with equation (2.3.1). Again, there is duality:

$$A_+' p \leq p, \quad p \geq 0$$
$$A_+ x \leq x, \quad x \geq 0$$

What we now need is a so-called "distribution rule".[31] At this point the Classical model defines an institutional framework[32], which is called "capitalism":

A.10: All means of production (capital goods) are privately owned by "capitalists". It is this property which gives the capitalist the exclusive right to organize and control the production process, i.e. he acts as the "entrepreneur".
A.11: The surplus is entirely distributed to the capitalists in the form of profit income.[33]
A.12: Each capitalist maximizes his *rate* of profit, i.e. his rate of return on invested capital (including the wage fund[34]), under a technology

---

[31] This question does not arise in a WGE-model because there is no concept of surplus. Of course it could be defined in a Walrasian model as pure profit, i.e. as the difference between revenues and costs. But there is no "distribution problem" in a Classical sense: profit is *by definition* distributed to the producer.

[32] It is frequently said that a WGE-model is 'pure' theory without institutional specification. This is not true. The Walrasian theory focuses on individual, decision-making agents. Their optimizing choices are brought into balance in markets. "Agents" and "markets" are institutions. Moreover: the *rules of the game*, like the initial distribution of endowments owned by individual, decision-making agents, are institutional arrangements because they are arbitrary.

[33] Keynes would call assumptions A.10 and A.11 the main "Classical" assumptions. As is well-known, he defined "Classical" as the theoretical system in which Say's Law holds (in which case both the CGE and WGE model are "Classical"). In Keynes' view it is the *independent* decision to invest of the entrepreneurs (no longer equivalent with the decision to save of the capitalists) which destroys Say's Law.

[34] The Ricardian theory of the *wage fund* states that a capitalist has to pay workers a wage before production can take place, because firstly workers have no other means of existence than their labour time and secondly production takes time. This wage fund has to be included in the invested capital. Morishima calls the theory of the wage fund the most important distinction between Ricardo's and Walras' own models, but he overlooks the importance of the theory of the uniform rate of profit (see below). See: Morishima 1989.

constraint only[35] (*profit maximization*).
A.13: The investment fund (or money capital, the value of capital goods) is mobile between industries.

There are no financial markets in the model. It is simply assumed that the owner of the means of production, the stock of capital goods, is also the coordinator of the production process. This institional framework obviously neglects the Walrasian distinction between the producer (the "entrepreneur") and the capital owner who hires his capital goods to the firm (the "capitalist"). In Classical analysis an economic agent is identified only by its source of income. In fact, a theory of the firm is missing altogether, an omission concealed by most Classical authors.[36]

Assumption A.12 is essential to present the Classical model of capitalist production as a general equilibrium model (but not sufficient). Profit maximization is considered an important characteristic of capitalism, but capitalists maximize their *rate* of return. The Classical approach is especially to be found in the technology constraint (instead of the resources constraint in Walrasian analysis). As we have seen, the price system of production with a surplus is: $A_+'p \leq p$. So, instead of (2.2.4):

$$(2.3.3) \qquad p = [I + \hat{r}]\, A_+'p, \quad p \geq 0 \qquad \text{with: } \hat{r} \equiv \text{diag}(r_1,..,r_n)$$

where r_i is the rate of profit in industry i (a scalar), while: $r_i \geq 0$. If we define r as the largest r_i the following inequality follows:

$$p \leq A_+'p\,(1+r)$$

If strict inequality holds for industry i, then $r_i < r$, so capitalists will not enter industry i. The capitalist can invest in the industry she wants (because of A.13). Competition ensures that only those industries are put in

---

[35] *Profit* in Ricardian analysis is equivalent to *interest* in Walrasian analysis; in both cases the return on invested capital is meant. *Profit* as defined in a WGE model is a "super profit" (a quasi-rent) which does not exist in the CGE model presented in this paper (because we abstract from non-produced resources). In this model prices cover exactly the production costs and nothing else; they are *production prices*.

