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BY MEANS OF MULTIDIMENSIONAL SCALING TECHNIQUES

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NEW MULTICRITERIA METHODS FOR PHYSICAL PLANNING BY MEANS OF MULTIDIMENSIONAL SCALING TECHNIQUES

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Abstract. This article aims at providing an integrated and operational framework for evaluating the quantitative and qualitative aspects of alternative projects or plans. After a brief survey of modern multidimensional methods, special attention is paid to evaluation problems characterized by qualitative and ordinal information. Next, multidimensional (geometric) scaling methods are introduced as an important analytical tool to treat soft information. A new geometric scaling algorithm for mixed ordinal-cardinal input data will be developed. This approach will be illustrated by means of an empirical application to plans to construct an artificial industrial island in the North Sea.

Keywords. Decision theory; plan evaluation; multivariable systems; multidimensional scaling; water resources.

INTRODUCTION

During the last decade a great deal of scientific attention has been paid to the multidimensional nature of many phenomena. Multidimensional analyses are based on the fact that many objects (for example, urban renewal plans, water resource systems and public facilities) cannot be characterized and represented in a meaningful way by means of single (unidimensional) indicators. Objects are usually characterized by multiple attributes, multiple components or multiple facets, so that a multidimensional profile is necessary to provide an adequate representation of all relevant aspects of the objects concerned; see Lancaster (1971) and Paelinck and Nijkamp (1976).

This multidimensional thinking has been induced among others by the increasing complexity of our present world (cf. Perloff, 1969), the strong influences of intangibles, spillovers and externalities (cf. Nijkamp, 1977), and the conflictual diversity and multi-component structure of regional, urban and physical planning processes (cf. Faludi, 1973; Friend and others, 1974; Isard, 1969; and Lichfield and others, 1975).

At present, there is a wide variety of multidimensional analytical techniques (see for a survey Nijkamp, 1979). These multidimensional methods may be used for two purposes:

- multivariate data analysis aiming at uncovering a systematic structure in a multivariate data set. Examples are:
  - correspondence analysis (to detect similar patterns among attributes of objects; see Benzécri, 1971).
  - canonical correlation analysis (to identify correlations among sets of variables, see Dhrymes, 1970).
  - interdependence analysis (to select representative subsets of variables from a multidimensional data structure; see Nijkamp, 1978).
  - partial least squares (to assess the degree of mutual impacts among a series of multi-attribute subprofiles; see Wold, 1977).
- multidimensional decision analysis aiming at identifying optimal or compromise solutions for conflictual planning and policy problems (see among others the books written by Bell and others, 1977; Blair, 1979; Cochrane and Zeleny, 1973; Cohon, 1978; van Delft and Nijkamp, 1977; Fandel, 1972; Guigou, 1974; Haimes, 1979; Haimes and others, 1975; Hill, 1973; Johnsen, 1968; Keeney and Raiffa, 1976; Nijkamp, 1977, 1979; Starr and Zeleny, 1977; Thiriez and Zionts, 1976; Wallenius, 1975; Wilhelm, 1975; Zeleny, 1974, 1976). Multidimensional decision analysis can be classified among others into:
  - multicriteria evaluation methods aiming at identifying the best alternative from a set of distinct alternatives.
  - multiobjective programming methods aiming at finding an optimal (compromise) solution for optimization models with multiple conflicting objective functions.

Multidimensional analyses have led to a substantial operationalization and enrichment of modern policy research, but the applicability of these methods is often hampered by the lack of reliable metric information. It turns out that many phenomena cannot be measured by means of the cardinal metric of a geometric system: many variables and attributes are only measured or measurable on an ordinal or qualitative scale, so that then the application of the above-mentioned multidimensional decision analysis is fraught with difficulties and uncertainties.

The present paper aims at overcoming the limitations inherent in the availability of ordinal or qualitative information for discrete evaluation problems by developing adjusted multidimensional scaling techniques which are appropriate for tackling this type of "soft" information. By incorporating such soft information, several important aspects of decision problems (incommensurables, social consequences etc.) can be taken into account, so that an operational framework for integrated interdisciplinary policy judgements may be obtained. This paper is a follow-up of an earlier published paper on ordinal evaluation problems (see Nijkamp and Voogd, 1979). After a brief introduction to multidimensional scaling analysis, some formal aspects of the related techniques will be discussed. Next, a new variant of multidimensional scaling techniques will be presented, which is capable of dealing with soft information about both the preference structure and the impact structure of a discrete multicriteria evaluation problem. A mixed situation with both ordinal and cardinal information will also be dealt with. Some attention will also be paid to computer algorithms. The applicability of this new approach for planning and policy problems will be illustrated by means of an integrated evaluation of recently developed plans to construct an artificial island in the North Sea as a main future location for heavy industry in the Netherlands.

MULTIDIMENSIONAL SCALING ANALYSIS

As indicated above, many phenomena are characterized by soft (non-metric) information, so that ordinal multidimensional profiles are associated with these phenomena. In such cases, (non-metric) multidimensional scaling (MDS) methods (also called: ordinal geometric scaling methods) provide the tools to assign metric (cardinal) values to the attributes or aspects of the phenomenon at hand, such that these values reflect the differences in the attributes or aspects of the phenomenon being scaled. In other works, (non-metric) MDS analysis aims at uncovering the metric properties and variations of attributes or aspects measured in an ordinal sense.

Assume a set of objects; each object can be characterized by a K-dimensional ordinal attribute profile. Then each object can only be represented as a point in a geometric (Euclidean) space, if the ordinal data input is transformed into cardinal information with less than K dimensions. MDS analysis attempts to construct such cardinal information by identifying a geometric space of minimum dimensionality such that the interpoint distances between the co-ordinates of the (attributes of the) objects reflect the ordinal differences between the attributes of the successive objects. The number of attributes is rather flexible, but the number of dimensions of the resulting geometric space has to be specified by the analyst, who has also the task to interpret each dimension in terms of the underlying attributes.

The appealing feature of (non-metric) MDS methods is their capability to infer metric information on objects from an underlying ordinal data structure such that the positions of the objects in a Euclidean space reflect a maximum correspondence to the ordinal rankings of these objects. In other words, the distances between the geometric points should be in agreement (in the sense of a monotone relationship) with the observed ordinal rankings. Despite a wide variety of current MDS methods, a common property of all these methods is that they aim at recovering the latent metric structures in ordinal proximity-type data.

The basic ideas of MDS techniques were mainly developed in mathematical psychology (see among others Torgerson, 1958; Shepard, 1962; Coombs, 1964; Kruskal, 1964a, 1964b; Guttman, 1968; McGee, 1968; Carroll and Chang, 1970; Lingoes and Roskam, 1971; Young, 1972; and Roskam, 1975). After several successful attempts in the field of psychometrics, MDS methods were also introduced into other disciplines such as geography (see Golledge and others, 1969; Rushton, 1969a, 1969b; Clark and Rushton, 1970; Demko and Briggs, 1970; Tobler and others, 1970; and Schwind, 1971), economics (see Adelman and Morris, 1974), marketing analysis (see Green and Carmone, 1970; Green and Rao, 1972; and Schocker and Srinivasan, 1974), spatial planning (see Voogd, 1978; Voogd, 1979; and Voogd and Van Setten, 1979), regional science (see Nijkamp and Van Veenendaal, 1978; Blommestein and others, 1979; and Nijkamp, 1979), operations research (see Bertier and Bouroche, 1970; and Green and others, 1969), and evaluation theory (see Nijkamp, 1979; and Nijkamp and Voogd, 1979).

MDS techniques can be used for any kind of ordinal information. Consequently, both proximity and preference data can be dealt with. Proximity data are related to ordinal (dis)-similarities between objects or attributes of objects (for example, in the form of a paired comparison table or an ordinal effectiveness matrix), while preference data reflect ordinal priority rankings of judges regarding objects or attributes (for example, a set of ordinal weights attached to the criteria of a discrete evaluation problem). The capability of MDS methods to deal with both kinds of data makes these methods extremely useful in the field of plan evaluation problems with soft information on both the effectiveness scores and the preference scores.

The extraction of metric inferences from nonmetric multidimensional data is based on a set of complicated fitting procedures and permissable transformations of ordinal (dis)-similarities or preferences into the cardinal metric of the normal measurement model. In the next section, a brief introduction to MDS algorithms will be given, while in a subsequent section the use of MDS techniques for soft (ordinal) plan evaluation problems will be discussed.

MULTIDIMENSIONAL SCALING ALGORITHMS

The standard structure of MDS algorithms will be exposed here by way of a series of successive steps.

(1) Ordinal input data (Δ). The ordinal input data may be paired comparisons data and/or ordinal rankings of objects (or attributes) reflecting the perceptions or preferences concerning these objects (or attributes). This information may be included in an ordinal input matrix Δ with elements $\delta_{nn'}$, $(n,n'=1,\ldots,N)$, representing the ordinal differences between N objects with regard to a certain attribute. An ordinal number 1 may, for example, indicate the highest agreement among a certain pair of objects, while an ordinal number J may indicate a maximum discrepancy between a certain pair. The elements $\delta_{nn'}$ will be called (dis)similarities.

(2) Initial configuration (X). An initial configuration of the N objects in a geometric space requires that the dimensionality of this space is fixed *a priori* (say K). Then the N objects may be provisionally depicted in this K-dimensional space. This tentative configuration of N points can be based *inter alia* on a principal components analysis of Δ such that this NxN matrix is reduced to a KxN matrix. The tentative initial configuration serves as a frame of reference for the next steps in order to judge the increase in consistency between Δ and subsequent metric configurations X_2, X_3,... . The co-ordinates of any n^{th} point $(n=1,\ldots,N)$ can be represented by means of a K-dimensional vector $\underline{x}_k = (x_{n1},\ldots,x_{nK})'$.

(3) Interpoint distances (D). The next step is the calculation of the Euclidean distances between the N points of the above-mentioned tentative configuration X_1. These metric distances can be included in a NxN matrix D_1. The elements of this matrix (denoted by $d_{nn'}$) can be calculated as:

$$d_{nn'} = \{ \sum_{k=1}^{k} (x_{nk} - x_{n'k})^2 \}^{\frac{1}{2}} \qquad (1)$$

(4) Order-isomorph values or disparities ($\hat{D}$)
The measurement of correspondence between Δ and D is formally precluded due to the non-metric nature of Δ. In order to measure whether the geometric configuration X (and its resulting distance matrix D) do not violate the (dis)similarity conditions from the matrix Δ, an auxiliary or intermediate variable has to be introduced, which has metric properties but which is in agreement with the ordinal (dis)similarities $\delta_{nn'}$. This variable is named an order-isomorph value or disparity. This variable, denoted by $\hat{d}_{nn'}$, has to be determined such that it does not contradict the ordinal conditions. In other words, there should be a monotone relationship between $\hat{d}_{nn'}$ and $\delta_{nn'}$:

$$\hat{d}_{nn'} \leq \hat{d}_{nn''} \text{ , whenever } \delta_{nn'} \leq \delta_{nn''} \qquad (2)$$

Such order-isomorph values can be assessed among others by means of a monotone regression (Kruskal, 1964a) or a rank-image procedure (Guttman, 1968). The initial configuration X_1 can be used to estimate the variables $\hat{d}_{nn'}$.