[36] But a theory of the firm is also missing in Walrasian models. The goal of production in the WGE model is the same as the goal of exchange itself, namely the fulfillment of the desires of the consumer. The producer is therefore not as fundamental as the consumer in the WGE model. The producer acts as a mediator: he or she coordinates the transformation of inputs supplied by the consumers into outputs demanded by the consumers, and he or she is not even paid for this mediation: in a competitive equilibrium all profit (a quasi-rent) disappears. Walras himself makes this clear in his famous statement: "Les entrepreneurs ne font ni bénéfices ni pertes". As Ellerman shows "the Arrow-Debreu type models fail as models of an idealized perfectly competitive capitalist economy" where positive profits exist structurally (Ellerman 1980).

operation, which give the maximum rate of profit r.[37] So in equilibrium we have:

$$p = A_+' p\ (1+r)$$

The theorems of Perron-Frobenius guarantee that this equilibrium is unique. The Classical data (the subsistence wage, the technology) are sufficient to determine all prices and a uniform rate of profit. A CRS assumption is therefore still not necessary.[38] We now state:

Proposition 2: In a Classical general equilibrium the *rate* of return on invested capital, i.e. the ratio between profit and invested capital, is equal in all sectors of production.

An important Ricardian theory is the theory of the *uniform rate of profit.* Ricardo states that "whilst every man is free to employ his capital where he pleases, he will naturally seek for it that employment which is most advantageous (...). This restless desire on the part of all the employers of stock to quit a less profitable for a more advantageous business has a strong tendency to equalise the rate of profits of all."[39] In the long run competition ensures equal profit rates, i.e. "the remaining value or overplus will in each trade be in proportion to the value of the capital employed".[40] We have seen that in equilibrium this equal profit rate is ensured, but nothing has been said about the adjustment process to this equilibrium. A lot of research is still needed here.

Because of proposition 2 equation (2.3.3) can be simplified to:

(2.3.4) $$p = A_+' p\ (1+r), \quad p \geq 0$$

where r is the uniform rate of profit. To illustrate the meaning of the uniform rate of profit in Classical analysis equation (2.3.2) is written in a different form:

(2.3.2') $$x = [I + \hat{R}]\, A_+ x \qquad \text{with: } \hat{R} \equiv \text{diag}(R_1,..,R_n),\ R_i \equiv z_i/(x_i - z_i)$$

where R_i is the rate of physical surplus in sector i. In general these rates are not equal, but if we notionally make them all equal to, say, R^*, we get:

---

[37] In a more complex CGE model with a spectrum of techniques this argument is reformulated as a choice of technique problem.

[38] The procedure adopted by Roemer 1980 to obtain the uniform rate of profit in a Marxian CGE model, profit maximization under capital constraints, is Walrasian in nature, because it uses resources of capital goods as constraints. It is no surprise that a CRS assumption is necessary in this procedure (because capitalists have to *change* their scale of production to operate all processes to the limit of their capital constraints). In our view, such a procedure is alien to the nature of the Classical approach.

[39] Ricardo 1978, p. 48.

[40] Ricardo 1978, p. 50.

$$x = A_{+}x\,(1+R^{*}), \quad x \geq 0$$

This is the dual problem of (2.3.4), so: $r = R^{*}$. We can now state:

Proposition 3: In a Classical model with subsistence wages the uniform rate of profit is equal to a hypothetical uniform rate of surplus. The mechanism of maximization of the rate of profit, under a technology constraint, leads to a redistribution of surplus between industries.

Price system (2.3.4) is consistent with (2.3.2). Premultiply (2.3.2) by p' and premultiply (2.3.4) by x' to get $p'z = p'Kr$, where $K \equiv A_{+}x$ is capital invested. Of course this is assumptions A.10-A.12 in a nut shell. The price system shows that the surplus cannot be alloted before the prices are determined. In Sraffa's words: "This is because the surplus (...) must be distributed in proportion to the means of production (or capital) advanced in each industry; and such a proportion between two aggregates of heterogeneous goods (in other words, the rate of profits) cannot be determined before we know the prices of the goods. (...) The result is that the distribution of the surplus must be determined through the same mechanism and at the same time as are the prices of commodities."[41] Prices are still determined by technology, but competition adjusts prices to obtain a uniform rate of profit. As proposition 3 states, the result of the uniform rate of profit is a redistribution of surplus between industries.

The system consisting of equations (2.3.2), (2.3.4), (2.2.7) and (2.2.8) is not a full general equilibrium model of production with a surplus because it is not closed. To do this we have to include a demand system. Only out of profit income demand can arise, not out of wage income, because workers have nothing to choose in this model. If we want to describe the Ricardian CGE model we should assume:

A.14: All profit income is reinvested (*Classical saving hypothesis*).
A.15: The rate of growth in all sectors is equal (*Balanced growth*).