(5) Badness-of-fit function or stress (φ)
The badness-of-fit function measures the disagreement between the order-isomorph values ($\hat{D}$) and the metric distances (D): the closer the agreement between $\hat{D}$ and D, the lower the badness-of-fit. This function attempts to minimize the residual variance between all distances $d_{nn'}$ and all order-isomorph values $\hat{d}_{nn'}$. This function may be specified among others as:

$$\varphi = \left\{ \sum_{n,n'=1}^{N} (d_{nn'} - \hat{d}_{nn'})^2 \Big/ \sum_{n,n'=1}^{N} d_{nn'}^2 \right\}^{\frac{1}{2}} , \qquad (3)$$

where the $\hat{d}_{nn'}$'s are known from step (4). The unknown variables in (3) are $d_{nn'}$ (and ultimately the co-ordinates x_{nk}; see (1)).

Therefore, the essential features of an MDS algorithm can be summarized as follows:

$$\left. \begin{array}{l} \min \varphi \\ \text{subject to (1), (2) and (3)} \end{array} \right\} \qquad (4)$$

This optimization model can be solved numerically (via a gradient procedure, e.g.). The solutions of (4) (in terms of co-ordinates x_{nk}) can be used to determine a second tentative configuration X_2, so that the whole procedure can be repeated again and again, until finally a converging equilibrium occurs.

The above-mentioned method can be used for rectangular input matrices (so-called conditional matrices) in an analogous manner. The general algorithmic structure of an MDS method is represented in Fig. 1.

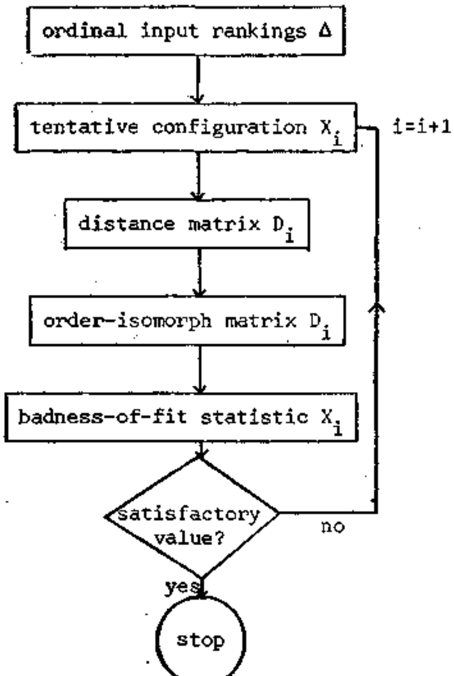

Fig. 1. Flow chart of MDS algorithm.

MULTIDIMENSIONAL SCALING FOR PLAN EVALUATION

In this section attention will be paid to an MDS method which is appropriate for a judgment of discrete alternatives (plans, projects, policy proposals etc.). The evaluation of alternatives is usually based on a plan impact matrix and on a set of preference scores for the evaluation criteria. As exposed before, this is the subject of multicriteria analysis. In the case of ordinal information, both the matrix of plan impacts (or effectiveness scores) and the set of weights have an ordinal structure. The combination of both types of ordinal data leads to complications for a traditional MDS procedure. Therefore, a new MDS method has to be devised which takes account of two different sets of ordinal data (viz. effectiveness scores and preference scores) and which is capable of linking the preference scores to the effectiveness scores, so that the weighted values of alternatives can be calculated. In this way, the alternatives can be evaluated with regard to their relative contribution to the judgement criteria concerned. This new approach is called ordinal geometric evaluation.

It is clear that the final judgement of all criteria has to be influenced by all criteria which are considered to be relevant. The extent of their influence is determined by the preference scores attached to them. This requires, in the framework of MDS techniques, that all alternatives and all criteria are to be transformed simultaneously to the same geometric space. The Euclidean distances between the alternatives in this geometric space which represent the differences between the alternatives, have to be valued according to the weights (preference scores) for the criteria. In other words, the distance function for the alternatives incorporates the (scaled metric) weights as arguments in order to allow inferences about weighted differences between alternatives.

The ordinal geometric evaluation method has several specific features:
- A so-called overall ideal point (a reference point for the evaluation) is constructed. This point reflects a (hypothetical) plan that is preferred to all other plans, given the information on plan impacts and criteria. Given this overall point, all alternatives may be ranked in a preference order according to their (weighted) geometric distances to the ideal point.
- A new algorithmic technique is developed which starts from a bottom-up procedure by trying to find a satisfactory solution in one dimension and, next, to improve the goodness-of-fit by taking account of more dimensions in a stepwise way.
- A new optimization technique is applied which combines a first-order gradient approach with a single-variable optimization method and which is also extended with a more efficient method to determine initial trial values for the iterative solution process.