Assumption A.14 means that preference maximization is missing in the present model: there are no free consumers (the workers are paid a subsistence wage, while the capitalists do not consume at all). Assumption A.15 is necessary in a Classical general equilibrium system, which is made dynamic by reinvestment of profit income. A system in which industries are growing at different rates cannot be in equilibrium. The assumption of balanced growth is a strong assumption, but it is necessary if we want to place general equilibrium in a dynamical setting. The assumptions modify (2.3.2) to:

$$x = A_{+}x(1+g), \quad x \geq 0 \tag{2.3.5}$$

where g is the accumulation rate, i.e. the uniform growth rate of the economy (a scalar). Now we have a full general equilibrium model. Equations (2.3.4) and (2.3.5) are a dual problem:

---

[41] Sraffa 1960, p. 6.

$$p = A_+'p(1+r), \quad p \geq 0$$
$$x = A_+ x(1+g), \quad x \geq 0$$

or:

$$p'[\lambda I - A_+] = 0, \quad p \geq 0 \qquad \text{where: } \lambda \equiv 1/(1+r)$$
$$[\mu I - A_+] x = 0, \quad x \geq 0 \qquad \text{where: } \mu \equiv 1/(1+g)$$

This leads to a familiar Classical result, the so-called *golden rule* of accumulation: $r = g$ ($= R$). We see that while prices form an eigenvector of the augmented input-output matrix, the rate of profit and the rate of growth are associated with an eigenvalue of this matrix. The Perron-Frobenius theorems ensure that a unique, positive price vector p exists which supports a uniform rate of profit r, and a unique, positive output vector x exists which supports a uniform rate of growth g. Furthermore, in that case λ and μ are the Frobenius roots (the dominant eigenvalues) of A_+.[42] Equations (2.3.4) and (2.3.5) form a simple but full general equilibrium model. All variables are determined, due to (2.2.7) and (2.2.8) also in scale. We have a system in which technology determines prices, while demand determines scale and relative quantities. This is what could be called the *Classical dichotomy*. Furthermore, this dichotomy means we can determine prices without assuming CRS, but this assumption is certainly necessary in a full general equilibrium specification.[43] So:

Proposition 4: In a Classical productive economy with subsistence wages prices are determined by the conditions of production. No CRS assumption is necessary.

2.4 Domestic labour

If we define in a Classical model the proportions of commodities necessary to produce a unit of labour (the subsistence vector b) and the proportions of labour units necessary to produce a unit of a commodity (the labour coefficient vector a_o), why not define the proportion of labour units necessary to produce a unit of labour? In our model with homogeneous labour there is exactly one such coefficient, say h, the *domestic labour coefficient* (a scalar). A hidden assumption in the above Ricardian system is that there is no household production, i.e. $h = 0$. In fact, the whole technology of the system could be represented in the following partitioned matrix:

$$A_{++} \equiv \left[\begin{array}{c|c} h & a_o \\ \hline b & A \end{array}\right]$$

---

[42] These highest eigenvalues λ and μ are associated with the lowest values of r and g, respectively. Higher values of r and g give negative prices. See: Woods 1978, ch. 2.

[43] Eatwell writes: "A great strength of the Classical framework derives from this separation between the determination of output and the theory of value. Not only does this separation free us from the need to assume constant returns, but also it embodies great potential for theoretical development" (Eatwell 1977, p. 66).

where b the subsistence vector (a column vector), h the domestic labour coefficient (a scalar), a_o the labour coefficient vector (a row vector), and A the input-output matrix proper. If we assume A.8 we obtain the closed, homogeneous system of production:

(2.4.1) $\quad x_{++} = A_{++} x_{++}$

or, equivalently:

(2.4.1′) $\quad x_o = h\, x_o + a_o x$

$\quad x = b\, x_o + A\, x$

where x_o is the fully employed labour supply (a scalar), and x_{++} is the output vector including x_o. In the above Ricardian system we assumed: $x_o = 1$ and $h = 0$. We now state:

Proposition 5: Implicit in Ricardian analysis is a zero domestic labour coefficient.

The introduction of a positive domestic labour coefficient does not change the Ricardian analysis fundamentally, except that the analysis is extended to the production of labour by labour, i.e. domestic labour (like housewives and butlers). Of course we assume, in correspondence with early Ricardian thinking, that the "wage" of the domestic worker is a subsistence wage. But if we should regard this as realistic is another matter.

3. A MODERN CGE MODEL

3.1 Introduction

The above Ricardian analysis is old-fashioned in several ways, especially with respect to the assumption of subsistence labour and as a consequence the distribution of the whole surplus as profit. In a more modern CGE model the wage is no longer a subsistence payment but a surplus payment. This means there is a distribution *conflict*: both wages and profit are paid from the surplus, and there are no "objective" means to settle this conflict (as in the Neoclassical marginal productivity theory of distribution, where the factor payments are equal to their marginal productivity[44]).

In early Ricardian thinking the real wage as a subsistence "bundle of commodities" was an exogenous variable. Sraffa on the other hand assumed a surplus wage. He showed that in a Classical system a price model can be specified with a profit and a wage rate, in which prices and one of these

[44] As J.B. Clark put it: "What a social class gets, is what it contributes to the general output" (Clark 1891).

two rates are simultaneously determined when the other one is given exogenously. Instead of a bundle of commodities a money wage is given, or alternatively, a rate of profit equal to a rate of interest determined in the monetary sector of the economy. The question arises: do we still have a general equilibrium model if an exogenous variable like the surplus wage rate is introduced? In our opinion, yes. A general equilibrium model is not a general equilibrium model because everything is explained. After all, in a WGE model there are some distributional data too, like the distribution of initial endowments between individuals. An important distinction between the WGE and the CGE model is that not the distribution of endowments between individuals is a data in the analysis, but a socially or historically determined wage rate.[45] This makes sense in an analysis focused on the reproducibility of the economic system. As will be shown below, profit maximization and even preference maximization can be assumed in a Classical context; this increases the complexity without destroying the main characteristics of the model.

3.2 A Sraffian GE model

Although Sraffa only specified a price system, we can speak of a Sraffian GE model when we close his system. In this section we also generalize the analysis and account for both fixed capital and preference maximization. We assume:

A.7′: The stock of each type of capital good depreciates over time at a given uniform rate δ, i.e. this rate is independent both of the industry in which the capital good is used and of the age of the capital good[46], with:
$0 < \delta \leq 1$.
A.9′: Labour is paid *at least* a subsistence basket of wage goods. The wage is paid at the end of the production period (*post factum* wages).
A.11′: The surplus is entirely distributed between profit and wage income.

For convenience we do not split up the wage in a subsistence and a surplus part but treat the whole wage as variable, exactly as Sraffa did.[47] This means we neglect the subsistence wage and assume there is a minimum level below which the wage cannot fall (or equivalently assume that subsistence is already discounted for in the input-output matrix). Now we have to following

---

[45] This is of course in concordance with Marx' analysis who is after all Ricardo's follower. The main difference between Marxian analysis as a particular form of Classical analysis and Classical analysis proper is the exploitation theory for which it is necessary to formulate a labour theory of value. All of this is missing in Classical analysis proper. See: Morishima 1973.

[46] This is a convenient way to formalize fixed capital. By abstracting from joint production we are giving up the possibility to regard a capital good, becoming one period older, as a *joint product* of the industry in which it is used. This is a more satisfactory method of dealing with fixed capital because depreciation is seen as an economic phenomenon (and not as a technical phenomenon). The joint production approach of fixed capital is introduced by both Von Neumann and Sraffa. See: Von Neumann 1945 and Sraffa 1960, ch. 10.

[47] See: Sraffa 1960, p. 10.

system:

(3.2.1) $p = A'p\,(\delta+r) + a_o'w, \qquad p \geq 0$

(3.2.2) $x = A\,x\,(\delta+g) + c, \qquad x \geq 0$

where c is the consumption vector (a column vector). There is again duality in this system, that is: $r=\varphi(w)$, while $g=\varphi(c)$, when the Classical saving hypothesis is assumed. In that case: $w = p'c$, since we assume (2.2.8). Premultiply (3.2.1) by x' and premultiply (3.2.2) by p' to find:

(3.2.3) $p'Kr + w = p'Kg + p'c \qquad$ with: $K \equiv Ax$

In plain English, profits plus wages equals investment plus consumption.[48] To close the model we have to include a demand theory. If we would use the Classical saving hypothesis as a crude and simple demand theory (in fact, more a method to avoid specifying a demand theory), we see immediately that: $r = g$. A trivial case is the case where $w = 0$ and $c = 0$, i.e. the case where both the profit rate and the growth rate are at their technically determined *maximum* values (equivalent to the model of section 2.3).