This new MDS technique includes 2 stages, viz. (1) a geometric scaling of all alternatives and all judgement criteria and (2) the calculation of a reference point for the evaluation (overall ideal point). The first stage can formally be described as:

$$\begin{aligned} &\min \varphi = f(D-\hat{D}) \\ &\text{s.t.} \\ &\hat{D} \overset{m}{=} R \\ &D = g(X,Y) \end{aligned} \qquad (5)$$

where:
- D = (unknown) rectangular distance matrix with elements d_{ij} (between criterion i and alternative j);
- $\hat{D}$ = (unknown) rectangular order-isomorph matrix with elements $\hat{d}_{ij}$ corresponding to the original rankings of the alternatives;
- R = (known) rectangular effectiveness matrix with rankings r_{ij};
- $\overset{m}{=}$ = monotonicity relationship, i.e. $\hat{d}_{ij} \leq \hat{d}_{ij'}$, whenever $r_{ij} \leq r_{ij'}$;
- X = (unknown) rectangular matrix with co-ordinates x_{jk} for alternative j and dimension k;
- Y = (unknown) rectangular matrix with co-ordinates y_{ik} for criterion i and dimension k.

The second stage is concerned with the calculation of the co-ordinates of the ideal point.

This ideal point is calculated as a (hypothetical) point with co-ordinates reflecting the ideal values of all relevant decision criteria. These ideal values should correspond to the most favourable outcomes for a particular criterion (and hence lead to a definite choice in favour of this ideal alternative, should this alternative be feasible). Next, one has to calculate the weighted geometric distances between all alternatives and the ideal point. It is clear that the alternative with the minimum distance to the ideal point has to be selected as the best alternative.

In the next sections some technical aspects of this ordinal geometric evaluation method will be dealt with in greater detail.

SPECIFICATION OF THE SCALING MODEL

Given a finite set of criteria i ($i=1,2,\dots,I$), our aim is to evaluate a finite set of alternatives j ($j=1,2,\dots J$). This requires that the alternatives are measured such that each choice possibility has one valuation or effectiveness score for each criterion. This is denoted by a matrix R (of order I x J) with elements r_{ij}, which indicate the degree at which a certain criterion has been reached by an alternative. Depending on the specific nature of the criteria, these elements (or effectiveness scores) can be measured both on an ordinal and a cardinal scale. In this section, it will be shown that the geometric evaluation approach is very appropriate to treat both types of information simultaneously. In other words, geometric evaluation techniques offer interesting possibilities to analyze so-called mixed evaluation problems, in which some criteria are measured on a cardinal scale, whilst others are measured on an ordinal scale. The following scaling model can be used for these purposes (see also (5)):

$$\min_{X,Y} \varphi = \sum_{i=1}^{I} n_i \sum_{j=1}^{J} (d_{ij} - \hat{d}_{ij})^2 \qquad (6)$$

subject to:

$$d_{ij} = \left\{ \sum_{k=1}^{K} (|y_{ik} - x_{jk}|)^c \right\}^{1/c} \quad (c \geq 1) \qquad (7)$$

$$n_i = \left\{ I \sum_{j=1}^{J} (\hat{d}_{ij} - \bar{d}_i)^2 \right\}^{-1} \qquad (8)$$

$$\bar{d}_i = \sum_{j=1}^{J} \hat{d}_{ij} / J \qquad (9)$$

$$\hat{d}_{ij} = f(d_{ij}, r_{ij}) \qquad (10)$$

Relationship (7) is a Minkowski distance metric in which any value of $c > 1$ may be chosen.

The definition of the auxiliary function in equation (10) provides the key for our mixed evaluation procedure. If we have a criterion which is measured in a cardinal way, then the following linear function is used:

$$\hat{d}_{ij} = \alpha + \beta \, r_{ij} \quad (\beta > 0) \qquad (11)$$

where α and β can be found by means of a conventional linear regression analysis of $\hat{D}$ upon R. It should be noted, however, that for reasons of interpretation of X and Y, it is not permitted to substitute a negative gradient of the regression line into (11). In such cases, the parameter β is assumed to be equal to 0.

It is easy to see that function (11) cannot be used, when the criterion concerned is measured on a qualitative scale. For those 'soft' criteria a monotone regression procedure (see Kruskal, 1964a) is used. This procedure implies a constrained minimization problem, written as:

$$\min_{\hat{d}_{ij}} \psi_i = \sum_{j=1}^{J} |d_{ij} - \hat{d}_{ij}| \qquad (12)$$

subject to:

$$r_{ij} > r_{ij'} \rightarrow d_{ij} > d_{ij'} \quad (\forall\, i,j) \qquad (13)$$

The principle of mixed evaluation of multiple criteria can be considered in several ways. For instance, if a large number of evaluation criteria is used, the resulting effectiveness matrix R might provide too much information to be digested by the decision-makers, so that criterion weights are hard to specify. Under these circumstances a two-step evaluation procedure is recommended. At first, a partioning of the effectiveness matrix into sub-matrices is carried out, each sub-matrix representing an effectiveness matrix with respect to a main criterion (e.g., economics, social aspects, environmental quality, etc.). This main criterion includes all information about the various aspects with regard to that particular main criterion. Usually it is possible to collect information about the weights attached to these main criteria. Secondly, the scaling model outlined in (6) - (10), can be applied to each sub-matrix, so that, instead of a sub-matrix, for each main criterion a vector with aggregated metric effectiveness scores can be derived (by assuming the sub-criteria to be equally important). This can be done by means of the same procedure via which the overall ideal point will be determined (see the next section). Thus, the two-step evaluation procedure embodies in fact a very complicated mixed evaluation strategy: some sub-matrices might contain completely qualitative information, while others are completely cardinal or mixed cardinal-ordinal. The resulting aggregated effectiveness matrix, however, is entirely cardinal, due to the specific qualities of the geometric scaling approach. This enables us us to use only relationship (11) in the second step.

The co-ordinates of the overall point (denoted by $\hat{x}_k$, k=1,...,K) can be considered as a function of the co-ordinates of the points y_{ik}, which reflect the ideal values of criterion i with regard to dimension k, and the weights of the evaluation criteria. Let us, for the moment, suppose that the relative priorities assigned to criterion i are expressed on a ratio scale (criterion i may be either a subcriterion or a main criterion). This can be denoted as:

$$\underline{w}' = (w_1, w_2, \ldots, w_I) \tag{14}$$

where w_i (i=1,2,...,I) represents the weight attached to criterion i. The co-ordinates of the overall ideal point can now be defined such that the more important a certain criterion is, the smaller the geometric distance between that particular criterion point and the overall ideal point should be. Therefore, $\hat{x}_k$ can be regarded as a set of co-ordinates which minimizes the following function:

$$\min_{\hat{x}_k} \xi = \sum_{i=1}^{I} w_i(\hat{x}_k - y_{ik}) \tag{15}$$

If we assume that the criterion weights add up to one:

$$\sum_{i=1}^{I} w_i = 1 \tag{16}$$

it follows from (15) and (16) that the co-ordinates of the overall ideal point are equal to

$$\hat{x}_k = \sum_{i=1}^{I} w_i \cdot y_{ik} \tag{17}$$

The closer the geometric distance from a certain alternative to this overall ideal point is, the more preferred this alternative is with respect to the criteria used. So we are now able to specify a <u>preference score</u> s_j for alternative j as:

$$s_j = \sum_{k=1}^{K} (|x_{jk} - \hat{x}_k|)^{1/c} \quad (c \geq 1) \tag{18}$$

In fact, equation (18) embodies a conventional Minkowski metric. It should be noted that this metric must equal the distance metric given in equation (7). However, it is clear from (18) that alternative j is more preferable, the lower s_j becomes.

It is often very difficult to assess the criterion weights on a cardinal scale. Preferences and priorities can frequently only be measured on an ordinal scale. This means that there is no sufficient information for a precise calculation of the $\hat{x}_k$ values. However, if we have multiple ordinal weights, there are several ways to arrive at a cardinal weight vector (14). This is outlined in Nijkamp and Voogd (1979). If we have only one single (ordinal) ranking of the criteria, the only way out is to examine the area in which the overall ideal point may be situated. This area is defined by the extreme (cardinal) values of the weights, which are in accordance with the rankings which reflect the importance of the criteria. Suppose there are three criteria for which the following ranking holds: $w_1 > w_2 > w_3$. Because of condition (16), the following extreme weight sets can now be distinguished: (1,0,0), (½,½,0) and (1/3,1/3,1/3). For each extreme weight vector, we may now proceed analogously to (17) and (18). A combined interpretation of the results of the various extreme weights will provide insight into the preferability of the alternatives (see for similar treatments of ordinal weights Nijkamp and Voogd, 1979; Paelinck, 1976; Pearman, 1979; Voogd, 1979).

THE ALGORITHM

The purpose of model (5) is to find a set of co-ordinates X and Y such that the rankings of the geometric distances d_{ij} between the ideal points of criteria i and alternatives j correspond as good as possible to the effectiveness scores e_{ij}. This can be realized by means of a multidimensional scaling approach based on several nested optimization procedures. This is outlined in Fig. 2.

The first step is to choose an initial set of co-ordinates X and Y in one dimension. The selection of adequate starting values is considered in detail in Nijkamp and Voogd (1979). Next, the geometric interpoint distances D are calculated, followed by the determination of the auxiliary values $\hat{D}$. This step has been outlined in the previous section. We are now able to assess the loss function ϕ, so that we can evaluate the goodness-of-fit of the initial set of co-ordinates.

The next step is to improve the position of these points in the geometric space by finding a better set of co-ordinates. This is done by minimizing the loss function ϕ in X and Y. It is practically impossible - due to the large number of variables - to use in this case advanced numerical optimization techniques based on the Hessian matrix of second-order partial derivatives. Therefore, gradient techniques may be more appropriate. Most multidimensional scaling algorithms are based on a steepest-descent method (see, for instance, Kruskal, 1964a, 1964b, and Guttman, 1968). However, a well-known characteristic of this method is its decreasing efficiency as the function ϕ approaches its minimum value.