To obtain a full general equilibrium model we include a more advanced demand system which determines c. We assume:

A.14′: Profit income is partly consumed, partly saved (and reinvested), while the wage income above the subsistence level is only consumed (*Kalecki's saving hypothesis*).
A.16: Capitalists and workers in choosing consumption goods maximize their preferences, under an income constraint (*preference maximization*).

Assumption A.14′ means that only capitalists saves and invests; they are the only ones who receive the fruits of investment: profit income. The propensity to save forms part of the preferences of the capitalists. What they not save, is used to pay for consumption. We have:

$$w = p'c_w, \qquad s\,p'Kr = p'Kg, \qquad (1-s)\,p'Kr = p'c_p$$

where c_w is the consumption vector chosen from wage income, c_p is the

---

[48] From this equation it can be seen that in a CGE model factor rewards are in general not equal to their marginal productivity. Rewrite the left-hand side of (3.2.3) as: $y = kr + w$, with $k \equiv p'K$. Scalars y and k are the net output per worker and the capital value per worker, i.e. the "capital intensity" in value terms, respectively. Consider the total differential: $dy = rdk + kdr + dw$. Now, only if $k = -dw/dr$ it follows that: $r = dy/dk$ (see: Woods 1978, p. 108). The condition is a fluke case, which will only hold in a multisectoral model with equal "capital intensities" in all sectors. Ironically, the Neoclassical theory of distribution is valid if the Marxian case of "equal organic composition of capital" holds, in which case the prices of commodities are equal to the labour value embodied in these commodities! The marginal productivity theory of distribution was the subject of a hot debate in the fifties and sixties. For a survey, see: Harcourt 1969.

consumption vector chosen from profit income ($c \equiv c_w + c_p$), and s is the uniform savings propensity out of profit income ($0 \leq s \leq 1$).

From this the well-known *Cambridge equation* is derived:

(3.2.4) $\qquad r = g/s$

In general we have: $r \geq g$.

Nothing essential hinges on the assumption that workers do not save. As the *Pasinetti Theorem* shows, even if workers would save part of their income, the profit rate (and thus the prices of production) in a Classical model is not influenced by savings from wage income: "When any individual saves a part of his income, he must be allowed to own it, otherwise he would not save at all. This means that the stock of capital which exists in the system is owned by those people (capitalists or workers) who in the past made the corresponding savings."[49] In other words, in a Classical model where savings and investments are equivalent, savings out of wage income means ownership of capital stock by the workers, in which case they are partly capitalists: they receive both wage and profit income (see also p. 11-12). The profit-capital ratio of both classes is equal (because of the uniform rate of profit), while the savings-capital ratio of both classes is also equal (because of balanced growth). But then the profit-savings ratio of both classes is also equal; "it simply and logically follows from the institutional principle that profits are distributed in proportion to ownership of capital".[50] The result is that:

$$s_c P_w = s_w (W + P_w)$$

where s_c and s_w are the propensity to save of the capitalists and the workers, P_c and P_w are the received profits of capitalists and workers, and W is the total of wages of the workers. Savings out of worker income are equal to the savings the capitalists would have done if they received the profit income of the workers. It follows that the Cambridge equation holds, even if workers do save.[51] Without loss of generality in the derivation of production prices we can assume A.14'.

Assumption A.16 boils down to two maximization problems. If we assume (for simplicity) that all workers have the same preferences and all capitalists too, we can introduce One Big Worker and One Big Capitalist. Let u_w and u_p be the utility functions for consumption goods of the Worker and the Capitalist, respectively. The preference maximization problem of the Worker

---

[49] Pasinetti 1962, p. 106.

[50] Pasinetti 1962, p. 110.

[51] For a full proof: see Pasinetti 1962. Meade 1966 shows that the Pasinetti Theorem depends on the crucial assumption that capitalists receive profit income and thus own capital goods. He regards this not as a general case and considers the case in which the workers dominate the accumulation process. Of course, a situation in which capitalists do not own capital goods is not in concordance with the Classical conception of capitalism.

is:

(3.2.5) $\max_{c_w} u_w(c_w)$ subject to: $w = p'c_w$

while the preference maximization problem of the Capitalist is:

(3.2.6) $\max_{c_p} u_p(c_p)$ subject to: $(1-s)\, p'Kr = p'c_p$

We assume that the utility functions are continuous and that the chosen consumption bundles are unique. For instance, we can assume CES functions.