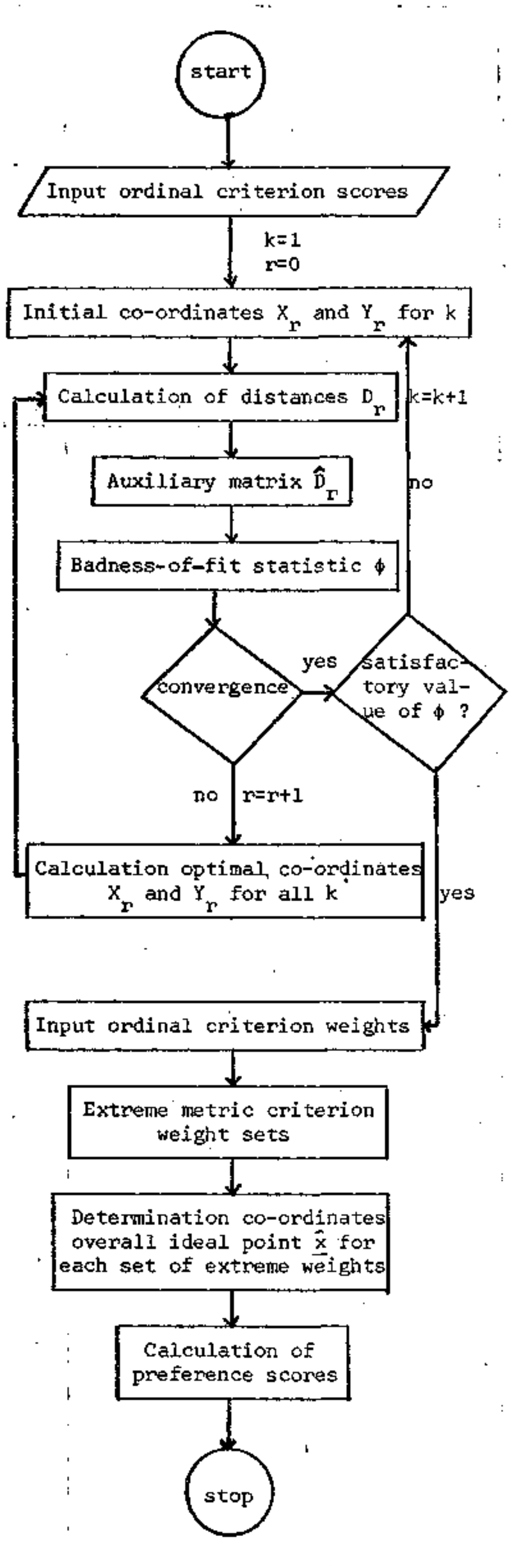

Fig. 2. Brief flow chart of the scaling algorithm.

As we can see from Fig. 2, this method is part of a main iteration process. Consequently, this optimization method starts each time very closely to the minimum value of function φ for that particular main iteration. For this reason a normal steepest-descent approach is not very efficient. Therefore, an adjusted optimization technique has to be employed, which is particularly appropriate for the minimization of a function with a large number of variables. In our geometric evaluation algorithm, we use a technique called the modified conjugate-gradient (MCG) method (cf. Van Setten and Voogd, 1978, and Voogd and Van Setten, 1979). In contrast to the conventional steepest-descent method, this technique is not a one-dimensional optimization on a <u>line</u> defined by the gradient, but a two-dimensional optimization in a <u>plane</u> given by the gradient and the preceding search direction.

This MCG method can - with respect to the scaling model outlined before - formally be denoted in the following brief manner:

$$\Delta y_{ik}\Big|_{t+1} = \alpha . g_{ik}\Big|_{t} + \beta . \Delta y_{ik}\Big|_{t} \qquad (19)$$

and

$$\Delta x_{jk}\Big|_{t+1} = \alpha . g_{jk}\Big|_{t} + \beta . \Delta x_{jk}\Big|_{t} \qquad (20)$$

where g_{ik} and g_{jk} represent the partial derivatives of φ with respect to y_{ik} and x_{jk} respectively. Each iteration step of the optimization process of this method is denoted by the subscript t, while α and β are parameters which can be estimated by using a Gauss-Newton related method. The interested reader is referred to Van Setten and Voogd (1978) for a broader exposition of the principles and elements of this method.

After the calculation of a new set of co-ordinates X and Y by means of the modified conjugate gradient method, a new distance matrix D is determined. In addition, the matrix $\hat{D}$ of auxiliary values is updated with respect to these new distances, after which (19) and (20) are again applied. This main iteration process proceeds until φ has reached either a satisfactory low value or becomes stationary. In the first case, a suitable solution for (5) has been found, whereas in the second case an extra dimension (k=k+1) is added for which new starting values are calculated and the whole process described above is repeated.

After the determination of a solution for model (5), the next step is to determine the extreme metric indicator weights, which are in accordance with the observed priority structure measured on an ordinal level. An "overall ideal point" can now be determined for each set of extreme weights. Consequently, the preference scores of the alternatives can be assessed with respect to the various "overall ideal points". In the next sections an application of this ordinal geometric scaling model will be presented.

PLANS FOR CONSTRUCTING AN INDUSTRIAL ISLAND

The usefulness of geometric scaling techniques for ordinal or qualitative evaluation problems will now be illustrated by means of a big research project undertaken in the Netherlands. This project concerns the appraisal of the proposals to construct an artificial island in the North Sea. This island has been suggested as a favourable location for heavy industry, especially since the high population density and the high degree of industrialization in the Netherlands precludes a further expansion of heavy industry on the mainland (see North Sea Island Group, 1976). A location of new industry on an artificial island in the North Sea would prevent a further environmental deterioration on the mainland and would offer a good opportunity to combat industrial waste discharges in a concentrated way by making use of scale advantages in the abatement sector.

It is clear that a social evaluation of such an ambitious project should take into account a wide variety of relevant physical planning aspects such as the impacts on the labour market, the infrastructural repercussions, the environmental impacts, etc.

Furthermore, a meaningful social judgement of the project at hand requires also a careful examination of all other spatial alternatives for locating new industries in the Netherlands on existing unoccupied industrial areas. Therefore, instead of a zero-one assessment of this North Sea island it is necessary to take into account also other industrial locations (see Kutsch Lojenga and Nijkamp, 1977).

An additional complication is the fact that there is no reliable information available on the activities to be located on the island. The major part of these activities will be private companies, so that the locational decisions of these firms cannot be controlled by public decision-making. These decisions will mainly be based on profitability criteria, growth targets etc. This implies that the impacts of the island are overloaded with uncertainty, especially because the project itself will not be completed before 1990.

Given the above-mentioned elements, it is no surprise that most impacts of the alternatives on the values of all relevant decision criteria can not be assessed on a cardinal scale. Therefore, most impacts are only assessed via ordinal effectiveness scores. Consequently, this planning problem is a glaring example of a data analysis for which geometric scaling techniques are useful.

The evidence of using geometric scaling techniques is even greater, because also the weights attached to the diverse decision criteria are a source of much uncertainty. Instead of assessing only one set of weights, seven different sets of weights are used. Each set corresponds, to a certain extent, to a certain future policy scenario for economic, spatial and urban developments in the Netherlands (see Ten Broek, 1979). Each scenario also is associated with general judgement criteria for future developments. In consequence, each scenario provides a set of preference scores for these criteria to be used for judging the industrial island. The main criteria used in our study are: (1) micro-economic costs, (2) macro-economic impacts, (3) environmental effects, (4) energy effects, (5) spatial conditions, (6) social conditions, and administration/management. The priority scores for the criteria are binary ordinal numbers, viz. 2 (a high priority and 1 (a normal or neutral priority).

The scenarios used in our study are:

1. An entrepreneurial scenario with much emphasis on micro-economic consequences.
2. A scenario with a high priority for macro-economic impacts, energy effects and spatial structure.
3. A scenario with much emphasis on macro-economic effects and environmental repercussions.
4. A scenario focussing on energy effects and spatial aspects.
5. A scenario associated with micro-economic costs and spatial elements.
6. A scenario with much emphasis on micro-economic costs and environmental aspects.
7. A scenario related to environmental aspects, spatial conditions and social/administrative aspects.

The seven sets of priority scores associated with these scenarios are represented in Table 1.

TABLE 1. Weights associated with scenarios.

scenario / criterion	1	2	3	4	5	6	7
(1)	2	1	1	1	2	2	1
(2)	1	2	2	1	1	1	1
(3)	1	1	2	1	1	2	2
(4)	1	2	1	2	1	1	1
(5)	1	2	1	2	2	1	2
(6)	1	1	1	1	1	1	2

The following alternatives - size and location - for future developments of heavy industry in the Netherlands were distinguished (Ten Broek, 1979):

(i) An industrial island in the North Sea, located north of Ameland (a small island in the Wadden Sea).
(ii) An industrial island in the North Sea, located 40 miles west of Rotterdam.
(iii) An industrial island in the North Sea, located west of Walcheren (a part of the southern province of Zeeland).

TABLE 2 Effectiveness Matrix.

c \ a	(i)	(ii)	(iii)	(iv)	(v)	(vi)
(1)						
-a	1	2/3	2/3	6	4/5	4/5
-b	1	3	2	4/6	4/6	4/6
-c	1/3	1/3	1/3	4/6	4/6	4/6
-d	1	2/3	2/3	6	5	4
-e	1	3	2	4/6	4/6	4/6
(2)						
-a	2010	2010	2010	0	0	0
-b	5760	5760	5760	4800	4800	4800
-c	+	+	+	0	0	0
-d	570	570	570	0	0	0
-e	3360	3360	3360	2800	2940	3080
-f	1180	1180	1180	980	980	980
-g	-360	-360	-360	0	0	0
-h	-12000	-12000	-12000	0	0	0
-i	62000	62000	62000	56000	56000	56000
-j	22000	22000	22000	19400	19400	19400
-k	++	+/0	+++	0	0	0
-l	-	-	-	-	-	-
-m	6	3/4	5	3/4	2	1
-n	3	1/2	4	1/2	5	6
-o	++	+	++	0	0	0
-p	+	-	+	-	++	++
(3)						
-a	---	-	--	-	--	--
-b	---	-	---	-	---	---
-c	---	-	--	-	---	---
-d	--	-	-	-	--	---
-e	-	-	-	--	-	-
-f	0	-	-	--	--	--
-g	0	0	0	---	--	--
-h	0	-	0	---	--	--
-i	0	0	0	-	-	-
-j	-	-	-	0	0	0
(4)						
-a	---	--	--	0	--	-
-b	+	+	+	++	0	0
-c	-	-	-	+	+	+
(5)						
-a	-	-	-	+	++	+++
-b	+	-	+	-	++	+++
-c	0	-	-	0	--	--
-d	-	-	-	+	+	+
(6)						
-a	-	+	0	+	+	+
-b	--	-	-	0	0	0
-c	-	-	-	0	0	0
-d	--	-	-	--	+	++
-e	-	+	+	+	-	--
-f	---	---	---	-	+	++

a : alternative
c : criterion

TABLE 3 Meaning of Symbols.

(1)	
-a	construction costs industrial area
-b	construction costs industrial equipment
-c	wage costs
-d	transportation costs
-e	costs public facilities, electricity, etc.
(2)	
-a	T.V.A. from building the area (mln. Dfl.)
-b	T.V.A. from building equipment (mln Dfl.)
-c	T.V.A. from production activities(mln Dfl)
-d	wages from building the area (mln. Dfl.)
-e	wages from building equipment (mln. Dfl.)
-f	wages from production activities(mln Dfl.)
-g	impacts on balance of payment (mln. Dfl.)
-h	jobs created by building the area
-i	jobs created by constructing equipment
-j	jobs created by production activities
-k	regional employment from building the area
-l	regional employment from building equipment
-m	regional employment from prod. activities
-n	differentiation in demand for labour
-o	increase in investments
-p	impact on regional income discrepancies
(3)	
-a	air pollution
-b	water pollution
-c	thermal pollution
-d	soil pollution
-e	risk for employees
-f	risk during transport
-g	noise annoyance
-h	stench annoyance
-i	visual pollution
-j	impact on stability of ecosystem
(4)	
-a	energy use from transports
-b	energy saving from complementary activities
-c	link to existing energy infrastructure
(5)	
-a	impact on efficient land use
-b	impact on regional policy
-c	impact on physical-natural landscape
-d	impact on existing trans. infrastructure
(6)	
-a	qual. equilibrium demand-supply labour
-b	quant. equilibrium demand-supply labour
-c	quality-of-labour conditions
-d	impact on strategic national safety
-e	military means to defend the area
-f	military means to restore sovereignty