We now have a full general equilibrium model. Equation (3.2.1) can be solved if we set the value of w. We have n equations in n+1 unknowns (n-1 relative prices and the two distributional variables r and w), so the system operates with one degree of freedom. In Classical analysis the wage rate is part of the data, i.e. is determined outside the system, which means p and r are simultaneously determined by (3.2.1), by a given w. It can be proved that in the case where $w > 0$, a positive and unique solution exists if and only if all eigenvalues of matrix A are less than 1.[52] This is a quite familiar condition in input-output analysis, equivalent with several other conditions, like the Hawkins-Simon condition (all principle minors of A positive) and the Nikaido "productivity" condition (there is a nonnegative output vector such that the surplus vector is semi-positive).[53]

We have seen that equation (3.2.1) determines a unique p (up to scale) and r, by a given w. Now relative prices are known, the quantity system can be introduced to determine the output. The Cambridge equation (3.2.4) determines a unique g, now we have a unique r. The demand system in which preference maximization is introduced is only relevant for the quantity system. It is assumed that the preference maximization problems (3.2.5) and

---

[52] *Proof:*

Let ξ^* be the Frobenius root of A and R the technically determined maximum rate of profit. Then: $R = 1/\xi^* - \delta$. Consider $w > 0$. Then:

$$p = [\lambda I - A']^{-1} \lambda a_o' w \qquad \text{where: } \lambda \equiv 1/(\delta+r)$$

The inverse matrix exists if λ is not equal to any of the eigenvalues of A.
$p \geq 0$ for $\lambda a_o' w \geq 0$ if and only if: $[\lambda I - A']^{-1} \geq 0$, which is true if and only if: $\lambda > \xi^*$, or: $1/(\delta+r) > 1/(\delta+R)$, but then: $r < R$

We see that in the case where $w > 0$ a positive and unique solution for p and r exists if and only if the rate of profit is less than the technically determined maximum rate. But is this maximum rate nonnegative itself?
For $r > 0$, we require $\xi^* < \delta$, because: $\lambda > \xi^*$, so: $1/(\delta+r) > \xi^*$,
But then: $1/\xi^* - \delta > r$. Because $0 < \delta \leq 1$, we have: $\xi^* < 1$

QED

[53] See: Woods 1978, p. 7.

(3.2.6) uniquely determine a set of c_w and c_p consumption bundles:

(3.2.5′) $\quad c_w = c_w(p,w)$

(3.2.6′) $\quad c_p = c_p(p,r,x)$

This, combined with (3.2.2), gives a unique solution for x and c. The proof for existence, positiveness, and uniqueness is similar to the above proof.

A Walrasian model would of course determine the wage rate in the labour market in which case an additional equation is introduced: the equilibrium condition of the labour market derived from a work-leisure choice problem of the workers. Such a model would look something like this:

$$x = Ax(1+g) + c(p,w,r,x)$$
$$p = A'q + a_o'w \qquad \text{with: } q = p(1+r)$$
$$a_o x = L(p,w)$$

from which we can derive Walras' Law:

$$p'Axg + p'c(p,w,r,x) = p'Axr + wL(p,w)$$

The wage rate is not given: it is determined in the labour market. In a Classical model the wage rate is part of the data of the system, determined socially or historically: the price system moves with one degree of freedom. It is possible to let another variable, like the rate of profit, be determined outside the system. For instance, it is not in contradiction with the Classical approach to let the rate of profit be determined by a rate of interest derived from household's time preferences. In the Postkeynesian version of the Classical model the rate of profit is determined by the rate of growth (via the Cambridge equation), while the rate of growth is exogenously determined by the so-called "animal spirits" of independent capital investors. Essential therefore is not the choice of the exogenous variable, but the residual nature of one of the distributional variables.

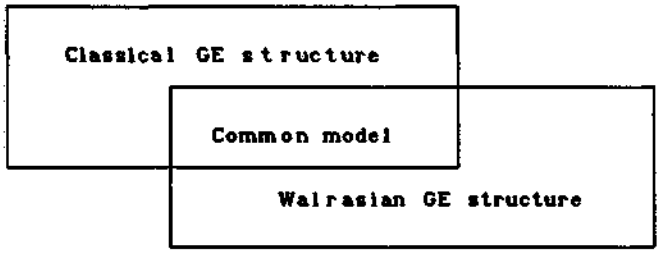
Proposition 6: In a modernized Classical productive economy, with profit and preference maximization, prices are determined by the conditions of production and one of the distributional variables. No CRS assumption is necessary.

The main conclusions of the CGE model do not necessarily change when the subsistence concept is replaced by preference maximization. The separation of the determination of prices and output still exists if we do this, because it is the consequence of other things (like the uniform rate of profit as a result of the maximization of the profit rate).

Proposition 7: In a modernized Classical productive economy, with profit and preference maximization, the Classical dichotomy still holds.

CONCLUSIONS

This paper shows that equilibrium prices can be determined in a quite different framework of general equilibrium analysis. In a Classical general equilibrium model prices can be determined without a quantity system, even if a demand system is included in the model. The assumption of constant returns to scale is not necessary to determine prices. Profit maximization and even preference maximization in a Classical context can be assumed. So the Walrasian methods can be used to study non-Walrasian problems, such as the maximization of profit under a technology constraint in a production system where scarce resources are not important. This increases the complexity of the Classical general equilibrium model without destroying the main characteristics. Nevertheless, a full general equilibrium model which also determines the output is mathematically equivalent with a special Walrasian general equilibrium model, under restrictive assumptions like constant returns to scale, but only if the conceptual differences between the two types of models are ignored.



Mathematically a common model can be identified, which looks something like this:

$$p = A'q + a_o'w, \qquad \text{where } q = p(1+r)$$

but the meaning of this equation in both theoretical systems is different. For a Walrasian it is an equation of exchange which is satisfied when all markets clear. For a Neoricardian it is an equation of production which must be satisfied for the system to reproduce itself. In Classical analysis everything focuses on *reproducibility*, which leads to a stronger equilibrium concept than the more usual Walrasian concept. Market clearing is necessary but not sufficient for reproducibility.

Although a common model can be identified, the Classical general equilibrium model is theoretically quite distinct from the Walrasian general equilibrium model. Behind the mathematical façade we find very different concepts of commodities, profit, equilibrium, etc. This is not surprising; after all the agenda of research of both systems is dissimilar.

An examination of these concepts shows that in Classical analysis a surplus approach is imminent: one of the categories of income is a residual, i.e. part of the surplus is distributed while no corresponding exchange takes place. In the Walrasian model the surplus has vanished. The flow of commodities in one direction is matched by a corresponding flow in the other: all income results from a payment for productive services. Such a theoretical difference cannot be settled on mathematical grounds. The bottom

line is: not all general equilibrium models are Walrasian.

LITERATURE

Abraham-Frois, G. & Berrebi, E., 1979, *Theory of Value, Prices and Accumulation, A Mathematical Integration of Marx, Von Neumann and Sraffa*, Cambridge University Press, Cambridge

Arrow, K.J. & Debreu, G, 1954, Existence of an Equilibrium for a Competitive Economy, *Econometrica*, Vol. 22, p. 265-290

Arrow, K.J. & Starrett, D.A., 1973, Cost- and Demand-Theoretical Approaches to the Theory of Price Determination, in: Hicks, J.R. & Weber, W. (ed.), *Carl Menger and the Austrian School of Economics*, Oxford University Press, Oxford

Barna, T, 1975, Quesnay's Tableau in modern guise, *The Economic Journal*, p. 485-496

Bharadwaj, K. & Schefold B. (ed.), 1989, *Essays in Honour of Piero Sraffa*, London

Bidard, C., 1988, The Falling Rate of Profit and Joint Production, *Cambridge Journal of Economics*, p. 355-360

Bidard, C., 1990, An Algorithmic Theory of the Choice of Techniques, *Econometrica*, p. 839-859

Clark, J.B., 1891, Distribution as Determined by Rent, *Quarterly Journal of Economics*, Vol. 5, p. 313

Debreu, G., 1959, *Theory of Value*, John Wiley and Sons, New York

Dobb, M., 1973, *Theories of Value and Distribution since Adam Smith*, Cambridge University Press, Cambridge

Eatwell, J., 1977, The Irrelevance of Return to Scale in Sraffa's Analysis, *Journal of Economic Literature*, Vol. 15, March, p. 61-88

Ellerman, D.P., 1980, Property theory and Orthodox economics, in: Nell, E.J. (ed.), *Growth, Profits, & Property, Essays in the Revival of Political Economy*, Cambridge University Press, Cambridge