(iv) A further industrial land use of the Maasvlakte (an artificial peninsula at the entrance of the port of Rotterdam to the North Sea).

(v) A spatial dispersion of future industrial activities over three existing industrial areas in the Netherlands.

(vi) A spatial dispersion of future industrial activities over five existing industrial areas in the Netherlands.

For all these alternative industrial solutions a set of effectiveness scores on all relevant judgement criteria has been assessed. These effectiveness scores were not calculated for the above-mentioned six main criteria, but rather for a large number of subcriteria related to the main criteria. This matrix of effectiveness scores has a mixed character: both cardinal and ordinal figures have been used simultaneously, so that this matrix contains the most accurate information available. This matrix is represented in Table 2. The meaning of the criteria is explained in Table 3. The elements of Table 2 are composed of cardinal numbers, ordinal numbers varying between 1 (a bad outcome) and 6 (a favourable outcome), and qualitative indicators varying from --- (a very bad outcome) to +++ (a very good outcome). These qualitative indicators were also transformed into ordinal numbers. The meaning of an effectiveness score 2/3 is that the alternative at hand has an undetermined position in the second or third rank order.

RESULTS OF THE SCALING METHOD

The data included in Tables 1 and 2 have been used as ordinal inputs for a geometric scaling procedure for the evaluation problem at hand. The results of this mixed evaluation problem were obtained in two steps. In the first step, 6 times a scaling method was used to aggregate the (unweighted) sub-criteria within the 6 main criteria from Table 2 to cardinal (matrix) impact scores (see Table 4). The mixed geometric scaling technique was used here for aggregating metric and non-metric outcomes. Thus, a cardinal effectiveness matrix of order 6 x 6 was constructed which formed the basis for the next step of the analysis. In the next step, the

TABLE 4 Aggregate Cardinal Effectiveness Matrix.

criterion \ scenario	(i)	(ii)	(iii)	(iv)	(v)	(vi)
(1)	1.326	.658	1.314	.219	.247	.481
(2)	.273	.498	.362	1.289	.725	.698
(3)	.682	.414	.685	.726	1.392	1.578
(4)	1.227	.645	.649	.145	.397	.385
(5)	.477	.989	.761	.477	.324	.315
(6)	.829	.564	.571	.357	.318	.302

results from Table 4 were combined with the ordinal preference scores from Table 1, in order to arrive at 7 sets of outcomes, each corresponding to a certain scenario. In this way, one may examine the sensitivity of the rankings of the alternative plans with regard to shifts in weights (or preference scores). Because the ordinal information on the preference scores is rather poor, there is not sufficient information available to identify a precise location of the 'ideal choice possibility', so that the only possible way to proceed is to identify a feasible area in which this ideal point can be positioned. By calculating for various points within this area a preference score, one may draw more or less definite inferences concerning the relative preferability of the alternatives, so that one may derive a rank order of preferred plans. In this case it is sufficient to investigate only the edges and corner points of the feasible solution area. In other words, one has to identify the extreme values of the weights corresponding to the corner points which are in accordance with the rankings of the criteria of this decision problem. The general results of this procedure are presented below in Table 5.

TABLE 5 Results of the Evaluation Procedure.

≈ : equally important
> : preferred to

scenario	
1	(vi) ≈ (iv) ≈ (v) > (ii) ≈ (i) >(iii)
2	(i) ≈ (ii) ≈ (iii) > (iv) > (v) > (vi)
3	(i) > (ii) > (iii) > (iv) ≈ (vi) > (v)
4	(iv) > (v) ≈ (vi) > (ii) ≈ (iii) ≈ (i)
5	(vi) ≈ (iv) ≈ (v) > (ii) ≈ (i) ≈(iii)
6	(iv) ≈ (v) > (vi) > (ii) ≈ (i) >(iii)
7	(v) ≈ (iv) ≈ (vi) > (ii) ≈ (i) ≈(iii)

The conclusions from the last results are rather straightforward. The island alternatives have a low ranking compared to the remaining plans, so that there is hardly any scenario which would favour a choice for the island alternatives. There are only two exceptions, viz. scenario 2 and 3. Scenario 2 focuses mainly on macro-economic aspects such as employment and wages, while scenario 3 implies macro-economic and environmental interests. For these two scenarios the island plans appear to score rather high, although one may doubt the feasibility of these scenarios in view of our present economic recession. Therefore, the mainland alternatives seem to be more realistic. The particular choice in favour of one of these alternatives depends clearly on the future policies reflected by the scenarios 1, 4, 5, 6 and 7.

Therefore, the final conclusion is that the island alternatives are less realistic due to the micro-economic costs, the energy repercussions and the spatial distributive impacts (see also Stunet, 1979). Only in the case of a rapid economic growth with a scarcity of industrial areas might the plans to build an industrial island become more plausible.

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