Harcourt, G.C., 1969, Some Cambridge Controversies in the Theory of Capital, *Journal of Economic Literature*, p. 369-405

Levine, A.L., 1974, This Age of Leontief... and who? An Interpretation, *Journal of Economic Literature*, p. 872-881

Levine, A.L., 1985, Sraffa's "Production of Commodities by Means of Commodities", Returns to Scale, Relevance, and Other Matters: A Note, *Journal of Post Keynesian Economics*, p. 342-349

Malinvaud, E, 1972, *Lectures on Microeconomic Theory*, North-Holland, Amsterdam

Meade, J.E., 1966, The Outcome of the Pasinetti-Proces: A Note, *Economic Journal*, p. 161-165, reprinted in: Harcourt, G.C. & Laing, N.F. (ed.), 1971, *Capital and Growth*, Penquin Books, Harmondsworth

Morishima, M., 1973, *Marx's Economics, A Dual Theory of Value and Growth*, Cambridge University Press, Cambridge

Morishima, M., 1989, *Ricardo's Economics, A General Equilibrium Theory of Distibution and Growth*, Cambridge University Press, Cambridge

Neumann, J. von, 1945, A Model of General Equilibrium, *Review of Economic Studies*, p. 1-9, reprinted in: Hahn, F.H. (red), 1971, *Readings in the Theory of Growth*, Macmillan, London

Noord, P.J. van den, 1990, *Een Post-Keynesiaanse Analyse van de Kleine Open Economie, op basis van een Jaargangenmodel*, SEO-rapport 259, Amsterdam

Nuti, D.M., 1970, "Vulgar Economy" in the Theory of Income Distribution, *De Economist*, p. 363-369, reprinted in: Hunt, E.K. & Schwartz, J.G. (ed.), 1972, *A Critique of Economic Theory*, Penquin Books, Harmondsworth

Pasinetti, L.L., 1962, Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth, *Review of Economic Studies*, p. 267-279, reprinted in: Pasinetti, L.L., 1974, *Growth and Income Distribution, Essays in Economic Theory*, Cambridge University Press, Cambridge

Pasinetti, L.L., 1977, *Lectures on the Theory of Production*, Macmillan, London

Pasinetti, L.L. (ed.), 1980, *Essays on the Theory of Joint Production*, Macmillan, London

Pasinetti, L.L., 1981, *Structural Change and Economic Growth*, Cambridge University Press, Cambridge

Pasinetti, L.L., 1989, Ricardian Debt/Taxation Equivalence in the Kaldor Theory of Profits and Income Distribution, *Cambridge Journal of Economics*, p. 25-36

Phillips, A., 1955, The Tableau Economique as a simple Leontief model, *The Quarterly Journal of Economics*, p. 137-144

Ricardo, D., 1817, reprinted in 1978, *The Principles of Political Economy and Taxation*, Everyman's Library, New York

Robinson, J., 1956, *The Accumulation of Capital*, Macmillan, London

Robinson, J., 1961, Prelude to a Critique of Economic Theory, reprinted in: Hunt, E.K. & Schwartz, J.G. (ed.), 1972, *A Critique of Economic Theory*, Penquin Books, Harmondsworth

Robinson, J., 1970, Capital Theory Up to Date, *Canadian Journal of Economics*, p. 309-317, reprinted in: Hunt, E.K. & Schwartz, J.G. (ed.), 1972, *A Critique of Economic Theory*, Penquin Books, Harmondsworth

Roemer, J.E., 1980, A General Equilibrium Approach to Marxian Economics, *Econometrica*, Vol. 48, No. 2 (March)

Roncaglia, A., 1978, *Sraffa and the Theory of Prices*, John Wiley & Sons, Chichester

Salvadori, N., 1988, Fixed Capital within the Sraffa framework, *Zeitschrift fur Nationalokonomie*, p. 1-17

Schefold, B., 1989, *Mr Sraffa on Joint Production and Other Essays*, London

Sraffa, P., 1960, *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge

Steedman, I., 1977, *Marx after Sraffa*, London

Steenge, A.E., 1980, *Stability and Standard Commodities in Multisector Input-Output Models*, Rijksuniversiteit Groningen, Groningen

Woods, J.E., 1978, *Mathematical Economics, Topics in Multi-sectoral Economics*, Longman, London